Self Practice Paper (SPP)

Find the derivative of functions using quotient rule.

1. g(x) = $\frac{x^2 - 4}{x + 0.5}$

2. Suppose u and v are differentiable functions of x and that
u (1) = 2, u'(1) = 0 v (1) = 5 v' (1) = – 1.
Find the values of the following derivatives at x = 1.

(a) $\frac{d}{dx}$ (uv) (b) $\frac{d}{dx}\left(\frac{u}{v}\right)$

(c) $\frac{d}{dx}\left(\frac{v}{u}\right)$ (d) $\frac{d}{dx}$ (7v – 2u).]

3. s = $\frac{1 + \text{cosec } t}{1 - \text{cosec } t}$

4. s = $\frac{\sin t}{1 - \cos t}$

5. r = – $(\sec\theta + \tan\theta)^{-1}$

6. (1 +2 cos x)$_2$

Evaluating integrals
Check your answers by differentiation.

7. $\int 4\sin^2 y \, dy$

8. $\int \frac{\cos^2 y}{7} \, dy$

9. $\int \frac{\csc\theta}{\csc\theta - \sin\theta} \, d\theta$

10. r = $(\text{cosec}\theta + \cot\theta)^{-1}$

11. The sum of the magnitudes of two forces acting at a point is 16 N. The resultant of these force is perpendicular to the smaller force and has a magnitude of 8 N. If the smaller force is of magnitude x, then the value of x is
(1) 2 N (2) 4N (3) 6 N (4) 7N

12. The resultant of two forces 3 P & 2 P is R, if first force is doubled, the resultant is also doubled. Then the angle between the forces is :
(1) 30º (2) 60º (3) 120º (4) 150º

13. The resultant of two forces acting at an angle of 150º is 10 kg wt and is perpendicular to one of the forces . The other force is :
(1) $10\sqrt{3}$ kg wt (2) $20\sqrt{3}$ kg wt (3) 20 kg wt (4) $\frac{20}{\sqrt{3}}$ kg wt

14. The resultant of two equal forces is double of either of the forces. The angle between them is :
(1) 120º (2) 90º (3) 60º (4) 0º

15. A force of 6 kg wt. and another of 8 kg wt. can be applied together to produce the effect of a single force of:
(1) 1 kg wt. (2) 11 kg wt. (3) 15 kg wt. (4) 20 kg wt.

16. Let $\vec{u}$ be a constant vector and $\vec{v}$ be a vector of constant magnitude such that $|\vec{v}| = \frac{1}{2}|\vec{u}|$ and $|\vec{u}| \neq 0$. Then the maximum possible angle between $\vec{u}$ and $\vec{u}+\vec{v}$ is :

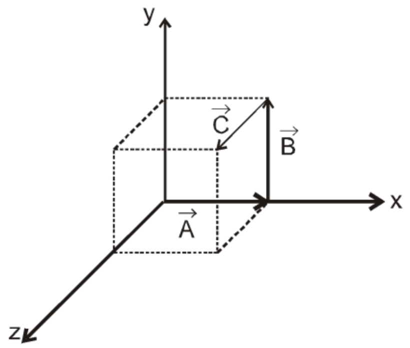
(1) 30° (2) 60° (3) 120° (4) 150°

17. A sheet of area 40 m^2 in used to make an open tank with a square base, then find the dimensions of the base such that volume of this tank is maximum.

(1) $x = \sqrt{\frac{35}{3}}$ m (2) $x = \sqrt{\frac{55}{2}}$ m (3) $x = \sqrt{\frac{27}{2}}$ m (4) $x = \sqrt{\frac{40}{3}}$ m

18. **Fill in the blanks**

(i) The magnitude of sum of three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ representing the sides of a cube of length A is equal to ....



(ii) If $\vec{A} = 3\hat{i}+4\hat{j}$ and $\vec{B} = 7\hat{i}+24\hat{j}$, then the vector having the same magnitude as $\vec{B}$ and parallel to $\vec{A}$ is ....

(iii) If $\vec{A} \parallel \vec{B}$ then $\vec{A} \times \vec{B}$ = ...

(iv) The magnitude of area of the parallelogram formed by the adjacent sides of vectors $\vec{A} = 3\hat{i} + 2\hat{j}$ and $\vec{B} = 2\hat{i} - 4\hat{k}$ is ....

(v) Sum of two opposite vector to each other is a .... vector.

(vi) The unit vector along vector $\hat{i} + \hat{j} + \hat{k}$ is ....

(vii) If $\vec{A}$ is .... to $\vec{B}$, then $\vec{A} \cdot \vec{B} = 0$

(viii) The vector $\vec{A} = \hat{i} + \hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors along x-axis and y-axis respectively, makes an angle of .... degree with x-axis.

(ix) If $\vec{A} + \vec{B} + \vec{C} = \vec{0}$, then $\vec{A} \cdot (\vec{B} \times \vec{C})$ = ...

SPP Answers

1. $g'(x) = \frac{x^2+x+4}{(x+0.5)^2}$ **2.** (a) –2 (b) $\frac{2}{25}$ (c) $-\frac{1}{2}$ (d) –7

3. $\frac{-2\,\text{cosec t cot t}}{(1-\text{cosec t})^2}$ **4.** $\frac{ds}{dt} = \frac{1}{\cos t - 1}$ **5.** $\frac{\sec\theta}{\sec\theta+\tan\theta}$

6. 3x + sin 2x + 4 sin x + C **7.** 2y – sin 2y +C **8.** $\frac{y}{14} + \frac{\sin 2y}{28} + C$

9. tan θ + C **10.** $\frac{\text{cosec}\theta}{\cot\theta+\text{cosec}\theta}$ **11.** (3) **12.** (3) **13.** (3) **14.** (4)

15. (2) **16.** (1) **17.** (4) **18.** (i) $(\sqrt{3})A$ (ii) $15\hat{i}+20\hat{j}$. (iii) Null vector

(iv) $\sqrt{224}$ units (v) Negative, Positive or Zero (vi) $\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}$.

(vii) Perpendicular (viii) 45º (ix) Zero

SPP Solutions

1. $g'(x) = \frac{(x+0.5)\frac{d}{dx}(x^2-4)-(x^2-4)\frac{d}{dx}(x+0.5)}{(x+0.5)^2}$

$= \frac{(x+.5)(2x)-(x^2-4)}{(x+0.5)^2} = \frac{2x^2+x-x^2+4}{(x+0.5)^2} = \frac{x^2+x+4}{(x+0.5)^2}$

2. (a) $\frac{d}{dx}(uv) = uV' + V u'$

at x = 1 = u(1) v′ (1) + v (1) u′(1) = 2x (– 1) + 5 (0) = **– 2**

(b) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu'-uv'}{v^2}$

at x = 1 $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v(1)u'(1)-u(1)v'(1)}{[v(1)]^2} = \frac{5(0)-(2)(-1)}{(5)^2} = \frac{2}{25}$

(c) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{uv'-vu'}{u^2} = \frac{uv'(1)-v(1)u'(1)}{[u(1)]^2}$

(d) $\frac{d}{dx}(7v-2u) = 7v' - 2u'$

at x = 1 = 7 v'(1) – 2u' (1)

7 (– 1) – 2 (0) = **– 7**

3. $S = \frac{1+\text{cosec t}}{1-\text{cosec t}}$

$\frac{ds}{dt} = \frac{(1-\text{cosec t})(-\text{cosec t cot t})-(1+\text{cosec t})(+\text{cosec t cot t})}{(1-\text{cosec t})^2} = \frac{-2\,\text{cosec t cot t}}{(1-\text{cosec t})^2}$

4. $s = \frac{\sin t}{1-\cos t}$

$\frac{ds}{dt} = \frac{(1-\cos t)\cos t-\sin t(\sin t)}{(1-\cos t)^2} = \frac{\cos t-(\cos^2 t+\sin^2 t)}{(1-\cos t)^2} = \frac{\cos t-1}{(1-\cos t)^2} = \frac{1}{\cos t-1}$

5. r = – (secθ + tanθ)$^{-1}$

$r = \frac{-1}{\sec\theta+\tan\theta} \Rightarrow \frac{dr}{d\theta} = \frac{1(\sec\theta\tan\theta+\sec^2\theta)}{(\sec\theta+\tan\theta)^2} = \frac{\sec\theta}{\sec\theta+\tan\theta}$

6. $\int (1+2\cos x)^2 dx$

$= \int (1+4\cos^2 x + 4\cos x)dx = \int 1.dx + \int 4\cos x dx + \int 4\cos^2 x dx$

$= x + 4\sin x + \int 4\left(\frac{\cos 2x + 1}{2}\right)dx = x + 4\sin x + \int 2\cos 2x dx + \int 2.dx$

$= x + 4\sin x + \frac{2\sin 2x}{2} + 2x = 3x + 4\sin x + \sin 2x + C$

7. $\int 4\sin^2 y\, dy$ $\quad \therefore \quad \sin^2 y = \frac{1-\cos 2y}{2}$

$\int 4\left(\frac{1-\cos 2y}{2}\right)dy = \int 2dy - \int 2\cos 2y\, dy = 2y - \frac{2\sin 2y}{2} = 2y - \sin 2y + c$

8. $\int \frac{\cos^2 y}{7} dy$

$\because \quad \cos^2 y = \frac{1+\cos 2y}{2}$

$= \int \frac{1+\cos 2y}{14} dy = \int \frac{1}{14} dy + \frac{1}{14}\int \cos 2y\, dy = \frac{1}{14} y + \frac{1}{28} \sin 2y + c$

9. $\int \frac{\csc\theta}{\csc\theta - \sin\theta} d\theta = \int \frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta} - \sin\theta} d\theta$

$= \int \frac{1}{1-\sin^2\theta} d\theta = \int \sec^2\theta \cdot d\theta = \tan\theta + c$

10. $r = (\csc\theta + \cot\theta)^{-1}$

$r = \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

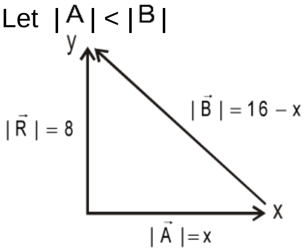
$\frac{dr}{d\theta} = \frac{(\cot\theta + \operatorname{cosec}\theta)(0) - 1(-\operatorname{cosec}^2\theta - \operatorname{cosec}\theta \cot\theta)}{(\cot\theta + \operatorname{cosec}\theta)^2}$

$\frac{dr}{d\theta} = \frac{\operatorname{cosec}\theta}{\cot\theta + \operatorname{cosec}\theta}$

11. Let $\vec{A}$ & $\vec{B}$ are two vector

give that $|\vec{A}| + |\vec{B}| = 16$

Let $|\vec{A}| < |\vec{B}|$



and $\vec{R} = \vec{A} + \vec{B}$

from given problem

and $\vec{R} \perp \vec{A}$ & $|\vec{R}| = 8$

Let $|\vec{A}| = x$

from triangle $x^2 + 8^2 = (16 - x)^2$ by solving this we get $x = 6$

12. Given that $\vec{R} = \vec{3P} + \vec{2P}$ .... (1)

If first force is doubled

then $2\vec{R} = \vec{6P} + \vec{2P}$ .... (2)

from equation $R^2 = (3P)^2 + (2P)^2 + 2(3P)(2P)\cos\theta$ .... (3)

from equation $(2R)^2 = (6P)^2 + (2P)^2 + 2(6P)(2P)\cos\theta$ .... (4)

from equation (3) and (4)

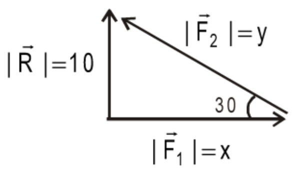
$0 = 12P^2 + 24P^2\cos\theta$

$\Rightarrow \cos\theta = -1/2$

$\theta = 120°$

13. Let $\vec{F_1}$ & $\vec{F_2}$ are two forces

$\vec{R} = \vec{F_1} + \vec{F_2}$ & $|\vec{R}|$ = 10 kg wt



from diagram

$\sin 30° = \dfrac{10}{y}$

$y = \dfrac{10}{\sin 30} = \dfrac{10}{1/2}$ = 20 kg wt

14. If $\vec{A}$ & $\vec{B}$ are two vectors give that $|\vec{A}| = |\vec{B}|$

Let $|\vec{A}| = |\vec{B}| = x$

If $\vec{R} = \vec{A} + \vec{B}$

then $|\vec{R}| = 2x$

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$(2x)^2 = (x)^2 + (x)^2 + 2x^2\cos\theta$

$\cos\theta = 1$ $\qquad \theta = 0$ **Ans.**

15. Let $|\vec{F_1}|$ = 6 kg wt

$|\vec{F_2}|$ = 8 kg wt

and $\vec{R} = \vec{F_1} + \vec{F_2}$ then $|\vec{A}| \sim |\vec{B}| \le |\vec{R}| \le |\vec{A}| + |\vec{B}|$

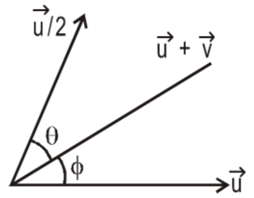
$2 \le |\vec{R}| \le 14$ so, from given option ans. is 11 kg wt.

16. $\tan\phi = \dfrac{u/2\sin\theta}{u + u/2\cos\theta} \Rightarrow \tan\phi = \dfrac{\sin\theta}{2+\cos\theta}$

$$\frac{d}{d\theta}(\tan\phi) = \frac{d}{d\theta}\left(\frac{\sin\theta}{2+\cos\theta}\right)$$

$$\sec^2\phi \frac{d\phi}{d\theta} = \frac{(2+\cos\theta)(\cos\theta) - \sin\theta(-\sin\theta)}{(2+\cos\theta)^2}$$

$$\sec^2\phi \frac{d\phi}{dt} = \frac{2\cos\theta + 1}{(2+\cos\theta)^2}$$



for maxima or minima $\frac{d\phi}{dt} = 0$

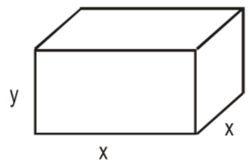
$2\cos\theta + 1 = 0$

$\cos\theta = -1/2 \quad \Rightarrow \quad \theta = 120°$

For $\theta = 120°$ $\quad \tan\phi = \frac{\sin 120}{2 + \cos 120} = \frac{\sqrt{3}/2}{3/2} = \frac{1}{\sqrt{3}}$

$\phi = 30°$

17. Let the dimensions of the tank be x and y area of the open tank = $x^2 + 4xy$.



again x & y are related to surface area of this tank which is equal to 40 m^2

$x^2 + 4xy = 40$

y =

volume $v = x^2$ =

for maximum volume

v' (x) = = 0

x =

and v '' (x) = –

v '' = – < 0

so volume is maximum at x = m

18. (i) $\vec{A} = A\hat{i}$ $\quad \vec{B} = A\hat{j}$ $\quad \vec{C} = A\hat{k}$

$\vec{A} + \vec{B} + \vec{C} = A\hat{i} + A\hat{j} + A\hat{k}$

$|\vec{A} + \vec{B} + \vec{C}| = \sqrt{A^2 + A^2 + A^2} = \sqrt{3}A$

(ii) Given $\vec{A} = 3\hat{i} + 4\hat{j}$ $\quad \vec{B} = 7\hat{j} + 24\hat{j}$

Let $\vec{C} = |\vec{C}|\vec{C}$

Given that $|\vec{C}| = |\vec{B}| = \sqrt{7^2 + (24)^2} = 25$

and $\hat{C} = \hat{A} = \frac{3\hat{i} + 4\hat{j}}{5}$

$\vec{C} = \frac{25 \times (3\hat{i} + 4\hat{j})}{5}$ $\quad \vec{C} = 15\hat{i} + 20\hat{j}$

(iii) If $|\vec{A}\vec{B}|$

then angle between $\vec{A}$ and $\vec{B}$ is equal to zero

so $\vec{A} \times \vec{B} = AB\sin\theta\, \hat{n} = 0$

(iv) Area of parallelogram = $|\vec{A} \times \vec{B}|$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 0 \\ 2 & 0 & -4 \end{vmatrix} = |-8\hat{i} + 12\hat{j} - 4\hat{k}| = \sqrt{224}$$