Differential Equation

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

This Section is not meant for classroom discussion. It is being given to promote self-study and self testing amongst the Resonance students.

Max. Marks : 120 **Max. Time : 1 Hr.**

Important Instructions :

1. The test is of **1 hour** duration and max. marks 120.
2. The test consists **30** questions, **4 marks** each.
3. Only one choice is correct **1 mark** will be deducted for incorrect response. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 3 above.

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1. The order and degree of the differential equation $\frac{d^3y}{dx^3} = \left\{2 + \left(\frac{d^4y}{dx^4}\right)^3\right\}^{5/2}$ are respectively

(1) 4, 1 (2) 3, 3 (3) 4, 15 (4) 4, 3

2. The degree of the differential equation $xy + \frac{d^3y}{dx^3} = \sin\left(\frac{d^4y}{dx^4}\right)$ is

(1) 3 (2) 4 (3) 0 (4) Not defined

3. The order of differential equation whose general solution is given by

$y = k_1.2^{x+k_2} + \frac{\tan(\pi/4 + k_3x)}{\cot(\pi/4 - k_3x)} + 2$ is

(1) 4 (2) 3 (3) 1 (4) 2

4. Differential equation of the family of circle touching the line x =1 at (1,0) is

(1) $y^2=2(x-1) + 2y(x-1)\frac{dy}{dx}$ (2) $y^2=2(x-1)^2 + y(x-1)\frac{dy}{dx}$

(3) $y^2=(x-1)^2 + 2y(x-1)\frac{dy}{dx}$ (4) $y^2=2(x-1) + y(x-1)\frac{dy}{dx}$

5. The differential equation of family of curve $y = k(x+k)^3$ where k is parameter is

(1) $y\left(\frac{dy}{dx}\right)^4 = 3\left(3y - x\frac{dy}{dx}\right)$ (2) $\left(\frac{dy}{dx}\right)^2 = (3y)^2\left(3y - x\frac{dy}{dx}\right)$

(3) $\left(\frac{dy}{dx}\right)^4 = 27y^2\left(3y - x\frac{dy}{dx}\right)$ (4) $\left(\frac{dy}{dx}\right)^4 = 3\left(3y - x\frac{dy}{dx}\right)$

6. The differential equation of all parabolas whose axis are parallel to the y-axis and have latus rectum a is

(1) $a\frac{d^2y}{dx^2} = 2$ (2) $\frac{d^2y}{dx^2} + a\frac{dy}{dx} = 0$ (3) $\frac{d^2y}{dx^2} = \frac{a}{2}$ (4) $\frac{xd^2y}{dx^2} = \frac{2}{a}$

Differential Equation

7. The general solution of the differential equation $\frac{dy}{dx}$ = cotx.coty is

(1) cos x = c cosec y (2) sin x = c sec y (3) sin x = c cos y (4) cos x = c sin y

8. The solution of the differential equation $\frac{dy}{dx}=\frac{y^2-y-2}{x^2+2x-3}$ is

(1) $\frac{1}{3}\ell n\left|\frac{y-2}{y+1}\right|=\frac{1}{2}\ell n\left|\frac{x+3}{x-1}\right|+c$ (2) $\frac{1}{3}\ell n\left|\frac{y+1}{y-2}\right|=\frac{1}{4}\ell n\left|\frac{x-1}{x+3}\right|+c$

(3) $2\ell n\left|\frac{y+1}{y-2}\right|=3\ell n\left|\frac{x+3}{x-1}\right|+c$ (4) $4\ell n\left|\frac{y-2}{y+1}\right|=3\ell n\left|\frac{x-1}{x+3}\right|+c$

9. The solution of differential equation $e^{dy/dx}$ =x+1, y(0) = 3 is

(1) y =xℓnx– x+2 (2) y =(x+1)ℓn(x+1)– x+3

(3) y = xℓnx + x+3 (4) y =– (x+1)ℓn|x+1|+ x+3

10. The general solution of the differential equation $\ell n\left(\frac{dy}{dx}\right)=x+y$ is

(1) $e^{-x}+e^{-y}=c$ (2) $e^{x}+e^{-y}=c$ (3) $e^{x}+e^{y}=c$ (4) $e^{-x}+e^{y}=c$

11. The solution of the differential equation $\frac{dy}{dx}=(2x+y)^2$ is

(1) $\frac{1}{2\sqrt{2}}\ell n\left|\frac{2x+y-\sqrt{2}}{2x+y-\sqrt{2}}\right|=x+c$ (2) $\frac{1}{2\sqrt{2}}\ell n\left|\frac{2x+y+\sqrt{2}}{2x+y-\sqrt{2}}\right|=x+c$

(3) $\frac{1}{\sqrt{2}}\tan^{-1}\frac{(2x+y)}{\sqrt{2}}=x+c$ (4) $\tan^{-1}\frac{(2x+y)}{\sqrt{2}}=x+c$

12. The solution of the differential equation $\frac{dy}{dy}\sqrt{1+x+y}$ = x+y –1 is 2t+$k_1$ln|t–1|+k_2 ℓn|t + 2| = x + c,

where t = $\sqrt{x+y+1}$, then $|k_1 + k_2|$ =

(1) 10/3 (2) 2 (3) 3 (4) 4/3

13. The solution of the differential equation $\frac{dy}{dx}=\frac{1}{x+y+1}$ is

(1) x = ce^{y}– y–2 (2) y = x+ ce^{y}–2 (3) x + ce^{-y}– y=2 (4) x + ce^{y}= y+2

14. The solution of the differential equation $\left(2\sqrt{xy}-x\right)$ dy + ydx = 0 is

(1) $\ell ny+\sqrt{x/y}=c$ (2) $e^{y}=\sqrt{x/y}+c$ (3) $\ell ny=\sqrt{x/y}+c$ (4) $e^{y}+\sqrt{x/y}=c$

15. The solution of differential equation $\left(x\sin\frac{y}{x}\right)dy=\left(y\sin\frac{y}{x}-2x\right)dx$ is

(1) $\sin\left(\frac{y}{x}\right)=2\ell nx+c$ (2) $\cos\left(\frac{y}{x}\right)=\ell nx+c$ (3) $\sin\left(\frac{y}{x}\right)=\ell nx+c$ (4) $\cos\left(\frac{y}{x}\right)=2\ell nx+c$

Differential Equation

16. The solution of the differential equation $\frac{dy}{dx}=\frac{x-2y+3}{2x+y+4}$ is 2xy+ $\frac{y^2}{2}=\frac{x^2}{2}-\lambda_1 y+\lambda_2 x+c$ then $|\lambda_1+\lambda_2|=$

(1) 4 (2) 3 (3) 7 (4) 1

17. The general solution of $y^2dx+(x^2-xy+y^2)dy = 0$

(1) $\tan^{-1}\left(\frac{x}{y}\right)+\ell ny+c=0$ (2) 2 $\tan^{-1}\left(\frac{x}{y}\right)+\ell ny+c=0$

(3) $\ell n\left(y+\sqrt{x^2+y^2}\right)+\ell ny=c$ (4) $\sin^{-1}\left(\frac{x}{y}\right)+\ell ny=c$

18. The solution of differential equation $\frac{dy}{dx}+\frac{1}{x}=\frac{e^y}{x^2}$ is

(1) $2xe^{-y}=cx^2-1$ (2) $2xe^{y}=x^2+c$ (3) $2xe^{-y}=cx^2+1$ (4) $xe^{-y}=cx^2+1$

19. The solution of the differential equation, $\frac{dx}{dy}=\frac{y}{2y\ell ny+y-x}$ is

(1) $xy = y^3\ell ny+c$ (2) $xy = y^2\ell ny+c$ (3) $xy = y^5\ell ny+c$ (4) $xy = y^4\ell ny+c$

20. The solution of the differential equation $\frac{dy}{dx}$ +ycotx =sinx is ysinx = k (2x –sin2x) +c then k is

(1) 1/2 (2) 1/3 (3) 1/4 (4) 1/5

21. The solution of the differential equation $\frac{dy}{dx}=\frac{-(y+y^3)}{1+x+xy^2}$ is

(1) $xy+\frac{1}{2}\ln\left|\frac{1-y}{1+y}\right|=c$ (2) $xy+\tan^{-1}y = c$ (3) $y =x +\tan^{-1}y + c$ (4) $xy+\sin^{-1}y = c$

22. The solution of the differential equation $\frac{dx}{dy}+\frac{x}{y}=x^3$ is

(1) $2xy^2+cx^2y^2=1$ (2) $2x^2y+cx^2y^2=1$ (3) $xy^2+cx^2y^2=1$ (4) $x^2y+cx^2y^2=1$

23. The solution of the differential equation $\frac{dy}{dx}=\frac{2xy}{x^2-1-2y}$ is

(1) $\frac{x^2}{y}=\frac{1}{y}-2\ell ny+c$ (2) $\frac{x^2}{y^2}=\frac{1}{y}-2\ell ny+c$ (3) $x^2y=\ell ny+c$ (4) $\frac{x^2}{y}=2\ell ny+c$

24. The solution of the differential equation $(xy^4+y)dx = xdy$ is

(1) $3x^3y^3+4x^3=cy^3$ (2) $3x^4y^3+4x^3=cy^3$ (3) $4x^4y^3+3x^3=cy^3$ (4) $x^2y^3+4x^3=cy^3$

25. The solution of the differential equation $\frac{x+y\frac{dy}{dx}}{y-x\frac{dy}{dx}}=x^2+2y^2+\frac{y^4}{x^2}$ is

(1) $\frac{2y}{x}=x^2+y^2+c$ (2) $\frac{y}{x}=\frac{1}{x^2+y^2}+c$ (3) $\frac{2y}{x}=\frac{1}{x^2+y^2}+c$ (4) $2y=\frac{x^2+y^2}{x}+c$

26. Tangent to a curve intersect the y-axis at a point P. A line perpendicular to this tangent through point 'P' also passes through another point (1,0). The differential equation of the curve is

(1) $y\left(\frac{dy}{dx}\right)=x\left(\frac{dy}{dx}\right)^2+1$ (2) $x\left(\frac{dy}{dx}\right)^2=y+c$

(3) $y\frac{dy}{dx}+x=1$ (4) $x\frac{dy}{dx}+y=1$

27. A normal at any point (x,y) to the curve y=f(x) cut a triangle of unit area with the coordinate axes. The differential equation of the curve is

(1) $y^2-x^2\left(\frac{dy}{dx}\right)^2=4\frac{dy}{dx}$ (2) $y^2-x^2\left(\frac{dy}{dx}\right)^2=\frac{dy}{dx}$

(3) $x+y\frac{dy}{dx}=y$ (4) $y^2\left(\frac{dy}{dx}\right)^2+2(xy-1)\frac{dy}{dx}+x^2=0$

28. Spherical rain drop evaporates at a rate proportional to its surface area. The differential equation corresponding to the rate of change of the radius of the rain drop if the constant of proportionality is $k > 0$, is

(1) $\frac{dr}{dt}+k=0$ (2) $\frac{dr}{dt}-k=0$ (3) $\frac{dr}{dt}=kr$ (4) $\frac{dr}{dt}=2kr$

29. Let I be the purchase value of an equipment and v(t) be the value after it has used for t years. The v(t) depreciates at a rate given by differential quation $\frac{dv(t)}{dt}=-K(T-t^2)$, where k>0 is a constant and T is the total life in years of the equipment. Then the scrap value v(T) of the equipment is.

(1) $I-k(T^2-T^3)$ (2) $I-k\left(T^2-\frac{T^3}{3}\right)$ (3) $I+k\left(T^2-\frac{T^3}{3}\right)$ (4) $I+\frac{T^3}{3}$

30. Solution of $y=x\frac{dy}{dx}+3\frac{dy}{dx}-4\left(\frac{dy}{dx}\right)^2$ is

(1) $y = cx + 3c- c^2$ (2) $y = cx- c^3$ (3) $y = cx+ c^3$ (4) $y = cx + 3c- 4c^2$

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

Marked questions may have for revision questions.

*** Marked Questions may have more than one correct option.**

1. Let f be a real-valued differentiable function on **R** (the set of all real numbers) such that f(1) = 1. If the y-intercept of the tangent at any point P(x, y) on the curve y = f(x) is equal to the cube of the abscissa of P, then the value of f(–3) is equal to

(1) 9 (2) –9 (3) 0 (4) 4

Differential Equation

2.* Let f(x) be a function such that $f''(x) = f'(x) + e^x$ and $f'(0) = 1$, $f(0) = 0$, then

(1) $\ell n\left(\frac{(f(2))^2}{4}\right) = 4$

(2) Range of f(x) is $\left[-\frac{1}{e}, \infty\right)$

(3) for $x > -2$, tangent at any point of f(x) lies below the curve

(4) for $x < -1$, f(x) is decreasing function.

3.* The function $f(\theta) = \frac{d}{d\theta}\int_0^{\theta}\frac{dx}{(1-\cos 2\theta)(1+\cos 2x)}$ satisfies the differential equation

(1) $\frac{df}{d\theta} + 4f(\theta).\cot 2\theta = 0$ (2) $\frac{df}{d\theta} - 2f(\theta).\tan 2\theta = 4\operatorname{cosec}^3 2\theta$

(3) $f(\theta) = \operatorname{cosec}^2 2\theta$ (4) $f(\theta) = -(\operatorname{cosec} 2\theta).(\cot 2\theta)$

4.* Consider the family of all circles whose centers lie on the straight line y = x. If this family of circles is represented by the differential equation $Py'' + Qy' + 1 = 0$, where P, Q are functions of x, y and y' (here $y' = \frac{dy}{dx}$, $y'' = \frac{d^2y}{dx^2}$), then which of the following statements is (are) true?

(1) $P = y + x$ (2) $P = y - x$

(3) $P + Q = 1 - x + y + y' + (y')^2$ (4) $P - Q = x + y - y' - (y')^2$

5. An right circular cone of height H and radius R is pointed at bottom. It is filled with a volatile liquid completely. If the rate of evaporation is directly proportional to the surface area of the liquid in contact with air (constant of proportionality k > 0). The time in which whole liquid evaporates is.

(1) H/k (2) 2H/k (3) 3H/k (4) 4H/k

6*. A tangent drawn to the curve y = f(x) at P(x, y) cuts the x-axis and y-axis at A and B respectively such that BP : AP = 3 : 1, given that f(1) = 1, then

(1) equation of curve is $x\frac{dy}{dx} - 3y = 0$ (2) normal at (1, 1) is $3y - x = 2$

(3) curve passes through (2, 1/8) (4) equation of curve is $x\frac{dy}{dx} + 3y = 0$

7. The solution of $x^2 y_1^2 + xy\, y_1 - 6y^2 = 0$ are

(1) $y = Cx^2$ (2) $x^3 y = C$ (3) $\frac{1}{2}\ell n\, y = C + \ell n\, x$ (4) All of these

8. The solution of the differential equation $\frac{d^2y}{dx^2} = \frac{dy}{dx}$ is -

(1) $y = c_1e^x + c_2$ (2) $y = c_1e^{-x} + c_2$ (3) $y = c_1e^{2x} + c_2$ (4) $y = c_1e^{-2x} + c_2$

9. The solution of the differential equation $\frac{d^3y}{dx^3} = 8\frac{d^2y}{dx^2}$ satisfying $y(0) = \frac{1}{8}$, $y_1(0) = 0$ and $y_2(0) = 1$ is -

(1) $32y = (e^{8x} - 8x) + 7$ (2) $64y = (e^{8x} - 8x) + 7$ (3) $48y = (e^{8x} - 8x) - 7$ (4) $56y = (e^{8x} + 8x) + 7$

10. The solution of the differential equation $y_1 y_3 = 3y_2^2$ is

(1) $x = A_1y^2 + A_2 y + A_3$ (2) $x = A_1 y + A_2$ (3) $x = A_1 y^2 + A_2 y$ (4) $y = A_1 x + A_2$

Differential Equation

11. The solution of differential equation $\frac{x\ \ dx-y\ \ dy}{x\ \ dy-y\ \ dx}=\sqrt{\frac{1+x^2-y^2}{x^2-y^2}}$ is

(1) $\sqrt{x^2-y^2}+\sqrt{1+x^2-y^2}=\frac{c(x+y)}{\sqrt{x^2-y^2}}$

(2) $\sqrt{x^2-y^2}+\sqrt{1+x^2-y^2}=\frac{c(x+y)}{x^2+y^2}$

(3) $x^3y + y^5 = 5$

(4) $x^3y - y^5 = c$

12. The differential equations of all conics whose centre lie at the origin is of order :

(1) 2 (2) 3 (3) 4 (4) 5

13. If gradient of a curve at any point P(x, y) is $\frac{x+y+1}{2y+2x+1}$ and it passes through origin, then curve is

(1) $6y + 3x = \ell n \left|\frac{3x+3y+2}{2}\right|$

(2) $6y - 3x = \ell n \left|\frac{3x+3y+2}{2}\right|$

(3) $5y - 3x = \ell n \left|\frac{3x+3y+2}{2}\right|$

(4) $6y - 5x = \ell n \left|\frac{3x+3y+2}{2}\right|$

14. The degree of the differential equation $e^{\left(d^3y/dx^3\right)^2} + x\frac{d^2y}{dx^2} + y = 0$ is

(1) 1 (2) 2 (3) 3 (4) not defined

15. The equation of curve passing through (3, 4) and satisfying the differential equation

$y\left(\frac{dy}{dx}\right)^2 + (x-y)\frac{dy}{dx} - x = 0$ is

(1) $x - y + 1 = 0$ (2) $x + y + 1 = 0$ (3) $x + y - 1 = 0$ (4) $2x + y - 1 = 0$

16. The differential equation of all conics whose axes coincide with the axes of coordinates is of order

(1) 2 (2) 3 (3) 4 (4) 1

17. The differential equation $\frac{d^2y}{dx^2} + y + \cot^2 x = 0$ must be satisfied by

(1) $y = 2 + c_1 \cos x + \sqrt{c_2}\ \sin x$

(2) $y = \cos x \,.\, \ell n \left(\tan \frac{x}{2}\right) + 2$

(3) $y = \sin x + \cos x$

(4) all the above

18. A curve passes through (2, 0) and slope at point P(x, y) is $\frac{(x+1)^2+(y-3)}{(x+1)}$. The area between curve and x-axis in 4th quadrant is

(1) 2 / 3 (2) 1 / 3 (3) 2 (4) 4 / 3

APSP Answers

PART-I

1.	(3)	**2.**	(4)	**3.**	(3)	**4.**	(3)	**5.**	(3)	**6.**	(1)	**7.**	(2)
8.	(4)	**9.**	(2)	**10.**	(2)	**11.**	(3)	**12.**	(2)	**13.**	(1)	**14.**	(1)
15.	(4)	**16.**	(3)	**17.**	(1)	**18.**	(3)	**19.**	(2)	**20.**	(3)	**21.**	(2)
22.	(2)	**23.**	(1)	**24.**	(2)	**25.**	(3)	**26.**	(1)	**27.**	(4)	**28.**	(1)
29.	(2)	**30.**	(4)										

PART-II

1.	(1)	**2.***	(1,2,3,4)	**3.***	(2,4)	**4.***	(2,3)	**5.**	(1)	**6*.**	(2,3,4)
7.	(4)	**8.**	(1)	**9.**	(2)	**10.**	(1)	**11.**	(1)	**12.**	(2)
13.	(2)	**14.**	(4)	**15.**	(1)	**16.**	(1)	**17.**	(2)	**18.**	(4)