Exercise-1

OBJECTIVE QUESTIONS

Section (A) : Integration using standard Integral :

A-1. $\int (e^{a\ell nx} + e^{x\ell na})$ dx, where x > 0, a > 0

(1) $x_{a+1} + \frac{a^x}{\ell na} + c$ (2) $\frac{x^{a+1}}{a+1} + a_x \ell n\, a + c$ (3*) $\frac{x^{a+1}}{a+1} + \frac{a^x}{\ell na} + c$ (4) None of these

(1) $x_{a+1} + \frac{a^x}{\ell na} + c$ (2) $\frac{x^{a+1}}{a+1} + a_x \ell n\, a + c$ (3*) $\frac{x^{a+1}}{a+1} + \frac{a^x}{\ell na} + c$ (4) buesa ls dksbZ ugha

Sol. $\int (e^{a\ln x} + e^{x\ln a})$ dx = $\int (x^a + a^x)$ dx = $\frac{x^{a+1}}{a+1} + \frac{a^x}{\ell na} + C$

A-2. If f'(x) = x_2 + 5 and f(0) = –1, then f(x) =

;fn f'(x) = x_2 + 5 rFkk f(0) = –1 gks, rks f(x) =

(1) $x_3 + 5x - 1$ (2) $x_3 + 5x + 1$ (3*) $\frac{1}{3}x^3 + 5x - 1$ (4) $\frac{1}{3}x^3 + 5x + 1$

Sol. f'(x) = x_2 + 5

$\int f'(x)dx = \int (x^2 + 5)dx$

f (x) = $\frac{x^3}{3} + 5x + C$

Put x = 0

f(0) = C ⇒ C = – 1 ⇒ f(x) = $\frac{x^3}{3} + 5x - 1$.

A-3. $\int \frac{\cos 2x}{\cos x}$ dx is equal to

(1*) 2 sin x – ℓn (sec x + tan x) + c (2) 2 sin x – ℓn (sec x – tan x) + c

(3) 2 sin x + ℓn (sec x + tan x) + c (4) None of these

$\int \frac{\cos 2x}{\cos x}$ dx dk eku gS&

(1*) 2 sin x – ℓn (sec x + tan x) + c (2) 2 sin x – ℓn (sec x – tan x) + c

(3) 2 sin x + ℓn (sec x + tan x) + c (4) buesa ls dksbZ ugha

Sol. I = $\int \frac{\cos 2x}{\cos x}$ dx = $\int \frac{2\cos^2 x - 1}{\cos x}$ dx = 2 sin x – $\int \sec x\, dx$ = 2 sin x – ℓn |sec x + tan x| + C

A-4. $\int$ sin x. cos x. cos 2x. cos 4x. cos 8x. cos 16 x dx is equal to

$\int$ sin x. cos x. cos 2x. cos 4x. cos 8x. cos 16 x dx dk eku gS &

(1) $\frac{\sin 16x}{1024} + c$ (2*) $-\frac{\cos 32x}{1024} + c$ (3) $\frac{\cos 32x}{1096} + c$ (4) $-\frac{\cos 32x}{1096} + c$

Sol. $I = \frac{1}{2}\int \sin 2x \cos 2x \cos 4x \cos 8x \cos 16x \, dx$

$= \frac{1}{32}\int \sin 32x \, dx = -\frac{1}{1024}\cos 32x + c$

A-5. $\int \sqrt{1-\sin 2x}\, dx$ where x ∈ (0, π/4) is equal to

$\int \sqrt{1-\sin 2x}\, dx$ x ∈ (0, π/4), cjkcj gS&

(1) – sinx + cosx + c (2) sinx – cosx + c (3) tanx + secx + c (4*) sinx + cosx + c

Sol. $\int \sqrt{1-\sin 2x}\, dx$, $x \in \left(0, \frac{\pi}{4}\right)$ $= \int (\cos x - \sin x)\, dx$ = sin x + cos x + C

A-6. $\int \frac{1+\cos^2 x}{\sin^2 x} dx =$

(1) –cot x – 2x + c (2) –2cot x – 2x + c (3*) –2cot x – x + c (4) –2cot x + x + c

Sol. $\int \text{cosec}^2 x \, dx + \int \cot^2 x \, dx = \int (2\,\text{cosec}^2 x - 1)\, dx$ = – 2 cot x – x + C

A-7. $\int \frac{dx}{\tan x + \cot x} =$

(1) $\frac{\cos 2x}{4} + c$ (2) $\frac{\sin 2x}{4} + c$ (3) $-\frac{\sin 2x}{4} + c$ (4*) $-\frac{\cos 2x}{4} + c$

Sol. $\int \frac{dx}{\tan x + \cot x} = \int \frac{dx}{\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}} = \frac{1}{2}\int \sin 2x \, dx = -\frac{1}{4}\cos 2x + C$

A-8. **Sol.** $\int \tan^{-1}(\tan x)\, dx = \int x \, dx = \frac{x^2}{2} + C$

A-9. **Sol.** $\int x^{51}(\tan^{-1} x + \cot^{-1} x)$

$\int \frac{\pi}{2} x^{51} dx = \frac{x^{52}}{52} \times \frac{\pi}{2} + C$

$= \frac{x^{52}}{52}(\tan^{-1} x + \cot^{-1} x) + C$

Section (B) : Integration by substitution

B-1. **Sol.** $I = \int \frac{a^{\sqrt{x}}}{\sqrt{x}} dx$ Put $\sqrt{x} = t$

$\Rightarrow \frac{dx}{2\sqrt{x}} = dt$ $\Rightarrow I = 2\int a^t\, dt = 2\frac{a^{\sqrt{x}}}{\ell na} + C$

B-2. **Sol.** Let $5^{5^x} = t$ $\Rightarrow$ $5^{5^x}.\ \ell n 5 . 5^x . \ \ell n 5\, dx = dt$

$I = \int \frac{5^t\, dt}{(\ell n 5)^2} = \frac{5^t}{(\ell n 5)^3} + c = \frac{5^{5^{5^x}}}{(\ell n\ 5)^3} + c$

B-3. **Sol.** Let $2^x = t$ $2^x \log 2\ dx = dt$

$I = \frac{1}{\log 2}\int \frac{dt}{\sqrt{1-t^2}} = \frac{1}{\ell n 2}\sin^{-1}(t) + c = \frac{1}{\ell n 2}\sin^{-1}(2^x) + c \Rightarrow$ $k = \frac{1}{\ell n\ 2}$

B-4. **Sol.** $\int \tan^3 2x . \sec 2x\ dx$

$I = \int (\sec^2 2x - 1) \sec 2x . \tan 2x\ dx$ Put $\sec 2x = t \Rightarrow 2 \sec 2x \tan 2x\ dx = dt$

$\Rightarrow$ $I = \frac{1}{2}\int (t^2 - 1) dt$

$= \frac{t^3}{6} - \frac{t}{2} + C = \frac{1}{6}\sec^3 2x - \frac{1}{2}\sec 2x + C$

B-5. **Sol.** $I = \int \frac{\cos 2x\, dx}{(\sin x + \cos x)^2} = \int \frac{\cos 2x\, dx}{1 + \sin 2x} = \frac{1}{2}\int \frac{dt}{t}$

when $t = 1 + \sin 2x$ $\Rightarrow$ $dt = 2\cos 2x\, dx$

$I = \frac{1}{2}\ell n\ t = \frac{1}{2}\ell n(\sin x + \cos x)^2 + c$ $= \ell n(\sin x + \cos x) + c$

B-6. **Sol.** Let $I = \int \frac{\ell n\left(1+\frac{1}{x}\right)}{x(1+x)} dx = \int \frac{\ell n\left(1+\frac{1}{x}\right)}{x^2\left(1+\frac{1}{x}\right)} dx$

put $\ell n\left(1+\frac{1}{x}\right) = t \Rightarrow \frac{1}{1+\frac{1}{x}}\left(\frac{-1}{x^2}\right) dx = dt$

$\Rightarrow I = \int -t\ dt = -\frac{1}{2}t^2 + C = -\frac{1}{2}\left(\ell n\left(1+\frac{1}{x}\right)\right)^2 + C$

B-7. **Sol.** Let $I = \int \frac{d(x^2+1)}{\sqrt{x^2+2}} = \int \frac{2x}{\sqrt{x^2+2}} dx$ Put $x^2 + 2 = t \Rightarrow 2x\ dx = dt$

$\Rightarrow I = \int \frac{dt}{\sqrt{t}} = 2t^{1/2} + C = 2\sqrt{x^2+2} + C$

B-8. Sol. $I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx = \int \frac{\sqrt{\tan x}}{\tan x} \sec^2 x\, dx$ Put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$\Rightarrow I = \int \frac{1}{\sqrt{t}} dt = 2\sqrt{t} + C = 2\sqrt{\tan x} + C$

B-9. Sol. $y = \int \frac{dx}{x^3\left(1+\frac{1}{x^2}\right)^{3/2}}$ put $1 + \frac{1}{x^2} = t^2 \Rightarrow -\frac{2}{x^3} dx = 2t\, dt$

$\Rightarrow y = \int \frac{-t\, dt}{t^3} = -\int \frac{dt}{t^2} = \frac{1}{t} + C = \frac{x}{\sqrt{1+x^2}} + C$

$\because y(0) = 0 \Rightarrow C = 0 \quad \therefore y(1) = \frac{1}{\sqrt{2}}$

Section (C) : Integration by parts

C-1. Sol. $\int (x-1)e^{-x} dx = (x-1)\int e^{-x} dx - \int\left(\frac{d}{dx}(x-1)\int e^{-x} dx\right) dx$

$= -(x-1)e^{-x} - e^{-x} + C = -xe^{-x} + C$

C-2. Sol. Let () $\tan^{-1}x = t \Rightarrow \frac{1}{1+x^2} dx = dt$

$I = \int e^t(\sec^2 t + \tan t)dt = e^t \tan t + c = x e^{\tan^{-1} x} + c$

C-3. Sol. $I = \int e^{\tan\theta}(\sec\theta - \sin\theta)d\theta = \int e^{\tan\theta}\left(\frac{\sec\theta - \sin\theta}{\sec^2\theta}\right)\sec^2\theta\, d\theta$

$I = \int e^{\tan\theta}\left(\frac{1}{\sec\theta} - \frac{\sin\theta}{\sec^2\theta}\right)\sec^2\theta\, d\theta = \int e^{\tan\theta}\left(\frac{1}{\sqrt{1+\tan^2\theta}} - \frac{\tan\theta}{(1+\tan^2\theta)^{3/2}}\right)\sec^2\theta\, d\theta$

$= e^{\tan\theta} \cdot \frac{1}{\sqrt{1+\tan^2\theta}} = e^{\tan\theta} \cdot \cos\theta + C$

C-4. Sol. $\int \left(xe^{\ell n \sin x} - \cos x\right)dx = \int x \sin x - \int \cos x\, dx$

$= -x\cos x + \int \cos x\, dx - \int \cos x\, dx = -e^{\ell nx}\cos x + C$

C-5. Sol. $\int \left(f(x)g''(x) - f''(x)g(x)\right)dx = \int f(x)g''(x)dx - \int f''(x)g(x)dx$

$= f(x)\, g'(x) - \int f'(x)g'(x) - \int f''(x)g(x) = f(x)\, g'(x) - f'(x)\, g(x) + \int f''(x)g(x) - \int f''(x)g(x)$

$= f(x)\, g'(x) - f'(x)\, g(x) + C$

C-6. Sol. $I = \int e^{3x}\cos 4x\, dx = e^{3x}\left(A\sin 4x + B\cos 4x\right) + C$ ....(i)

$I = \frac{1}{4}e^{3x}\sin 4x - \int \frac{3}{4}e^{3x}\sin 4x\, dx = \frac{1}{4}e^{3x}\sin 4x + \frac{3}{16}e^{3x}\cos 4x - \int \frac{9}{16}e^{3x}\cos 4x\, dx$

$\frac{25}{16} I = \frac{1}{16}\left(4e^{3x}\sin 4x + 3e^{3x}\cos 4x\right)$

comparing with equation (i)

$\Rightarrow \quad A = \frac{4}{25},\ B = \frac{3}{25} \quad \Rightarrow \quad \frac{A}{B} = \frac{4}{3} \Rightarrow 3A = 4B \quad \Rightarrow \quad 4A + 3B = 1$

C-7. Sol. $F(x) = \int \frac{x + \sin x}{1 + \cos x}\, dx = \int \left(x\, \frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right) dx = \frac{2x.\tan\frac{x}{2}}{2} - \int \tan\frac{x}{2}dx + \int \tan\frac{x}{2}dx$

$= x \tan x/2 + C$

Since $0 = F(0)$

$\therefore \quad C = 0$ and $F(\pi/2) = \pi/2$

C-8. Sol. $F(x) = \int e^{\sin^{-1}x}\left(1 - \frac{x}{\sqrt{1-x^2}}\right) dx$

$= \int e^{\sin^{-1}x}\left(\frac{1}{\sqrt{1-x^2}}\ \sqrt{1-x^2} + \frac{-x}{\sqrt{1-x^2}}\right) dx$ put $\sin^{-1}x = t \Rightarrow \frac{dx}{\sqrt{1-x^2}} = dt$

$\Rightarrow \quad F(x) = \int e^{t}\ \left(\sqrt{1-\sin^2 t} - \sin t\right)\, dt = e^{t}\cos t + C = e^{\sin^{-1}x}\ \sqrt{1-x^2} + C$

$\because \quad 1 = F(0) \quad \Rightarrow \quad C = 0$

Hence, $F(1/2) = e^{\pi/6}.\ \frac{\sqrt{3}}{2} = \frac{k\sqrt{3}}{\pi}\ e^{\pi/6}$ (given) $\therefore \quad k = \frac{\pi}{2}$

C-9. Sol. $I = \int \frac{e^{\sqrt{x}}(x + \sqrt{x})}{\sqrt{x}}dx$ put $\sqrt{x} = t \quad \Rightarrow \quad \frac{1}{\sqrt{x}}dx = 2\, dt$

$\Rightarrow I = 2\int e^{t}\left(t^2 + t\right) dt = 2\left[e^{t}.t^2 - \int 2t.e^{t}\, dt + \int e^{t}.t\, dt\right] = 2\left[e^{t}.t^2 - \int t.e^{t}\, dt\right] = 2\left[e^{t}.t^2 - t.e^{t} + e^{t}\right] + C$

$= 2e^{t}\left[t^2 - t + 1\right] + C = 2\, e^{\sqrt{x}}\ [x - \sqrt{x} + 1] + C$

C-10. Ans. $\frac{\sec x \tan x}{2} + \frac{1}{2}\ \ell n\, |\sec x + \tan x| + C$

Sol. Let $t = \tan x$

$dt = \sec^2 x\, dx$

$I = \int \sqrt{1 + \tan^2 x}\ \sec^2 x\, dx = \int \sqrt{1+t^2}\, dt = \frac{t}{2}\sqrt{1+t^2} + \frac{1}{2}\ \ell n\, |t + \sqrt{1+t^2}| + C$

$= \frac{\sec x \tan x}{2} + \frac{1}{2} \ln|\sec x + \tan x| + C$

Section (D) : Algebraic Integral

D-1. **Sol.** $I = \int \frac{dx}{(x+1/2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$

D-2. **Sol.** $\int \frac{dx}{2x^2+x+1} = \frac{1}{2}\int \frac{dx}{\left(x+\frac{1}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2} = \frac{2}{\sqrt{7}} \tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right) + C$

D-3. **Sol.** $\int \frac{x+1}{x^2+x+3}dx = \frac{1}{2}\left[\int \frac{2x+1}{x^2+x+3}dx + \int \frac{dx}{x^2+x+3}\right]$

$= \frac{1}{2}\left[\int \frac{2x+1}{x^2+x+3}dx + \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{11}}{2}\right)^2}\right]$

$= \frac{1}{2}\left[\ell n\,|x^2+x+3| + \frac{2}{\sqrt{11}}\tan^{-1}\left(\frac{2x+1}{\sqrt{11}}\right)\right] + k$; $a = \frac{1}{2}$, $b = 11$, $c = 1$

D-4. **Sol.** $\int \frac{2x+3}{x^2-5x+6}dx$

$= \int \frac{2x-5}{x^2-5x+6}dx + \int \frac{8}{x^2-5x+6}dx = \ell n\,|x^2-5x+6| + \int \frac{8}{(x-2)(x-3)}dx$

$= \ell n\,|x^2-5x+6| + 8\int \left(\frac{1}{(x-3)} - \frac{1}{(x-2)}\right)dx = \ell n\,|x^2-5x+6| + 8\,\ell n\,|x-3| - 8\,\ell n\,|x-2| + C$

$= 9\,\ell n\,|x-3| - 7\,\ell n\,|x-2| + C \Rightarrow A = 9,\ B = -7$

D-5. **Sol.** $I = \int \frac{e^x-1}{\sqrt{e^{2x}-1}}dx$

$= \int \frac{e^x}{\sqrt{e^{2x}-1}}dx - \int \frac{e^x}{e^x\sqrt{e^{2x}-1}}dx$ put $e^x = t \Rightarrow e^x\,dx = dt$

$\Rightarrow I = \int \frac{dt}{\sqrt{t^2-1}} - \int \frac{dt}{t\sqrt{t^2-1}}$

$= \ell n\left|t+\sqrt{t^2-1}\right| - \sec^{-1}(t) + C$, where () $t = e^x$

D-6. **Sol.** Put $1 - x^3 = t^2 \Rightarrow -3x^2\,dx = 2t\,dt$

$I = \int \frac{dx}{x\sqrt{1-x^3}} = \int \frac{x^2dx}{x^3\sqrt{1-x^3}} = -\frac{1}{3}\int \frac{2tdt}{(1-t^2).t} = -\frac{2}{3}\int \frac{dt}{1-t^2} = \frac{2}{3}\int \frac{dt}{t^2-1}$

$= \frac{2.1}{3.2}\ell n\left|\frac{1-t}{1+t}\right| = \frac{1}{3}\ell n\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right| + c$

D-7. **Sol.** $I = \int \frac{dx}{x^3(x+1)} = \frac{A}{x^2} + \frac{B}{x} + \ell n\left(\frac{x}{x+1}\right) + C$

let $\frac{1}{x^3(x+1)} = \frac{a}{x} + \frac{b}{x^2} + \frac{c}{x^3} + \frac{d}{(x+1)}$

$1 = a(x + 1)x^2 + bx(x + 1) + c(x + 1)) + dx^3 = a(x^3 + x^2) + b(x^2 + x) + c(x + 1) + dx^3$

$0 = a + d$

$0 = a + b$

$0 = b + c$

$c = 1, b = –1, a = 1, d = –1$

$\Rightarrow \int \frac{dx}{x^3(x+1)} = \int \frac{dx}{x} - \int \frac{dx}{x^2} + \int \frac{dx}{x^3} - \int \frac{1}{x+1}dx = \ell n|x| + \frac{1}{x} - \frac{1}{2x^2} - \ell n|x + 1| + C$

Comparing $B = 1, A = -\frac{1}{2}$

D-8. **Sol.** $\int \frac{dx}{(x+1)(x+2)} = \int\left(\frac{1}{x+1} - \frac{1}{x+2}\right)dx = \ell n\,|x+1| - \ell n\,|x+2| + C = \ell n\left|\frac{x+1}{x+2}\right| + C$

D-9. **Sol.** $x^4 + 4 = x^4 + 4 + 4x^2 - 4x^2 = (x^2 + 2)^2 - (2x)^2 = (x^2 + 2x + 2)(x^2 - 2x + 2)$

$\therefore$ $I = \int (x^2 + 2x + 2)dx = \frac{x^3}{3} + x^2 + 2x + c$

D-10. **Sol.** $\int \frac{dx}{(x^2+1)(x^2+4)} = \frac{1}{3}\int\left(\frac{1}{x^2+1} - \frac{1}{x^2+4}\right)dx = \frac{1}{3}\left(\tan^{-1}x - \frac{1}{2}\tan^{-1}\frac{x}{2}\right) + C$

D-11. Sol. $\int \frac{1-x^7}{x(1+x^7)}dx = \int \frac{(1+x^7)-2x^7}{x(1+x^7)}dx = \int \frac{1}{x}dx - 2\int \frac{x^6}{(1+x^7)}dx = \ell n\,|x| - \frac{2}{7}\ell n\,|1 + x^7| + c$

D-12. **Sol.** $\int \frac{x^2+2}{x^4+4}dx = \int \frac{1+\frac{2}{x^2}}{\left(x-\frac{2}{x}\right)^2+4}dx = \frac{1}{2}\tan^{-1}\left(\frac{x-\frac{2}{x}}{2}\right) + c$

D-13. **Sol.** $I = \int \frac{x^{n-1}}{x^n(x^n+1)}dx$ Let $x^n = t$ $n.\,x^{n-1}\,dx = dt$

I = $\frac{1}{n}\int \frac{1}{t(t+1)}dt$ = $\frac{1}{n}\int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt$ = $\frac{1}{n}[\ell nt - \ell n(t+1)]+c$ = $\frac{1}{n}\ell n\left(\frac{x^n}{x^n+1}\right)+c$

D-14. **Sol.** $\int \frac{1}{x^5(1+\frac{1}{x^4})^{3/4}}$ dx

Let $1+\frac{1}{x^4}=t$ ⇒ $-\frac{4}{x^5}dx = dt$ = $-\frac{1}{4}\int\frac{dt}{t^{3/4}}$ = $-\frac{1}{4}.\frac{t^{1/4}}{1/4}+c$ = $-\left(1+\frac{1}{x^4}\right)^{1/4}$ + c

D-15. **Sol.** $\int\frac{dx}{(x+1)\sqrt{x-2}}$

let () x–2 = t^2

dx = 2tdt

$\int\frac{2t\,dt}{(t^2+3)t}$ = $\frac{2}{\sqrt{3}}$ $\tan^{-1}\frac{t}{\sqrt{3}}$ + C

= $\frac{2}{\sqrt{3}}$ $\tan^{-1}\left(\sqrt{\frac{x-2}{3}}\right)$ + C.

Section (E) : Integration of trigonometric functions, Reduction formulae, Miscellaneous

E-1. **Sol.** $\int\frac{dx}{4\sin^2x+5\cos^2x}$ = $\int\frac{\sec^2x\,dx}{4\tan^2x+5}$

let () tan x = t

$\sec^2x$ dx = dt

= $\int\frac{dt}{4t^2+5}$ = $\frac{1}{2\sqrt{5}}$ $\tan^{-1}\left(\frac{2t}{\sqrt{5}}\right)$ + C = $\frac{1}{2\sqrt{5}}$ $\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right)$ + C

E-2. **Sol.** $\int\frac{\sin^2x}{1+\sin^2x}dx$ = $\int\left(1-\frac{1}{1+\sin^2x}\right)dx$ = x – $\int\frac{\sec^2x\,dx}{2\tan^2x+1}$ = x – $\frac{1}{2}\int\frac{\sec^2x\,dx}{\tan^2x+\left(\frac{1}{\sqrt{2}}\right)^2}$

= x – $\frac{\sqrt{2}}{2}\tan^{-1}(\sqrt{2}\tan x)+c$

E-3. **Sol.** I = $\int\frac{1}{1+\cos\left(\frac{\pi}{2}-x\right)}dx$

= $\int\frac{1}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}dx$ = $\frac{1}{2}\int\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)dx$ = $\tan\left(\frac{x}{2}-\frac{\pi}{4}\right)+b$ a = $-\frac{\pi}{4}$, b ∈ R

E-4. Sol. Let $I = \int \frac{dx}{5+4\cos x} = k \tan^{-1}\left(\tan\frac{x}{2}\right) + C$

$\int \frac{dx}{1+4(1+\cos x)} = \int \frac{\sec^2\frac{x}{2}dx}{5+5\tan^2\frac{x}{2}+4-4\tan^2\frac{x}{2}}$

$= \int \frac{\sec^2\frac{x}{2}dx}{9+\tan^2\frac{x}{2}}$ Put $\tan\frac{x}{2} = t \Rightarrow \sec^2\frac{x}{2}\,dx = 2dt$

$\Rightarrow I = 2\int \frac{dt}{9+t^2} = \frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right) + C \Rightarrow k = \frac{2}{3}, m = \frac{1}{3}$

E-5. Sol. $\int \frac{1}{\sqrt{3}\cos x+\sin x} = \frac{1}{2}\int \frac{1}{\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x}dx$

$= \frac{1}{2}\int \frac{1}{\cos\left(x-\frac{\pi}{6}\right)}dx = \frac{1}{2}\int \sec\left(x-\frac{\pi}{6}\right)dx = \frac{1}{2}\ell n\left|\sec\left(x-\frac{\pi}{6}\right)+\tan\left(x-\frac{\pi}{6}\right)\right|+C$

E-6. Sol. $\int \frac{2\sin x+3\cos x}{2\cos x+3\sin x}dx \Rightarrow 2\sin x + 3\cos x = A\frac{d}{dx}(2\cos x + 3\sin x) + B(2\cos x + 3\sin x)$

$\left.\begin{matrix}-2A+3B=2\\ 3A+2B=3\end{matrix}\right\} \Rightarrow A = \frac{5}{13} \quad B = \frac{12}{13}$

so $\int \frac{2\sin x+3\cos x}{2\cos x+3\sin x}dx = \frac{5}{13}\int \frac{3\cos x-2\sin x}{2\cos x+3\sin x}dx + \frac{12}{13}\int 1.dx$

$\frac{5}{13}\ell n\,|2\cos x + 3\sin x| + \frac{12}{13}x + C$

E-7. Sol. $\int \sin^3 x\cos^3 x\,dx = \int \sin^3 x(1-\sin^2 x)\cos x\,dx$

let () sin x = t

cos x dx = dt

$= \int (t^3 - t^5)dt = \frac{\sin^4 x}{4} - \frac{\sin^6 x}{6} + C$

E-8. Sol. $\int \frac{dx}{\sqrt{\sin^3 x\cos^5 x}} = \int \frac{dx}{\sqrt{\tan^3 x\cos^8 x}} = \int \frac{\sec^4 x\,dx}{\sqrt{\tan^3 x}} = \int \frac{(1+\tan^2 x)\sec^2 x}{\sqrt{\tan^3 x}}dx$

Let () $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$\Rightarrow \int \frac{(1+t^2)}{t^{3/2}}dt = \frac{-2}{\sqrt{t}}+\frac{2t^{3/2}}{3}+C = -2\sqrt{\cot x}+\frac{2}{3}\sqrt{\tan^3 x}+C$

E-9. **Sol.** Let $I = \int \frac{dx}{\sqrt{\sin^3 x.\cos x}} = \int \sin^{-3/2} x \cos^{-1/2} x\, dx$

$= \int \tan^{-3/2} x.\sec^2 x\, dx$ put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$\Rightarrow \quad I = \int t^{-3/2} dt = \frac{-2}{\sqrt{\tan x}} + C$

E-10. **Sol.** $I = \int \sqrt{\frac{1}{(\tan x)^{11}(\cos^8 x)}}\, dx = \int (\tan x)^{-11/2} \sec^4 x dx$

$= \int (\tan x)^{-11/2}(1+\tan^2 x)\sec^2 x dx$ put $\tan x = t$ $\sec^2 x dx = dt$

$I = \int (t^{-11/2} + t^{-7/2}) dt$

$= \frac{-2}{9} t^{-9/2} + \frac{-2}{5} t^{-5/2} + C = \frac{-2}{9}(\tan x)^{-9/2} + \frac{-2}{5}(\tan x)^{-5/2} + C \Rightarrow A = \frac{1}{9}, B = \frac{1}{5}$

E-11. Sol. $I = \int \frac{dx}{\cos^3 x \sqrt{\sin 2x}}$

$= \int \frac{\sec^2 x.\sec^2 x\, dx}{\sqrt{2\tan x}}$ put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$\Rightarrow \quad I = \frac{1}{\sqrt{2}} \int \frac{1+t^2}{\sqrt{t}} dt = \frac{1}{\sqrt{2}} \int \left[t^{-\frac{1}{2}} + t^{\frac{3}{2}} \right] dt = \sqrt{2} \left(\sqrt{\tan x} + \frac{1}{5}(\tan x)^{5/2} \right) + C$

E.12. **Sol.** $\int \frac{(\sin x + \cos x)dx}{12 - 5(\sin x - \cos x)^2}$, put $\sin x - \cos x = t$

$\Rightarrow \int \frac{1}{12 - 5t^2} dt = \frac{1}{4\sqrt{15}} \ell n \left| \frac{t + \sqrt{\frac{12}{5}}}{t - \sqrt{\frac{12}{5}}} \right| + C \Rightarrow \frac{1}{4\sqrt{15}} . \ell n \left| \frac{\sin x - \cos x + \sqrt{\frac{12}{5}}}{\sin x - \cos x - \sqrt{\frac{12}{5}}} \right| + C$

E-13. **Sol.** $\int \frac{\sin x - \cos x}{26 + \sin 2x} dx = \int \frac{(\sin x - \cos x)dx}{(\sin x + \cos x)^2 + 25} = \int \frac{-dt}{t^2 + 25} = -\frac{1}{5} \tan^{-1}\left(\frac{t}{5}\right) + C$

$-\frac{1}{5}\tan^{-1}\frac{(\sin x + \cos x)}{5} + C$

E-14. **Sol.** Let $I = \int \frac{1}{1 - \cot x} dx = \int \frac{\sin x dx}{\sin x - \cos x}$

Let $\sin x \equiv A(\cos x + \sin x) + B(\sin x - \cos x)$

then $\left.\begin{matrix} A + B = 1 \\ A - B = 0 \end{matrix}\right\}$ $\therefore$ $A = \frac{1}{2}$, $B = \frac{1}{2}$

$I = \int \frac{\frac{1}{2}(\cos x + \sin x) + \frac{1}{2}(\sin x - \cos x)}{(\sin x - \cos x)} dx = \frac{1}{2}\int \frac{\cos x + \sin x}{\sin x - \cos x} dx + \frac{1}{2}\int dx + C$

$= \frac{1}{2}\log|\sin x - \cos x| + \frac{1}{2}x + C$

E-15._ Sol. We known that the reduction formula of cosx is

$I_n = \int \cos^n x dx = \frac{n-1}{n} I_{n-2} + \frac{1}{n}\cos^{n-1}x\sin x$

$\therefore I_6 = \frac{5}{6} I_4 + \frac{1}{6}\cos^5 x \sin x \Rightarrow I_6 = \frac{5}{6}\left[\frac{3}{4}I_2 + \frac{1}{4}\cos^3 x \sin x\right] \frac{1}{6}\cos^5 x \sin x$

$\Rightarrow I_6 = \frac{5}{8} I_2 + \frac{5}{24}\cos^3 x \sin x + \frac{1}{6}\cos^5 \sin x$

$\Rightarrow I_6 = \frac{5}{8}\left[\frac{1}{2}x + \frac{1}{2}\cos x \sin x\right] + \frac{5}{24} x \cos^3 x \sin x + \frac{1}{6}\cos^5 x \sin x + C$

$\Rightarrow I_6 = \frac{5}{16}[x + \cos x \sin x] + \frac{5}{24}\cos^3 x \sin x + \frac{1}{6}\cos^5 x \sin x + C$

E-16. Sol. $I_n = \int \cot^n x\, dx = \int \cot^2 x \,.\, \cot^{n-2} x\, dx = \int (\operatorname{cosec}^2 x - 1)\cot^{n-2} x\, dx$

$\Rightarrow I_n = \int \operatorname{cosec}^2 x \cot^{n-2} x\, dx - I_{n-2}$

$\Rightarrow I_n = -\frac{\cot^{n-1} x}{n-1} - I_{n-2}, \quad n \geq 2$