Exercise-1

Marked Questions may have for Revision Questions.

OBJECTIVE QUESTIONS

Section (A) : Expansion of $(x + a)^n$, general term, middle term and coefficient of x^k

A-1. Sol. ${}^{15}C_{3r} = {}^{15}C_{r+3}$ $\Rightarrow 3r = r + 3$ or $3r + r + 3 = 15$

$\Rightarrow r = 3/2$ or $r = 3$

But $3r,\ r + 3 \in W$ $\Rightarrow r = 3$

A-2. Sol. $\dfrac{{}^{2n}C_3}{{}^{n}C_2} = \dfrac{44}{3}$ $\Rightarrow$ $3.\ {}^{2n}C_3 = 44.\ {}^{2}C_2$ $\Rightarrow$ $3.\dfrac{2n(2n-1)(2n-2)}{3!} = 44.\dfrac{n(n-1)}{2!}$

$\Rightarrow$ $4n(n-1)(2n-1) - 44\,n(n-1) = 0$ $\Rightarrow$ $4n(n-1)(2n-1-11) = 0$

$\Rightarrow$ $n = 6$

A-3. Sol. ${}^{15}C_3 + {}^{15}C_{13} = {}^{15}C_3 + {}^{15}C_2 = {}^{16}C_3$

A-4. Sol. $T_7 = T_{6+1} = {}^{17}C_6 (3x)^{11} (4y)^6$

A-5. Sol. 6th term from end = $(11 - 6 + 2)^{th}$ = 7^{th} term from begining

$T_7 = {}^{11}C_6\,(2a)^5 \left(\dfrac{b}{2}\right)^6 = \dfrac{1}{2}\ {}^{11}C_5\, a^5 b^6$

A-6. Sol. $T_{r+1} = {}^{13}C_r\,(3x)^{13-x} \left(\dfrac{1}{x}\right)^r = {}^{13}C_r\,3^{13-r}\,x^{-2r}$

Put r = 3

$\Rightarrow$ coefficient of x^7 is = ${}^{13}C_3\,(3)^{10}$

A-7. Sol. $T_{r+1} = {}^{12}C_r\,2^{12-r}(3x)^r$

r = 5

$T_6 = {}^{12}C_5\,2^7\,3^5\,.\,x^5$

$\therefore$ coefficient of $x^5 = {}^{12}C_5\,.\,2^7\,.\,3^5$

A-8. Sol. Let its come in T_{r+1}

then $T_{r+1} = {}^{5}C_r\,(x^2)^{5-r}\,.\,x^{-r}$

$\Rightarrow$ $10 - 3r = 1$

$3r = 9$

$r = 3$

Hence $T_{3+1} = T_4$

A-9. Sol. $T_{r+1} = {}^{10}C_r \left(\sqrt{\dfrac{x}{3}}\right)^{10-r} \left(\dfrac{3}{2x^2}\right)^r$

$\dfrac{10-r}{2} - 2r = 0$

$10 - r - 4r = 0$

$r = 2$

hence term = ${}^{10}C_2 \times \dfrac{1}{3^4} \times \dfrac{3^2}{2^2} = \dfrac{5}{4}$

A-10. Sol. ${}^{2m+1}C_m \left(\dfrac{x}{y}\right)^{m+1} \left(\dfrac{y}{x}\right)^m = {}^{2m+1}C_m \left(\dfrac{x}{y}\right)$

Dependent upon the ratio $\frac{x}{y}$ and m.

A-11. Sol. $(x + a)^{100} + (x - a)^{100}$

$= 2\left({}^{100}C_0\ x^{100} + {}^{100}C_2\ x^{98}a^2 + ... + {}^{100}C_{100}\ a^{100}\right)$

Number of terms = 51 terms

A-12. Sol. $T_6 = {}^8C_5\left(\frac{1}{x^{8/3}}\right)^3 (x^2 \log_{10} x)^5 = 5600 \Rightarrow \frac{1}{x^8}\ x^{10} (\log_{10} x)^5 = 100 \Rightarrow x = 10$

A-13. Sol. $T_{11} = {}^{15}C_{10}\ (3)^5 \left(-\sqrt{\frac{17}{4} + 3\sqrt{2}}\right)^{10}$

$= {}^{15}C_{10}(3)^5 \left(\frac{17}{4} + 3\sqrt{2}\right)^5$ = a positive irrational number

A-14. Sol. $T_2 = {}^nC_1\ (a^{1/13})^{n-1}\ (a^{3/2}) = 14a^{5/2}$

$\Rightarrow n = 14$

$\therefore \frac{{}^nC_3}{{}^nC_2} = 4$

A-15. Sol. ${}^{6561}C_r (7)^{\frac{6561-r}{3}} (11^{1/9})^r$

Here r should be multiple of 9

r = 0, 9,18...............6561

Number of terms = 730

A-16. Sol. $(1 - 3x + 3x^2 - x^3)^6 = (1 - x)^{18}$

A-17. Sol. middle term = T_5

$T_5 = T_{4+1} = {}^8C_4 \cdot k^4 = 1120$

$\Rightarrow$ k = 2

A-18. Sol. Obviously it comes in $T_{2+1} = T_3$

and ${}^6C_2 = {}^6C_4$

So also

comes in $T_{4+1} = T_5$

A-19. Sol. $(1 - 2x^3 + 3x^5)\left(1 + \frac{1}{x}\right)^8$

Co-efficient of x = $-2.\ {}^8C_2 + 3.{}^8C_4 = 154$

A-20. Sol. $(x^{1/3} - x^{-1/2})^{15}$

$T_{r+1} = {}^{15}C_r\ x^{\left(\frac{15-r}{3}\right)}\ (-x^{-1/2})^r$

For constant term $\frac{15-r}{3} - \frac{r}{2} = 0 \Rightarrow r = 6$

Co-efficient of $x^0 = {}^{15}C_6 = 5 \times 1001 \Rightarrow m = 1001$

A-21. Sol. $\left(x-\frac{1}{x}\right)\left(x^2-\frac{1}{x^2}\right)^3 = \left(x-\frac{1}{x}\right)$ $({}_3C_0\, x^6 - {}_3C_1\, x^2 + {}_3C_2\, x^{-2} - {}_3C_3\, x^{-6})$

There is no term independent of x

A-22. Sol. $P = {}_{2n}C_n$ and $Q = {}_{2n-1}C_n \Rightarrow \frac{P}{Q} = 2$

$\left(1+\frac{P}{Q}\right)^5 = (1 + 2)^5 = 3^5$

A-23. Sol. $(1 + by)^n = 1 + 8y + 24y^2 + ...$

$(1 + by)^n = 1 + n\,.\,by + \frac{n(n-1)}{2}\cdot b^2y^2 + ...$

by comparison

$bn = 8$ .... (1)

$\frac{n(n-1)}{2}\; b^2 = 24$ .... (2)

$4n - 4 = 3n$

$n = 4$

and $b = 2$

A-24. Sol. $\frac{(18+7)^3}{(3+2)^6} = \frac{25^3}{5^6} = 1$

A-25. Sol. $S = \sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} 2^m$

$S = {}_{100}C_0\, (x - 3)^{100} + {}_{100}C_1\, (x - 3)^{99}\, .\, 2 + ... + {}_{100}C_{100}\, .\, 2^{100}$

$S = (2 + (x - 3))^{100} = (x - 1)^{100}$

Co-efficient of $x^{52} = {}_{100}C_{52} = {}_{100}C_{48}$

A-26. Sol. $(1 + x)^{21}\, [1 + (1 + x) + ... (1 + x)^9] = (1 + x)^{21} = \left[\frac{(1+x)^{10}-1}{x}\right] = \frac{(1+x)^{31}-(1+x)^{21}}{x}$

Coefficient of $x^5 = {}_{31}C_6 - {}_{21}C_6$

A-27. Sol. $(1 + x)^m \left(1+\frac{1}{x}\right)^n = \frac{(1+x)^{m+n}}{x^n}$

Independent term of x = ${}_{m+n}C_n$

A-28. Sol. $(1 + x)\, (1 + x + x^2)......\, (1 + x + ... + x^{100})$

Highest exponent of x = 1 + 2 + ...+ 100

= 5050

Section (B) : Remainder and Divisibility problems

B-1. Sol. $17^{10} = (18 - 1)^{10} = 18\lambda + 1$

B-2. Sol. $7^{98} = (50 - 1)^{49} = {}_{49}C_0(50)^{49} - {}_{49}C_1(50)^{48} + ...+ {}_{49}C_{48} \times 50 - {}_{49}C_{49}$

Remainder = 5 – 1 = 4

B-3. **Sol.** $2^{2003} = 8.(16)^{500}$
$= 8\ (17-1)^{500}$
$\therefore$ Remainder = 8

B-4. **Sol.** $\left\{\frac{3^{1001}}{82}\right\} = \left\{\frac{3.(82-1)^{250}}{82}\right\} = \left\{\frac{3.[{}^{250}C_0(82)^{250} + {}^{250}C_1(82)^{249}(-1) + ... + {}^{250}C_{250}]}{82}\right\} = \frac{3}{82}$

B-5. **Sol.** $3^{50} = 9^{25} = (10-1)^{25}$
$(10-1)^{25} = {}^{25}C_0\ 10^{25} - {}^{25}C_1\ 10^{24} + ... - {}^{25}C_{23}\ 10^2 + {}^{25}C_{24}\ 10 - 1 = 1000\lambda + 249$
$\therefore$ last three digits = 249

B-6. **Sol.** $3^{400} = (10-1)^{200}$
${}^{200}C_0\,(10)^{200} + ... + {}^{200}C_{199}\,(10)\,(-1) + {}^{200}C_{200}$
Last two digits = 01

B-7. **Sol.** Last two digits in 10! are 00 and third digit = 8

B-8. **sol.** $10^n + 4 = (9+1)^n + 5 = 9\lambda + 1 + 5 = 9\lambda + 6$

Section (C) : Sum of series, Product and division of binomial coefficients, Reverse Expansion

C-1. **Sol.** $(1+x)^n = C_0 + C_1 x + C_2 x^2 + ... + C_n\ .\ x^n$
Put $x = 1$ both sides
$2^n = C_0 + C_1 + C_2 + ... + C_n$

C-2. **Sol.** $(1+x)^n = C_0 + C_1\ .\ x + C_2\ x^2 + ...C_n\ .\ x^n$
Now $C_0 + 3C_1 + 5C_2 + ... + (2n+1)\ C_n$

$= \sum_{r=0}^{n} (2r+1)\ \ C_r$

$= 2n\sum_{r=0}^{n} {}^{n-1}C_{r-1} + \sum_{r=0}^{n} {}^{n}C_r$

$= 2n\ .\ 2^{n-1} + (2^n)$
$= 2^n\ (n+1)$

C-3. **Sol.** $\sum_{r=1}^{10} r.\ \frac{{}^nC_r}{{}^nC_{r-1}} = \sum_{r=1}^{10} (n-r+1)$

$= (n+1) \times 10 - \frac{10 \times 11}{2}$
$= 10n - 45$

C-4. **Sol.** $(1+x)^n = \sum_{r=0}^{n} a_r\ x^r = a_0 + a_1 x + ... + a_n x^n$

$b_r = 1 + \frac{a_r}{a_{r-1}} = 1 + \frac{n-r+1}{r} = \frac{n+1}{r}$

$\prod_{n=1}^{n} b_r = b_1 b_2 .... b_n = \frac{(n+1)^n}{1,\ 2,3......n} = \frac{(101)^{100}}{100!}$

$\Rightarrow n = 100$

C-5. **Sol.** $\sum_{r=0}^{n-1} \frac{^nC_r}{^nC_r + ^nC_{r+1}} = \sum_{r=0}^{n-1} \frac{r+1}{n+1}$

$= \frac{1}{n+1}$ [1 + 2 + ...+n] = $\frac{1}{n+1} \times \frac{n(n+1)}{2} = \frac{n}{2}$

C-6. **Sol.** $\frac{^{11}C_0}{1} + \frac{^{11}C_1}{2} + \frac{^{11}C_2}{3} + ... + \frac{^{11}C_{10}}{11}$

$= \frac{1}{12}\left[\frac{12}{1} \cdot {^{11}C_0} + \frac{12}{2} \cdot {^{11}C_1} + \frac{12}{3} \cdot {^{11}C_2} + ... + \frac{12}{11} \cdot {^{11}C_{10}}\right]$

$= \frac{1}{12}\left[{^{12}C_1} + {^{12}C_2} + {^{12}C_3} + ... + {^{12}C_{11}}\right]$

$= \frac{1}{12}$ $(2^{12} - 2) = \frac{2^{11} - 1}{6}$

C-7. **Sol.** $\int_0^1 (1-x)^n\,dx = \int_0^1 \left(C_0 - C_1x + C_2x^2 - C_3x^3 + ... + (-1)^n C_nx^n\right)dx$

$\Rightarrow \frac{1}{n+1} = \left[C_0 - \frac{C_1}{2} + \frac{C_2}{3} - .... + (-1)^n \frac{C_n}{n+1}\right]$

$\Rightarrow \frac{1}{3}\left(\frac{1}{n+1}\right) = \frac{1}{3}\left[C_0 - \frac{C_1}{2} + \frac{C_2}{3} - .... + (-1)^n \frac{C_n}{n+1}\right]$

C-8. **Sol.** Let $(1 + x + x^2 + x^3)^5 = a_0 + a_1x + a_2x^2 + ...+$ ...... (i)
put x = 1 $\quad 4^5 = a_0 + a_1 + a_2 + ...$
put $\quad x = -1, 0 \quad = a_0 - a_1 + a_2 + ...$
adding $\quad 2(a_0 + a_2 + ...) = 4^5$

$a_0 + a_2 + ... = \frac{1024}{2} = 512$

C-9. **Sol.** $^{47}C_4 + {^{51}C_3} + {^{50}C_3} + {^{49}C_3} + {^{48}C_3} + {^{47}C_3} = {^{52}C_4}$

C-10. **Sol.** $\left(\sum_{r=0}^{10} {^{10}C_r}\right)\left(\sum_{k=0}^{10} (-1)^k \frac{^{10}C_k}{2^k}\right)$

$= (^{10}C_0 + ... + {^{10}C_{10}}) \left({^{10}C_0} - \frac{^{10}C_1}{2} + \frac{^{10}C_2}{2^2} .... + \frac{^{10}C_{10}}{2^{10}}\right)$

$= 2^{10} \times \left(1 - \frac{1}{2}\right)^{10} = 1$

C-11. **Sol.** $^{50}C_0 \times {^{50}C_1} + {^{50}C_1} \times {^{50}C_2} + ...+ {^{50}C_{49}} \times {^{50}C_{50}}$
$= {^{50}C_0} \times {^{50}C_{49}} + {^{50}C_1} \times {^{50}C_{48}} + ...+ {^{50}C_{49}} \times {^{50}C_0}$
= co-eff. of x^{49} in $(1 + x)^{100}$ $= {^{100}C_{49}}$

C-12. **Sol.** $(1 + x)^n \left(1 + \frac{1}{x}\right)^n$

$= [C_0 + C_1 . x + C_2 . x^2 + ...+ C_n . x^n] \left[C_0 + C_1 \cdot \frac{1}{x} + C_2 \cdot \frac{1}{x^2} + ... + C_n \cdot \frac{1}{x^n}\right]$

Coeff. of x^0 = $C_0^2 + C_1^2 + C_2^2 + ... + C_n^2$ = $\frac{2n!}{(n!)^2}$ .....(1)

$(1 + x)^n (1 - x)^n = (C_0 x^n + C_1 x^{n-1} + ... + C_n)(C_0 - C_1 x + ... + (-1)^n C_n x^n)$

coefficient of x^n

$C_0^2 - C_1^2 + C_2^2 - C_3^2 + ... + (-1)^n C_n^2$ = coefficient of x^n in $(1 - x^2)^n$

$C_0^2 - C_1^2 + C_2^2 - C_3^2 + ... + (-1)^n C_n^2 = 0$, n is odd .....(2)

$\therefore$ subtracting (2) from (1)

$2(C_1^2 + C_3^2 + C_5^2 ....) = \frac{2n!}{(n!)^2}$

C-13. **Sol.** $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$

$S = \sum_{r=0}^{n} \frac{n-2r}{{}^nC_r} = \sum_{r=0}^{n}\left(\frac{n-r}{{}^nC_r} - \frac{r}{{}^nC_r}\right)$

$S = \sum_{r=0}^{n} \frac{n-r}{{}^nC_{n-r}} - \sum_{r=0}^{n} \frac{r}{{}^nC_r} = 0$

C-14. **Sol.** $(1 + x)^n \left(1+\frac{1}{x}\right)^n$

$= [C_0 + C_1 .x + C_2 . x^2 + ...+ C_n . x^n] \left[C_0 + C_1 \cdot \frac{1}{x} + C_2 \cdot \frac{1}{x^2} + ... + C_n \cdot \frac{1}{x^n}\right]$

Coeff. of x^0 = $C_0^2 + C_1^2 + C_2^2 + ... + C_n^2$

Section (D) : Binomial Theorem for negative and fractional index

D-1. **Sol.** $T_{r+1} = {}^{2+r-1}C_r x^r = {}^{r+1}C_r x^r$ $\Rightarrow$ $T_7 = {}^7C_6 x^6 = 7x^6$

D-2. **Sol.** $\left(x^2 + \frac{1}{x}\right)^{-4/3} = x^{-8/3}\left(1+\frac{1}{x^3}\right)^{-4/3}$ $\Rightarrow$ Hence $\left|\frac{1}{x^3}\right| < 1$ $\Rightarrow$ $|x| > 1$

D-3. **Sol.** Co-efficient of x^n in $(1 - x)^{-2} = {}^{2+n-1}C_1 = n + 1$

D-4. **Sol.** We have : $(1 + 2x + 3x^2 + 4x^3 + ...)^{1/2}$

$= [(1 - x)^{-2}]^{1/2} = (1 - x)^{-1} = 1 + x + x^2 + ... + x^n + ...$

Hence, coefficient of $x^4 = 1$

D-5. **Sol.** General term in the expansion of $(1 - x)^{-3}$ is ${}^{3+r-1}C_r x^r = {}^{r+2}C_r x^r$

D-6. **Sol.** $\frac{(1+x)^{3/2} - (1+\frac{1}{2}x)^3}{(1-x)^{1/2}} = \left\{\left(1+\frac{3}{2}x + \frac{\frac{3}{2}.\frac{1}{2}x^2}{2!}\right) - \left(1+\frac{3}{2}x+\frac{3}{4}x^2\right)\right\}(1-x)^{-1/2}$

$= \left(-\frac{3}{8}x^2\right)\left(1+\frac{1}{2}x+...\right) = -\frac{3}{8}x^2$

Exercise-2

Marked Questions may have for Revision Questions.

PART - I : OBJECTIVE QUESTIONS

1. **Sol.** $^{18}C_{r-2} + 2.{}^{18}C_{r-1} + {}^{18}C_r \geq {}^{20}C_{13}$

or $^{19}C_{r-1} + {}^{19}C_r \geq {}^{20}C_{13}$

or $^{20}C_r \geq {}^{20}C_{13}$

r = 7,8,9,10,11,12,13

2. **Sol.** $^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r} \Rightarrow {}^{39}C_{3r-1} + {}^{39}C_{3r} = {}^{39}C_{r^2-1} + {}^{39}C_{r^2}$

$^{40}C_{3r} = {}^{40}C_{r^2}$

r_2 = 3r or r = 0, 3

3. **Sol.** $\left(x^3 - \frac{1}{x^2}\right)^n$

General term = $\frac{n!}{r!(n-r)!}$ $(-1)_{n-r}\ x_{5r-2n}$

If 5r – 2n = 5, then 5r = 2n + 5 ⇒ r = $\frac{2n}{5}$ + 1

If 5r – 2n = 10, then 5r = 2n + 10⇒ r = $\frac{2n}{5}$ + 2

Let n = 5k

Now $\frac{5k!}{(2k+1)!(3k-1)!} - \frac{5k!}{(2k+2)!(3k-2)!} = 0$

⇒ $\frac{1}{3k-1} - \frac{1}{2k+2} = 0$

⇒ k = 3 ⇒ n = 15

4. **Sol.** $\left(4^{1/3} + \frac{1}{6^{1/4}}\right)^{20}$

$T_{r+1} = {}^{20}C_r(4_{1/3})_{20-r}(6_{-1/4})_r$

For rational terms

20 – r = 3k & r = 4k, where k, p ∈ n

⇒ r = 20 & r = 8

no. of rational terms = 2

no. of irrational terms = 19

5. **Sol.** $\left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}}\right)^{10} = \left(x^{1/3} + 1 - 1 - \frac{1}{\sqrt{x}}\right)^{10}$

$T_{r+1} = {}^{10}C_r(x_{1/3})_{10-r}\left(-\frac{1}{\sqrt{x}}\right)^r$

For independent term $\frac{10-r}{3} - \frac{r}{2} = 0$ ⇒ r = 4

Coefficient of the term independent of x = ${}_{10}C_4$

6. **Sol.** $(2x - 5)^6$

Greatest binomical Co-efficient is of middle term = $T_{\frac{6}{2}+1}$ = T_4

7. **Sol.** $7^9 + 9^7 = (8 - 1)^9 + (8 + 1)^7$

$= {}_9C_0(8)^9 - {}_9C_1(8)^8 + {}_9C_2(8)^r .....+ {}_9C_8(8) - {}_9C_9 + {}_7C_0(8)^7 +......+ {}_7C_6(8) + {}_7C_7$

This is divisible by 64

8. **Sol.** $(27)^{27} = 3^{81} = 3.(9)^{40}$

$= 3(10 - 1)^{40} = 3(10^{40} - {}_{40}C_1.10^{39} + ... + {}_{40}C_{38}.10^2 - {}_{40}C_{39}.10 +1)$

$= 3(1000\lambda - 400 + 1)$

Last 3 digits of this number = 803.

9. **Sol.** $f(n) = 10^n + 3.4^{n+2} + 5$

put n = 1

f(1) = 10 + 192 + 5 = 207 this is divisible by 3 and 9

10. **Sol.** $\frac{1}{1!(n-1)!} + \frac{1}{2!(n-2)!} + \frac{1}{1!(n-1)!} .... + = \frac{1}{n!}[{}_nC_1 + {}_nC_2 + ... + {}_nC_{n-1}]$

(multiply and divde by n!)

$= \frac{1}{n!}[2^n - 2] = \frac{2}{n!}(2^{n-1} - 1)$

11. **Sol.** ${}_{10}C_3 + {}_{11}C_3 + {}_{12}C_3 + ... + {}_{20}C_3$

$= {}_{10}C_4 + {}_{10}C_3 + {}_{11}C_3 + 12_{C3} + ... + {}_{20}C_3 - {}_{10}C_4$

$= {}_{11}C_4 + {}_{11}C_3 + {}_{12}C_3 + ... + {}_{20}C_3 - {}_{10}C_4$

$= {}_{21}C_4 - {}_{10}C_4 = {}_{21}C_{17} - {}_{10}C_6.$

12. **Sol.** $= a\sum_{r=1}^{n}(-1)^{r-1}.\ {}^nC_r - \sum_{r=1}^{n} r.\ {}^nC_r(-1)^{r-1}$

$= a[{}_nC_1 - {}_nC_2 + {}_nC_3 .....+(-1)^{n-1}\,{}_nC_n] - a[{}^nC_0 - {}^nC_1 + {}^nC_2.....(-1)^n\ {}^nC_n.....{}^nC_0] = a$

13. **Sol.** $3 . {}_nC_0 - 8 . {}_nC_1 + 13 . {}_nC_2 - 18\ {}_nC_3 + ...$ up to (n + 1) terms

$(1 + x^5)^n = C_0 + C_1x^5 + C_2x^{10} + ... + C_nx^{5n}$

Multiplying by x^3 and differentiating w.r.t. x

$x^3 . n(1 + x^5)^{n-1} . 5x^4 + 3x^2 (1 + x^5)^n = 3C_0\, x^2 + 8C_1\, x^7 + 13C_2\, x^{12} + ... + (5n + 3)\, C_n\, x^{5n+2}$

Now put x = – 1

$3C_0 - 8C_1 + 13\, C_2 + ...+$ (n + 1) terms = 0

14. **Sol.** $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + ...+ a_{2n}x^{2n}$

put x = 1

$3^n = a_0 + a_1 + a_2 +.......+ a_{2n}$ ....(i)

x = – 1

$1 = a_0 - a_1 + a_2 + ...+ a_{2n}$ ....(ii)

adding (i) & (ii)

$\frac{3^n + 1}{2} = a_0 + a_2 + ...+ a_{2n}.$

15. **Sol.** $\left(1+2\sqrt{x}\right)^{40}$ $= {}_{40}C_0 + {}_{40}C_1\ 2\sqrt{x}$ +.....+ ${}_{40}C_{40}\ (2\sqrt{x})^{40}$

$(1-2\sqrt{x})^{40} = {}_{40}C_0 - {}_{40}C_1\ 2\sqrt{x}$ +......+ ${}_{40}C_{40}\ (2\sqrt{x})^{40}$

$(1+2\sqrt{x})^{40} + (1-2\sqrt{x})^{40}$

$= 2\ [{}_{40}C_0 + {}_{40}C_2\ (2\sqrt{x})_2 + ... + {}_{40}C_{40}\ ]\ (2\sqrt{x})^{40}$

Putting x = 1

${}_{40}C_0 + {}_{40}C_2(2)_2 + ... + {}_{40}C_{40}\ (2)_{40} = \dfrac{3^{40}+1}{2}$

16. **Sol.** $(9x_2 + x - 8)_6 = a_0 + a_1x + a_2x_2 + ... + a_{12}\ x_{12})$

x = 1 $2_6 = a_0 + a_1 + a_2 + ... + a_{12}$

x = –1 $0 = a_0 - a_1 + a_2 + ... + a_{12}$

$2_6 = 2(a_1 + a_3 + a_5 + ... + a_{11})$

subtracting (1) from (2)

∴ $a_1 + a_3 + ... + a_{11} = 2_5 = 32$

17. **Sol.** $\sum_{r=0}^{n} (r+1)\ C_{r2}$

$(1+x)_n = C_0 + C_1x + ... + C_nx_n$

Multiply by x & then differentiate

$(1+x)_n + x.\ n(1+x)_{n-1} = C_0 + 2C_1x + ... +$

$(n+1)C_nx_n$ ....(i)

$(x+1)_n = C_0x_n + C_1x_{n-1} + ... + C_n$ ....(ii)

Multiply (i) & (ii) & equate the coefficient of x_n on both side

$C_{02} + 2C_{12} + ... + (n+1)\ C_{n2} = {}_{2n}C_n + n.\ {}_{2n-1}C_{n-1} = 2.{}_{2n-1}C_{n-1} + n.{}_{2n-1}C_{n-1} = \dfrac{(n+2)(2n-1)!}{n\ !\ \ (n-1)!}$

18. **Sol.** $\left(\left(x+\dfrac{1}{x}\right)^2 - 1\right)^n = {}_nC_0\left(x+\dfrac{1}{x}\right)^{2n} - {}_nC_1\left(x+\dfrac{1}{x}\right)^{2n-2} + ... + {}_nC_n\ (-1)_n$

Total number of terms = 2n + 1

19. **Sol.** Coeff. of x_{10} in $(1-x_4)_5\ (1-x)_{-5}$

$(1 - {}^5C_1 x_4 + {}^5C_2\ x_8 + ...)\ (1-x)_{-5}$

Coeff. of x_{10} in $(1-x)_{-5}$ – 5.coeff. of x_6 in $(1-x)_{-5}$ + 10 coeff. of x_2 in $(1-x)_{-5}$

${}^{14}C_4 - 5 \times {}^{10}C_4 + 10.{}^6C_4 = 101$

20. **Sol.** $(x+3)_n + (x+3)_{n-1}\ (x+2) + ... + (x+2)_n = (x+3)_n \left(\dfrac{1-\left(\dfrac{x+2}{x+3}\right)^{n+1}}{1-\dfrac{x+2}{x+3}}\right) = [(x+3)_{n+1} - (x+2)_{n+1}]$

Coefficient of $x_{n-1} = {}_{n+1}C_{n-1}\ (3)_2 - {}_{n+1}C_{n-1} \times 4$

21. **Sol.** $(1+x)_2\ (1-x)_{-2}$

$= (1 + x_2 + 2x)\ (1-x)_{-2}$

Co-efficient of $x_4 = {}_5C_4 + {}_3C_2 + 2\ {}_4C_3 = 16$

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true.

(2) Statement-I is true, but Statement-II is false.

(3) Statement-I is false, but Statement-II is true.

(4) Both the statements are false.

A-1. Ans. (1)

Sol. Statement-1 : $\left(x+\frac{1}{x}+2\right)^m = \frac{(x+1)^{2m}}{x^m}$

$\therefore$ co-efficient of $x^0 = {}^{2m}C_m = \frac{(2m)!}{(m!)^2}$

True

Statement-2 : Obviously true and correct explanation of statement-1

A-2. Ans. (3)

Sol. Statement-1 : $({}^{2n}C_1 + {}^{2n}C_3 + {}^{2n}C_5 + ... + {}^{2n}C_{n-1}) + ({}^{2n}C_{n+1} + ... + {}^{2n}C_{2n-1}) = 2^{2n-1}$

$\Rightarrow$ $2({}^{2n}C_1 + {}^{2n}C_3 + {}^{2n}C_5 + ... + {}^{2n}C_{n-1}) = 2^{2n-1}$

$\Rightarrow$ ${}^{2n}C_1 + {}^{2n}C_3 + {}^{2n}C_5 + ... + {}^{2n}C_{n-1} = 2^{2n-2}$

Statement-1 : false

Statement-2 : ${}^{2n}C_1 + {}^{2n}C_3 + ... + {}^{2n}C_{2n-1} = (2)^{2n-1}$, True

Section (B) : MATCH THE COLUMN

B-1. Sol. (A) ${}^nC_0 + {}^nC_1 + {}^nC_2 = 46$ $\Rightarrow$ $1 + n + \frac{n(n-1)}{2} = 46$

$\Rightarrow$ $n^2 + n - 90 = 0$

$\Rightarrow$ $(n + 10)(n - 9) = 0$

$\Rightarrow$ $n = 9$

(B) The general term $t_{r+1} = {}^{1024}C_r\ 5^{\frac{1024-r}{2}}\ 7^{\frac{r}{8}}$ where $0 \le r \le 1024$

t_{r+1} is an integer $\Rightarrow r = 8\lambda$

$\Rightarrow$ $r = 0, 8, 16,......, 1024$ $\Rightarrow$ 129 terms

(C) $(1+ x + x^2 + x^3 + x^4)^{199} (x-1)^{201} = -\frac{(1-x^5)^{199}}{(1-x)^{199}}.\ (1-x)^{201} = -(1 - x^5)^{199}\ (1 - x)^2$

$= -(1 - 2x + x^2) \sum {}^{199}C_r(-x^5)^r$

Hence coeff of $x^{103} = 0$

(D) $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $2^7 = 128$, $2^8 = 256$,..............

Last digit is repeted with period 4

$\Rightarrow$ last digit of the number 2^{999} is = 8

B-2. Sol. (A) $\sum_{r=0}^{n}(r+1)C_r = \sum_{r=0}^{n} rC_r + \sum_{r=0}^{n} C_r = n.\ 2^{n-1} + 2^n$

(B) $C_0 + C_1 + C_2 .....+ C_n = 2^n$

$\Rightarrow$ $2(C_0 + C_1 +... .....+ C_{n/2}) = 2^n + C_{n/2}$

$\Rightarrow$ $C_0 + C_1 + \ldots\ \ldots + C_{n/2} = \frac{1}{2}\left(2^n + C_{n/2}\right)$

(C) $\therefore xC_0 + \frac{x^2}{2}C_1 + \frac{x^3}{3}C_2 + \ldots\ldots + \frac{x^{n+1}}{n+1}C_n = \frac{(1+x)^{n+1}}{n+1}$

put x = 2

(D) ${}_nC_n\ {}_nC_2 + {}_nC_{n-1}\ {}_nC_3 + \ldots\ldots\ldots + {}_nC_2\ {}_nC_n$ = Selection of (n+2) persons out of n boy and n girls

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. Sol. $a_n = \frac{(1000)(1000)\ldots\ldots\ldots(1000)}{1.2\ldots\ldots n}$

$a_{999} = a_{1000}$

a_n is maximum for n = 999 and n = 1000

C-2. Sol. Total term = (2n + 1)

Middle term = $\frac{(2n+1)\ \ +1}{2}$ = (n + 1)th

$T_{n+1} = {}_{2n}C_n x_n$

Coefficient of $x_n = {}_{2n}C_n$

$x_n = {}_{2n}C_n$

$= \frac{2n\ !}{n\ !\ n\ !} = \frac{2^n n\ !\{1.3.5\ldots\ldots(2n-1)\}}{n\ !\ n\ !} = \left\{\frac{1.3.5\ldots\ldots(2n-1)}{n\ !}\right\} 2_n$

C-3. Sol. $(101)_{50} - (99)_{50} = (100+1)_{50} - (100-1)_{50}$

$= 2\ ({}_{50}C_1\ 100_{49} + {}_{50}C_3\ 100_{47} + {}_{50}C_5\ 100_{45} + \ldots\ldots + {}_{50}C_{49}\ 100]$

$= 2.50\ .\ 100_{49} + 2[{}_{50}C_3\ 100_{49} + {}_{50}C_3\ 100_{47} + {}_{50}C_5\ 100_{45} + \ldots\ldots\ldots + {}_{50}C_{49}\ 100]$

$= 100_{50} + 2[{}_{50}C_3\ 100_{49} + {}_{50}C_3\ 100_{47} + {}_{50}C_5\ 100_{45} + \ldots\ldots\ldots + {}_{50}C_{49}\ 100]$

$\Rightarrow (101)_{50} - (99)_{50} > (100)_{50}$

or ;k $(101)_{50} - (99)_{50} > (100)_{50}$

or ;k $(101)_{50} - (100)_{50} > (99)_{50}$

Again = $\left(\frac{1001}{1000}\right)^{999} = \left(1 + \frac{1}{1000}\right)^{999}$ $1 + {}_{999}C_1\left(\frac{1}{1000}\right) + {}^{999}C_2\left(\frac{1}{1000}\right)^2 + {}^{999}C_3\left(\frac{1}{1000}\right)^3 + \ldots..$ < 1 + 1 + 1 + 1 +......1000 terms

$\left(\frac{1001}{1000}\right)^{999} < 1000 \Rightarrow (1001)^{999} < (1000)^{1000}$

C-4. Sol. $T_{r+1} = {}_{20}C_r\ 4^{\frac{20-r}{3}}\ 6^{-\frac{r}{4}}$

T_{r+1} is rational if r = 8, 20

Here only 2 terms are rational and 19 terms are inrrational

also middle term is t_{11} which is irrational

C-5. Sol. $(101)_{100} - 1 = (1+100)_{100} - 1$

$= 1 + {}_{100}C_1\,(100) + {}_{100}C_2(100)_2 + \ldots. + {}_{100}C_{100}(100)_{100} - 1$

$= 10_4\lambda\ \forall\lambda \in N$

Exercise-3

*** Marked Questions may have more than one correct option.**

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Sol. $\because$ $(1 + 2x + 3x^2 + ...)^{-3/2} = [(1 - x)^{-2}]^{-3/2} = (1 - x)^3$

So, coefficient of x^5 in $(1 + 2x + 3x^2 + ...)^{-3/2}$

= coefficient of x^5 in $(1 - x)^3 = 0$.

2. Sol. $(r + 1)^{th}$ term of

i.e., $T_{r+1} = {}^{256}C_r\ (3)^{(256-r)/2}\ (5)^{r/8}$

The terms are integral, if $\frac{256-r}{2}$ and $\frac{r}{8}$ are both positive integer.

$\Rightarrow$ r = 0, 8, 16, 24, 32,....., 256

Hence total terms are 33.

3. Sol. $\because$ $(r + 1)^{th}$ term in the expansion of $(1 + x)^{27/5}$

$$= \frac{\frac{27}{5}\left(\frac{27}{5}-1\right)....\left(\frac{27}{5}-r+1\right)}{r\ !}x^r$$

Now this term will be negative, if the last factor in numerator is the only negative factor.

$\Rightarrow \frac{27}{5} - r + 1 < 0 \Rightarrow \frac{32}{5} < r$

$\Rightarrow 6.4 < r \Rightarrow$ least value of r is 7.

Thus first negative term will be 8th.

4. Sol. Coefficient of middle term in $(1 + \alpha x)^4 = {}^4C_2\ \alpha^2$

coefficient of middle term in $(1 - \alpha x)^6 = {}^6C_3\ (-\alpha)^3$

${}^4C_2\ \alpha^2 = -\ {}^6C_3\ \alpha^3$

$-\frac{6}{20} = \alpha$

$\alpha = \frac{-3}{10}$

5. Sol. $(1 - x)\ (1 - x)^n = (1 - x)^n + x\ (1 - x)^n$

Coefficient of $x^n = (-1)^n + (-1)^{n-1} . n$

$= (-1)^n (1 - n)$

6. Sol. $s_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$

$t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$

$t_n = \sum_{r=0}^{n} \frac{n-r}{{}^nC_r}$

$\therefore$ $2t_n = n \sum_{r=0}^{n} \frac{1}{{}^nC_r} = ns_n$

$\Rightarrow \quad \frac{t_n}{s_n} = \frac{n}{2}$

7. Sol. $(1 + y)^m$

$T_r = {}^mC_{r-1} \, . \, y^{r-1}$

$T_{r+1} = {}^mC_r \, . \, y^r$

$T_{r+2} = {}^mC_{r+1} \, . \, y^{r+1}$

$\because \quad {}^mC_{r-1} + {}^mC_{r+1} = 2\, {}^mC_r \quad \Rightarrow \quad \frac{{}^mC_{r-1}}{{}^mC_r} + \frac{{}^mC_{r+1}}{{}^mC_r} = 2 \quad \Rightarrow \quad m^2 - m(4r + 1) + 4r^2 - 2 = 0$

8. Sol. ${}^{50}C_4 + {}^{55}C_3 + {}^{54}C_3 + \ldots\ldots\ldots + {}^{50}C_3$

$= {}^{56}C_4$

9. Sol. $\frac{(1+x)^{3/2} - \left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}} \Rightarrow \frac{\left(1+\frac{3}{2}x+\frac{3}{2}.\,\frac{1}{2}\,\frac{x^2}{2!}\right)-\left(1+\frac{3x}{2}+\frac{3.2}{2}.\,\frac{x^2}{4}\right)}{(1-x)^{1/2}}$

$= \left(-\frac{3}{8}x^2\right)(1 - x)^{-1/2} = -\frac{3}{8}x^2$

10. Sol. $(1 - ax)^{-1} (1 - bx)^{-1}$

$= (1 + ax + (ax)^2 + \ldots\ldots.)\,(1 + bx + (bx)^2 + \ldots\ldots\ldots\ldots)$

so, () $a_n = a^n + a^{n-1} b + a^{n-2} b^2 + \ldots\ldots. + b^n$

$= a^n \frac{\left(1-\left(\frac{b}{a}\right)^{n+1}\right)}{1-\frac{b}{a}} = \frac{a^{n+1} - b^{n+1}}{b - a}$

11. Sol. $(1 - y)^m (1 + y)^n = 1 + a_1 y + a_2 y^2 + \ldots\ldots..$

$(1 - my + {}^mC_2 y^2 \ldots\ldots..)\,(1 + ny + {}^nC_2 y^2 \ldots\ldots...) = 1 + a_1 y + a_2 y^2 + \ldots\ldots..$

$a_1 = n - m = 10$ ..... (1)

$a_2 = {}^mC_2 + {}^nC_2 - mn = 10$ ..... (2)

solving (1) & (2), we get $(m, n) \equiv (35, 45)$

12. Sol. $S = {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2$ ..... $+ {}^{20}C_{10}$

We know, ${}^{20}C_0 - {}^{20}C_1 + \ldots\ldots\ldots.. + {}^{20}C_{20} = 0$

$2({}^{20}C_0 - {}^{20}C_1 + \ldots\ldots\ldots.. - {}^{20}C_9) + {}^{20}C_{10} = 0$

$\therefore \quad {}^{20}C_0 - {}^{20}C_1 + \ldots\ldots... - {}^{20}C_9 = \frac{1}{2} - {}^{20}C_{10}$

$\Rightarrow \quad {}^{20}C_0 - {}^{20}C_1 + \ldots\ldots... - {}^{20}C_9 + {}^{20}C_{10} = \frac{1}{2}\, {}^{20}C_{10}$

so, () $S = \frac{1}{2}\, {}^{20}C_{10}$

13. Sol. $T_5 + T_6 = 0$

${}^nC_4 a^{n-4}.\ b^4 - {}^nC_5 a^{n-5}.\ b^5 = 0$

$\Rightarrow \frac{a^{n-4}\ b^4}{a^{n-5}\ b^5} = \frac{{}^nC_5}{{}^nC_4}$

$\Rightarrow \frac{a}{b} = \frac{n!}{5!(n-5)!} \times \frac{4!(n-4)!}{n!} = \frac{n-4}{5}$

14. **Sol.** **Statement -1** : $\sum_{r=0}^{n}(r+1)\,{}^nC_r = \sum_{r=0}^{n} r.{}^nC_r + \sum_{r=0}^{n}{}^nC_r$

$= n \cdot 2^{n-1} + 2^n = (n+2)\,2^{n-1}$

Statement-2 : $\sum_{r=0}^{n}(r+1)\,{}^nC_r x^r = \sum_{r=0}^{n} r.\,{}^nC_r\,x^r + \sum_{r=0}^{n}{}^nC_r\,x^r$

$= xn(1+x)^{n-1} + (1+x)^n$

15. **Sol.** $S_1 = \sum_{j=1}^{10} j(j-1) \cdot \frac{10(10-1)}{j\,(j-1)}\,{}^8C_{j-2}$

⇒ $S_1 = 9 \times 10 \sum_{j=2}^{10}{}^8C_{j-2}$

⇒ $S_1 = 90 \cdot 2^8$

$S_2 = \sum_{j=1}^{10} j \cdot \frac{10}{j}\,{}^9C_{j-1} = 10.2^9$

$S_3 = \sum_{j=1}^{10}(j(j-1)+j)\,{}^{10}C_j = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j + \sum_{j=1}^{10} j\,{}^{10}C_j = 90\sum_{j=2}^{10}{}^8C_{j-2} + 10\sum_{j=1}^{10}{}^9C_{j-1}$

$= 90 \times 2^8 + 10 \times 2^9 = (45+10) \cdot 2^9 = (45+10) \cdot 2^9 = 55.2^9$

so statement-1 is true and statement 2 is false.

Hence correct option is (2)

16. **Sol.** **(3)**

$(1 - x - x^2 + x^3)^6$

$(1-x)^6\,(1-x^2)^6$

$({}^6C_0 - {}^6C_1\,x^1 + {}^6C_2\,x^2 - {}^6C_3x^3 + {}^6C_4x^4 - {}^6C_5x^5 + {}^6C_6x^6)\,({}^6C_0 - {}^6C_1x^2 + {}^6C_2x^4 - {}^6C_3x^6 + {}^6C_4x^8 + \ldots\ldots\ldots\ldots + {}^6C_6x^{12})$

Now coefficient of $x^7 = {}^6C_1{}^6C_3 - {}^6C_3{}^6C_2 + {}^6C_5{}^6C_1$

$= 6 \times 20 - 20 \times 15 + 36$

$= 120 - 300 + 36$

$= 156 - 300$

$= -144$ **Ans.**

17. **Sol.** **Ans. (1)**

$(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n}$

$= 2[{}^{2n}C_1(\sqrt{3})^{2n-1} + {}^{2n}C_3(\sqrt{3})^{2n-3} + {}^{2n}C_5(\sqrt{3})^{2n-5} + \ldots\ldots\ldots]$

= which is an irrational number

18. **Sol.** (3)

$\left((x^{1/3}+1) - \left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$

$(x^{1/3} - x^{-1/2})^{10}$

$T_{r+1} = {}^{10}C_r\,(x^{1/3})^{10-r}\,(-x^{-1/2})^r$

$\frac{10-r}{3} - \frac{r}{2} = 0$ ⇒ $20 - 2r - 3r = 0$

⇒ $r = 4$

$T_5 = {}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$

19. **Ans.** **(1)**

Sol. $\left(1-2\sqrt{x}\right)^{50} = C_0 - C_1\,2\sqrt{x} + C_2\left(2\sqrt{x}\right)^2 + \ldots\ldots + C_{50}\left(2\sqrt{x}\right)^{50}$

$\left(1+2\sqrt{x}\right)^{50} = C_0 + C_1\left(2\sqrt{x}\right) + C_2\left(2\sqrt{x}\right)^2 + \ldots\ldots + C_{50}\left(2\sqrt{x}\right)^{50}$

Put x = 1

$\therefore \quad \frac{1+3^{50}}{2} = C_0 + C_2(2)^2 + . .$

20. Ans. (3) or Bonus

Sol. Theortically the number of terms are 2N + 1 (i.e. odd) But As the number of terms being odd hence considering that number clubbing of terms is done hence the solutions follwos :

Number of terms = ${}^{n+2}C_2 = 28 \quad \therefore \quad n = 6$

sum of cofficient = $3^n = 3^6 = 729$

put x = 1

21. Ans. (4)

Sol. $({}^{21}C_1 + {}^{21}C_2 + {}^{21}C_3 + \ldots\ldots {}^{21}C_{10}) - ({}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \ldots\ldots {}^{10}C_{10}) = S_1 - S_2$

$S_1 = {}^{21}C_1 + {}^{21}C_2 + {}^{21}C_3 + \ldots\ldots {}^{21}C_{10}$

$S_1 = \frac{1}{2}({}^{21}C_1 + {}^{21}C_2 + \ldots\ldots + {}^{21}C_{20}) = \frac{1}{2}({}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + \ldots\ldots + {}^{21}C_{20} + {}^{21}C_{21} - 2)$

$S_1 = 2^{20} - 1$

$S_2 = ({}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \ldots\ldots {}^{10}C_{10}) = 2^{10} - 1$

Therefore, $S_1 - S_2 = 2^{20} - 2^{10}$

22. Sol. (2)

$\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$

$= (T_1 + T_2 + T_3 + T_4 + T_5 + T_6) + (T_1 - T_2 + T_3 - T_4 + T_5 - T_6)$

$= 2(T_1 + T_3 + T_5)$

$= 2({}^5C_0(x)^5 + {}^5C_2(x)^3 \left(\sqrt{x^3-1}\right)^2 + {}^5C_4(x)^1 \left(\sqrt{x^3-1}\right)^4$

$= 2(x^5 + 10x^3(x^3 - 1) + 5x(x^6 + 1 - 2x^3)]$

$= 2(x^5 + 10x^6 - 10x^3 + 5x^7 + 5x - 10x^4)$

$= 2(5x^7 + 10x^6 + x^5 - 10x^4 - 10x^3 + 5x)$

sum of odd degree terms = 10 + 2 –20 + 10 = 2

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. Sol. $S = \sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$

$= {}^{10}C_0 . {}^{20}C_m + {}^{10}C_1 . {}^{20}C_{m-1} + \ldots\ldots\ldots\ldots$

$\Rightarrow$ S = coefficient of x^m in $(1+x)^{10}(1+x)^{20} = {}^{30}C_m$

S is maximum when m = 15

2. Sol. $(1 + t^2)^{12}(1 + t^{12} + t^{24} + t^{36}) = (1 + t^{12} + t^{24})(1 + t^2)^{12}$

coefficient of $t^{24} = {}^{12}C_{12} + {}^{12}C_6 + {}^{12}C_0 = {}^{12}C_6 + 2$

3. Sol. ${}^{(n-1)}C_r = (k^2 - 3)\, {}^nC_{r+1}$

or ${}^{(n-1)}C_{n-(r+1)} = (k^2 - 3)\, {}^nC_{n-(r+1)}$

$1 \geq k^2 - 3 > 0 \Rightarrow k \in [-2, -\sqrt{3}) \cup (\sqrt{3}, 2]$

4. Sol. $S = {}^{30}C_0\, {}^{30}C_{20} - {}^{30}C_1\, {}^{30}C_{19} + {}^{30}C_2\, {}^{30}C_{18} \ldots\ldots\ldots\ldots\ldots$

S = Co-efficient of x^{20} in $(1-x)^{30}(1+x)^{30}$

S = Co-efficient of x^{20} in $(1-x^2)^{30} = {}^{30}C_{10}$

5. Sol. $B_{10}\sum_{r=1}^{10} A_rB_r - C_{10}\sum_{r=1}^{10}(A_r)^2 = {}^{20}C_{10}({}^{30}C_{20} - 1) - {}^{30}C_{10}({}^{20}C_{10} - 1) = {}^{30}C_{10} - {}^{20}C_{10} = C_{10} - B_{10}$

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

1. Sol. $T_{r+1} = {}^{10}C_r (2x^2)^{10-r} \left(\frac{1}{3x^2}\right)^r$

$\therefore T_{5+1} = T_6 = {}^{10}C_5 (2x^2)^5 \left(\frac{1}{3x^2}\right)^5$

$= \frac{896}{27} = \frac{a}{b}$ $\therefore$ a + b = 923

2. Sol. $T_{25} = T_{26} \Rightarrow {}^{44}C_{24}(-x)^{24} = {}^{44}C_{25}(-x)^{25}$

$\Rightarrow x = -\frac{{}^{44}C_{24}}{{}^{44}C_{25}} = -\frac{25}{44-25+1} \Rightarrow x = -\frac{25}{20} = -\frac{5}{4}$

3. Sol. Given ${}^mC_0 + {}^mC_1 + {}^mC_2 = 46$

$\Rightarrow 1 + m + \frac{m(m-1)}{2} = 46$

$\Rightarrow m^2 + m - 90 = 0 \Rightarrow m = 9$

$\therefore T_{r+1}\, {}^9C_r (x^2)^{9-r}\left(\frac{1}{x}\right)^r = {}^9C_r x^{18-3r}$

For constant term, 18–3r = 0 ⇒ 6

$\therefore$ constant term = ${}^9C_6 = \frac{9.8.7}{3.2.1} = 84$

4. Sol. $T_{r+1} = {}^nC_r \left(\sqrt[3]{2}\right)^{n-r}\left(\frac{1}{\sqrt[3]{3}}\right)^r$

$\therefore T_{6+1} = {}^nC_6 \left(\sqrt[3]{2}\right)^{n-6}\left(\frac{1}{\sqrt[3]{3}}\right)^6 = T_7$ from begining, 7^{th} term from the end is $T_{7'} = {}^nC_6 \left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}\left(\sqrt[3]{2}\right)^6$

given $\frac{T_7}{T_7'} = \frac{1}{6} \Rightarrow 6T_7 = T_{7'}$

$\Rightarrow 6.\, {}^nC_6 . \left(\sqrt[3]{2}\right)^{n-6}\left(\frac{1}{\sqrt[3]{3}}\right)^6 = {}^nC_6 \left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}\left(\sqrt[3]{2}\right)^6$

$\Rightarrow 6.\ 2^{\frac{n-6}{3}}.3^{\frac{-6}{3}} = 3^{-\frac{n-6}{3}}.2^{\frac{6}{3}}$

$\Rightarrow 2\times3\ .\ 2^{\frac{n-6}{3}}.3^{-2} = 3^{-\frac{n-6}{3}}2^2$

$\Rightarrow 2^{\frac{n-6}{3}-1} = 3^{-\frac{n-6}{3}+1}$

$\Rightarrow 2^{\frac{n-9}{3}} = 3^{\frac{9-n}{3}} \Rightarrow 9$

5. Sol. Here $T_{r+1} = {}^{15}C_r x^r$

$\therefore T_{2r-1} = T_{(2r-2)+1} = {}^{15}C_{2r-2}\, x^{2r+2}$

& $T_{r-1} = T_{(r-2)+1} = {}^{15}C_{r-2}\, x^{r-2}$

given ${}^{15}C_{2r-2} = {}^{15}C_{r+2}$

$\Rightarrow 2r+2 = r-2$ or $2r+2+r-2 = 15$
$\Rightarrow r = 4$ or $3r = 15 \Rightarrow r = 4$ or 5

6. **Sol.** $(ab + bc + ca)^6 = a^6 b^6 c^6 (a^{-1} + b^{-1} + c^{-1})^6$

General term $= a^6 b^6 c^6 \dfrac{!6(a^{-1})^{k_1}.(b^{-1})^{k_2}.(c^{-1})^{k_3}}{k_1!\, k_2!\, k_3!}$

$\therefore k_1 = 3, k_2 = 2, k_3 = 1$

$\therefore$ Coefficient of $a^3 b^4 c^5$ is $\dfrac{6!}{3!\ 2!\ 1!} = 60$

$\therefore a^3 b^4 c^5 \ \dfrac{6!}{3!\ 2!\ 1!} = 60$

7. **Sol.** Let $S = \dfrac{1}{1!(n-1)!} + \dfrac{1}{3!(n-3)} + \dfrac{1}{5!(n-5)!} + ...$

$= \dfrac{1}{n!}\left(\dfrac{n!}{1!\ (n-1)!} + \dfrac{n!}{3!\ (n-3)!} + \dfrac{1}{5!(n-5)!} + ...\right)$

$= \dfrac{1}{n!} ({}^nC_1 + {}^nC_3 + {}^nC_5 + ...)$

$= \dfrac{1}{n!} . \dfrac{2^n}{2} \Rightarrow \dfrac{2^{n-1}}{n!}$

8. **Sol.** sum of coefficient is zero

$\therefore a^3 - 2a^2 + 1 = 0$
$\Rightarrow a^3 - a^2 - a^2 + a - a + 1 = 0$
$\Rightarrow a^2 (a-1) - a(a-1) - 1(a-1) = 0$
$\Rightarrow (a-1)(a^2 - a - 1) = 0$
$\Rightarrow a = 1, a = \dfrac{1 \pm \sqrt{1+4}}{2}$

9. **Sol.** n is even here

$\therefore$ middle term $= {}^nC_{n/2}(x^2)^{\frac{n}{2}}.\left(\dfrac{1}{x}\right)^{\frac{n}{2}} = 924x^6$

$\Rightarrow {}^nC_{n/2} . x^{n/2} = 924x^6$

$\therefore \dfrac{n}{2} = 6 \Rightarrow n = 12$

10. **Sol.** Here $(1-2+3)^n = 128$
$\Rightarrow 2^n = 2^7 \Rightarrow n = 7$
$\therefore$ greatest coefficient of $(1+x)^{14}$ is ${}^{14}C_7$

11. **Sol.** Let ${}^nC_r = 165$, ${}^nC_{r+1} = 330$ and ${}^nC_{r+2} = 462$

$\therefore \dfrac{{}^nC_{r+1}}{{}^nC_r} = \dfrac{330}{165} = 2 \Rightarrow r = \dfrac{1}{3}(n-2)$

and $\dfrac{{}^nC_{r+2}}{{}^nC_{r+1}} = \dfrac{n-r-1}{r+2} = \dfrac{231}{165} \Rightarrow 165n - 627 = 396r$

$\therefore 165n - 627 = 132(n-2) \Rightarrow n = 11$

12. **Sol.** $17^{256} = (17^2)^{128} = (290-1)^{128}$

$= 1000\,m + {}_{128}C_2(290)^2 - {}_{128}C_1(290) + 1$
$= 1000(m+683527) + 681$
$\therefore$ last two digits = 81

13. **Sol.** Here $3^{2003} = 9\times3^{2001} = 9\,(28-1)^{667}$
$= 9[{}_{667}C_0 28^{667} - {}_{667}C_1 28^{666} + \ldots\ldots - {}_{667}C_{667}]$
$= 9\times 28k - 9 + 28 - 28$
$= (9\times28k - 28) + 19$
$\therefore \left\{\frac{3^{2003}}{28}\right\} = \frac{19}{28}$

14. **Sol.** given = $\frac{2}{2^7\sqrt{4x+1}}\left\{{}^7C_1\sqrt{4x+1} + {}^7C_3\left(\sqrt{4x+1}\right)^3 + {}^7C_5\left(\sqrt{4x+1}\right)^5 + {}^7C_7\left(\sqrt{4x+1}\right)^7\right\}$
$= \frac{1}{2^6}\,[{}_7C_1 + {}_7C_3(4x+1) + {}_7C_5(4x+1)^2 + {}_7C_7(4x+1)^3$
$\therefore$ degree = 3

15. **Sol.** $p(n) = 49^n + 16n - 1$
$p(1) = 49 + 16 - 1 = 64$ which is divisible by 16, 32, 64

16. **Sol.** $(x + a)^n = {}_nC_0x^n + {}_nC_1x^{n-1}a + \ldots + {}_nC_na^n$
$= T_0 + T_1 + T_2 + \ldots + T_n$
Replace a by ai
$(x + ai)^n = ({}_nC_0x^n - {}_nC_2x^{n-2}a^2 + \ldots\ldots) + i\,({}_nC_1x^{n-1}a - {}_nC_3x^{n-3}a^3 + \ldots\ldots)$
$= (T_0 - T_2 + T_4 - T_6 + \ldots) + i(T_1 - T_3 + T_5 - T_7 + \ldots)$
taking mod both sides
$(T_0 - T_2 + T_4 - T_6 + \ldots)^2 + (T_1 - T_3 + T_5 - T_7 + \ldots)^2 = (x^2 + a^2)^n$

17. **Sol.** Here $\frac{n+1}{1+\left|\frac{x}{y}\right|} = \frac{15+1}{1+\left|\frac{3}{1}\right|} = 4 = r$
$\therefore$ T_4 & T_5 are numerically gratest terms
$\therefore |T_4| = |T_5| = {}_{15}C_4.3^{11} = 455 \times 3^{12}$
$\therefore n = 12 \qquad \therefore \frac{{}^nC_2}{2} = \frac{12.11}{2.2} = 3$

18. **Sol.** $\sum_{r=0}^{n} r\ {}^nC_r x^r y^{n-r} = \sum_{r=1}^{n} n\ {}^{n-1}C_{r-1}x^{r-1}.xy^{n-r}$
$= nx\sum_{r=1}^{n} {}^{n-1}C_{r-1}x^{r-1}.y^{n-1-(r-1)}$
$= nx\,.\,(x+y)^{n-1}$
$= nx$

19. **Sol.** $(1 - 2x^3 + 3x^5)\left(1+\frac{1}{x}\right)^8$ Co-efficient of x ¼x ½ = $-2.\,{}_8C_2 + 3.{}_8C_4 = 154$

20. **Sol.** Given = ${}^nC_0 + {}_{n+1}C_1 + {}_{n+2}C_2 + \ldots\ldots + {}_{2n}C_n$
$= {}_nC_n + {}_{n+1}C_n + {}_{n+2}C_n + \ldots\ldots + {}_{2n}C_n$
$= {}_{2n+1}C_{n+1}$ (by pascal rule ${}_nC_r + {}_nC_{r-1} = {}_{n+1}C_r$)

21. **Sol.** Let $1 - \frac{1}{8} + \frac{1.3}{8.16} - \frac{1.3.5}{8.16.24} + \ldots\ldots = (1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \ldots$

$\therefore nx = -\frac{1}{8}, \frac{n(n-1)}{2}x^2 = \frac{3}{8.16}$

$\Rightarrow x = \frac{1}{4}$, $n = -\frac{1}{2}$ $\therefore$ sum = $\left(1+\frac{1}{4}\right)^{-\frac{1}{2}} = \left(\frac{5}{4}\right)^{-\frac{1}{2}} = \frac{2}{\sqrt{5}}$

22. **Sol.** $(1 - 9x + 20x_2)_{-1} = ((1 - 5x)(1 - 4x))_{-1}$

$= \frac{1}{(1-5x)(1-4x)} = \frac{5}{1-5x} - \frac{4}{1-4x}$

$= 5(1 - 5x)_{-1} - 4(1 - 4x)_{-1}$

$\therefore$ coefficient of x_n is $5_{n+1} - 4_{n+1}$

$\therefore x_n \; 5_{n+1} - 4_{n+1}$

23. **Sol.** Here $T_{r+1} = {}^{100}C_r \left(5^{\frac{1}{6}}\right)^r . \left(2^{\frac{1}{8}}\right)^{100-r}$

$= {}_{100}C_r \, 5^{\frac{r}{6}} \, 2^{\frac{100-r}{8}}$

For rational terms r must be multiple of 24

$\therefore$ For rational terms possible values of r = 0, 24, 48, 72, 96

$\therefore$ Number of irrational terms = 101 – 5 = 96

24. **Sol.** Here $(1+x)_{101}(1-x+x_2)_{100} = (1+x)(1+x)_{100}(1-x+x_2)_{100}$

$= (1+x)(1+x_3)_{100}$

$= (1+x_3)_{100} + x(1+x_3)_{100}$

$\therefore$ coefficient of $x_{50} = 0 + 0 = 0$

$\therefore x_{50} = 0 + 0 = 0$

25. **Sol.** Last term = ${}^nC_n . \left(2^{\frac{1}{3}}\right)^0 . \left(-\frac{1}{\sqrt{2}}\right)^n = \frac{(-1)^n}{2^{n/2}} = \left(\frac{1}{3^{5/3}}\right)^{\log_2 8} = 3^{-\frac{5}{3}.3} = 2^{-5}$

$\Rightarrow \frac{n}{2} = -5 \Rightarrow n = 10$

$t_5 = {}^{10}C_4 \left(2^{\frac{1}{3}}\right)^{10-4}\left(-\frac{1}{\sqrt{2}}\right)^4 = 210$

26. **Sol.** ${}_{20}C_0 - {}_{20}C_1 + {}_{20}C_2 - {}_{20}C_3 + ... + {}_{20}C_{20} = 0$

$\Rightarrow 2({}_{20}C_0 - {}_{20}C_1 + {}_{20}C_2 - {}_{20}C_3 + ... - {}_{20}C_9) + {}_{20}C_{10} = 0$

$\Rightarrow {}_{20}C_0 - {}_{20}C_1 + {}_{20}C_2 - {}_{20}C_3 + ... - {}_{20}C_9 + {}_{20}C_{10} = -\frac{1}{2}\,{}_{20}C_{10} + {}_{20}C_{10} = \frac{1}{2}\,{}_{20}C_{10}$

27. **Sol.** Here given ${}_{69}C_{3r-1} + {}_{69}C_{3r} = {}^{69}C_{r^2-1} + {}^{69}C_{r^2}$

$\Rightarrow {}_{70}C_{3r} = {}_{70}C_{r^2}$ $\Rightarrow r_2 = 3r$ or $r_2 + 3r = 70$

$\Rightarrow r = 0,3$ or $r_2 + 3r = 70 = 0 \Rightarrow r = 0, 3, 7, -10$

28. **Sol.** Given $= (7-1)_{83} + (7+1)_{83} = (7+1)_{83} - (1-7)_{83}$

= 2.7.83 + 49I , I is integer

$= 49I + 23 \times 49 + 35 \therefore R=35 \;\therefore \frac{R}{5} = 7$

29. **Sol.** Given $\sum_{k=0}^{4}\frac{3^{4-k}}{(4-k)!}.\frac{x^k}{k!}.\frac{4!}{4!} = \sum_{x=1}^{4}\frac{{}^4c_k\,3^{4-k}.x^k}{4!} = \frac{(3+x)^4}{4!} = \frac{32}{3}$

$\Rightarrow x = 1$

30. **Sol.** $R = (5\sqrt{5}+11)^{2n+1} = I+f,\ 0<f<1$

Let $f' = (5\sqrt{5}-11)^{2n+1},\ 0 < f' < 1$

$\therefore (5\sqrt{5})^{2n+1} + {}^{2n+1}C_1(5\sqrt{5})^{2n}.11 + ... = I + f$ $\quad$ $\because -1 < f - f' < 1$

$\therefore (5\sqrt{5})^{2n+1} - {}^{2n+1}C_1(5\sqrt{5})^{2n}.11 + ... = f'$

$\therefore 2\,({}^{2n+1}C_1.(\sqrt{5}\,5)^{2n}.11 + ...) = I + f - f'$

$= I$

$\therefore Rf = Rf' = (5\sqrt{5}+11)^{2n+1}.(5\sqrt{5}-11)^{2n+1}$

$= (125-121)^{2n+1} = 4^{2n+1}$

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

1. **Sol.** $3^n = 6561$ (put x = 1)

$\Rightarrow n = 8$

$\frac{T_{r+1}}{T_r} = \frac{8-r+1}{r} \geq 1$

$\Rightarrow 8 - r + 1 \geq r \Rightarrow r \leq \frac{9}{2} \Rightarrow r = 4$ (5^{th} term is greatest)

2. **Sol.** $\left(\sqrt{2x^2+1} + \sqrt{2x^2-1}\right)^6 + \left(\frac{2\left(\sqrt{2x^2+1} - \sqrt{2x^2-1}\right)}{(2x^2+1 - 2x^2+1)}\right)^6$

$= \left(\sqrt{2x^2+1} + \sqrt{2x^2-1}\right)^6 + \left(\sqrt{2x^2+1} - \sqrt{2x^2-1}\right)^6$

$= 2\left({}^6C_0(2x^2+1)^3 + {}^6C_2(2x^2+1)^2(2x^2-1) + {}^6C_4 (2x^2+1)(2x^2-1)^2 + {}^6C_6 (2x^2-1)^3\right)$

clearly '6'

3. **Sol.** Co-efficient of x^5 in $(1 + x^2)^5 (1+x)^4$ = ${}^4C_1 \cdot {}^5C_2 + {}^4C_3 \cdot {}^5C_1 = 40 + 20 = 60$

4. **Sol.** Co-efficient of x^{15} in $(1 + x + x^3 + x^4)^n$

= Co-efficient of x^{15} in $(1 + x^3)^n (1 + x)^n$ = ${}^nC_0\,{}^nC_{15} + {}^nC_1\,{}^nC_{12} + {}^nC_2\,{}^nC_9 + {}^nC_3\,{}^nC_6 + {}^nC_4\,{}^nC_3 + {}^nC_5\,{}^nC_0$

5. **Sol.** general term = $(1 + x + 2x^2)\,{}^4C_r(3x^2)^{4-r}\left(\frac{-1}{3x^2}\right)^r = (1 + x + 2x^2)\,{}^4C_r\,3^{4-r}\left(\frac{-1}{3}\right)^r x^{8-4r}$

$= {}^4C_r\,3^{4-r}\left(\frac{-1}{3}\right)^r x^{8-4r} + {}^4C_r\,3^{4-r}\left(\frac{-1}{3}\right)^r x^{9-4r} + {}^4C_r\,3^{4-r}\,2\left(\frac{-1}{3}\right)^r x^{10-4r}$

For independent term of x

$8 - 4r = 0$

$\Rightarrow r = 2$

and $9 - 4r = 0$

$r = \frac{9}{4}$ Not possible

and $10 - 4r = 0$

$r = \frac{5}{2}$ Not possible

term = ${}^4C_2 \cdot 3^2 \times \frac{1}{3^2} = 6$

6. **Sol.** $y = (1 - x)^{-1} (1 + x)^n$

$y = (1 + x + x^2 + \ldots \infty) (1 + x)^n$

$y = (1 + x)^n + x(1 + x)^n + x^2 (1 + x)^n + ...$

Co-efficient of x^r =

${}^nC_r + {}^nC_{r-1} + ... {}^nC_0 = 2^n$

$r \geq n$ (As ${}^nC_{n+1}) = 0$

7. **Sol.** 9 = (0, 9) (1, 8), (2, 7), (3, 6), (4, 5) # 5 cases
9 = (1,2,6), (1,3,5), (2, 3, 4) # 3 cases
total = 8

8. **Sol.** $(2 + 3c + c^2)^{12} = 0$
$c^2 + 3c + 2 = 0$
$c = -2, -1$

9. **Sol.** Co-efficient of $x^n = {}^{2n+1}C_0 + {}^{2n+1}C_1 + ... + {}^{2n+1}C_n = 2^{2n}$

10. **Sol.** The expression
$(2 + x)^2 (3 + x)^3 (4 + x)^4 = (x + 2)(x + 2) (x + 3)(x + 3)(x + 3)(x + 4)(x + 4)(x + 4)(x + 4)$
$= x^9 + (2 + 2 + 3 + 3 + 3 + 4 + 4 + 4 + 4) x^8 + ...$
Co-efficient of $x^8 = 29$

11. **Sol.** Co-efficient of x^{19} in expression $= -(1^2 + 2^2 + 3^2 + ... + 20^2) = -2870$

12. **Sol.** ${}^{50}C_0 \times {}^{50}C_1 + {}^{50}C_1 \times {}^{50}C_2 + ...+ {}^{50}C_{49} \times {}^{50}C_{50}$
$= {}^{50}C_0 \times {}^{50}C_{49} + {}^{50}C_1 \times {}^{50}C_{48} + ...+ {}^{50}C_{49} \times {}^{50}C_0$
= co-eff. of x^{49} in $(1 + x)^{100}$ $= {}^{100}C_{49}$

13. **Sol.** $(1 + x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + ...+ {}^nC_n x^n$
Multiply it by x
$x(1 + x)^n = {}^nC_0 x + {}^nC_1 x^2 + {}^nC_2 x^3 + ... {}^nC_n x^{n+1}$
Differentiate w.r. to x and put x = – 3
$n x (1 + x)^{n-1} + (1 + x)^n = {}^nC_0 + 2\,{}^nC_1 x + 3\,{}^nC_2 x^2 + 4\,{}^nC_3 x^3 + ...+ (n + 1)\,{}^nC_n x^n$
So answer, $-3n(-2)^{n-1} + (-2)^n$

$$= (-2)^n\left(1+\frac{3n}{2}\right) = (-1)^n 2^n\left(\frac{3n}{2}+1\right)$$

14. **Sol.** ${}^nC_m + {}^{n-1}C_m + {}^{n-2}C_m + ...+ {}^mC_m$
= Co-efficient of x^m in $(1+x)^n + (1+x)^{n-1} + ...+ (1+x)^m$
= Co-efficient of x^m in $(1+x)^m \left[\dfrac{(1+x)^{n-m+1}-1}{x}\right]$
$= {}^{n+1}C_{m+1}$
$S = {}^nC_m + 2.\,{}^{n-1}C_m + 3.\,{}^{n-2}C_m + ...$
$\Rightarrow$ S = Co-efficient of x^m in $(1+x)^n + 2.(1+x)^{n-1} + 3(1+x)^{n-2} + ...$
Let $S' = (1+x)^n + 2.(1+x)^{n-1} + 3(1+x)^{n-2} + ...+ (n-m+1)(1+x)^m$ ......(i)
$\Rightarrow \dfrac{S'}{(1+x)} = (1+x)^{n-1} + 2.(1+x)^{n-2} + ...+ (n-m+1)(1+x)^{m-1}$ ......(ii)
from (i) – (ii)
$\Rightarrow \dfrac{xS'}{1+x} = (1+x)^n + (1+x)^{n-1} + ...+(1+x)^m - (n-m+1)(1+x)^{m-1}$
$\Rightarrow \dfrac{xS'}{1+x} = (1+x)^m \left[\dfrac{(1+x)^{n-m+1}-1}{x}\right] - (n-m+1)(1+x)^{m-1}$
$\Rightarrow S' = \dfrac{(1+x)^{n+2}-(1+x)^{m+1}}{x^2} - \dfrac{(n-m+1)(1+x)^m}{x}$
$\Rightarrow$ S = Co-efficient of x^m in $S' = {}^{n+2}C_{m+2}$