Complex Numbers

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

This Section is not meant for classroom discussion. It is being given to promote self-study and self testing amongst the Resonance students.

Max. Marks : 120 **Max. Time : 1 Hr.**

Important Instructions :

1. The test is of **1 hour** duration and max. marks 120.

2. The test consists **30** questions, **4 marks** each.

3. Only one choice is correct **1 mark** will be deducted for incorrect response. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

4. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 3 above.

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1. The amplitude of $\frac{1+\sqrt{3}\,i}{\sqrt{3}+i}$ is

(1) $\frac{\pi}{6}$ (2) $\frac{-\pi}{6}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

2. $\sqrt{-8-6i}$ =

(1) 1± 3i (2) ± (1–3i) (3) ± (1+3i) (4) ± (3 – i)

3. The value of $(-i)^{\frac{1}{3}}$ is

(1) $\frac{1+\sqrt{3}i}{2}$ (2) $\frac{1-\sqrt{3}i}{2}$ (3) $\frac{-\sqrt{3}-i}{2}$ (4) $\frac{\sqrt{3}+i}{2}$

4. If |z| = 2, then the points representing the complex numbers –1+5z will lie on a

(1) circle (2) straight line (3) parabola (4) hyperbola

5. Let z_1 = 2 + 3i and z_2 = 2 – 3i be two points on the complex plane . Then the set of complex numbers z satisfiyng $|z-z_1|_2 + |z-z_2|_2 = |z_1-z_2|_2$ represents

(1) A straight line (2) A point (3) A circle (4) A pair of stright lines

6. If n is a positive integer not multiple of 3, then $1+\omega_n + \omega_{2n}$ = (where ω is imaginary cube root of unity)

(1) 3 (2) 1 (3) 0 (4) 2

7. If ω is imaginary cube root of unity, then a root of the equation $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$ is

(1) x = 1 (2) x = ω (3) x = ω_2 (4) x = 0

8. If ω is imaginary cube root of unity then value of $(1+\omega-\omega_2)_2 + (1-\omega+\omega_2)_2 + 1$ is

(1) 1 (2) – 3 (3) – 1 (4) 7

Complex Numbers

9. If $z + \frac{1}{z} = 1$ then $z^{100} + z^{-100}$ is equal to

(1) i (2) – i (3) 1 (4) – 1

10. $\frac{\left(-1+i\sqrt{3}\right)^{15}}{(1-i)^{20}} + \frac{\left(-1-i\sqrt{3}\right)^{15}}{(1+i)^{20}}$ is equal to

(1) – 64 (2) – 32 (3) – 16 (4) $\frac{1}{16}$

11. Locus of z such that $\arg\left(\frac{z-i}{z+i}\right) = \frac{\pi}{2}$ is

(1) Circle (2) Minor arc of circle

(3) Major arc of circle (4) Semi circle

12. If α and β are imaginary cube roots of unity, then $\alpha^4 + \beta^4 + \frac{1}{\alpha\beta} =$

(1) 3 (2) 0 (3) 1 (4) 2

13. $(\sin\theta + i\cos\theta)^n$ is equal to

(1) $\cos n\theta + i \sin n\theta$ (2) $\sin n\theta + i \cos n\theta$

(3) $\cos n\left(\frac{\pi}{2}-\theta\right) + i\sin n\left(\frac{\pi}{2}-\theta\right)$ (4) $\sin\theta + i\cos\theta$

14. $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n =$

(1) $\cos\left(\frac{n\pi}{2}-n\theta\right) + i\sin\left(\frac{n\pi}{2}-n\theta\right)$ (2) $\cos\left(\frac{n\pi}{2}+n\theta\right) + i\sin\left(\frac{n\pi}{2}+n\theta\right)$

(3) $\sin\left(\frac{n\pi}{2}-n\theta\right) + i\cos\left(\frac{n\pi}{2}-n\theta\right)$ (4) $\sin n\left(\frac{\pi}{2}-2\theta\right) + i\cos n\left(\frac{\pi}{2}-2\theta\right)$

15. If z = x + iy is a complex number satisfying $|z+\frac{i}{2}|^2 = |z-\frac{i}{2}|^2$, then the locus of z is

(1) 2y = x (2) y = x (3) y – axis (4) x-axis

16. If z = x + iy and |z – iz| = 1, then

(1) z , lies x-axis (2) z , lies y-axis (3) z , lies on a circle (4) z , lies on a parabola

17. If ω is a complex number satisfying $\left|\omega + \frac{1}{\omega}\right| = 2$, then maximum distance of ω from orgin is

(1) $2 + \sqrt{3}$ (2) $1 + \sqrt{2}$ (3) $1 + \sqrt{3}$ (4) $1 - \sqrt{2}$

18. If a and b are real numbers between 0 and 1 such that the points $z_1 = a + i$, $z_2 = 1 + bi$, and $z_3 = 0$ form an equilateral triangle, then

(1) a = b = $2+\sqrt{3}$ (2) a = b = $2-\sqrt{3}$ (3) a = $2-\sqrt{3}$ (4) a = b = $3+\sqrt{3}$

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19. $\left(-7-24i\right)^{\frac{1}{2}} = x - iy$ then $x^2 + y^2 =$

(1) 15 (2) 25 (3) – 25 (4) 20

20. If z = $\frac{7-i}{3-4i}$, then z^{14}=

(1) 2^7 (2) $2^7 i$ (3) $2^{14} i$ (4) $-2^7 i$

21. If $|z_1+z_2| = |z_1 - z_2|$, then the difference in the amplitude of z_1 and z_2 is

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{2}$ (4) 0

22. Argument and modulus of $\frac{1+i}{1-i}$ are respectively

(1) $\frac{-\pi}{2}$ and 1 (2) $\frac{\pi}{2}$ and $\sqrt{2}$ (3) 0 and $\sqrt{2}$ (4) $\frac{\pi}{2}$ and 1

23. The principle argument of $(\sin 40° + i\cos 40°)^5$ is

(1) 70° (2) – 110° (3) 110° (4) – 70°

24. Which of the following equation can represent a triangle

(1) $|z-1| = |z-2|$ (2) $|z-1| = |z-2| = |z-i|$ (3) $|z-1| - |z-2| = 29$ (4) $|z-1|^2 + |z-2|^2 = 4$

25. The modulus of $\frac{1-i}{3+i} + \frac{4i}{5}$ is

(1) $\sqrt{5}$ units (2) $\frac{\sqrt{11}}{5}$ units (3) $\frac{1}{\sqrt{5}}$ units (4) $\frac{\sqrt{12}}{5}$ units

26. Suppose z_1, z_2, z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1+i$, then values of z_3 and z_2 respectively

(1) – 2, $1-i\sqrt{3}$ (2) $2, i\sqrt{3}$ (3) $1+i\sqrt{3}, -2$ (4) $-2, i\sqrt{3}$

27. If z satisfies the inequality $|z - 1 - 2i| \le 1$, then which of the following are true.

(1) maximum value of $|z| = \sqrt{5}+1$ (2) minimum value of $|z| = \sqrt{5}+1$

(3) minimum value of arg(z) = π/2 (4) maximum value of arg(z) = $\tan^{-1}\left(\frac{3}{4}\right)$

28. If z is a complex number then $(z + 5)(\bar{z} + 5)$ is

(1) $(z+5)^2$ (2) $|z+5|^2$ (3) $|z+5i|^2$ (4) $|z-5|^2$

29. The complex number $\frac{(-\sqrt{3}+3i)(1-i)}{(3+\sqrt{3}i)\, i\, (\sqrt{3}+\sqrt{3}i)}$ when represented in the argand plane is

(1) In the 2^{nd} quadrant (2) In the I^{st} quadrant (3) on the y-axis (4) on the x-axis

30. If cube root of unity be $1, \omega, \omega^2$, then the roots of the equation $(x-2)^3 + 27 = 0$ are

(1) $-1,\ 3-2\omega,\ 3-2\omega^2$ (2) $-1,\ 1-2\omega,\ 1-2\omega^2$

(3) $-1,\ 1+2\omega,\ 1+2\omega^2$ (4) $-1,\ 2-3\omega,\ 2-3\omega^2$

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Complex Numbers

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

1. Values of z satisfying the equation $z^2 - (1+i)\, zz_1 + iz_1^2 = 0$ (where z_1 is a complex no.) are two vertices of a triangle having one vertex as origin then the area of this triangle is

(1) $\frac{1}{3}|z_1|^2$ (2) $2|z_1|^2$ (3) $\frac{1}{2}|z_1|^2$ (4) $3|z_1|^2$

2. Let z be non real number such that $\frac{1+z+z^2}{1-z+z^2} \in R$, then value of 7|z| is

(1) 1 (2) 3 (3) 5 (4) 7

3. Given z is a complex number with modulus 1. Then the equation $[(1 + ia)/(1 - ia)]^4 = z$ in 'a' has

(1) all roots real and distinct (2) two real and two imaginary

(3) three roots real and one imaginary (4) one root real and three imaginary

4. If $\log_{1/2}\left(\frac{|z-1| + 4}{3|z-1| - 2}\right) > 1$, then the locus of z is

(1) Exterior to circle with center 1 + i0 and radius 10

(2) Interior to circle with center 1 + i0 and radius 10

(3) Circle with center 1 + i0 and radius 10

(4) Circle with center 2 + i0 and radius 10

5. Let O = (0, 0) ; A = (3, 0) ; B = (0, 1) and C = (3, 2), then minimum value of $|z| + |z-3| + |z-i| + |z-3-2i|$ occur at

(1) intersection point of AB and CO (2) intersection point of AC and BO

(3) intersection point of CB and AO (4) mean of O, A, B, C

6. Locus of z such that Arg (z + i) − Arg (z − i) = π/2 is semicircle $x^2 + y^2 = k^2$ in first and fourth quadrants, then k =

(1) 3 (2) 2 (3) 4 (4) 1

7. If z = x + iy and $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{6}$, then locus of z is

(1) A straight line (2) Arc of a circle (3) A parabola (4) An ellipse

8. The vertices B and D of a parallelogram are 1 – 2i and 4 + 2i. If the diagonals are at right angles and AC = 2BD, the complex number representing A is

(1) $\frac{5}{2}$ (2) $-\frac{3}{2}$ (3) $\frac{3}{2}+2i$ (4) $-\frac{3}{2}+3i$

9. If z_1 & z_2 are two complex numbers & if $\arg\frac{z_1+z_2}{z_1-z_2}=\frac{\pi}{2}$ but $|z_1+z_2| \neq |z_1-z_2|$ then the figure formed by the points represented by 0, z_1, z_2 & z_1+z_2 is :
(1) a parallelogram but not a rectangle or a rhombous
(2) a rectangle but not a square
(3) a rhombous but not a square
(4) a square

10. The real values of the parameter 'a' for which at least one complex number z = x + iy satisfies both the equality $|z - ai| = a + 4$ and the inequality $|z - 2| < 1$.
(1) $\left(-\frac{21}{10},-\frac{5}{6}\right)$ (2) $\left(-\frac{7}{2},-\frac{5}{6}\right)$ (3) $\left(\frac{5}{6},\frac{7}{2}\right)$ (4) $\left(-\frac{21}{10},\frac{7}{2}\right)$

11. If z_1 , z_2 and $z_1 + z_2$ are vertices of an equilateral triangle (where z_1 and z_2 are complex numbers) and $z_1^2 + z_2^2 = \lambda z_1 z_2$ ($\lambda \in R$) then λ is equal to
(1) 2 (2) 1 (3) –2 (4) –1

12. If $4 \le |z - 2 - 3i| \le 5$ and maximum and minimum values of $|z - 1 - i|$ are M and m respectively then M + m is equal to
(1) 8 (2) $\sqrt{5}+4$ (3) $8+\sqrt{5}$ (4) 9

APSP Answers

PART - I

1.	(1)	**2.**	(2)	**3.**	(3)	**4.**	(1)	**5.**	(3)	**6.**	(3)	**7.**	(4)
8.	(2)	**9.**	(4)	**10.**	(1)	**11.**	(4)	**12.**	(2)	**13.**	(3)	**14.**	(1)
15.	(4)	**16.**	(3)	**17.**	(2)	**18.**	(2)	**19.**	(2)	**20.**	(4)	**21.**	(3)
22.	(4)	**23.**	(2)	**24.**	(2)	**25.**	(3)	**26.**	(1)	**27.**	(1)	**28.**	(2)
29.	(3)	**30.**	(4)										

PART - II

1.	(3)	**2.**	(4)	**3.**	(1)	**4.**	(1)	**5.**	(1)	**6.**	(4)	**7.**	(2)
8.	(4)	**9.**	(3)	**10.**	(1)	**11.**	(2)	**12.**	(4)				