Exercise-1

Section (A) : Equation of Tangent and Normal and angle of intersection of two curves[

A-1. **Sol.** $y = 2\cos x$ At $x = \frac{\pi}{4}$, $y = \frac{2}{\sqrt{2}} = \sqrt{2}$

and $\frac{dy}{dx} = -2.\sin x \therefore \left(\frac{dy}{dx}\right)_{x=\pi/4} = -\sqrt{2}$

$\therefore$ Equation of tangent at $\left(\frac{\pi}{2},\sqrt{2}\right)$ is $y - \sqrt{2} = -\sqrt{2}\left(x - \frac{\pi}{4}\right)$

$\therefore \left(\frac{\pi}{2},\sqrt{2}\right)$ $y - \sqrt{2} = -\sqrt{2}\left(x - \frac{\pi}{4}\right)$

A-2. **Sol.** $x_2 = -4y \Rightarrow 2x = -4\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{-x}{2}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(-4,4)} = 2$

We known that equation of tangent is,

$(y - y_1) = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} (x - x_1)$

$\Rightarrow y + 4 = 2(x + 4) \Rightarrow 2x - y + 4 = 0$

A-3. **Sol.** $x = t_2$ and $y = 2t \Rightarrow$ At t = , x = 1 and y = 2

Now $\left(\frac{dy}{dx}\right) = \frac{dy/dt}{dx/dt} = \frac{2}{2t} = \frac{1}{t} \Rightarrow \left(\frac{dy}{dx}\right)_{t=1} = 1$

$\therefore$ Equation of the normal at (1,2) is $y - 2 = \frac{1}{\frac{dy}{dx}} (x - 1)$

$\Rightarrow y - 2 = -1(x - 1) \Rightarrow x + y - 3 = 0$

A-4. **Sol.** Given curve is $y_4 = ax_3 \Rightarrow 4y_3 = \frac{dy}{dx} 3ax_2$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(a,a)} = \frac{3a^3}{4a^3} = \frac{3}{4}$

$\therefore$ Equation of normal at point (a,a) is

$y - a = -\frac{4}{3} (x - a) \Rightarrow 4x + 3y = 7a$

A-5. **Sol.** $y - e_{xy} + x = 0$

Differentiating w.r.t. to y

$1 - e_{xy}\left(\frac{dx}{dy}.\ y + x\right) + \frac{dx}{dy} = 0$

$\frac{dx}{dy} = 0$

$1 - xe_{xy} = 0$

$xe_{xy} = 1 \Rightarrow x = 1 , y = 0$

Point is (1, 0)

A-6. **Sol.** $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$= \frac{a\ \ (-\sin\theta)}{a\ \ (1+\cos\theta)}$

$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{-\sqrt{3}}{3} = -\frac{1}{\sqrt{3}}$

$\tan\alpha = -\frac{1}{\sqrt{3}} \Rightarrow \alpha = \pi - \frac{\pi}{6}$

$\alpha = \frac{5\pi}{6}$

A-7. **Sol.** Equation of tangent is

$y - 4/h = -4/h^2 (x - h)$

It passes through (0, 1) $\Rightarrow 1 - 4/h = 4h/h^2$

$\Rightarrow h - 4 = 4 \Rightarrow h = 8 \Rightarrow$ tangent is $y - 1/2 = -1/16 (x - 8)$.

A-8. **Sol.** $y = \tan(\tan^{-1} x)$

$\Rightarrow y = x$

$\Rightarrow x = -\sqrt{x} + 2$

$x + \sqrt{x} - 2 = 0$

$\sqrt{x} = 1 \Rightarrow x = 1,\quad y = 1$

$\frac{dy}{dx} = -\frac{1}{2\sqrt{x}} \Rightarrow \left.\frac{dy}{dx}\right|_{(1,\,1)} = -\frac{1}{2}$

Slope of normal = 2

Equation of normal is $2x - y = 1$

A-9. **Sol.** Given curves are $y^2 = 4x + 4$ and $y^2 = 36(9 - x)$ ......(i)

On solving , we get the point (8,6) and (8,–6)

On differentiating equation (i) , we get

$2y\frac{dy}{dx} = 4$ and $2y\frac{dy}{dx} = -36$

$\Rightarrow \frac{dy}{dx} = \frac{2}{y}$ and $\frac{dy}{dx} = \frac{-18}{y}$

At point (8,6) , $m_1 = \frac{dy}{dx} = \frac{1}{3}$ and $m_2 = \frac{dy}{dx} = -3$

$m_1m_2 = -1$

A-10. **Sol.** If $\sin x = \cos x \Rightarrow x = \frac{\pi}{4}$

$y = \sin x \Rightarrow \left(\frac{dy}{dx}\right)_{x=\pi/4} = \frac{1}{\sqrt{2}}$

If $y = \cos x \Rightarrow \left(\frac{dy}{dx}\right)_{x=\pi/4} = -\frac{1}{\sqrt{2}}$

$\tan\theta = \frac{m_1 - m_2}{1 + m_1m_2} = 2\sqrt{2} \Rightarrow \theta = \tan^{-1}\left(2\sqrt{2}\right)$

A-11. **Sol.** Subtangent = $2x_1$

ordinate = y_1
subnormal = 2a

A-12. **Sol.** Length of normal = y $\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

Now $\frac{dy}{dx}$ = $\frac{dy/d\theta}{dx/d\theta}$ = $\frac{a\sin\theta}{a(1+\cos\theta)}$ = $\frac{\sin\theta}{1+\cos\theta}$ = $\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$

∴ $\left(\frac{dy}{dx}\right)_{\left(\theta=\frac{\pi}{2}\right)}$ = $\left(\tan\frac{\theta}{2}\right)_{\left(\theta=\frac{\pi}{2}\right)}$ = 1 and y = $a\left(1-\cos\frac{\pi}{2}\right)$ = a

∴ Length of normal = a $\sqrt{1+(1)^2}$ = $\sqrt{2}a$

Section(B) : Rate of change, Error and Approximation

B-1. **Sol.** V = $\frac{1}{3}$ $\pi r^2 h$

⇒ $\frac{dv}{dr}$ = $\frac{2}{3}$ πrh

B-2. **Sol.** A = x^2, $\frac{dx}{dt}$ = 4 cm/min

$\frac{dA}{dt}$ = 2x $\frac{dx}{dt}$ = 8x

⇒ at x = 8 cm

$\frac{dA}{dt}$ = 64 cm^2/min.

B-3. **Sol.** $\frac{dr}{dt}$ = 0.5 cm/s

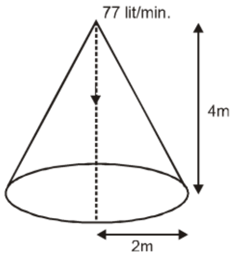
v = $\frac{4}{3}\pi r^3$, $\frac{dv}{dt} = 4\pi r^2$. $\frac{dr}{dt}$

$\frac{dv}{dt}$ = 4 × π × 1 × 1 × 0.5 = 2πcm^3/s

B-4. **Sol.** V = $\frac{1}{3}\pi r^2 h$ $\left(\because \quad \frac{r}{h} = \frac{2}{4} = \frac{1}{2}\right)$

V = $\frac{1}{3}\pi\frac{h^3}{4}$ = $\frac{\pi}{12}h^3$

$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$



$77 \times 10^3 = \frac{22}{7} \times \frac{1}{4} \times 70 \times 70 \times \frac{dh}{dt}$ (∵ 1 litre = 10^3 c.c.)

$\therefore \frac{dh}{dt} = 20$ cm/min.

B-5. **Sol.** $\frac{dy}{dx} = 2x + 2$

If x & y coordinates of the particle are changing at the same rate then

$\frac{dy}{dx} = 1 \Rightarrow x = \frac{-1}{2}$, $y = \frac{-3}{4}$

B-6. **Sol.** $2y \frac{dy}{dx} = 8$

If ordinate & abscissa changes at same rate then $\frac{dy}{dx} = 1$

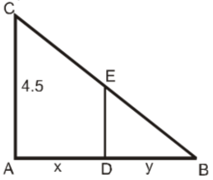
$\Rightarrow y = 4, x = 2$

B-7. **Sol.** Let AC be pole, DE be man and B be farther end of shadow as shown in figure

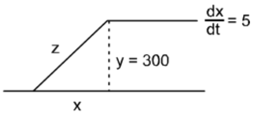
From triangles ABC and DBE

$\frac{4.5}{x+y} = \frac{1.5}{y}$

$3y = 1.5\ x$



$\frac{dy}{dt} = 2$, $(x + y) = \frac{dx}{dt} + \frac{dy}{dt}$



Figure

B-8. **Sol.**

From figure $z^2 = x^2 + y^2$

$z \frac{dz}{dt} = x \frac{dx}{dt}$

If z = 500 then x = 400

$\Rightarrow \quad 500 \frac{dz}{dt} = 400(5)$

$\Rightarrow \quad \frac{dz}{dt} = 4$

$\Rightarrow \quad \frac{dz}{dt} = 4$

B-9. **Sol.** Let r be the radius of the sphere and Δr be the error in measuring the radius. Then r = 8 cm, and Δr = 0.03 cm, Now the voume V of the sphere is given by

$V = \frac{4}{3}\pi r_3$

$\frac{dV}{dr} = 4\pi r_2$

$\Delta V = \left(\frac{dV}{dr}\right)\Delta r = (4\pi r_2)\,\Delta r = 4\pi(8)_2 \times 0.03 = 7.68\ \pi cm_3$

B-10. **Sol.** $\sqrt{25.2} = \sqrt{25+0.2}$

Let x = 25 and Δx = 0.2 such that $f(x) = \sqrt{x}$

$\therefore\ f'(x) = \frac{1}{2\sqrt{x}}$

$\therefore\ f(x + \Delta x) = f(x) + f'(x).\ \Delta x$

$\sqrt{x+\Delta x} = \sqrt{x} + \frac{1}{2\sqrt{x}}.\ \Delta x$

$\sqrt{25+0.2} = \sqrt{25} + \frac{1}{2\sqrt{25}} \times 0.2$

$\sqrt{25.2} = 5 + \frac{0.2}{2\times 5} = 5 + 0.02 = 5.02$

B-11. **Sol.** $V = x_3$

$\Delta V = \left(\frac{dV}{dx}\right)\Delta x = (3x_2)\,\Delta x$

$= (3x_2)\,(0.04x) = 0.12x_3m_3$

Section(C) : Monotonicity

C-1. **Sol.** $f'(x) = 1 - \frac{1}{x^2} = \frac{(x-1)\ \ (x+1)}{x^2}$

$f'(x) > 0 \Rightarrow x_2 > 1 \Rightarrow |x| > 1$

C-2. **Sol.** $f'(x) = 2(x - 1)$

for decreasing, $f'(x) < 0 \Rightarrow x < 1$

C-3. **Sol.** $f(x) = x_3$

$f'(x) = 3x_2 \geq 0$

hence always increasing

C-4. **Sol.** $f'(x) = e^{-\frac{1}{x}}.\frac{1}{x^2}$ (> 0 for x > 0)

Hence increasing

C-5. **Sol.** $y = \frac{2X^2-1}{X^4}$ is even funciton.

Even function is nonmonotonic.

C-6. **Sol.** $f'(x) = 6(x^2 - 3x + 2) = 6(x - 2)(x - 1)$
for monotonically increasing, $f'(x) > 0$
$\Rightarrow x \in (-\infty, 1) \cup (2, \infty)$

C-7. **Sol.** $f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x}$
for decreasing, $f'(x) < 0 \Rightarrow \frac{x-1}{x} < 0 \quad \Rightarrow x \in (0, 1)$

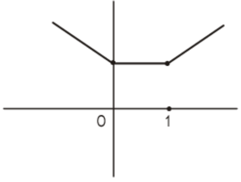
C-8. **Sol.** $f'(x) = \frac{-1}{2x\sqrt{x}}$
for $x \in (0, 1)$, $f'(x) < 0$
Hence decreasing.

C-9. **Sol.** $f'(x) \geq 0$
$\Rightarrow \quad -\sin x + x \geq 0 \quad \Rightarrow \quad x \geq \sin x \quad \Rightarrow \quad x \in [0, \infty)$

C-10. **Sol.** $f(x) = \frac{e^{2x}-1}{e^{2x}+1}$
$f'(x) = \frac{(e^{2x}+1)2e^{2x} - (e^{2x}-1)2.e^{2x}}{(e^{2x}+1)^2}$
$\frac{e^{2x}(2)}{(e^{2x}+1)^2} > 0$
hence f(x) is increasing

C-11. **Sol.** $f'(x) = \cot x$
monotonically increasing ,$\Rightarrow f'(x) > 0 \quad \Rightarrow x \in \left(0, \frac{\pi}{2}\right)$

C-12. **Sol.** $f(x) = |x| + |x - 1|, \quad 0 \leq x \leq 1$
From graph it is constant function $0 \leq x \leq 1$

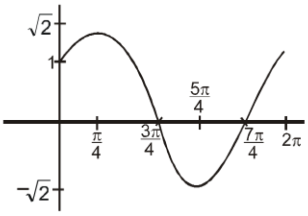


C-13. **Sol.** $f'(x) = 1 - \sin x \geq 0 \ \forall \ x \in R \quad \Rightarrow \quad f(x)$ is M.I.
f(x)

C-14. **Sol.** $f'(x) = 3x^2 + 2ax + b + 5 \sin 2x \geq \forall \ 0 \ x \in R$
$\because \quad \sin 2x \geq -1$
$\Rightarrow \quad f'(x) \geq 3x^2 + 2ax + b - 5 \ \forall \ x \in R \quad \Rightarrow \quad 3x^2 + 2ax + b - 5 \geq 0 \ \forall \ x \in R$
$\Rightarrow \quad 4a^2 - 4 .3 . (b - 5) \leq 0 \quad \Rightarrow \quad a^2 - 3b + 15 \leq 0$

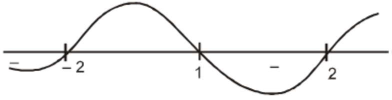
C-15. **Ans.** $(-\infty, -3]$
Sol. $f'(x) = 3(a + 2)x^2 - 6ax + 9a \leq 0 \ \forall \ x \in R$
$\Rightarrow \quad a + 2 < 0 \quad$ and $\quad D \leq 0$
$\Rightarrow \quad a < -2 \quad$ and $\quad a \in (-\infty, -3] \cup [0, \infty) \quad \Rightarrow \quad a \in (-\infty, -3]$

C-16. **Sol.** $f(x) = \sin x + \cos x = \sqrt{2} \ \sin\left(\frac{\pi}{4} + x\right)$



C-17. **Sol.** $f(x) = \log(x-2) - \frac{1}{x}$

$$f'(x) = \frac{1}{x-2} + \frac{1}{x^2} = \frac{x^2+x-2}{x^2(x-2)} = \frac{x^2+2x-x-2}{x^2(x-2)} = \frac{(x+2)\ (x-1)}{x^2(x-2)}$$



but $\log(x-2)$ is defined when $x > 2$

⇒ f(x) is M.I. for $x \in (2, \infty)$

C-18. **Sol.** The function $f(x) = x_3$ increases ∀ x and the function

$6x_2 + 15x + 5$ increases is

$g'(x) > 0 \quad \Rightarrow \quad 12x + 15 > 0 \quad \Rightarrow \quad x > -5/4$

It is given that f(x) increases less rapidly than g(x),

therefore function $\varphi(x) = f(x) - g(x)$ is

decreasing function, which implies that $\varphi'(x) < 0$

$\Rightarrow \quad 3x_2 - 12x - 15 < 0 \quad \Rightarrow \quad (x-5)(x+1) < 0 \quad \Rightarrow \quad -1 < x < 5$

Hence, x_3 increases less rapidly than

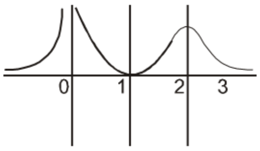
$6x_2 + 15x + 5$

in the interval (– 1, 5)

C-19. **Sol.**

$$f(x) = \begin{cases} \frac{1-x}{x^2} & x < 1, \quad x \neq 0 \\ \frac{x-1}{x^2}, & x \geq 1, \end{cases}$$

The given function is not differentiable at x = 1



$$f'(x) = \begin{cases} \frac{1}{x^2} - \frac{2}{x^3}, & x < 1, \quad x \neq 0 \\ \frac{2}{x^3} - \frac{1}{x^2}, & x > 1 \end{cases}$$

Now $f'(x) < 0 \Rightarrow \begin{cases} \frac{x-2}{x^3} < 0 & \text{given} \quad x < 1 \\ \frac{2-x}{x^3} < 0 & \text{when} \quad x > 1 \end{cases}$

f(x) decreasing $\forall\ x \in (0, 1) \cup (2, \infty)$ and f(x) increases $\forall\ x \in (-\infty, 0) \cup (1, 2)$

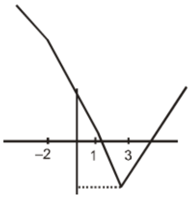
here f(x) is decreasing at all points in $x \in (0, 1) \cup (2, \infty)$ so will also be decreasing at x = 3

at x = 1 minima and at x = 2 maxima

C-20. **Sol.** at x = – 2 decreasing

at x = 0 decreasing

at x = 3 neither increasing nor decreasing at x = 5 increasing



Section(D) : Local maxima and minima

D-1. **Sol.** $f(x) = x^3 – 3x + 4$

$f'(x) = 3(x^2 – 1)$

Hence minima at x = 1

D-2. **Sol.** $f(x) = 2x^3 – 9x^2 + 100$

$\therefore$ $f'(x) = 6x^2 – 18x = 0$

$x = 0, 3$

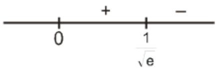
$f(0) = 100$

$f(3) = 54 – 81 + 100 = 73$

$\therefore$ maximum $f(x) = 100$

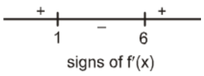
D-3. **Sol.** $f(x) = x^2 \ell n\left(\frac{1}{x}\right) = –x^2 \ell n\, x$

$f'(x) = –[2x\, \ell n\, x + x] = –x(2lnx + 1)$



$\Rightarrow$ maximum at $x = \frac{1}{\sqrt{e}}$

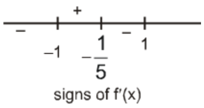
D-4. **Sol.** $f'(x) = 6(x – 1)(x – 6)$



Local maxima at x = 1

Local minima at x = 6

D-5. **Sol.** $f'(x) = –(x – 1)^2 (x + 1)(5x + 1)$



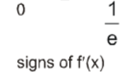
Local minima at x = – 1

Local maxima at $x = –\frac{1}{5}$

Neither local minima nor local maxima at x = 1 .

D-6. Sol. $f'(x) = \ell nx + 1$

Local minima at $x = \frac{1}{e}$



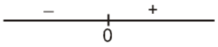
No local maxima

D-7. **Sol.** $f(x) = a \sin x + \frac{1}{3} \sin 3x$

$f'(x) = a \cos x + \cos 3x$

at $x = \frac{\pi}{3}$, $f'(x) = 0 \Rightarrow \frac{a}{2} - 1 = 0 \Rightarrow a = 2$

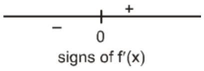
D-8. **Sol.** $f'(x) = e^x - e^{-x} = \frac{e^{2x} - 1}{e^x}$



only one point of extrema (point of minima)

D-9. **Sol.** $f'(x) = (2^2 + 4^2x^2 + 6^2 x^4 + \ldots\ldots + 100^2 x^{98})\, x$

Minimum at $x = 0$



D-10. **Sol.** $f(x) = \sum_{k=1}^{5} (x - k)^2$

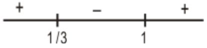
$f'(x) = 2\sum_{k=1}^{5} (x - k) = 2[5x - 15] = 0 \quad \Rightarrow x = 3$ (point of minima)

D-12. **Sol.** $f(x) = x(1 - x)^2$

$f'(x) = -2x(1 - x) + (1 - x)^2 = 0$

$(1 - x)(-2x + 1 - x) = 0$

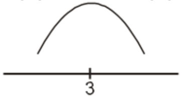
$\Rightarrow \quad (1 - x)(1 - 3x) = 0, \quad x = 1, \frac{1}{3}$



Local max. at $x = \frac{1}{3}$

D-13. **Sol.** for maxima at $x = 3$

$f'(3) = 0$ & $f''(3) < 0$



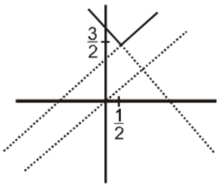
D-14. **Sol.** $x = 2$ is the point of inflection.

D-15. **Sol.** $f(1^-) \leq f(1)$ and $f(1^+) \leq f(1)$

$-2 + \log_2 (b^2 - 2) \leq 5$

$0 < b^2 - 2 \leq 128 \; 2 < b^2 \leq 130$

D-16. **Sol.** Minimum value of $f(x)$ is $\frac{3}{2}$ at $x = \frac{1}{2}$



Section(E) : Global maxima & minima

E-1. **Sol.** $f'(x) = 3x^3$

$f'(x) = 0 \Rightarrow x = 0$

$x = -2, \quad f(-2) = -8$

$x = 0, \quad f(0) = 0$

$x = 2, \quad f(2) = 8$

Minimum = – 8, maximum = 8

E-2. **Sol.** $f'(x) = \cos x - \sin x$

$f'(x) = 0 \Rightarrow x = \frac{\pi}{4}$

$x = 0, \quad f(0) = 1$

$x = \frac{\pi}{4}, \quad f\left(\frac{\pi}{4}\right) = \sqrt{2}$

$x = \pi, \quad f(\pi) = -1$

Minimum = –1, Maximum = $\sqrt{2}$

E-3. **Sol.** $f'(x) = 4 - x$

$f'(x) = 0 \Rightarrow x = 4$

$x = -2, f(-2) = -10$

$x = 4, \quad f(4) = 8$

$x = \frac{9}{2}, f\left(\frac{9}{2}\right) = \frac{63}{8}$

Minimum = –10, Maximum = 8

E-4. **Sol.** $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$

$\therefore \quad f'(x) = 12x^3 - 24x^2 + 24x - 48 = 0 \Rightarrow x^3 - 2x^2 + 2x - 4 = 0 \Rightarrow (x^2 + 2)(x - 2) = 0$

$\Rightarrow \quad x = 2 \in [0, 3]$

$\therefore \quad f(0) = 25$

$f(2) = 48 - 64 + 48 - 96 + 25 = -39$

$f(3) = 243 - 216 + 108 - 144 + 25 = 16$

E-5. **Sol.** $f'(x) = \cos x - \sin 2x$

$f'(x) = 0 \Rightarrow \cos x = 0, \sin x = \frac{1}{2}$

$\Rightarrow x = \frac{\pi}{2}, \; x = \frac{\pi}{6}$

$x = 0, \quad f(0) = \frac{1}{2}$

$x = \frac{\pi}{6}, f\left(\frac{\pi}{6}\right) = \frac{3}{4}$

$x = \frac{\pi}{2}, \quad f\left(\frac{\pi}{2}\right) = \frac{1}{2}$

Minimum = $\frac{1}{2}$, Maximum = $\frac{3}{4}$

E-6. **Sol.** $f'(x) = 1 > 0$

f(x) is increasing

f(0), f(1) is not defined. Hence no local maxima/minima.

E-7. **Sol.** $5 \sin\theta + 3 \sin\left(\theta + \frac{\pi}{3}\right) + 3$

$= 5\sin\theta + 3\left(\sin\theta\ .\ \frac{1}{2} + \cos\theta\ .\ \frac{\sqrt{3}}{2}\right) + 3$

$\frac{13}{2}\sin\theta + \frac{3\sqrt{3}}{2}\cos\theta + 3$

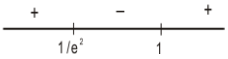
max value $\sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2} + 3$

$= \sqrt{\frac{169 + 27}{4}} + 3$

$= 7 + 3 = 10$

E-8. **Sol.** $y = x(\ell nx)^2$

$y' = x\ .\ 2\ \ell nx\ .\ \frac{1}{x} + (\ell nx)^2$



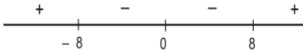
y is min at x = 1

∴ $y_{min} = 0$

E-9. **Sol.** xy = 4, x < 0

Let S = x + 16y

S = x + 64/x



$S = \frac{x^2 + 64}{x}$

$\frac{dS}{dx} = \frac{(x-8)(x+8)}{x^2}$

S is max at x = – 8

S vf/kdre gS x = – 8 ij

∴ $S_{max} = -16$

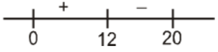
Section(F) : Application of maxima and minima

F-1. **Sol.** x + y = 20

$x^3 y^2 = x^3(20 - x)^2 = f(x)$

$f'(x) = 3x^2(20 - x)^2 - 2x^3(20 - x)$

$= (20 - x)x^2 (60 - 5x)$



⇒ maximum at x = 12

F-2. **Sol.** h = R(sin θ + 1)

$v = \pi\frac{1}{3}(R\cos\theta)^2 h = \frac{\pi R^3}{3}\cos^2\theta\,(1 + \sin\theta)$

$\frac{dh}{d\theta} = \frac{\pi R^3}{3}(\cos^3\theta - 2\sin\theta\cos\theta\,(1 + \sin\theta)]$

$= \frac{\pi R^3}{3}\cos\theta\,(\cos^2\theta - 2\sin\theta - 2\sin^2\theta)$

$= \frac{\pi R^3 \cos\theta}{3}$ (1 – 2 sin θ – 3 $\sin^2$ θ)

= (1 – 3 sin θ) (1 + sin θ) $\frac{1}{3}$

⇒ maximum when sin θ = $\frac{1}{3}$

⇒ $\frac{h}{2R} = \frac{2}{3}$

F-3. **Sol.** A = Area = $\frac{1}{2}$ (2R cos θ).R(sin θ + 1)

$\frac{dA}{d\theta}$ = R^2[$\cos^2$ θ – sin θ (sin θ + 1)]

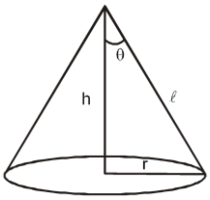
= R^2 [1 – sin θ – 2 $\sin^2$ θ]

= R^2 (1 – 2 sin θ) (1 + sin θ)

⇒ maximum when sin θ = $\frac{1}{2}$ ⇒ equilateral triangle.

F-4. **Sol.** h = ℓcosθ

r = ℓsin θ



Figure

V = $\frac{1}{3}$ πr^2 h

V = $\frac{1}{3}$ π ℓ^3 $\sin^2\theta$ cosθ

$\frac{dV}{d\theta} = \frac{1}{3}$ πℓ^3 (2sinθ $\cos^2\theta$ – $\sin^3\theta$)

$\frac{dV}{d\theta} = \frac{1}{3}$ πℓ^3 sinθ (2 – 3 $\sin^2\theta$) = 0 at

sinθ = $\sqrt{\frac{2}{3}}$ ⇒ tanθ = $\sqrt{2}$

F-5. **Sol.** $R^2 + r^2 = h^2$

$R^2 = h^2 - r^2$

volume of cylinder ,

V = πR^2 (2h) = π (2h) ($\sqrt{r^2 - h^2}$)^2

$\frac{dV}{dh}$ = 2π ($r^2 - h^2$) + 2πh(–2h) = 0

⇒ $r^2 = 3h^2$ ⇒ h = $\frac{r}{\sqrt{3}}$

$\frac{d^2V}{dh^2} < 0$ at h = $\frac{r}{\sqrt{3}}$ ⇒ $V_{max} = 2\pi \frac{r}{\sqrt{3}}\left(r^2 - \frac{r^2}{3}\right) = \frac{4\pi r^3}{3\sqrt{3}}$

Section(G) : Inequalities using monotonicity

G-1. **Sol.** Let f(x) = $\frac{\tan x}{x}$, x ∈ $\left(0\ ,\ \frac{\pi}{2}\right)$

f′(x) = $\frac{x\ \ \sec^2 x - \tan x}{x^2}$.

Let $g(x) = x \sec^2 x - \tan x$

$g'(x) = 2x \sec^2 x \tan x > 0$

$x > 0$

⇒ $g(x) > g(0)$

$g(x) > 0$

⇒ $f'(x) > 0$ ⇒ f(x) is M.I.

$x_1 < x_2$

$f(x_1) < f(x_2)$

$$\frac{\tan x_1}{x_1} < \frac{\tan x_2}{x_2}$$

$$\frac{x_2}{x_1} < \frac{\tan x_2}{\tan x_1}$$

G-2. **Sol.** Let $f(x) = 2 \sin x + \tan x - 3x$

$f'(x)\ 2 \cos x + \sec^2 x - 3$

$$= \frac{(\cos x - 1)^2 \quad (2\cos x + 1)}{\cos^2 x} > 0$$

f(x) is M.I.

$x > 0$

$f(x) > f(0)$

$2 \sin x + \tan x > 3x$

$3x < 2 \sin x + \tan x$

⇒ $\frac{3x}{2\sin x + \tan x} < 1$ for $x \in \left(0 , \frac{\pi}{2}\right)$ and $\displaystyle \operatorname{Lt}_{x \to 0^+} \frac{3x}{3\sin x + \tan x} = 1$

⇒ $\displaystyle \operatorname{Lt}_{x \to 0^+} \left[\frac{3x}{3\sin x + \tan x}\right] = 0$

G-3. **Sol.** $f'(x) = 1 + \ell n\, x - 1 = \ell n\, x$

for $x > 1$, $f'(x) > 0$

⇒ f(x) is increasing

⇒ $f(x) > f(1)$

⇒ $f(x) > 0$

for $0 < x < 1$

$f'(x) < 0$

⇒ f(x) is decreasing

⇒ $f(x) > f(1)$

⇒ $f(x) > 0$

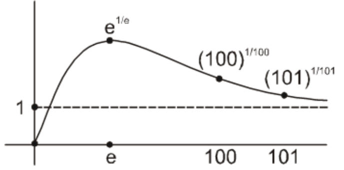
G-4. **Sol.** Assume $f(x) = x^{1/x}$ and let us examine monotonic nature of f(x)

$f'(x) = x^{1/x} . \left(\frac{1 - \ell nx}{x^2}\right)$

$f'(x) > 0$ ⇒ $x \in (0,e)$

and $f'(x) < 0$ ⇒ $x \in (e,\infty)$

Hence f(x) is M.D. for $x \geq e$



and since 100 < 101

⇒ $f(100) > f(101)$

⇒ $(100)^{1/100} > (101)^{1/101}$

Section(H) : Rolle's theorem & LMVT

H-1. **Sol.** $f'(x) = 0$ ⇒ $x = -2, 3$

$x = -2 \in (-3, 0)$

$\therefore \quad c = -2$

H-2. **Sol.** $\because$ for $f(x) = x_2$, $f(-1) = f(1) = 1$

Also, $f(x) = x_2$ is continous in $[-1,1]$ and differentiable in $(-1,1)$.

$\therefore$ Rolle's theorem is applicable.

H-3. **Sol.** f(x) is not continuous at x = 1

H-4. **Sol.** at x = 0, f(x) is not differentiable.

H-5. **Sol.** at x = 0, f(x) is not differentiable.

H-6. **Sol.** $f'(c) = \frac{e-1}{1-0} = e_c \Rightarrow c = \ell n(e-1)$

H-7. **Sol.** $\frac{a-b}{ab} = \frac{1}{x_1^2}(b-a) \Rightarrow x_1 = \sqrt{ab}$

H-8. **Sol.** By Lagrange's Mean value theorem, we have,

$$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{f(6)-f(0)}{6} = 3, \; c \in (0,6)$$

$\therefore$ For some point between x = 0 and x = 6, $f'(x) = 3$

H-9. **Sol.** $f'\left(\frac{7}{4}\right) = \frac{f(2)-f(1)}{2-1}$

$\Rightarrow 3\left(\frac{7}{4}\right)^2 - 129\left(\frac{7}{4}\right) + 5 = (8 - 24a + 10) - (1 - 6a + 5)$

$\Rightarrow a = \frac{35}{48}$

Exercise-2

Marked Questions may have for Revision Questions.

PART - I : OBJECTIVE QUESTIONS

Single choice type

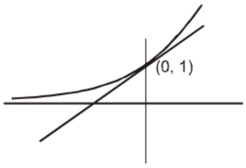
1. **Sol.** $\frac{y}{b} = 1 - \frac{x}{a}$

$\frac{y}{b} = e_{-x/a}$

$\Rightarrow \quad e_{-x/a} = 1 - \frac{x}{a}$

put $t = -\frac{x}{a}$

$e_t = 1 + t$



Draw graph of $y = e_t$, $\quad y = 1 + t$

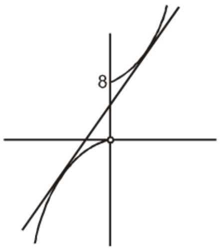
From graph it is clear that t = 0 is the only Solution

$\Rightarrow \quad x = 0 \quad \Rightarrow \quad y = b \quad (0, b)$

2. **Sol.** $f'(0) = \lim_{x \to 0} \frac{\frac{\sin x^2}{x} - 0}{x - 0} = 1$ (slope of tangent)
slope of normal is – 1
Equation of normal is y – 0 = – (x – 0)

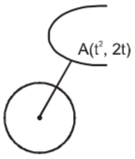
3. **Sol.** The tangent at $(x_1, \sin x_1)$ is $y - \sin x_1 = \cos x_1 (x - x_1)$
It passes through the origin.
$\sin x_1 = x_1 \cos x_1 = x_1 \sqrt{1 - \sin^2 x_1}$
$y_{12} = \sin_2 x_1 = x_{12}(1 - y_{12})$ ⇒ (x_1y_1) (x_1y_1) lies on the curve
$y_2 = x_2(1 - y_2)$ ⇒ $x_2 - y_2 = x_2y_2$

4. **Sol.** Let y = mx + c be tangent touching both branches.
$f(x) = -x_2$, $y = mx + c$, $x < 0$



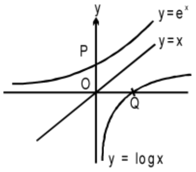
Figure

$x_2 + mx + c = 0$, $m > 0$ ($\because$ $x < 0$) (negative roots)
$D = 0 \Rightarrow m_2 = 4c$
$f(x) = x_2 + 8$, $y = mx + c$, $x > 0$
$x_2 - mx + 8 - c = 0$, $m > 0$ (positive roots)
$D = 0 \Rightarrow m_2 = 32 - 4c$
$\Rightarrow c = 4, m_2 = 16 \Rightarrow c = 4, m = 4$



5. **Sol.**
shortest distance always lie along the common normal
Equation of normal at $(t_2, 2t)$ to the parabola is
$y + xt = 2t + t_3$ ..... (i)
above equation passes through the center of the circle c(0, 12)
$\therefore$ $12 = 2t + t_3$
$t_3 + 2t - 12 = 0$
$t = 2$
$\therefore$ point is (4, 4)

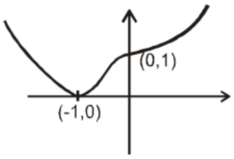
6. **Sol.** $f(x) = e_x$ & $g(x) = \ell nx$
are image of each other in line mirror y = x hence minimum distance between these will be equal to distance between parallel tangents of f(x) & g(x) which are parallel y = x.
⇒ $e_x = 1$ & $\frac{1}{x} = 1$
⇒ x = 0 & x = 1



P ≡ (0, 1) ; Q = (1 , 0)
PQ = $\sqrt{2}$

7. **Sol.** $\frac{dy}{dx} = 4ax_3 + 3bx_2 + c$
at (0, 1)

$\frac{dy}{dx} = 0 \Rightarrow c = 0$ & $d = 1$



It touches x-axis at (–1, 0) ⇒ $\left.\frac{dy}{dx}\right|_{(-1,\ 0)} = 0$
⇒ $-4a + 3b = 0$

so $\frac{dy}{dx} = 4a(x_3 + x_2)$ ⇒ two points of extrema

$\frac{d^2y}{dx^2} = 4a(3x_2 + 2x)$ ⇒ one point of inflection
Hence negative gradient for $x < -1$

8. **Sol.**

$\frac{da}{dt} = 2 \Rightarrow a = 2t + c$
∵ $c = 0$ {∵ $a = 0$, when $t = 0$}
∴ $a = 2t$
∴ distance of vertex from the origin = $2\sqrt{2}\ t$
∴ rate of change of distance of vertex from origin with respect to t = $2\sqrt{2}$

9. **Sol.** $f(x) = \begin{cases} \frac{1-x}{x^2} & x < 1,\ x \neq 0 \\ \frac{x-1}{x^2}, & x \geq 1 \end{cases}$

The given function is not differentiable at x = 1

$f'(x) = \begin{cases} \frac{1}{x^2} - \frac{2}{x^3}, & x < 1,\ x \neq 0 \\ \frac{2}{x^3} - \frac{1}{x^3}, & x > 1 \end{cases}$

Now $f'(x) < 0 \Rightarrow \begin{cases} \frac{x-2}{x^3} < 0 & \text{given} \quad x < 1 \\ \frac{2-x}{x^3} < 0 & \text{when} \quad x > 1 \end{cases}$

⇒ $x < 1$ or $x > 2$ ⇒ $x \in (-\infty, 1) \cup (2, \infty)$

10. **Sol.** $\frac{df(x)}{dx} = \left(\frac{1}{\sqrt{1-x^2}} + \frac{1}{1+x^2}\right)\frac{1}{\pi} + \frac{2}{2\sqrt{x}}$

Domain : $0 \leq x \leq 1$,
at x = 0 f(x) = 0,
at x = 1 $f(x) = (\sin_{-1} 1 + \tan_{-1} 1) / \pi + 2\sqrt{1}$

$= \frac{\frac{\pi}{2} + \frac{\pi}{4}}{\pi} + 2 = \frac{11}{4}$

$f(x) \in \left[0,\ \frac{11}{4}\right]$

11. **Sol.** If range of f(x) is not R and c does not belong to range of f(x) then it is not necessary to have one solution.

12. **Sol.** For strictly monotonic decreasing $f'(x) < 0$

$f'(x) = a^2 - 2a - 2 - \sin x < 0 \quad \forall x \in R$

$\Rightarrow (a - 1)^2 < 3 + \sin x \quad \forall x \in R$

$\Rightarrow (a - 1)^2 < 2$

$\Rightarrow 1 - \sqrt{2} < a < \sqrt{2} + 1$

13. **Sol.** Given that f is a real valued function s.t

$f(x) f'(x) < 0 \ \forall x \in R$

Now, $\frac{d}{dx} |f(x)| = \frac{f(x)}{|f(x)|} f'(x)$

since $f(x) f'(x) < 0$

$\Rightarrow \quad \frac{d}{dx} |f(x)| < 0$

$\Rightarrow$ |f(x)| is a decreasing function

14. **Sol.** g(x) is monotonically increasing

$\Rightarrow \quad g'(x) \geq 0 \quad \& \quad$ f(x) is M.D. $\quad \Rightarrow \quad f'(x) \leq 0$

$\frac{d}{dx}$ (fog) (x) = $\frac{d}{dx} \left((f(g(x)) \right) = f'((g \ (x) \ ) \ \ g'(x) \ \leq 0 \ f'((g \ (x) \ ) \ \ g'(x) \ \leq 0$

as $\quad f'(x) \leq 0 \quad \& \quad g'(x) \geq 0 \quad \Rightarrow \quad$ (fog) (x) is monotonically decreasing

Also $\quad x + 1 > x - 1 \quad \Rightarrow \quad f(x + 1) < f(x - 1) \quad$ as $\quad$ f(x) is M.D.

$\Rightarrow \quad g(f(x + 1) < g(f(x - 1))$ $\quad$ as $\quad$ g(x) is M.I.

15. **Sol.** Since $f(x) \geq 0$ and $g(x) \leq 0$, $x \in I$. Also f(x) is stricity decreasing on I, therefore $f'(x) < 0$ and g(x) is strictly increasing on I, therefore $g'(x) > 0$

Now, $\frac{d}{dx} [f(x) \ \ g(x)] = \underbrace{f'(x) \ \ g(x)}_{+ve} + \underbrace{f(x) \ \ g'(x)}_{+ve}$

$\Rightarrow \quad \frac{d}{dx} f(g(x)) = f'(g(x)) \, . \, g'(x) < 0$

$\Rightarrow$ (fog) (x) is decreasing function

16. **Sol.** $x > 1 \quad \Rightarrow \quad f(x) \geq f(1)$

$x > 1 \quad \Rightarrow \quad g(x) \leq g(1)$

$\Rightarrow \quad f(g(x)) \leq f\,(g(1))$

$\Rightarrow \quad h(x) \leq 1$ .... (i)

Range of h(x) is subset of [1, 10]

$\Rightarrow \quad h(x) \geq 1$ ... (ii)

By (i), (ii) we have $h(x) = 1 \quad \Rightarrow \quad h(2) = 1$

17. **Sol.** $f'(x) = (x - 1)^{n-1} (x + 1)^{n-1} (2(n + 1)x^3 + (2n + 1)x^2 + 2(n - 1)x - 1)$

At x = 1 $2(n + 1)x^3 + (2n + 1)x^2 + 2 (n - 1)$

$x - 1 \neq 0$

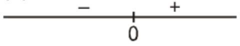
for $\quad n \in N$

$\therefore \quad n - 1$ must be odd

$\Rightarrow$ n is even

18. **Sol.** $f(x) = \frac{x^2 - 1}{x^2 + 1} = 1 - \frac{2}{x^2 + 1}$

$f'(x) = \frac{4x}{(x^2+1)^2}$



⇒ f(x) is minimum at x = 0

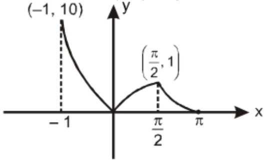
⇒ f(x), x = 0 ij U;wure gS

⇒ min (f(x)) = –1

19. **Sol.** $f'(x) = \begin{cases} a & ; \quad x < 0\ , \\ 2x & ; \quad x > 0. \end{cases}$

f′(x) > 0 ⇒ a > 0

20. **Sol.** By graph



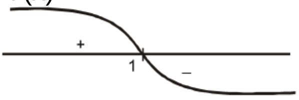
21. **Sol.** $f(x) = 3_{x + 1} + 3_{–(x + 1)}$

$f(x) = \frac{3^{x+1} + 3^{-(x+1)}}{2} \geq \sqrt{3^{x+1}.\ \ 3^{-(x+1)}}$

⇒ $3_{x + 1} + 3_{– (x + 1)} \geq 2$

22. **Sol.** $f'(x) = \begin{cases} \frac{x}{\sqrt{1-x^2}} & , \quad 0 < x < 1 \\ -1 & , \quad x > 1 \end{cases}$

graph of f′(x)



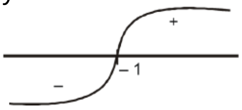
$y = -\sqrt{1-x^2}$

$x = -\sqrt{1-y^2} = f_{-1}(y)$ y > – 1 or y < 0

$x = – y = f_{-1}(y)$ y < – 1

$\frac{df^{-1}(y)}{dy} = \frac{2y}{\sqrt{1-y^2}}$

y = – 1



23. **Sol.** $f'(x) = \frac{a}{x} + 2bx + 1$

f′(– 1) = 0

– a – 2b + 1 = 0

a + 2b = 1

f′(2) = 0

$\frac{a}{2} + 4b + 1 = 0$ ⇒ a + 8b + 2 = 0

– 6b = 3 ⇒ $b = \frac{-1}{2}$, a = 2

24. **Sol.** $f'(x) = 3x^2 - 3p2x + 3p^2 - 3$

$= 3((x - p)^2 - 1)$

$= 3(x - (p + 1))\,(x - (p - 1))$

$\Rightarrow$ $p - 1 > -2$ and () $p + 1 < 4$

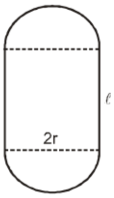
$\Rightarrow$ $p > -1$ and () $p < 3$

$\Rightarrow$ $-1 < p < 3$

25. **Sol.** $2\ell + 2\pi r = 440$

$A = \ell\, 2r = -2\pi r^2 + 440r$

$\frac{dA}{dr} = -4\pi r + 440 = 0$



Figure

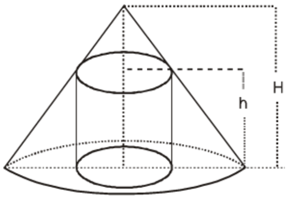
at r =

26. **Sol.** $\frac{H}{R} = \frac{H-h}{r}$

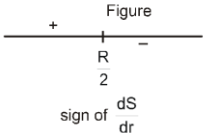
$S = 2\pi rh$

$= 2\pi H\left(r - \frac{r^2}{R}\right)$

$\frac{dS}{dr} = 2\pi H\left(1 - \frac{2r}{R}\right)$

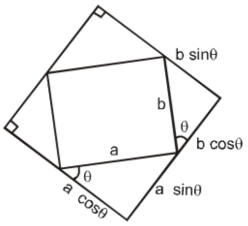


Figure



Maximum at $r = \frac{R}{2}$

27. **Sol.**



$\text{Area} = ab + \left(\frac{1}{2}a^2 \sin\theta \quad \cos\theta + \frac{1}{2}b^2 \sin\theta\cos\theta\right) \;.\; 2$

$= ab + \frac{(a^2 + b^2)}{2} \sin 2\theta$

Maximum area is ab + $\frac{(a^2+b^2)}{2}$

28. **Sol.** Let $f(x) = \sin x \tan x – x_2$

$f'(x) = \cos x \cdot \tan x + \sin x \cdot \sec_2 x – 2x$

⇒ $f'(x) = \sin x + \sin x \sec_2 x – 2x$

⇒ $f''(x) = \cos x + \cos x \sec_2 x + 2\sec_2 x \sin x \tan x – 2$

⇒ $f''(x) = (\cos x + \sec x – 2) + 2 \sec_2 x \sin x \tan x$

Now $\cos x + \sec x – 2 = \left(\sqrt{\cos x} - \sqrt{\sec x}\right)^2$ and $2 \sec_2 x \tan x \cdot \sin x > 0$ because $x \in \left(0, \frac{\pi}{2}\right)$

⇒ $f''(x) > 0$ ⇒ $f'(x)$ is M.I.

Hence $f'(x) > f'(0)$

⇒ $f'(x) > 0$ ⇒ $f(x)$ is M.I.

⇒ $f(x) > 0$ ⇒ $\sin x \tan x – x_2 > 0$

Hence $\sin x \tan x > x_2$

⇒ $\frac{\sin x \tan x}{x^2} > 1$ ⇒ $\lim_{x\to 0}\left[\frac{\sin x \tan x}{x^2}\right] = 1$

29. **Sol.** $f(x) = x_3 – 6x_2 + ax + b$ ⇒ $f'(x) = 3x_2 – 12x + a$

⇒ $f'(c) = 0$ ⇒ $f'\left(2+\frac{1}{\sqrt{3}}\right) = 0$

⇒ $3\left(2+\frac{1}{\sqrt{3}}\right)^2 – 12\left(2+\frac{1}{\sqrt{3}}\right) + a = 0$

⇒ $3\left(4+\frac{1}{3}+\frac{4}{\sqrt{3}}\right) – 12\left(2+\frac{1}{\sqrt{3}}\right) + a = 0$

$12 + 1 + 4\sqrt{3} – 24 –4\sqrt{3} + a = 0$ ⇒ $a = 11$

30. **Sol.** $f(x) = \frac{a_3x^4}{4}+\frac{\alpha_2x^3}{3}+\frac{a_1x^2}{2}+a_0x$

$f(0) = f(1) = 0$

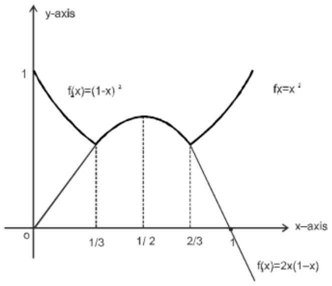
⇒ $f(x) = 0$ has one real root in $[0,1]$

(Rolle's theorem)

31. **Sol.** Draw the graph of $f_1(x) = x_2$, $f_2(x) = (1 – x)_2$ & $f_3 = 2x (1 – x)$

Now the bold part is the graph of f(x)

Hence



$$f(x) = \begin{cases} (1-x)^2 , & 0 \le x < \frac{1}{3} \\ 2x(1-x) , & \frac{1}{3} \le x \le \frac{2}{3} \\ x^2 , & \frac{2}{3} < x \le 1 \end{cases}$$

Clearly Rolle's theorem is applicable on $\left[\frac{1}{3}, \frac{2}{3}\right]$

where $f(x) = 2x(1 - x)$ ⇒ $f'(c) = 2 - 4c = 0$ ⇒ $c = \frac{1}{2}$

⇒ $a + b + c = \frac{1}{3} + \frac{2}{3} + \frac{1}{2}$ ⇒ $\frac{3}{2}$

32. **Sol.** Here f is a differentiable function then f is continuous function
So by L.M.V. theorem for any a ∈ (0, 4)

$f'(a) = \frac{f(4) - f(0)}{4 - 0}$ ...(1)

Again from mean value for any b ∈ (0, 4)

$f(b) = \frac{f(4) + f(0)}{2}$ ...(2)

Now multiplying (1) and (2), we get

$\frac{f^2(4) - f^2(0)}{8} = f'(a) \cdot f(b)$

⇒ $f_2(4) - f_2(0) = 8f'(a) \cdot f(b)$

Comprehension # (Q.33 to Q. 35)

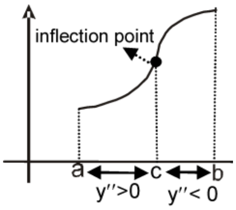
Concavity and convexity :

If f″(x) > 0 ∀ x ∈ (a, b), then the curve y = f(x) is concave up (or simply concave) in (a,b) and

If f″(x) < 0 ∀ x ∈ (a, b) then the curve y = f(x) is concave down (or simply convex) in (a, b).

Inflection point :

The point where concavity of the curve changes is known as point of inflection (at inflection point f″(x) is equal to 0 or undefined).

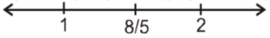


33. **Sol.** $y = (x - 1)_3 (x - 2)_2$

$\frac{dy}{dx} = 3(x - 1)_2 (x - 2)_2 + 2(x - 2)(x - 1)_3$

$= (x - 1)_2 (x - 2) [3(x - 2) + 2(x - 1)]$

$= (x - 1)_2 (x - 2)(5x - 8)$



$(x_2 - 2x + 1)(5x_2 - 18x + 16)$

$\frac{d^2y}{dx^2} = (2x - 2)(5x_2 - 18x + 16) + (10x - 18)(x_2 - 2x + 1) = 0$

$= 20x_3 - 42x_2 + 11x - 50 = 0$

$= 10x_3 - 42x_2 + 57x - 25 = 0$

$(x - 1)(10x_2 - 32x + 25) = 0$

x = 1 or $x = \frac{32 \pm \sqrt{24}}{20}$

no. of points of inflections = 3

34. **Sol.** $f(x) = x_4 + ax_3 + \frac{3x^2}{2} + 1$

$f'(x) = 4x_3 + 3ax_2 + 3x$
$f''(x) = 12x_2 + 6ax + 3$
Now f(x) will be concave upward along the entire
real line iff $f''(x) \geq 0 \ \forall \ x \in R$
$12x_2 + 6ax + 3 > 0 \quad \Rightarrow \quad D \leq 0$
$36a_2 - 144 \leq 0$
$a_2 - 4 \leq 0 \quad \Rightarrow \quad a \in [-2, 2]$

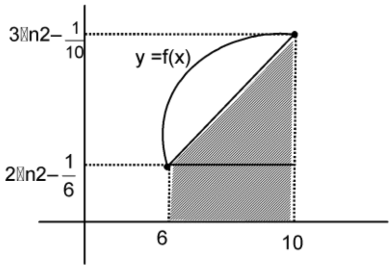
35. **Sol.** $f(x) = \ell n(x-2) - \frac{1}{x}$

$f'(x) = \frac{1}{x-2} + \frac{1}{x^2} = \frac{x^2+x-2}{x^2(x-2)} = \frac{x^2+2x-x-2}{x^2(x-2)}$

$= \frac{(x+2)\ \ (x-1)}{x^2(x-2)}$.

As $\ell n(x-2)$ is defined when x > 2
⇒ f(x) is M.I. for $x \in (2, \infty)$
⇒ $f_{-1}(x)$ is M.I. wherever defined

Also $f''(x) = \frac{-1}{(x-2)^2} - \frac{2}{x^3} < 0$
⇒ f(x) is always concave downward



area of shaded portion

$= \frac{1}{2}.4\left(\ell n2 - \frac{1}{10} + \frac{1}{6}\right) + 4\left(2\ell n2 - \frac{1}{6}\right)$

$= 10\ell n2 - \frac{8}{15}$

Required area is greater than $10\ell n2 - \frac{8}{15}$

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.
(1) Both the statements are true.
(2) Statement-I is true, but Statement-II is false.
(3) Statement-I is false, but Statement-II is true.
(4) Both the statements are false.

A-1. **Ans.** **(1)**
Sol. f(x) = 2 + cos x
f′(x) = – sin x

$f'(x) = 0$

$\Rightarrow \quad x = n\pi$, $n \in I$

Now, we can easily see that, the interval $[t, t + \pi]$ for all values of t, contain atleast one integral multiple of π

$\Rightarrow$ Statement - 1 is true

$f(t) = 2 + \cos t$

$f(t + 2\pi) = 2 + \cos (t + 2\pi) = 2 + \cos t = f(t)$

$\Rightarrow$ Statement - 2 is true

but we can see that Statement - 2 is not a correct explanation of Statement - 1

A-2. Ans. (2)

Sol. $f'(x) = \frac{x^{1/x}}{x^2} (1 - \ell nx)$

$f'(x) \leq 0$, when $x \geq e$

$\therefore$ f(x) is decreasing function, when $x \geq e$

$\pi > e \quad \Rightarrow \quad f(\pi) < f(e)$

$\pi_{1/\pi} < e_{1/e} \quad \Rightarrow \quad e_{\pi} > \pi_{e}$

$\therefore$ Statement-1 is True, Statement-2 is False

A-3. Ans. (1)

Sol. Area of ΔOPQ is minimum when (8,2) is midpoint of line. So, P(16, 0), Q(0,4)

$\Delta OPQ = \frac{1}{2} (16) (4) = 32.$

A-4. Ans. (1)

Sol. $f'(x) = 50x_{49} - 20x_{19}$

x = 0 is stationary point. Statement-2 is ture.

$f(0) = 0$

$f\left(\left(\frac{2}{5}\right)^{1/30}\right) = \left(\frac{2}{5}\right)^{5/3} - \left(\frac{2}{5}\right)^{2/3} < 0$

$f(1) = 0$

$\therefore$ Global maximum is 0. Statement-1 is true.

A-5. Ans. (3)

Sol. Let g(x) be the inverse function of f(x). Then f(g(x)) = x.

$\therefore \quad f'(g(x)).g'(x) = 1 \qquad$ i.e. $\quad g'(x) = \frac{1}{f'(g(x))}$

$\therefore \quad g''(x) = -\frac{1}{f''(g(x))}.g'(x)$

In statement-1 $f''(g(x)) > 0$ and $g'(x) > 0$

$\therefore \quad g''(x) < 0$ and so the concavity of $f_{-1}(x)$ is downwards

$\therefore$ statement-1 is false

In statement-2 $f''(g(x)) > 0$ and $g'(x) < 0$

$\therefore \quad g''(x) > 0$ and so the concavity of $f_{-1}(x)$ is upwards

$\therefore$ statement-2 is true

Section (B) : MATCH THE COLUMN

B-1. Sol. (1)

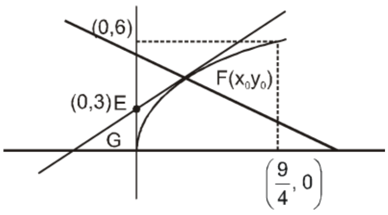
tangent at F $\quad yt = x + 4t_2$

a : x = 0y = 4t $\quad$ (0, 4t)

$(4t_2, 8t)$ satisfies the line

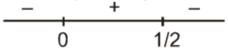
$8t = 4mt_2 + 3$

$4mt_2 - 8t + 3 = 0$



Area = $\frac{1}{2}\begin{vmatrix} 0 & 3 & 1 \\ 0 & 4t & 1 \\ 4t^2 & 8t & 1 \end{vmatrix}$

$= \frac{1}{2}$ $(4t_2 (3 - 4t))$
$= 2t_2 (3 - 4t)$
$A = 2[3t_2 - 4t_3]$
$\frac{dA}{dt} = 2[6t - 12t_2]$
$= 24\, t(1 - 2t)$



t = 1/2 maxima
G(0, 4t) ⇒ G(0, 2)
$y_1 = 2$
$(x_0, y_0) = (4t_2, 8t) = (1, 4)$
$y_0 = 4$
Area = $2\left(\frac{3}{4} - \frac{1}{2}\right) = 2\left(\frac{3-2}{4}\right) = \frac{1}{2}$

B-2. Hence condition in Rolle’s theorem and LMVT are satisfied.

(Q) $f(1_-) = -1, f(1) = 0, f(1_+) = 1$

f(x) is not continuous at x = 1, belonging to $\left[\frac{1}{2}, \frac{3}{2}\right]$
Hence, atleast one condition in LMVT and Rolle’s theorem is not satisfied

(R) $f'(x) = \frac{2}{5}$ $(x - 1)_{-3/5}, x \neq 1$
At x = 1, f(x) is not differentiable.
Hence at least one condition in LMVT and Rolle’s theorem is not satisfied.

(S) At x = 0

L.H.D. = $\lim\limits_{x\to 0^-} t \frac{x\left(\frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}\right) - 0}{x - 0} = \frac{0-1}{0+1} = -1$
R.H.D. = 1
At x = 0, f(x) is not differentiable
Hence at least one condition in LMVT and Rolle’s theorem is not satisfied.

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

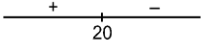
C-1. **Sol.** Solve by graph and we know tan x > x ∀ $\left(0, \frac{\pi}{2}\right)$.

C-2. **Sol.** $f'(x) = \begin{bmatrix} 6x + 12, & -1 \le x < 2 \\ -1, & 2 < x < 3 \end{bmatrix}$
Clearly f '(2) does not exist and f '(x) > 0 ∀ x ∈ [– 1, 2]
∴ increasing also f(x) is continuous at x = 2.

C-3. **Sol.** $h'(x) = f'(x)\,[1 - 2 f(x) + 3(f(x))_2]$
$= 3 f'(x) \left[\left(f(x) - \frac{1}{3}\right)^2 + \frac{2}{9}\right]$

So, (1) and (3) is true as $\left(f(x)\ -\ \frac{1}{3}\right)^2 + \frac{2}{9}$ > 0.

C-4. **Sol.** f '(x) = $\frac{-xe^{\frac{-x}{20}}}{20} + e^{-\frac{x}{20}}$ = $e^{-\frac{x}{20}}$. $\left[1 - \frac{x}{20}\right]$



∴ f(5) > f(4) and f(40) > f(60).

C-5. **Sol.** Since f(x) has local maxima at x = –1 and f′(x) has local minima at x = 0.

$f''(x) = \lambda x$

$f'(x) = \lambda\frac{x^2}{2} + c$ {f′(–1) = 0}

⇒ $\frac{\lambda}{2}$ + c = 0

⇒ λ = –2c ....(i)

again, Integrating both sides we get

f(x) = $\lambda\frac{x^3}{6}$ + cx + d ....(ii)

f(2) = $\lambda\left(\frac{8}{6}\right)$ + 2c + d = 18

and f(1) = $\frac{\lambda}{6}$ + c + d = – 1 ....(iii)

using (i),(ii) and (iii) we get

f(x) = $\frac{1}{4}$ ($19x_3$ – 57x + 34)

f′(x) = $\frac{1}{4}$ ($57x_2$ –57)

= $\frac{57}{4}$ (x –1)(x + 1), using number line rule

f(x) is increasing for [1, 2 $\sqrt{5}$]

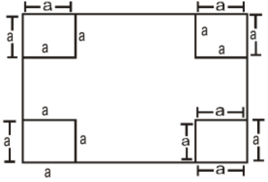
and f(x) has local maximum at x = –1 and

f(x) has local minimum at x = 1

also, f(0) = $\frac{34}{4}$

C-6. **Sol.** Obviously (2) and (3) are wrong.

C-7. **Sol.** **(1,3)**



Let ℓ = 8x, b = 15 x

∴ Volume = (8x – 2a) (15x – 2a) (a) = $4a_3 – 46a_2x + 120\ ax_2$

$\frac{dV}{da}$ = $6a_2 – 46ax + 60\ x_2$

$\left(\frac{dV}{da}\right)_{\text{at } x=5} = 0$

∴ x = 3 and $\frac{5}{6}$

$\frac{d^2V}{da^2} = 6a - 23x$

$\left(\frac{d^2V}{da^2}\right)_{\text{at } a=5 \,\&\, x=3} < 0,$

So, at x = 3 gives maxima

$\left(\frac{d^2V}{da^2}\right)_{\text{at } a=5 \,\&\, x=\frac{5}{6}} > 0$

So, at x = $\frac{5}{6}$ gives minima.

$\frac{dV}{da}$ = 0 when a = 5 given (∴ $4a_2$ = 100 given for maximum volume)

at a = 5

by $\frac{dV}{da}$ = 0

⇒ $6x_2 - 23x + 15 = 0$

x = 3 or 5/6

So by x = 3 (for max volume)

8x = 24, 15x = 45 **Ans. (A, C)**

C-8. **Ans.** **(2, 3)**

Sol. Let h(x) = f(x) – 3g(x)

$\left.\begin{matrix} h(-1) = 3 \\ h(0) = 3 \end{matrix}\right\}$ ⇒ h'(x) = 0 has atleast one root in (–1, 0) and atleast one root in (0, 2)

h(2) = 3

But since h"(x) = 0 has no root in (–1, 0) & (0, 2) therefore h'(x) = 0 has exactly 1 root in (–1, 0) & exactly 1 root in (0, 2)