Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

This Section is not meant for classroom discussion. It is being given to promote self-study and self testing amongst the Resonance students.

Max. Marks : 120 **Max. Time : 1 Hr.**

Important Instructions :

1. The test is of **1 hour** duration and max. marks 120.
2. The test consists **30** questions, **4 marks** each.
3. Only one choice is correct **1 mark** will be deducted for incorrect response. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 3 above.

1. Number of all possible skew symmetric matrices whose elements are taken from 0, 0, 0, 1, 1, 1, 1, –1, 1,–1,–1, are
(1) 8 (2) 9 (3) 10 (4) 11

2. If a ≠ b ≠ c and $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$ then value of $\frac{3abc(ab+bc+ca)}{a+b+c}$ is
(1) 1 (2) 2 (3) 3 (4) 4

3. Let A= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (a, b, c, d ≠ 0) is a matrix such that A^2 = A then |A| must be equal to
(1) 1 (2) 0 (3) –1 (4) abcd

4. If A = $\begin{bmatrix} 1 & a \\ b & -1 \end{bmatrix}$ is a matrix such that AA' = A'A then
(1) a = –b (2) a ≠ b (3) a =b (4) ab = 1

5. The largest value of a third order determinant whose elements are 0 or 2 only is
(1) 16 (2) 0 (3) 24 (4) 2

6. If $\Delta = \begin{vmatrix} abc & b^2c & c^2b \\ abc & c^2a & ca^2 \\ abc & a^2b & b^2a \end{vmatrix}$ = 0 (a, b, c∈R and a ≠ b ≠ c ≠ 0) then value of 3(a+b+c) is
(1) 3 (2) 2 (3) 0 (4) 1

7. Let Δ= $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ and a, b, c are the roots of $x^3 + 3x^2 + 4x + 1 = 0$ then value of Δ is
(1) 4 (2) 3 (3) 0 (4) 1

8. If $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$ = k (a–b) (b–c) (c–a) then

(1) 4 (2) –4 (3) 2 (4) –2

9. Let A = $\begin{bmatrix} 1 & x & x^2 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{bmatrix}$ be a matrix such that |A| = –2 then value of $\begin{vmatrix} x^3-1 & 0 & x-x^4 \\ 0 & x-x^4 & x^3-1 \\ x-x^4 & x^3-1 & 0 \end{vmatrix}$ is

(1) –4 (2) 2 (3) 8 (4) 4

10. If y = $\begin{vmatrix} \sin x & \cos x & \sin x \\ \cos x & -\sin x & \cos x \\ 2x & 3 & 4 \end{vmatrix}$ then $\frac{dy}{dx} =$

(1) 1 (2) 2 (3) 0 (4) sinx cosx

11. If $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} = 0$ then the line ax + by + c = 0 passes through a fixed point which is

(1) (1, 2) (2) (1, 1) (3) (–2, 1) (4) (1, 0)

12. If $\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix} = \begin{vmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{vmatrix}$ = 0 then x =

(1) 0 (2) –9 (3) 3 (4) –3

13. If $p\lambda_4 + q\lambda_3 + r\lambda_2 + s\lambda + t = \begin{vmatrix} \lambda^2+3\lambda & \lambda-1 & \lambda+3 \\ \lambda^2+1 & 2-\lambda & \lambda-3 \\ \lambda^2-3 & \lambda+4 & 3\lambda \end{vmatrix}$ then value of p is

(1) –2 (2) –3 (3) –4 (4) –5

14. If c < 1 and system of equations x + y = 1, 2x –y – c = 0 and –bx + 3by – c = 0 is consistant, then set of possible real values of b is

(1) $\left(-3, \frac{3}{4}\right)$ (2) $\left(-\frac{3}{2}, 4\right)$ (3) $\left(-\frac{3}{4}, 3\right)$ (4) (3, 4)

15. If ω(≠1) is a cube root of unity, then value of determinant $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix}$ is

(1) 0 (2) 1 (3) i (4) ω

16. Let A= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and B = $\begin{bmatrix} p \\ q \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that AB = B and a + d = 3 then |A| =

(1) 1 (2) 2 (3) 3 (4) 4

17. If A is a non singular matrix satisfying AB – BA = A and |B| = 3 then value of |B – I|+|B + I| is

(1) 0 (2) 3 (3) 6 (4) 2

18. The number of diagonal matrix A of order n for which A_3 = A is

(1) 1 (2) 0 (3) 2_n (4) 3_n

19. Let A be a 3_{rd} order square matrix and B be its adjoint matrix such that |A| = –2 then $|AB + 3I_3|$ =

(1) –2 (2) 4 (3) 2 (4) 1

20. Given $A = \begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$. If xyz = 60 and 8x + 4y + 3z = 20 then |A.adjA| =
(1) $(64)_3$ (2) $(88)_3$ (3) $(68)_3$ (4) $(34)_3$

21. If A is a diagonal matrix of order 3 × 3 and commutative with every square matrix of order 3 under multiplication and tr(A) = 12, then value of $|A|^{\frac{1}{2}}$ is
(1) 64 (2) 16 (3) 8 (4) 4

22. If A = $\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ be a matrix and $A_8 = \lambda A + \mu I$, λ, μ∈I then λ+ μ =
(1) 1 (2) 2 (3) 0 (4)3

23. Let f(x) = $\begin{vmatrix} 2\cos^2 x & \sin 2x & -\sin x \\ \sin 2x & 2\sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$ then value of $\int_0^{\pi/2} (f(x) + f'(x))dx$ is
(1) $\frac{\pi}{2}$ (2) π (3) $-\frac{\pi}{2}$ (4) 0

24. Let A and B be two n × n matrices such that A + B = AB then
(1) $AB = I_n$ (2) $A= I_n$ or $B= = I_n$ (3) AB = BA (4) A = B

25. If A and B are two square matrices such that $B = -A_{-1}BA$ then $(A+B)_2$ =
(1) 0 (2) $A_2 + 2AB + B_2$ (3) A + B (4) $A_2 + B_2$

26. The system of equation
λx + (λ+1) y + (λ–1) z = 0, (λ+1)x + λy + (λ+2)z = 0 , (λ–1)x + (λ+2)y + λz = 0 has a non trivial solution for
(1) Exactly there real values of λ (2) Exactly two real values of λ
(3) Exactly one real value of λ (4) Infinite number of values of λ

27. Let $a_k = k.\ {}_{10}C_k$, $b_k = (10-k)\ {}_{10}C_k$ and $A_k = \begin{bmatrix} a_k & 0 \\ 0 & b_k \end{bmatrix}$ If A= $\sum_{k=1}^{9} A_k = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ than a + b =
(1) 10200 (2) 10220 (3) 10100 (4) 10230

28. Let A= $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and X ≠ O be a column matrix such that AX = λX then sum of square of all possible values of λ is
(1) 0 (2) 1 (3) 4 (4) 6

29. If Δ = $\begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6} \\ \sqrt{12} & \sqrt{3}+\sqrt{8}i & 3\sqrt{2}+\sqrt{6}i \\ \sqrt{18} & \sqrt{2}+\sqrt{12}i & \sqrt{27}+2i \end{vmatrix}$ then value of Δ is
(1) $\sqrt{2}+\sqrt{3}$ (2) 4 + i $\sqrt{3}$ (3) 0 (4) –6

30. Let P = $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where α ∈ R. Suppose Q = $[q_{ij}]$ is a matrix such that PQ = k I, where k ∈ R, k ≠ 0 and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and det (Q) = $\frac{k^2}{2}$, then
(1) α = 0, k = 8 (2) 4α + k + 8 = 0 (3) det (P adj (Q)) = 2^9 (4) det (Q adj (P)) = 2^{13}

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

1. If f(x) = $\begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & 1 \end{vmatrix}$, then $\lim\limits_{x\to 0}\frac{f(x)}{x^2}$ is :

(1) 1 (2) – 1 (3) 0 (4) 2

2. For all values of $\theta \in \left[0, \frac{\pi}{2}\right]$, the determinant of the matrix $\begin{bmatrix} -2 & \tan\theta+\sec^2\theta & 3 \\ -\sin\theta & \cos\theta & \sin\theta \\ -3 & -4 & 3 \end{bmatrix}$ always lies in the interval :

(1) $\left[\frac{7}{2}, \frac{21}{4}\right]$ (2) [3, 5] (3) (4, 6) (4) $\left(\frac{5}{2}, \frac{19}{4}\right)$

3. Let k be a positive real number and letA = $\begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$.

If det (adj A) + det (adj B) = 10^6, then [k] is equal to

(**Note** : adj M denotes the adjoint of a square matrix M and [k] denotes the largest integer less than or equal to k].

(1) 4 (2) 2 (3) 0 (4) 1

4. The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations

$(y + z) \cos 3\theta = (xyz) \sin 3\theta$

$x \sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$

$(xyz) \sin 3\theta = (y + 2z) \cos 3\theta + y \sin 3\theta$

have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is

(1) 4 (2) 3 (3) 0 (4) infinite

5. The solution of the matrix equation $\begin{bmatrix} 3 & 0 & 3 \\ 2 & 1 & 0 \\ 4 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2y \\ z \\ 3y \end{bmatrix}$ is

(1) $x = 1, y = 3, z = 2$ (2) $x = 3, y = 2, z = 1$ (3) $x = 2, y = 3, z = 1$ (4) $x = 1, y = 2, z = 3$

6. The total number of distinct $x \in R$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is

(1) 0 (2) 1 (3) 2 (4) 3

7. Let m be a positive integer & $D_r = \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$ $(0 \le r \le m)$, then the value of $\sum_{r=0}^{m} D_r$ is

(1) –1 (2) 0 (3) 2 (4) 3

8. Let $a, \lambda, \mu \in R$. Consider the system of linear equations

$ax + 2y = \lambda$

$3x - 2y = \mu$

Which of the following statement(s) is(are) correct ?

(1) if $a = -3$, then the system has infinitely many solutions for all values of λ and μ

(2) If $a \neq -3$, then the system has a a unique solution for all values of λ and μ

(3) If $\lambda + \mu = 0$, the the system has infinitely many solutions for $a = -3$

(4) If $\lambda + \mu \neq 0$, then the system has no solution for $a = -3$

Comprehension # 1 (Q.9 to Q.11)

Some special square matrices are defined as follows :

Nilpotent matrix : A square matrix A is said to be nilpotent (of order 2) if, $A^2 = O$. A square matrix is said to be nilpotent of order p, if p is the least positive integer such that $A^p = O$.

Idempotent matrix : A square matrix A is said to be idempotent if, $A^2 = A$.

e.g. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an idempotent matrix.

Involutory matrix : A square matrix A is said to be involutory if $A^2 = I$, I being the identity matrix.

e.g. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an involutory matrix.

Orthogonal matrix : A square matrix A is said to be an orthogonal matrix if $AA' = I = A'A$.

9. If A and B are two square matrices such that AB = A & BA = B, then A & B are

(1) Idempotent matrices (2) Involutory matrices

(3) Orthogonal matrices (4) Nilpotent matrices

10. If the matrix $\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal, then

(1) $\alpha = \pm \frac{1}{\sqrt{2}}$ (2) $\beta = \pm \frac{1}{\sqrt{6}}$ (3) $\gamma = \pm \frac{1}{\sqrt{3}}$ (4) all of these

11. The matrix A = $\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ is

(1) idempotent matrix (2) involutory matrix
(3) nilpotent matrix (4) symmetric matrix

Comprehension # 2 (Q. 12 to 14)

A = $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, if U_1, U_2, and U_3 are columns matrices satisfying $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ and $AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$. If U is 3 × 3 matrix whose columns are U_1, U_2, U_3 then answer the following questions

12. The value of |U| is

(1) 3 (2) –3 (3) 3/2 (4) 2

13. The sum of the elements of U^{-1} is

(1) –1 (2) 0 (3) 1 (4) 3

14. The value of [3 2 0] U $\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ is

(1) [5] (2) $\left[\frac{5}{2}\right]$ (3) [4] (4) $\left[\frac{3}{2}\right]$

Comprehension # 3 (Q. 15 to 17)

Let a, b and c be three real numbers satisfying

$$[a\ \ b\ \ c] \begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0\ \ 0\ \ 0] \qquad \text{...........(E)}$$

15. If the point P(a, b, c), with reference to (E), lies on the plane 2x + y + z = 1, then the value of 7a + b + c is

(1) 0 (2) 12 (3) 7 (4) 6

16. Let ω be a solution of $x^3 - 1 = 0$ with Im (ω) > 0. if a = 2 with b and c satisfying (E), then the value of $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$ is equal to

(1) – 2 (2) 2 (3) 3 (4) – 3

17. Let b = 6, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty} \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n$ is

(1) 6 (2) 7 (3) $\frac{6}{7}$ (4) ∞

APSP Answers

PART - I

1.	(4)	**2.**	(3)	**3.**	(2)	**4.**	(3)	**5.**	(1)	**6.**	(3)	**7.**	(2)
8.	(2)	**9.**	(4)	**10.**	(2)	**11.**	(2)	**12.**	(2)	**13.**	(3)	**14.**	(3)
15.	(1)	**16.**	(2)	**17.**	(3)	**18.**	(4)	**19.**	(4)	**20.**	(3)	**21.**	(3)
22.	(1)	**23.**	(2)	**24.**	(3)	**25.**	(4)	**26.**	(3)	**27.**	(2)	**28.**	(4)
29.	(4)	**30.**	(3)										

PART - II

1.	(1)	**2.**	(2)	**3.**	(1)	**4.**	(2)	**5.**	(4)	**6.**	(3)	**7.**	(2)
8.	(2,3,4)	**9.**	(1)	**10.**	(4)	**11.**	(3)	**12.**	(1)	**13.**	(2)	**14.**	(1)
15.	(4)	**16.**	(1)	**17.**	(2)								