

Fundamental of Mathematics - II

Exercise-1

A-1. Sol. $|x| + 2 = 3 \Rightarrow |x| = 1 \Rightarrow x = \pm 1$
so sum of solutions = $1 - 1 = 0$

A-2. Sol. $|x|^2 + 3|x| + 2 = 0$
 $(|x| + 2)(|x| + 1) = 0$
 $|x| = -2 \quad |x| = -1$
 $x = \text{Not possible} \quad x = \text{Not possible}$
so no solution is possible

A-3. Sol. $3|x - 3| = 7 \Rightarrow x - 3 = \pm \frac{7}{3} \Rightarrow x = \frac{16}{3}, \frac{2}{3}$
Product of roots = $\frac{32}{9}$

A-4. Sol. $|x|^2 - |x| + 4 = 2x^2 - 3|x| + 1 \Rightarrow |x|^2 - 2|x| - 3 = 0 \Rightarrow |x| - 3 = 0 \quad \text{or} \quad |x| + 1 = 0 \text{ not possible}$
 $\Rightarrow x = \pm 3$ so sum of solutions = 0

A-5. Sol. $|x|^2 + |x| - 6 = 0$
assume that $|x| = t$
 $t^2 + t - 6 = 0$
 $(t - 2)(t + 3) = 0$
 $t = 2, -3$
so $|x| = 2$ and $|x| = -3$ not possible
 $\Rightarrow x = \pm 2$
roots are real & sum = 0

A-6. Sol. $||x - 1| - 2| = 1$
 $|x - 1| - 2 = 1 \quad \text{or} \quad |x - 1| - 2 = -1$
 $|x - 1| = 3 \quad |x - 1| = 1 \Rightarrow x - 1 = \pm 3 \Rightarrow x - 1 = \pm 1 \Rightarrow x = 4, -2 \Rightarrow x = 2, 0$
 $\therefore x = -2, 0, 2, 4$

A-7. Sol. $|4x + 3| + |3x - 4| = 12$

case-1 : $x < -\frac{3}{4}$

$$\begin{array}{c} | \\ \hline -3 \quad 4 \\ -4 \quad 3 \end{array}$$

$$\begin{aligned} -4x - 3 - 3x + 4 &= 12 \\ -7x &= 11 \\ x &= -\frac{11}{7} \end{aligned}$$

case-2 : $-\frac{3}{4} \leq x \leq \frac{4}{3}$

$$\begin{aligned} 4x + 3 - 3x + 4 &= 12 \\ x &= 5, \text{ not acceptable.} \end{aligned}$$

case-3 : $x \geq \frac{4}{3}$

$$\begin{aligned} 4x + 3 + 3x - 4 &= 12 \\ 7x &= 13 \end{aligned}$$

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$$x = \frac{13}{7}$$

$$\therefore x = -\frac{11}{7}, \frac{13}{7}.$$

A-8. Sol. $|x - 3| + 2|x + 1| = 4$

case (i) : $x < -1$

$$-x + 3 - 2x - 2 = 4$$

$$-3x = 3$$

$$x = -1$$

case (ii) : $-1 \leq x < 3$

$$-x + 3 + 2x + 2 = 4$$

$$x = -1$$

case (iii) : $x \geq 3$

$$x - 3 + 2x + 2 = 4$$

$$3x = 5$$

$$x = \frac{5}{3}$$

not possible

$$\therefore x = -1$$

so number of real solutions is one.

A-9. Sol. $|x| - 2x + 5 = 0$

case (i) : $x < 0$

$$-x - 2x + 5 = 0$$

$$\Rightarrow x = \frac{5}{3} \quad x < 0$$

not possible ($\because x < 0$)

case (ii) : $x \geq 0$

$$x - 2x + 5 = 0 \quad \therefore x = 5$$

so number of solutions is one.

A-10. Sol. $|x - 2| = x_2$

so $x - 2 = \pm x_2$

$x_2 - x + 2 = 0$ no real solution

or $x_2 + x - 2 = 0 \Rightarrow (x + 2)(x - 1) = 0$

$x = 1, -2$

A-11. Sol. $|x + 2| = 2(3 - x)$

if $3 - x \geq 0 \Rightarrow x \leq 3$

then $x + 2 = 6 - 2x$ or $x + 2 = 2x - 6$

$3x = 4$ or $x = 8$ (not possible)

$$x = \frac{4}{3} \text{ (possible)}$$

so number of solutions = 1

A-12. Sol. $x|x| = 4$

case (i) : $x < 0 \therefore -x_2 = 4$

no solution.

case (ii) : $x \geq 0$

$$x_2 = 4$$

$$x = \pm 2 \quad \therefore x = 2$$

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so number of solutions = 1

A-13. Sol. $|x_2 - 4x + 3| \Rightarrow x_2 - 4x + 3 \quad (x-1)(x-3) \geq 0$
 $\quad \quad \quad -(x_2 - 4x + 3) \quad (x-1)(x-3) < 0$

$$y = x_2 - 4x + 3 + x = 7$$

$$x_2 - 4x + 3 + 7 - x = 0$$

A-14. Sol. $f(x) = |x-1| + |x-2| + |x-3|$

$$= \begin{cases} -x+1-x+2-x+3=6-3x, & x \leq 1 \\ x-1-x+2-x+3=4-x & 1 < x \leq 2 \\ x-1+x-2-x+3=x & 2 < x \leq 3 \\ x-1+x-2+x-3=3x-6 & x > 3 \end{cases}$$

min $f(x) = 2$.

A-15. Sol. $||x-1|-2| = |x-3|$
 by using property
 $||a|-|b|| = |a-b| \Rightarrow a \cdot b \geq 0$
 $2(x-1) \geq 0 \Rightarrow x \geq 1 \Rightarrow x \in [1, \infty)$

Section (B) : Modulus Inequalities

B-1. Sol. $|x-3| \geq 2$
 $x-3 \geq 2$ or $x-3 \leq -2$
 $x \geq 5$ or $x \leq 1$

B-2. Sol. $|x-2|-3=0$
 $|x-2|=3$
 $x=5$ or $x=-1$
 sum of integral solutions = 4

B-3. Sol. $|x+3| > |2x-1| \Rightarrow x_2 + 9 + 6x > 4x_2 + 1 - 4x \Rightarrow 3x_2 - 10x - 8 < 0$
 $\Rightarrow \left(x + \frac{2}{3}\right)(x-4) < 0 \Rightarrow -\frac{2}{3} < x < 4$
 So integral solutions are $x = 0, 1, 2, 3$

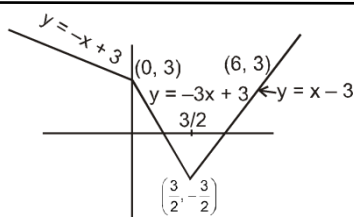
B-4. Sol. $-1 \leq |x-1| - 1 \leq 1 \Rightarrow 0 \leq |x-1| \leq 2$
 $\therefore 0 \leq |x-1| \Rightarrow x \in \mathbb{R} \dots(1)$
 and $|x-1| \leq 2 \Rightarrow -2 \leq x-1 \leq 2 \Rightarrow -1 \leq x \leq 3 \dots(2)$
 by (1) $\cap$ (2) $\Rightarrow x \in [-1, 3]$.

B-5. Sol. $|3x-9|+2 > 2$ or $|3x-9|+2 < -2$
 $|3x-9| > 0$ or $x \in \varnothing$
 $x \in \mathbb{R} - \{3\}$

B-6. Sol. $1 + \frac{3}{x} > 2$ or $1 + \frac{3}{x} < -2$
 $\Rightarrow \frac{3-x}{x} > 0$ or $\frac{x+1}{x} < 0 \Rightarrow 0 < x < 3$ or $-1 < x < 0$
 $\Rightarrow x \in (-1, 0) \cup (0, 3)$

B-7. Sol.

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for $y \leq 3$ $x \in [0, 6]$

Integral solutions $x = 0, 1, 2, 3, 4, 5, 6$

Number of integral solutions = 7

Comprehension # 1 (B-8 to B-10)

B-10. Sol. Case I when $1 + \frac{3}{x} \geq 0 \Rightarrow \frac{x+3}{x} \geq 0$
 then $x(x+3) \geq 0$ ($x \neq 0$)
 $\Rightarrow x \in (-\infty, -3] \cup (0, \infty)$ (i)

Now for above interval

$$\left| 1 + \frac{3}{x} \right| > 2 \Rightarrow 1 + \frac{3}{x} > 2$$

$$\Rightarrow 1 - \frac{3}{x} < 0 \Rightarrow x(x-3) < 0 \quad (x \neq 0) \Rightarrow x \in (0, 3) \quad \text{..... (ii)}$$

from (i) and (ii) $x \in (0, 3)$

Case II when $1 + \frac{3}{x} < 0$
 then $x \in (-3, 0)$ (iii)
 Now $\left| 1 + \frac{3}{x} \right| > 2 \Rightarrow -\left(1 + \frac{3}{x} \right) > 2$

$$\Rightarrow 3 + \frac{3}{x} < 0 \Rightarrow x(x+1) < 0 \quad (\because x \neq 0)$$

$$\Rightarrow x \in (-1, 0) \quad \text{..... (iv)}$$

from (iii) and (iv) $x \in (-1, 0)$

$\therefore$ Total solution set for

$$\left| 1 + \frac{3}{x} \right| > 2 \text{ is}$$

$$x \in (-1, 0) \cup (0, 3)$$

it with $(a, 0) \cup (0, b)$ we get $a = -1, b = 3$

8. $\therefore |a+b| = |-1+3| = 2$

9. $\therefore x^3 - kx^2 + x + 2$ is divisible by $x - a$ i.e. $(x+1)$

so $f(-1) = 0$

$$-1 - k - 1 + 2 = 0$$

$$\Rightarrow k = 0$$

10. $(x+1)^2 < (7x-3)$

$$\Rightarrow x^2 + 2x + 1 < (7x-3) \Rightarrow x^2 - 5x + 4 < 0 \Rightarrow (x-1)(x-4) < 0$$

$$\Rightarrow x \in (1, 4)$$

$$\therefore c = 1, d = 4$$

$$\therefore a + b + c + d = -1 + 3 + 1 + 4 = 7$$

B-11. Sol. $y = \begin{cases} -2x+1, & x \in (-\infty, -1) \\ 3 & x \in [-1, 2) \\ 2x-1 & x \in [2, \infty) \end{cases} \quad y \geq 3 \quad \forall x \in \mathbb{R}$

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Section (C) : Irrational inequalities

C-1. Sol. $\sqrt{6-x} > x-1$
Domain $6-x \geq 0 \Rightarrow x \leq 6$

Case I : $x \geq 1$

+ve $\geq$ +ve

$$6-x \geq x^2 - 2x + 1$$

$$x^2 - x - 5 \leq 0$$

$$\left(x - \frac{1}{2}\right)^2 - \frac{21}{4} \leq 0 \Rightarrow x \in \left[\frac{1-\sqrt{21}}{2}, \frac{1+\sqrt{21}}{2}\right]$$

$$\text{so } x \in \left[1, \frac{1+\sqrt{21}}{2}\right]$$

Case 2 : $x < 1$

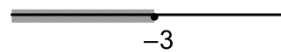
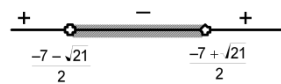
+ve $>$ -ve

always true

$$x \in (-\infty, 1)$$

$$\therefore x \in \left(-\infty, \frac{1+\sqrt{21}}{2}\right]$$

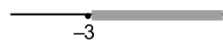
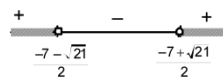
C-2. Sol. Case-I $x+3 < 0$ and $2-x \geq 0$ and $(x+3)^2 < 2-x \Rightarrow x^2 + 7x + 7 < 0$



$$x \in \left(\frac{-7-\sqrt{21}}{2} - 3\right)$$

Case - II $x+3 \geq 0$ and $2-x \geq 0$

$$(x+3)^2 > 2-x \Rightarrow x^2 + 7x + 7 > 0$$



$$x \in \left(\frac{-7+\sqrt{21}}{2}, 2\right]$$

$$\text{Finally } x \in \left(\frac{-7-\sqrt{21}}{2} - 3\right) \cup \left(\frac{-7+\sqrt{21}}{2}, 2\right]$$

C-3. Sol. $\sqrt{x^2 - x - 6} < 2x - 3$

$$\text{Domain } x^2 - x - 6 \geq 0 \Rightarrow (x-3)(x+2) \geq 0$$

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$$\Rightarrow x \in (-\infty, -2] \cup [3, \infty)$$

$$\text{Case : } x < \frac{3}{2}$$

$$(+ve) < (-ve)$$

Never true

$$\therefore x = \varnothing$$

$$\text{Case : } x \geq \frac{3}{2}$$

$$(+ve) < (+ve)$$

squaring

$$x^2 - x - 6 < 4x^2 + 9 - 12x$$

$$\Rightarrow 3x^2 - 11x + 15 > 0 \quad \Rightarrow x^2 - \frac{11}{3}x + 5 > 0 \quad \Rightarrow D = \frac{121}{9} - 20 < 0$$

$$\therefore x \in \mathbb{R} \Rightarrow x \in \left[\frac{3}{2}, \infty \right)$$

Now take intersection with domain

$$\therefore x \in [3, \infty)$$

C-4. Sol.

Domain

$$x^2 + 4x - 5 \geq 0 ;$$

$$x^2 + 5x - x - 5 \geq 0$$

$$(x - 1)(x + 5) \geq 0$$

$$x \in (-\infty, -5] \cup [1, \infty)$$

case-I $x - 3 < 0$

$$x < 3$$

$$-ve < +ve$$

always true

$$\therefore x \in (-\infty, -5] \cup [1, 3) \quad \dots(i)$$

case-II $x - 3 \geq 0$

$$x \geq 3$$

$$+ve < +ve$$

$$(x - 3)^2 < (x^2 + 4x - 5)$$

$$10x - 14 > 0$$

$$x > 7/5$$

$$\therefore x \in [3, \infty) \quad \dots(ii)$$

by (i) $\cup$ (ii)

$$x \in (-\infty, -5] \cup [1, \infty)$$

C-5. Sol.

Case-I $4 - x < 0$ and $2x - x^2 \geq 0 \Rightarrow x \in [0, 2]$ and $x > 4 \Rightarrow x \in \varnothing$

Case-II $4 - x \geq 0$ and $2x - x^2 \geq 0$ and $16 + x^2 - 8x < 2x - x^2$

$$\Rightarrow x \in [0, 2] \text{ and } x > 4 \text{ and } x^2 - 5x + 8 < 0 \Rightarrow x \in \varnothing$$

Section (D) : Transformation of curves

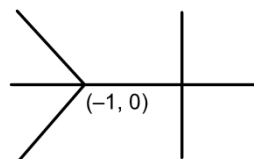
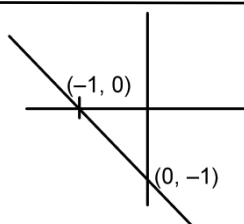
D-1. Sol.

$$|y| = -x - 1$$

$$y = -x - 1$$

$$|y| = -x - 1$$

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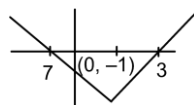
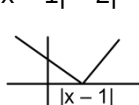
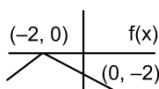


D-4.

Sol.

$$f_1(x) = -|x + 2|$$

$$f_2(x) = ||x - 1| - 2|$$



$$||x - 1| - 2|$$

$$f_3(x) = |x + 2| + |x - 3|$$

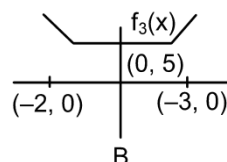
$$x \geq 3 \quad 2x - 1$$

$$f_3(x) = |x + 2| + |x - 3|$$

$$x \geq 3 \quad 2x - 1$$

$$-2 \leq x < 3$$

$$-2x + 1$$

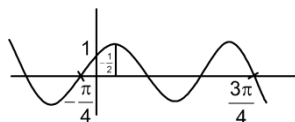
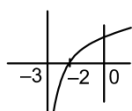


D-5.

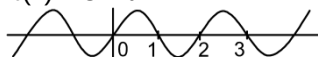
Sol.

$$f_1(x) = \ln(x + 3)$$

$$f_2(x) = \cos\left(x - \frac{\pi}{4}\right)$$



$$f_3(x) = \sin \pi x$$



Section (E) : Greatest Integer $[\cdot]$ Fractional part $\{\cdot\}$ and signum function

E-1.

Sol.

$$[e] - [-\pi] = [2.71] - [-3.14] = 2 + 4 = 6$$

E-2.

Sol.

$$-5 \leq [x + 1] < 2$$

$$-5 \leq [x + 1] \leq 1 \quad \Rightarrow \quad -5 \leq x + 1 < 2$$

$$-6 \leq x < 1 \quad \Rightarrow \quad x \in [-6, 1)$$

E-3.

Sol.

$$[x]^2 + 5[x] - 6 < 0$$

$$\Rightarrow ([x] + 6)([x] - 1) < 0$$

$$\Rightarrow -6 < [x] < 1$$

$$\Rightarrow -5 \leq [x] \leq 0$$

$$\Rightarrow -5 \leq x < 1 \quad \Rightarrow \quad x \in [-5, 1)$$

E-4.

Sol.

$$\{x\} = 0 \quad \text{or} \quad \{x\} = -1$$

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$x \in I$ (rejected)

E-5. Sol. $2\{x\}_2 - 5\{x\} + 2 = 0$
 $2f_2 - 4f - f + 2 = 0$
 $2f(f-2) - 1(f-2) = 0$
 $f = \frac{1}{2}, 2, f \neq 2 (0 \leq f < 1) \Rightarrow f = \frac{1}{2} \Rightarrow \infty \text{ solution}$

E-6. Sol. $\text{sgn}(x_2) = |x-2|$
 $1 = x-2 \quad x \geq 2$
 $x = 3$
 $1 = -(x-2) \quad x < 2$
 $-1 = x-2$
 $x = 1$ two solution

E-7. Sol. $\text{sgn } x = |1-x|$
 Case-I $0 > x$
 $-1 = 1-x \Rightarrow x = 2$ no solution
 Case-II $x = 0$
 $0 = 1$ not possible
 Case-III $0 < x \leq 1$
 $1 = 1-x \Rightarrow x = 0$ no solution
 Case-IV $x > 1$
 $1 = x-1 \Rightarrow x = 2$ Ans.

Exercise-2

Marked Questions may have for Revision Questions.

PART - I : OBJECTIVE QUESTIONS

1. **Sol.** Since $(x_2 + x - 2) - (x_2 - 2x - 8) = 3x + 6 = 3(x+2)$
 $\therefore (x_2 - 2x - 8)(x_2 + x - 2) \leq 0$
 i.e. $(x-4)(x+2)(x+2)(x-1) \leq 0$

$\begin{array}{ccccccc} & + & & + & & - & & + \\ \hline & -2E & & 1 & & 4 & & \end{array}$

 $\therefore$ Solution set is $[1,4] \cup \{-2\}$
2. **Sol.** $|x_3 - 9x_2 + 26x - 24| = |x-2| |x-3| |x-4|$
 When $x \in I$; above 3 are consecutive positive integers, hence multiplication can never be a prime number
3. **Sol.** $|a| + |b| = |a+b| \Rightarrow a.b. \geq 0$
 $x(x+5)(x)(1-x) \geq 0$
 $x_2(x+5)(x-1) \leq 0$

$\begin{array}{ccccccc} & + & & - & & - & & + \\ \hline & -5 & & 0 \in & & 1 & & \end{array}$

 $x \in [-5,1] \Rightarrow \{-5, -4, -3, -2, -1, 0, 1\}$
4. **Sol.**

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$$f(x) = \begin{cases} x-3 & -\infty < x < -1 \\ 3x-1 & -1 \leq x < 1 \\ -x+3 & 1 \leq x < \infty \end{cases}$$

$$\text{max at } x = 1 \quad f(1) = 2 - 2|1 - 1| = 2$$

5. **Sol.** $||x| - 2| = 2$

$$|x| - 2 = \pm 2$$

$$|x| = 4, 0$$

$$x = \pm 4, 0$$

$$\therefore \text{order pairs are } (4,3) (-4,3) (0,1)$$

6. **Sol.** $-1 \leq \frac{3x}{x^2 - 4} \leq 1$

$$\frac{3x + x^2 - 4}{x^2 - 4} \geq 0 \text{ and } \frac{3x - x^2 + 4}{x^2 - 4} \leq 0$$

$$\Rightarrow \frac{(x+4)(x-1)}{(x-2)(x+2)} \geq 0 \text{ and } \frac{(x-4)(x+1)}{(x-2)(x+2)} \geq 0$$

$$x \in (-\infty, -4] \cup (-2, 1] \cup (2, \infty) \text{ and } x \in (-\infty, -2) \cup [-1, 2) \cup [4, \infty)$$

Taking intersection we get

$$x \in (-\infty, -4] \cup [-1, 1] \cup [4, \infty)$$

7. **Sol.** $\therefore |a| + |b| \geq |a - b|$

$$|a| + |-b| \geq |a + (-b)|$$

$$\text{but given inequality is } |2x - 3| + |x + 5| \leq |x - 8|$$

$$|2x - 3| + |x + 5| \leq |(2x - 3) - (x + 8)| \Rightarrow |a| + |-b| = |a + (-b)|$$

$$\Rightarrow a(-b) \geq 0 \text{ i.e. } ab \leq 0$$

$$\therefore \text{solution set is given by } (2x - 3)(x + 5) \leq 0$$

$$\text{i.e. } -5 \leq x \leq 3/2$$

so positive integer solution is $x = 1$.

8. **Sol.** **case-I:** $x \geq -3$

$$\Rightarrow \frac{2x + 3 - x - 2}{x + 2} > 0 \Rightarrow \frac{x + 1}{x + 2} > 0 \Rightarrow x \in (-\infty, -2) \cup (-1, \infty)$$

$$\text{But } x \geq -3 \Rightarrow x \in [-3, -2) \cup (-1, \infty)$$

case-II: $x < -3$

$$\Rightarrow \frac{-3 - x - 2}{x + 2} > 0 \Rightarrow \frac{x + 5}{x + 2} < 0 \Rightarrow -5 < x < -2$$

$$\text{But } x < -3 \Rightarrow x \in (-5, -3) \therefore x \in (-5, -2) \cup (-1, \infty).$$

9. **Sol.** $(|x| - 3)(|x| - 5) < 0$

$$\text{Let } |x| = t \Rightarrow (t - 3)(t - 5) < 0$$

$$\Rightarrow 3 < t < 5 \Rightarrow 3 < |x| < 5$$

$$\Rightarrow x \in (-5, -3) \cup (3, 5)$$

10. **Sol.** $(|x - 1| - 3)(|x + 2| - 5) < 0$

Case-I $x \leq -2$

$$(x + 2)(x + 7) < 0 \Rightarrow x \in (-7, -2) \dots\dots(i)$$

Case-II $-2 < x \leq 1$

$$(x - 3)(-x - 2) < 0 \Rightarrow (x - 3)(x + 2) > 0$$

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$$\Rightarrow x \in (-\infty, -2) \cup (3, \infty)$$

No solution

Case-III II : $x > 1$

$$(x-4)(x-3) < 0$$

$$x \in (3, 4) \quad \dots\dots\dots(ii)$$

$$x \in (i) \cup (ii) \Rightarrow x \in (-7, -2) \cup (3, 4)$$

11. Sol. $x^2 - 16 \geq 0$
 $\therefore (x-4)(x+4) \geq 0 \quad \therefore x \in (-\infty, -4] \cup [4, \infty) \quad \dots\dots\dots(1)$

Now $\frac{(x^2+2)(\sqrt{x^2-16})}{(x^4+2)(x-3)(x+3)} \leq 0$

$x \in (-3, 3) \quad \dots\dots\dots(2)$
 By (1) and (2) $x \in \{-4, 4\}$
 (1) o (2) is $x \in \{-4, 4\}$

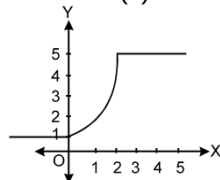
12. $\Rightarrow x \geq 0$ and $x+5 > 1+x+2\sqrt{x}$
 $\Rightarrow x \geq 0$ and $x < 4 \Rightarrow x \in [0, 4) \Rightarrow a+b=4$

13. Sol. Let $\sqrt{x-1}=t \quad \sqrt{2t} > 1-t$
 Case-I $t \geq 0$ and $1-t < 0 \Rightarrow (1, \infty)$
 Case-II $t \geq 0$ and $t \leq 1 \Rightarrow 2t > 1+t-2t \Rightarrow t^2-4t+1 < 0$

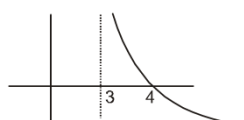


$(2-\sqrt{3}, 1]$
 so finally $x \in (2-\sqrt{3}, \infty)$

14. Sol. $f_1(x) = \begin{cases} 1 & , \quad x \leq 0 \\ x^2+1 & , \quad 0 < x < 2 \\ 5 & , \quad x \geq 2 \end{cases}$

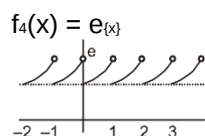
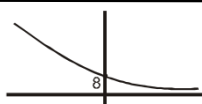


$f_2(x) = \log_{1/2}(x-3)$



$f_3(x) = 2_{3-x}$

Fundamental of Mathematics - II



16. Sol. $\sum_{n=1}^{49} f(n) = 0$; $\sum_{n=50}^{149} f(n) = 100$;

$f(150) + f(151) = 4$; $\sum_{n=1}^{151} f(n) = 104$

17. Sol. $[x]_2 = -[x]$

$x = I + f$

$I_2 = -I$

$I = 0$ or $I = -1$

Case-I $I = 0$

$x = I + f = f$

$x \in [0, 1)$ (i)

Case-II $I = -1$

$-1 \leq x < 0, x \in [-1, 0)$ (ii)

by (i) and (ii)

$x \in [-1, 1)$

18. Sol. (i) $-x^2 + 5x - 6 \geq 0$

(ii) $2\{x\} < 1 \Rightarrow x \in \left[2, \frac{5}{2}\right) \cup \{3\}$

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

A-1. Ans. (1)

Sol. If a & b are of same sign then $|a + b| = |a| + |b|$ $ab \geq 0$

$\therefore (x - 2)(x - 7) \geq 0 \Rightarrow x \leq 2$ or $x \geq 7$

$\therefore 2x - 9 = (x - 2) + (x - 7)$

A-2 Ans. (3)

Sol. $x > 3 \Rightarrow \frac{x-3}{x-3} + 5 > x \Rightarrow 3 < x < 6$

$x < 3 \Rightarrow \frac{-x+3}{x-3} + 5 > x \Rightarrow x < 4$

$\therefore x \in (-\infty, 3) \cup (3, 6)$

Section (B) : MATCH THE COLUMN

1. Ans+Sol. (A) $\rightarrow$ (r), (B) $\rightarrow$ (p), (C) $\rightarrow$ (q), (D) $\rightarrow$ (s)

Fundamental of Mathematics - II

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

1. **Sol.** $|x - 2|^{10x^2 - 1} = |x - 2|^{3x} \Rightarrow x - 2 = \pm 1 \text{ or } 10x^2 - 1 - 3x = 0 \Rightarrow x = 1, 3, \frac{1}{2}, -\frac{1}{5}$

2. **Sol.** $y = x + 2|x|$
and $y = 4 + x - |x|$
so, $x + 2|x| = 4 + x - |x|$
 $3|x| = 4$
 $x = \pm \frac{4}{3}$

(i) When $x = \frac{4}{3}$ then $y = 4$ (ii) When $x = -\frac{4}{3}$ then $y = \frac{4}{3}$

3. **Sol.** $-\infty < x < -2 \Rightarrow f(x) = 3$
 $-2 \leq x < 1 \Rightarrow f(x) = |2x + 1|$
 $1 \leq x < \infty \Rightarrow f(x) = 3$

$\left(-\frac{1}{2}, 0\right)$