Probability

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

This Section is not meant for classroom discussion. It is being given to promote self-study and self testing amongst the Resonance students.

Max. Marks : 120 **Max. Time : 1 Hr.**

Important Instructions :

1. The test is of **1 hour** duration and max. marks 120.

2. The test consists **30** questions, **4 marks** each.

3. Only one choice is correct **1 mark** will be deducted for incorrect response. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

4. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 3 above.

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1. When two fair coins are tossed, probability of getting one head and one tail is p, probability that it shows a tail in 1_{st} toss and a head in 2_{nd} is q then the value of $\frac{p}{q}$ is

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) 2 (4) 3

2. If odds against an event is $\frac{4}{11}$ then the probability of occurence of that event is ?

(1) $\frac{4}{11}$ (2) $\frac{11}{12}$ (3) $\frac{11}{15}$ (4) $\frac{4}{15}$

3. A fair die is rolled . The probability that it shows either a perfect square or an even number is ?

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) $\frac{5}{6}$ (4) $\frac{1}{2}$

4. Consider two independent event A and B such that P(A) = $\frac{1}{4}$ and P(B) = $\frac{1}{3}$, then the value of $P(A' \cap B')$ is

(1) $\frac{1}{12}$ (2) $\frac{11}{12}$ (3) $\frac{5}{12}$ (4) $\frac{1}{2}$

5. Gargy speaks the truth in 60% cases while Ashmeet speaks the truth in 90% of the cases. In what percentage of cases they are likely to contradict each other in stating the same fact ?

(1) 60% (2) 21% (3) 42% (4) 30%

6. The probability that police inspector Mark will catch the thief in a day is $\frac{1}{4}$ and the probability that he will catch a robber in a day is $\frac{1}{5}$ and the probability that he will catch both a thief and robber in a day is $\frac{1}{15}$ then the probability that He will catch at least 1 mischief is ?

(1) $\frac{23}{60}$ (2) $\frac{27}{60}$ (3) $\frac{31}{60}$ (4) $\frac{19}{60}$

7. A fair die is thrown then probability of geting an odd number if it is given that the number obtained is a prime number

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{5}{6}$ (4) $\frac{2}{3}$

8. Two dice are thrown together, then the probability that sum of these two number is a prime number if it is given that both the results are prime numbers

(1) $\frac{4}{9}$ (2) $\frac{1}{4}$ (3) $\frac{2}{3}$ (4) $\frac{5}{9}$

9. Three students are selected from group of 10 students. The probability that Hayden is selected if it is given that Adom is always one of the selected students ?

(1) $\frac{7}{36}$ (2) $\frac{11}{36}$ (3) $\frac{2}{9}$ (4) $\frac{5}{9}$

10. Mr. Thomas a famous liar is known to speak the truth 3 out of 4 times. His friend David throws a die blindfolded and asked Thomas the result who says that "it is a six", the probability that it is actually a six

(1) $\frac{3}{8}$ (2) $\frac{5}{8}$ (3) $\frac{1}{4}$ (4) $\frac{1}{8}$

11. If two cards are drawn simultaneously from a well shuffled pack of cards then the probability of both being cards king is

(1) $\frac{2}{52}$ (2) $\frac{^4C_2}{^{52}C_2}$ (3) $\frac{4}{52}$ (4) $\frac{^2C_2}{^{52}C_2}$

12. If four cards drawn at random from a well shuffled pack of cards then the probability that all of them are face cards is

(1) $\frac{^{16}C_4}{^{52}C_4}$ (2) $\frac{^4C_4}{^{52}C_4}$ (3) $\frac{^{16}C_2}{^{52}C_4}$ (4) $\frac{^{12}C_4}{^{52}C_4}$

13. If 7 boys and 8 girls are seated around a circular table then the probability that all the boys do not sit together is

(1) $\left(\frac{14!-8(7!)}{14!}\right)$ (2) $\frac{7!\times 8!}{14!}$ (3) $\frac{(7!)^2}{14!}$ (4) $\frac{(8!)^2}{14!}$

14. If two cards are drawn at random from a pack of cards the the probability that one of them is a numbered card and the other is a queen is

(1) $\frac{^{36}C_2}{^{52}C_2}$ (2) $\frac{^{40}C_1 . ^4C_1}{^{52}C_2}$ (3) $\frac{^{36}C_1 . ^4C_1}{^{52}C_2}$ (4) $\frac{^{40}C_2}{^{52}C_2}$

15. A bag contains 25 tickets, numbered from 1 to 25 two tickets are drawn from the bag without replacement, the probability that the 2_{nd} ticket is perfect square if it is known that 1_{st} ticket was perfect square number is

(1) 1/5 (2) 4/25 (3) 1/5 (4) 1/6

16. A die is thrown 5 times, the probability that a composite number will appear exactly three times is?

(1) $\frac{2^6}{6^6}$ (2) $\frac{2^2}{3^5}$ (3) $\frac{10.2^2}{3^5}$ (4) $\frac{10.2^3}{3^5}$

Probability

17. A bag contains 3 red and 2 black balls. One ball is drawn 4 times (with replacement) then the probability that the result is 2 red and 2 black balls is ?

(1) $\frac{3^2 2^2}{5^4}$ (2) $\frac{3^3 2^3}{5^4}$ (3) $\frac{3^2 2^3}{5^4}$ (4) $\frac{3^3 2^2}{5^4}$

18. If two dice are thrown 5 times then the probability that sum 10 will appear in exactly 3 times is

(1) ${}^5C_3\left(\frac{11}{12}\right)^3\left(\frac{1}{12}\right)^2$ (2) ${}^5C_2\left(\frac{11}{12}\right)^2\left(\frac{1}{12}\right)^3$

(3) ${}^5C_2\left(\frac{11}{12}\right)^2\left(\frac{1}{12}\right)^2$ (4) None of these

19. If three coins are tossed simultaneously then the probability of getting two heads and one tail is ?

(1) $\frac{1}{4}$ (2) $\frac{1}{8}$ (3) $\frac{3}{8}$ (4) $\frac{5}{8}$

20. The probability that a leap year selected at random will have 53 friday is ?

(1) $\frac{2}{7}$ (2) $\frac{1}{7}$ (3) $\frac{7}{52}$ (4) $\frac{8}{52}$

21. If P(A) = $\frac{1}{8}$, P(B) = $\frac{2}{5}$ and A & B are mutually exclusive events then the value of P(A∪B) is

(1) $\frac{23}{40}$ (2) $\frac{19}{40}$ (3) $\frac{21}{40}$ (4) $\frac{17}{40}$

22. If P(A/B) = $\frac{2}{3}$, P(B) = $\frac{3}{5}$ then the value of P (A'∩B) is

(1) $\frac{2}{5}$ (2) $\frac{1}{10}$ (3) $\frac{1}{5}$ (4) $\frac{3}{5}$

23. Urn A contains 6 red & 4 black balls and urn B contains 4 red & 6 black balls. One ball is drawn at random from urn A & placed in urn B. Then one ball of different colour is drawn at random from urn B & placed in urn A. If one ball is now drawn at random from urn A, then the probability of red ball is drawn is

(1) $\frac{32}{55}$ (2) $\frac{73}{275}$ (3) $\frac{9}{55}$ (4) $\frac{28}{275}$

24. In a single throw of three dice, the probability of the event ‘a total of 7 is

(1) 7/72 (2) 6/72 (3) 5/72 (4) 4/72

25. A car is parked by an owner in a parking lot of 24 cars in a row, including his car not at either end. On his return he finds that exactly 12 placed are still occupied. The probability that both the neighboring places are empty is

(1) $\frac{91}{276}$ (2) $\frac{15}{184}$ (3) $\frac{15}{92}$ (4) $\frac{6}{23}$

26. A number is chosen at random from the numbers 10 to 99. A number whose product of digits is 15 will be called a good number. If he choose three numbers with replacement then the probability that he will choose a good number at least once is

(1) 0.0578 (2) 0.127 (3) 0.562 (4) 0.461

Probability

27. Mean and variance of a Binomial variate of 6 trials of the experiment are in the ratio of 3 : 2, then the correct options are

(1) Probability of getting exactly 5 success is $\frac{1}{81}$ (2) Probability of getting exactly 6 success is $\frac{2}{81}$

(3) Probability of getting exactly 5 success is $\frac{2}{81}$ (4) Probability of getting exactly 4 success is $\frac{2}{81}$

28. An unbiased coin is tossed n times. Let X denote the number of times head occurs. If P(X = 4), P (X = 5) and P(X = 6) are in GP, then the value of n can be

(1) 7 (2)–1

(3) 2 (4) there is no value of n

29. A and B throw alternately with a pair of dice. A wins if he throws 6 (sum of numbers in pair of dice) before B throws 7, B wins if he throws 7 before A throws 6. then the ratio of winning chances of A and B is (A makes the first throw)

(1) $\frac{30}{31}$ (2) $\frac{30}{61}$ (3) $\frac{31}{61}$ (4) 1

30. The numbers 'a and b ' are randomly selected from the set of natural numbers. Probability that the number $3^a + 2^b$ has a digit equal to 1 at the units place, is $\frac{p}{q}$ then p + q is : (Where p & q are co-prime natural numbers)

(1) 13 (2) 19 (3) 5 (4) 20

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

Marked Questions may have for Revision Questions.

*** Marked Questions may have more than one correct option.**

1*. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$ For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denotes respectively the events that the engines E_1 E_2 and E_3 are functioning. Which of the following is (are) true ?

(1) $P\left[X_1^c \mid x\right] = \frac{3}{16}$

(2) P[Exactly two engines of the ship are functioning | X] = $\frac{7}{8}$

(3) $P[X \mid X_2] = \frac{5}{16}$

(4) $P[X \mid X_1] = \frac{7}{16}$

2. An integer x is chosen from the first 100 positive integers. The probability that, $x + \frac{100}{x} > 50$ is:

(1) $\frac{1}{10}$ (2) $\frac{11}{20}$ (3) $\frac{1}{2}$ (4) none

3. The sides of a rectangle are chosen at random, each less than 10 cm, all such lengths being equally likely. The chance that the diagonal of the rectangle is less than 10 cm is

(1) 1/10 (2) 1/20 (3) $\pi/4$ (4) $\pi/8$

4. Let p be the probability that a man aged x years will die in a year time. The probability that out of 'n' men $A_1, A_2, A_3, \ldots, A_n$ each aged 'x' years. A_1 will die & will be the first to die in a year time is:

(1) $\frac{1-p^n}{n}$ (2) $\frac{p}{n}$ (3) $\frac{p\ (1-p)^{n-1}}{n}$ (4) $\frac{1-(1-p)^n}{n}$

5. A person has to go through three successive tests. Probability of his passing first exam is P. Probability of passing successive test is P or P/2 according as he passed the last test or not. He is selected if he passes atleast two tests. The probability of his selection is

(1) $3P^2 - P^3$ (2) $2P^2 - P^3$ (3) $P^2 - P^3$ (4) None of these

6. In a combat, A targets B, and both B and C target A, The probabilities of A, B, C hitting their targets are 2/3, 1/2 and 1/3 respectively. They shoot simultaneously and A is hit. The probability that B hits his target whereas C does not is

(1) $\frac{1}{3}$ (2) $\frac{1}{4}$ (3) $\frac{1}{2}$ (4) $\frac{1}{6}$

7. Let $H_1, H_2, \ldots H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$ i = 1, 2, ..., n. Let E be any other event with $0 < P(E) < 1$.

STATEMENT-1 : $P(H_i / E) > P(E / H_i) \cdot P(H_i)$ for i = 1, 2, ..., n.

because

STATEMENT-2 : $\sum_{i=1}^{n} P(H_i) = 1$.

(1) Statement-1 is True, Statement-2 is True ; Statement-2 **is** a correct explanation for Statement-1

(2) Statement-1 is True, Statement-2 is True ; Statement-2 **is NOT** a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

8. **Statement 1 :** If p is chosen at random in the closed interval [0, 5], then the probability that the equation $x^2 + px + \frac{1}{4}(p + 2) = 0$ has real root is $\frac{3}{5}$.

Statement 2 : If discriminant ≥ 0 then roots of the quadratic equation of real coefficient are always real.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

9. **Statement 1 :** If A and B are independent events and P(C) is the probability of exactly one of A or B occurs, then $P(A \cup B)\, P\left(\bar{A} \cap \bar{B}\right) \le P(C)$.

Statement 2 : $P(A \cup B) \le P(A) + P(B)$ and $P(A) \le 1$.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(2) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

Comprehension # 1

If A and B are two events, then probability that atleast one of them is selected is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. For three events A, B, C the probability that atleast one of them is seleted is $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$.

10. Probability that none of the three events (A, B, C) occurs

(1) $P(\bar{A} \cup \bar{B} \cup \bar{C}) - P(A) - P(B) - P(C) + P(A \cap B) + P(B \cap C) + P(C \cap A)$

(2) $P(A \cup B \cup C) - P(A) - P(B) - P(C) + P(A \cap B) + P(B \cap C) + P(C \cap A) - P(A \cap B \cap C)$

(3) $P(\bar{A}) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

(4) none of these

11. Probability that exactly two events occurs

(1) $P(A) + P(B) + P(C) - P(A \cup B) - P(B \cup C) - P(C \cup A)$

(2) $2(P(A) + P(B) + P(C)) - P(A \cup B) - P(B \cup C) - P(C \cup A)$

(3) $P(A \cap B) + P(B \cap C) + P(C \cap A) - 3 \times P(A \cap B \cap C)$

(4) $P(\bar{A} \cap \bar{B}) + P(\bar{B} \cap \bar{C}) + P(\bar{C} \cap \bar{A})$

12. Probability that atmost two events happen

(1) $P(A \cap B) + P(B \cap C) + P(C \cap A) - 3 \times P(A \cap B \cap C)$

(2) $P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)$

(3) $P(\bar{A}) + P(\bar{B}) + P(\bar{C})$

(4) $P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cup B \cup C)^c$

Comprehension # 2

Probability

Consider the experiment of distribution of balls among urns. Suppose we are given M urns, numbered 1 to M, among which we are to distribute n balls (n < M). Let P(A) denote the probability that each of the urns numbered 1 to n will contain exactly one ball. Then answer the following questions.

13. If the balls are different and any number of balls can go to any urns, then P(A) is equal to

(1) $\frac{M!}{n^M}$ (2) $\frac{n!}{M^n}$ (3) $\frac{n!}{{}^MP_n}$ (4) $\frac{1}{M^n}$

14. If the balls are identical and any number of balls can go to any urns, then P(A) equals

(1) $\frac{1}{M^n}$ (2) $\frac{1}{{}^{M+n-1}C_{M-1}}$ (3) $\frac{1}{{}^{M+n-1}C_{n-1}}$ (4) $\frac{1}{{}^{M+n}P_{M-1}}$

15. If the balls are identical but atmost one ball can be put in any box, then P(A) is equal to

(1) $\frac{1}{{}^MP_n}$ (2) $\frac{n!}{{}^nC_M}$ (3) $\frac{n!}{{}^MC_n}$ (4) $\frac{1}{{}^MC_n}$

Comprehension # 3

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

16. The probability of the drawn ball from U_2 being white is

(1) $\frac{13}{30}$ (2) $\frac{23}{30}$ (3) $\frac{19}{30}$ (4) $\frac{11}{30}$

17. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(1) $\frac{17}{23}$ (2) $\frac{11}{23}$ (3) $\frac{15}{23}$ (4) $\frac{12}{23}$

18. Consider the system of equations
$ax + by = 0$; $cx + dy = 0$, where $a, b, c, d \in \{0, 1\}$

STATEMENT-1 : The probability that the system of equations has a unique solution is $\frac{3}{8}$.
and
STATEMENT-2 : The probability that the system of equations has a solution is 1.

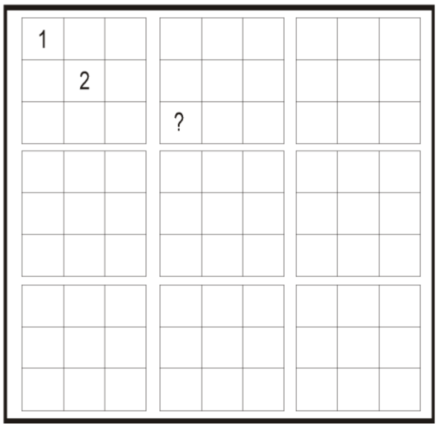
(1) STATEMENT-1 is True, STATEMENT-2 is True ; STATEMENT-2 is a correct explanation for STATEMENT-1
(2) STATEMENT-1 is True, STATEMENT-2 is True ; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(3) STATEMENT-1 is True, STATEMENT-2 is False
(4) STATEMENT-1 is False, STATEMENT-2 is True

19. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - 2\beta)\,p = \alpha\beta$ and $(\beta - 3\gamma)\,p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3} =$

(1) 4 (2) 5 (3) 6 (4) 7

20. A Sudoku matrix is defined as a 9 × 9 arrary with entries from {1, 2, 3 . . . . . 9} and with the constraint that each row, each column and each of the nine 3 × 3 boxes that tile the array contains each digit from 1 to 9 exactly once. A Sudoku matrix is chosen at random (so that every Sudoku matrix has equal probability of being chosen). We know two of square in this matrix as shown. Then the probability that the square marked by ? contains the digit 3 is



(1) $\frac{1}{21}$ (2) $\frac{2}{21}$ (3) $\frac{4}{21}$ (4) $\frac{8}{21}$

Probability

APSP Answers

PART - I

1.	(3)	**2.**	(3)	**3.**	(2)	**4.**	(4)	**5.**	(3)	**6.**	(1)	**7.**	(4)
8.	(1)	**9.**	(3)	**10.**	(1)	**11.**	(2)	**12.**	(4)	**13.**	(1)	**14.**	(2)
15.	(4)	**16.**	(3)	**17.**	(2)	**18.**	(2)	**19.**	(3)	**20.**	(1)	**21.**	(3)
22.	(3)	**23.**	(2)	**24.**	(3)	**25.**	(4)	**26.**	(1)	**27.**	(3)	**28.**	(4)
29.	(1)	**30.**	(2)										

PART - II

1.	(2,4)	**2.**	(2)	**3.**	(3)	**4.**	(4)	**5.**	(2)	**6.**	(3)	**7.**	(4)
8.	(1)	**9.**	(2)	**10.**	(1)	**11.**	(3)	**12.**	(4)	**13.**	(2)	**14.**	(2)
15.	(4)	**16.**	(2)	**17.**	(4)	**18.**	(2)	**19.**	(3)	**20.**	(2)		