Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

This Section is not meant for classroom discussion. It is being given to promote self-study and self testing amongst the Resonance students.

Max. Marks : 120 **Max. Time : 1 Hr.**

Important Instructions :

1. The test is of **1 hour** duration and max. marks 120.
2. The test consists **30** questions, **4 marks** each.
3. Only one choice is correct **1 mark** will be deducted for incorrect response. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 3 above.

1. Which of the following is correct ?(where [.] and {.} denotes are greatest integer function and fractional function)

(1) $\lim\limits_{x\to\pi/2}\{x\}=1$ (2) $\lim\limits_{x\to1}[x^2]=1$ (3) $\lim\limits_{x\to0}\sqrt{x}=0$ (4) $\lim\limits_{x\to0}[\cos x]=0$

2. Which of the following not in indeterminant form ?

(1) $\lim\limits_{x\to0}\dfrac{1-\cos x}{x}$ (2) $\lim\limits_{x\to2}\dfrac{x^2-4}{x-2}$ (3) $\lim\limits_{x\to0}\dfrac{[\sin x]}{2x}$ (4) $\lim\limits_{x\to\pi/2}\sin^{\frac{2}{\frac{\pi}{2}-x}}x$

3. $\lim\limits_{x\to1}\dfrac{(2x-3)(x^2-\sqrt{x})}{2x^2+x-3}=$

(1) $\dfrac{-1}{5}$ (2) $\dfrac{-2}{10}$ (3) $\dfrac{-4}{10}$ (4) $\dfrac{-3}{10}$

4. $\lim\limits_{x\to0}\dfrac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}=$

(1) $a^{\frac{3}{2}}$ (2) $-a^{\frac{1}{2}}$ (3) $a^{-\frac{3}{2}}$ (4) $a^{\frac{1}{2}}$

5. $\lim\limits_{x\to0}\dfrac{\tan 2x-2\sin x}{x^3}=$

(1) $\dfrac{1}{2}$ (2) 2 (3) 3 (4) 0

6. $\lim\limits_{x\to0}\dfrac{10^x-2^x-5^x+1}{\ell n\cos x}=$

(1) $\ell n2\ell n5$ (2) $-\ell n2\ell n5$ (3) $-2\ell n2\ell n5$ (4) $2\ell n2\ell n5$

7. $\lim\limits_{x\to\pi/6}\dfrac{2-\sqrt{3}\cos x-\sin x}{(6x-\pi)^2}=$

(1) $\dfrac{1}{6}$ (2) $\dfrac{1}{36}$ (3) $\dfrac{1}{2}$ (4) $\dfrac{1}{9}$

8. $\lim\limits_{x\to\pi/2}\dfrac{2^{-\cos x}-1}{x(x-\pi/2)}=$

(1) $\ell n4$ (2) $\dfrac{1}{4}\ell n2$ (3) $\pi\ell n2$ (4) $\dfrac{1}{\pi}\ell n4$

9. $\lim_{x\to 0}\left[\min(y^2-4y+13)\dfrac{\sin x}{x}\right]$ (where [.] denotes the greatest integer function) is

(1) 8 (2) 9 (3) 7 (4) 0

10. If $f(x)=\begin{cases}3x-2, & x<1\\ 2-x, & x\ge 1\end{cases}$, then

(1) $\lim_{x\to 1^-} f(f(x))=0$ (2) $\lim_{x\to 1^+} f(f(x))=-1$ (3) $\lim_{x\to 1} f(f(x))=1$ (4) $\lim_{x\to 1} f(f(x))=0$

11. If $\lim_{x\to\infty}\left[\dfrac{2x^2+3x+4}{3x+5}-(ax+b)\right]=2$, then a + b =

(1) $\dfrac{1}{10}$ (2) $\dfrac{2}{3}$ (3) $\dfrac{-19}{9}$ (4) $-\dfrac{13}{9}$

12. $\lim_{x\to 0}(\cos x)^{\frac{1}{\sin(\pi\cos^2 x)}}$ =

(1) 1 (2) $e^{1/2\pi}$ (3) $e^{-1/2\pi}$ (4) $e^{-2\pi}$

13. $\lim_{n\to\infty}\dfrac{1^2-2^2+3^2-4^2+5^2-....\text{upto n terms}}{n^2}$, (where n is odd) =

(1) $\dfrac{1}{2}$ (2) $\dfrac{1}{2}$ (3) –1 (4) 1

14. $\lim_{x\to 0}\left(\dfrac{1}{x^2}-\dfrac{1}{\sin^2 x}\right)=$

(1) $-\dfrac{1}{2}$ (2) $-\dfrac{1}{3}$ (3) $-\dfrac{1}{6}$ (4) $-\dfrac{1}{4}$

15. Which of the following function is not continuous ∀ x∈R ?

(1) $\dfrac{e^x+1}{e^x+3}$ (2) $\sqrt{2\cos x+3}$ (3) $\sqrt{\text{sgn}(x)+1}$ (4) $\sqrt{\dfrac{2^{2x}+1}{2^{3x}+5}}$

16. f(x) = $\begin{cases}2x\tan x-\dfrac{\pi}{\cos x}, & x\ne\dfrac{\pi}{2}\\ k, x=\dfrac{\pi}{2}, & \end{cases}$ is continuous x = $\dfrac{\pi}{2}$, then k equals to

(1) –2 (2) 2 (3) 0 (4) – 3

17. Number of integral values of x for which

f(x) = $\left[\dfrac{2}{1+x^2}\right]$ (where [.] denotes greatest integer function) is discontinuous is -

(1) 2 (2) 1 (3) 3 (4) 4

18. The number of values of a for which f(x) $=\lim_{n\to\infty}\dfrac{ax^{2n}+2}{x^{2n}+a+1}$ is continuous at x = 1, is

(1) 2 (2) 0 (3) 3 (4) 1

19. If f(x) $\begin{cases}x^2-ax+3, & x \text{ is rational}\\ 2-x, & x \text{ is irrational}\end{cases}$, is continuous at exactly two points, then possible values of a are.

(1) (2, ∞) (2) (–∞,3) (3) (–∞, –1) ∪ (3, ∞) (4) (–1,3)

20. If y = $\dfrac{1}{t^2+2t-15}$ where t = $\dfrac{1}{x-2}$, then number of points where f(x) is discontinuous is

(1) 1 (2) 3 (3) 2 (4) 0

21. If f(x) = $\begin{cases} x-2, & x \le 0 \\ 4-x^2, & x > 0 \end{cases}$, then number of values of x for which f(f(x)) is discontinuous is:
(1) 0 (2) 1 (3) 2 (4) 3

22. f(x) = xcosx – sinx, then which is correct
(1) f(x) = 0 has exactly one root in (0, $\frac{\pi}{2}$)
(2) f(x) is increasing in x∈ (0, $\frac{\pi}{2}$)
(3) Range of f(x) in x∈ (0, $\frac{\pi}{2}$) is (–1,1)
(4) f(x) = $\frac{-1}{2}$ has exactly one root in (0, $\frac{\pi}{2}$)

23. f(x) = $\begin{cases} \dfrac{|x|+1}{(2|x|+1)(|x|+1)} & ,x \ne 0 \\ 1, & x = 0 \end{cases}$ then f(x) is
(1) continuous but not differentiable at x = 0
(2) discontinuous at x = 0
(3) differentiable at x = 0
(4) continuous and differentiable at x = 0

24. Let f(x) = $\begin{cases} \min\{x,\ x^2\}, x \ge 0 \\ \max\{2x,\ x^2-1\}, x < 0 \end{cases}$
Then which of the following is incorrect.
(1) f(x) is continuous at x = 0
(2) f(x) is not differentiable at x = 1
(3) f(x) is not differentiable at exactly three points
(4) f(x) is differentiable exactly three points

25. If f(x) = $(x^2 - 4)\,|x^3 - 6x^2 + 11x - 6| + \frac{x}{1+|x|}$ then the set of points at which f(x) is not differentiable is
(1) {1, 3} (2) {–2, 2,0} (3) {–2, 0,3} (4) {–2, 2, 1,3}

26. The value of a and b if
$f(x) = \begin{cases} a + \tan^{-1}(x+b) & ;x \ge 1 \\ x & ;x < 1 \end{cases}$ is differentiable at x = 1 respectively is
(1) 1, –1 (2) 0, 1 (3) –1, 1 (4) 3, 2

27. A differentiable function f is satisfying the relation f(x+y) = f(x) + f(y) + 2xy(x+y) – $\frac{1}{3}$ $\forall x, y \in R$. and f'(0) = $\frac{4}{3}$, then f(x) =
(1) $\frac{4}{3} + x^2$ (2) $\frac{4x}{3} + x^2$ (3) $\frac{4x}{3} + \frac{2x^3}{3}$ (4) 0

28. If f(x) = $\begin{cases} ||x|-3|, & \text{for } |x| \le 4 \\ 5-|x| & \text{for } |x| > 4 \end{cases}$, then number of points where f(x) is non-differentiable is
(1) 3 (2) 0 (3) 7 (4) 5

29. If f'(3) = 4 and f'(1) = 2 then $\lim_{x\to 0} \frac{f(3h+3+h^2)-f(3)}{f(3h-h^2+1)-f(1)} =$
(1) 2 (2) –2 (3) 3 (4) 1

30. $\lim_{x\to\infty} \frac{[2x^3]}{x^3}$ equal to (where [.] denotes the gneatest integer function)

(1) 0	(2) 1	(3) 2	(4) 3

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

1. The value of $\ell im_{x\to\infty} \left\{ \cos\left(\sqrt{x+1}\right) - \cos\left(\sqrt{x}\right) \right\}$ is

(1) 0 (2) 1 (3) – 1 (4) limit does not exist

2. If f(x) = $\ell im_{n\to\infty} \dfrac{x^{2n}-1}{x^{2n}+1}$, then range of f(x) is

(1) {0, 1} (2) [–1, 1] (3) {–1, 0, 1} (4) (–1, ∞)

3. The value of $\ell im_{x\to 0^-} \dfrac{\ell n(1+\{x\})}{\{x\}}$ is (where {x} denotes the fractional part of x)

(1) 1 (2) 1 (3) $\ell n\, 2$ (4) 2

4. $\ell im_{x\to 0} \left[\left(1-e^{x}\right)\dfrac{\sin\ x}{|x|} \right]$ is (where [·] represents greatest integral part function)

(1) – 1 (2) 1 (3) 0 (4) does not exist

5. The value of $\ell im_{x\to 0} \left(\dfrac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}}$ is

(1) 1 (2) –1 (3) e (4) $\dfrac{1}{e}$

6. The polynomial function f(x) of degree 6 satisfying : $\ell im_{x\to 0} \left(1+\dfrac{f(x)}{x^3}\right)^{\frac{1}{x}}$ = e^2 , is -

(1) $2x^3 + a_5x^5 + a_6x^6$ (2) $2x^4 + a_5x^5 + a_6x^6$ (3) $4x^4 + a_5x^5 + a_6x^6$ (4) None of these

7. Let α, β ∈ R be such that $\lim_{x\to 0} \dfrac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$. Then 6(α + β) equals

(1) 7 (2) – 7 (3) 12 (4) 3

8. If $\ell im_{x\to 0}$ $(\cos x + a \sin b\, x)^{1/x} = e^2$, then the number of possible pairs (a, b) is / are

(1) 4 (2) 3 (3) 0 (4) infinite

9. If $f(x) = \begin{cases} \dfrac{\sin\{\cos x\}}{x-\pi/2} & , \ x \neq \dfrac{\pi}{2} \\ 1 & , \ x = \dfrac{\pi}{2} \end{cases}$, where {.} represents the fractional part function, then

(1) f(x) is continuous at $x = \dfrac{\pi}{2}$ (2) $\lim\limits_{x\to\frac{\pi}{2}} f(x)$ exists , but f is not continuous at $x = \dfrac{\pi}{2}$

(3) $\lim\limits_{x\to\frac{\pi}{2}} f(x)$ does not exist. (4) $\lim\limits_{x\to\frac{\pi}{2}} f(x) = 1$

10. The left-hand derivative of f(x) = [x] sin (πx) at x = k, (k is an integer), is

(1) $(-1)^k (k-1)\pi$ (2) $(-1)^{k-1} (k-1)\pi$ (3) $(-1)^k k\pi$ (4) $(-1)^{k-1} k\pi$

11. If a function f : [–2a, 2a] → R is an odd function such that f(x) = f(2a – x) for x ∈ [a, 2a] and the left hand derivative at x = a is 0, then find the left hand derivative at x = –a is

(1) 1 (2) –1 (3) 0 (4) not defined

12. Let $f(x) = x - x^2$ and $g(x) = \begin{cases} \max \ f(t), \ 0 \le t \le x, \ 0 \le x \le 1 \\ \sin \ \pi x \ , \ x > 1 \end{cases}$, then in the interval [0, ∞)

(1) g(x) is everywhere continuous except at two points
(2) g(x) is everywhere differentiable except at two points
(3) g(x) is everywhere differentiable except at x = 1
(4) none of these

13. Let [x] denote the integral part of x ∈ R and g(x) = x − [x]. Let f(x) be any continuous function with f(0) = f(1), then the function h(x) = f(g(x)) :

(1) has finitely many discontinuities (2) is continuous on R
(3) is discontinuous at some x = c (4) is a constant function.

14. A point (x, y), where function f(x) = [sin [x]] in (0, 2π) is not continuous, is ([.] denotes greatest integer ≤ x).

(1) (3, 0) (2) (2, 0) (3) (1, 0) (4) (4, –1)

15. If f(x) takes only rational values for all real x and is continuous, then the value of f'(10) is

(1) 0 (2) 5 (3) 10 (4) can't say

16. If $f(x) = [x^2] + \sqrt{\{x\}^2}$, where [.] and {.} denote the greatest integer and fractional part functions respectively, then-

(1) f(x) is continuous at all integral points (2) f(x) is continuous and differentiable at x = 0
(3) f(x) is discontinuous for all x ∈ I – {1} (4) f(x) is not differentiable for all x ∈ I .

17. Let $L = \lim\limits_{x\to0} \dfrac{a - \sqrt{a^2 - x^2} - \dfrac{x^2}{4}}{x^4}$, a > 0. If L is finite, then

(1) a = 2 (2) a = 1 (3) $L = \dfrac{1}{64}$ (4) $L = \dfrac{1}{32}$

18. A function f(x) is defined in the interval [1, 4] as follows :

$f(x) = \begin{cases} \log_e [x] \ , & 1 \le x < 3 \\ |\log_e x| , & 3 \le x \le 4 \end{cases}$

the graph of the function f(x) ([.] represents greatest integer function)

(1) is broken at two points

(2) is broken at exactly one point
(3) does not have a definite tangent at two points
(4) does not have a definite tangent at more than two points

19. If f(x) = $\begin{cases} -x-\frac{\pi}{2} & , \quad x \le -\frac{\pi}{2} \\ -\cos x & , \quad -\frac{\pi}{2} < x \le 0 \\ x-1 & , \quad 0 < x \le 1 \\ \ell n\ x & , \quad x > 1 \end{cases}$, then

(1) f(x) is continuous at x = $-\frac{\pi}{2}$
(2) f(x) is not differentiable at x = 0
(3) f(x) is differentiable at x = 1
(4) f(x) is differentiable at x = $-\frac{3}{2}$

20. Let f : R → R , g : R → R and h : R → R be differentiable functions such that f(x) = x^3 + 3x + 2, g(f(x)) = x and h(g(g(x))) = x for all x ∈ R. Then

(1) g'(2) = $\frac{1}{15}$
(2) h'(1) = 666
(3) h(0) = 16
(4) h(g(3)) = 36

21. If f(x) = min {1, x^2, x^3}, then
(1) f(x) is continuous ∀ x ∈ R
(2) f'(x) > 0, ∀ x > 1
(3) f(x) is not differentiable but continuous ∀ x ∈ R
(4) f(x) is not differentiable for two values of x

22. Let g: R → R be a differentiable function with g(0) = 0, g'(0)= 0 and g'(1) ≠ 0. Let f(x) = $\begin{cases} \frac{x}{|x|}g(x), & x \ne 0 \\ 0, & x = 0 \end{cases}$ and h(x) = $e^{|x|}$ for all x ∈ R. Let (foh)(x) denote f(h(x)) and (hof)(x) denote h(f(x)). Then which of the following is(are) true?

(1) f is differentiable at x = 0
(2) h is differentiable at x = 0
(3) foh is differentiable at x = 0
(4) hof is differentiable at x = 0

23. Let f(x) = $\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all x ∈ R and g(x) = $\frac{\pi}{2}$ sin x for all x ∈ R. Let (fog) (x) denote f(g(x)) and (gof) (x) denote g(f(x)). Then which of the following is(are)true?

(1) Range of f is $\left[-\frac{1}{2},\frac{1}{2}\right]$
(2) Range of fog is $\left[-\frac{1}{2},\frac{1}{2}\right]$
(3) $\lim\limits_{x\to0}\frac{f(x)}{g(x)} = \frac{\;}{6}$
(4) There is an x ∈ R such that (gof)(x) = 1

24. Let a, b ∈ R and f : R → R be defined by f(x) = a cos (|x^3 –x|) + b|x| sin(|x^3 +x|). Then f is
(1) differentiable at x = 0 if a = 0 and b = 1
(2) differentiable at x = 1 if a = 1 and b = 0
(3) NOT differentiable at x = 0 if a = 1 and b = 0
(4) NOT differentiable at x = 1 if a = 1 and b = 1

Comprehension # 1 (Q.25 and 26)

If $f(x) = \dfrac{\cos 2 - \cos 2x}{x^2 - |x|}$, then

25. $\lim\limits_{x \to 1} f(x) =$

(1) 2 sin 2 (2) –2 sin 2 (3) 2 cos 2 (4) –2 cos 2

26. $\lim\limits_{x \to -1} f(x) =$

(1) 2 sin 2 (2) –2 sin 2 (3) 2 cos 2 (4) –2 cos 2

Comprehension # 2 (Q.27 to 29)

Let $f(x) = \begin{cases} x + a & \text{if } x < 0 \\ |x - 1| & \text{if } x \ge 0 \end{cases}$ and $g(x) = \begin{cases} x + 1 & \text{if } x < 0 \\ (x - 1)^2 + b & \text{if } x \ge 0 \end{cases}$, where a and b are non-negative real numbers.

27. The composite function gof (x) =

(1) $\begin{cases} x + a + 1 & \text{if } x < -a \\ (x + a - 1)^2 + b & \text{if } -a \le x < 0 \\ x^2 + b & \text{if } 0 \le x \le 1 \\ (x - 2)^2 + b & \text{if } x > 1 \end{cases}$

(2) $\begin{cases} x - a + 1 & \text{if } x < -a \\ (x + a - 1)^2 + b & \text{if } -a \le x < 0 \\ x^2 + b & \text{if } 0 \le x \le 1 \\ (x - 2)^2 + b & \text{if } x > 1 \end{cases}$

(3) $\begin{cases} x + a - 1 & \text{if } x < -a \\ (x + a - 1)^2 + b & \text{if } -a \le x < 0 \\ x^2 + b & \text{if } 0 \le x \le 1 \\ (x - 2)^2 + b & \text{if } x > 1 \end{cases}$

(4) $\begin{cases} x + a + 1 & \text{if } x < -a \\ (x + a - 1)^2 + b & \text{if } -a \le x < 0 \\ x^2 + b & \text{if } 0 \le x \le 1 \\ (x + 2)^2 + b & \text{if } x > 1 \end{cases}$

28. If (gof) (x) is continuous for all real x, the values of a and b are respectively

(1) 0, 1 (2) 1, 0 (3) 2, 1 (4) 1, 2

29. For values of a and b obtained in previous question, gof at x = 0 is

(1) differentiable (2) continuous but not differentiable

(3) discontinuous (4) differentiable but not continuous

DIRECTIONS : (Q. 30 to Q. 33)

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true.

(2) Statement-I is true, but Statement-II is false.

(3) Statement-I is false, but Statement-II is true.

(4) Both the statements are false.

30. **Statement - 1 :** $\lim\limits_{x \to \infty} (1^x + 2^x + 3^x + \ldots\ldots\ldots\ldots. + n^x)^{1/x} = n$

Statement - 2 : $\lim\limits_{x \to \infty} (1 + h)^n = 0$

31. **Statement - 1 :** If $\lim\limits_{x \to 0} (1 + ax + bx^2)^{2/x} = e^3$, then a = 3/2 and b ∈ R .

Statement - 2 : $\lim\limits_{x \to 0} (1 + x)^{\frac{1}{x}} = e$

32. Let f and g be real valued functions defined on interval (–1, 1) such that g'' (x) is continuous, g(0) ≠ 0, g'(0) = 0, g'' (0) ≠ 0, and f(x) = g(x) sin x

Statement - 1 : $\lim\limits_{x \to 0} [g(x) \cot x - g(0) \operatorname{cosec} x] = f''(0)$

and

Statement - 2 : f'(0) = g(0)

33. **Statement - 1** $\lim_{x\to 0}\left[\frac{\sin x}{x}\right] \neq \left[\lim_{x\to 0}\frac{\sin x}{x}\right]$, where [.] represents greatest integer function.

Statement - 2 $\lim_{x\to a} h(g(x)) = h\left(\lim_{x\to a} g(x)\right)$, if $y = h(\lambda)$ is continuous at $\lambda = \lim_{x\to a} g(x)$.

APSP Answers

PART - I

1.	(4)	**2.**	(3)	**3.**	(4)	**4.**	(2)	**5.**	(3)	**6.**	(3)	**7.**	(1)
8.	(4)	**9.**	(1)	**10.**	(3)	**11.**	(4)	**12.**	(3)	**13.**	(1)	**14.**	(2)
15.	(3)	**16.**	(1)	**17.**	(3)	**18.**	(4)	**19.**	(3)	**20.**	(2)	**21.**	(3)
22.	(4)	**23.**	(1)	**24.**	(4)	**25.**	(1)	**26.**	(1)	**27.**	(3)	**28.**	(4)
29.	(1)	**30.**	(3)										

PART - II

1.	(1)	**2.**	(3)	**3.**	(3)	**4.**	(1)	**5.**	(4)	**6.**	(2)	**7.**	(1)
8.	(4)	**9.**	(3)	**10.**	(1)	**11.**	(3)	**12.**	(3)	**13.**	(2)	**14.**	(4)
15.	(1)	**16.**	(1)	**17.**	(1,3)	**18.**	(1,3)	**19.**	(1,2,3,4)	**20.**	(2,3)	**21.**	(1,3)
22.	(1,4)	**23.**	(1,2,3)	**24.**	(1,2)	**25.**	(1)	**26.**	(1)	**27.**	(1)	**28.**	(2)
29.	(1)	**30.**	(2)	**31.**	(2)	**32.**	(1)	**33.**	(1)				