Exercise-1

Marked Questions can be used as Revision Questions.

PART - I : OBJECTIVE QUESTION

Section (1) : Basic question

A-5.

Sol. Clear from the figure

A-6.

Sol. $V_{max} = A\omega = 5 \Rightarrow A\frac{2\pi}{4} = 5 \Rightarrow A = \frac{10}{\pi}$ cm.

A-7. Sol. Satisfy the standard equation of wave

A-8. Sol. **Key Ideal :** The standard wave equation is

$y = a \sin(\omega t - kx)$

The given wave equation is

$y = a \sin\left(100t - \frac{x}{10}\right)$

Compare it with the standard wave equation, we obtain

$\omega = 100,\ k = \frac{1}{10}$

Velocity of the wave,

$v = \frac{\omega}{k} = \frac{100}{\frac{1}{10}} = 100 \times 10 = 1000$ m/s

A-9. Sol. The given waves are

$y_1 = 10^{-6} \sin[100t + (x/50) + 0.5]$ m

and $y_2 = 10^{-6} \cos[100t + (x/50)]$ m

$\Rightarrow$ $y_2 = 10^{-6} \sin\left[100t + (x/50) + \frac{\pi}{2}\right]$m

$\left[\because \sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta\right]$

Hence, the phase difference between the waves is

$\Delta\phi = \left(\frac{\pi}{2} - 0.5\right)\text{rad} = \left(\frac{3.14}{2} - 0.5\right)\text{rad}$

$= (1.57 - 0.5)$ rad $= (1.07)$ rad

Note : The given waves are sine and cosine function so there are plane progressive harmonic waves.

A-10. Sol. Find the parameters and put in the general wave equation.

Here, $A = 2$ cm

direction = +ve x direction

$v = 128\ \text{ms}^{-1}$

and $5\lambda = 4$

Now, $k = \frac{2\pi}{\lambda} = \frac{2\pi \times 5}{4} = 7.85$

and $v = \frac{\omega}{k} = 128\ \text{ms}^{-1}$

$\Rightarrow$ $\omega = v \times k = 128 \times 7.85 = 1005$

As, $y = A \sin(kx - \omega t)$ $\therefore$ $y = 2\sin(7.85\,x - 1005\,t)$

= (0.02)m sin (7.85 x – 1005 t)

A-11. Sol. Equation of wave is

y = a sin $\left(400\pi t - \frac{\pi x}{0.85}\right)$ Comparing this equation with y = a sin $\left(\omega t - \frac{2\pi x}{\lambda}\right)$

ω = 400π, or 2πn = 400π ⇒ n = 200

and λ = 0.85 × 2 = 1.7

Velocity v = n λ = 1.7 × 200 = 340 m/s

A-12. Sol. Given equation is

y = 25 cos (2πt – px) …(1)

Standard equation is

y = A cos (ωt – kx) …(2)

Comparing (1) and (2),

Amplitude A = 25, ω = 2π, k = π

∴ Frequency n = $\frac{\omega}{2\pi} = \frac{2\pi}{2\pi} =$ 1 Hz

A-13. Sol. Here : Equation of the wave is

y = 0.3 sin (314 t – 1.57 x) ...(1)

The standard equation of wave is

y = a sin $2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right]$ ...(2)

Compairing the given equation (1) and standard equation (2), we get

y = 0.3 sin $2\pi\left[\frac{314t}{2\pi} - \frac{1.57x}{2\pi}\right]$

= 0.3 sin $2\pi\left(50t - \frac{x}{4}\right)$ or $\frac{1}{T}$ = 50 and λ = 4

Hence, the velocity of the wave is

v = nλ = $\frac{1}{T}$ × 4 = 50 × 4 = 200 m/s

Section (B) : Equation of travelling wave

B-1.

Sol. As $\frac{5\lambda}{2}$ = 20 ⇒ λ = 8 cm

K = $\frac{2\pi}{\lambda} = \frac{314}{4}$

ω = KV $\frac{2\pi}{8\times10^{-2}}$ × 350 = 27500

∴ y = 0.05 sin $\left(\frac{314}{4}x - 27500t\right)$

B-2. Sol. $V_{P_{max}}$ = Aω = Y_0 2πf = 4V_ω

Y_0 2πf = 4 $\frac{2\pi f}{\frac{2\pi}{\lambda}}$

$\therefore \quad \lambda = \frac{\pi Y_0}{2}$

B-3. **Sol.** Put α, β, A, x and t in the equation

$\frac{2\pi}{\lambda} = 0.56 \text{ cm}^{-1}$

$2\pi f = 12 \text{ sec}^{-1}$

$\alpha x + \beta t + \frac{\pi}{6} = \frac{12.56 \times 180}{3.14} + 30 = 750^o$

$y = 7.5 \text{ cm} \sin 750^o = 3.75$ cm.

$v = \frac{dy}{dt} = Ab \cos\left(\alpha x + \beta t + \frac{\pi}{6}\right) = 7.5 \times 12 \times \frac{\sqrt{3}}{2} = 77.94$ cm/sec.

B-4. **Sol.** $\omega = 2\pi f = 4\pi \text{ sec}^{-1}$

$K = \frac{2\pi}{\lambda} = 2\pi \text{ m}^{-1}$ $\therefore y = 0.5 \cos(2\pi x + 4\pi t)$

B-5.

Sol. $V_{CD} = \sqrt{\frac{3.2g}{8 \times 10^{-3}}} = \sqrt{4000} \cong 63 \frac{m}{sec}$

$V_{AB} = \sqrt{\frac{6.4g}{10 \times 10^{-3}}} \cong \sqrt{6400} \cong 79 \frac{m}{sec}$

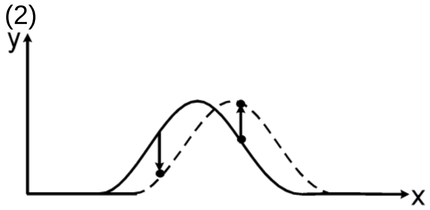
B-6.

Sol. $R_A = \frac{V}{V_A}$, $R_B = \frac{V}{V_B}$

as $V_A > V_B$, $R_A < R_B$

B-7.

Sol. (2)



Dotted shape shows pulse position after a short time interval. Direction of the velocities are decided according to direction of displacements of the particles.

at x = 1.5 slope is +ve

at x = 2.5 slope is –ve

B-8.

Sol. $V \propto \sqrt{T}$

$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{T/2}{T}} = \frac{1}{\sqrt{2}}$

Section (3) : Interference, Reflection, Transmission, Power Transmited along the string.

C-1. Sol. By defination

C-3. Sol. Amplitude varies between 0 and 2A

C-4.
Sol. Path difference is λ between B and G.

C-5. Sol. By defination

C-6. Sol. As $y = A \sin (Kx - \omega t + 30^o)$ for incident wave
Now for reflected wave : Energy α Amp^2
$\therefore Y = 0.8 A \sin (-Kx - \omega t + 30 + 180)$
$Y = 0.8 A \sin (-Kx - \omega t + 210)$
$Y = -0.8 A \sin (kx + \omega t - 210)$
$Y = -0.8 A \sin [Kx + \omega t - 30 - 180]$
$Y = 0.8 A \sin [180 - (Kx + \omega t - 30)]$
$Y = 0.8 A \sin (Kx + \omega t - 30)$

C-7. Sol. $V_{vel.} = 10 + 10 = 20 \frac{m}{sec}$
when string is flate $v = f\lambda$
$20 = \frac{1}{\Delta t} \lambda$
$\lambda = 20 \Delta t = 10$ m.

Section (4) : Standing wave and resonance

D-5. Sol. $\frac{n}{2\ell}\sqrt{\frac{T}{\mu}} = 350$ and $\frac{n+1}{2\ell}\sqrt{\frac{T}{\mu}} = 420$

$\therefore \quad \frac{n}{n+1} = \frac{350}{420} \Rightarrow n = 5 \therefore \quad \frac{5\lambda}{2} = \ell \Rightarrow \lambda = \frac{2\lambda}{5}$

$\frac{v}{f} = \frac{2\ell}{5} \quad \Rightarrow \frac{v}{2\ell} = \frac{f}{5} \quad \Rightarrow \quad f' = \frac{f}{5} = 70$ Hz

D-7. Sol. The frequency of vibrating wire is $n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$, where T is the tension in the wire.
We have $n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$
Here, m = mass per unit length = $\pi r^2 d$

$\therefore \quad n = \frac{1}{2\ell}\sqrt{\frac{T}{\pi r^2 d}}$

or $\quad n \propto \frac{1}{r}\left(\frac{T}{d}\right)^{1/2}$

$\therefore \quad \frac{n_1}{n_2} = \frac{r_2}{r_1}\left(\frac{T_1}{T_2} \times \frac{d_2}{d_1}\right)^{1/2}$

We have given,

$\frac{T_1}{T_2} = \frac{1}{2}$, $\frac{d_1}{d_2} = 2$, $\frac{r_1}{r_2} = \frac{1}{2}$

∴ $\frac{n_1}{n_2} = \frac{2}{1}\left(\frac{1}{2} \times \frac{1}{2}\right)^{1/2}$

or $\frac{n_1}{n_2} = \frac{2}{1} \times \frac{1}{2} = 1$

or $n_2 = n_1 = n$

Exercise-2

Marked Questions can be used as Revision Questions.

PART - I : ONLY ONE OPTION CORRECT TYPE

2. Sol. $\frac{\text{measure AB}}{\text{measure CD}} = \frac{\lambda}{T} = V_\Omega$

3. Sol. $V_\Omega = \frac{W}{K} = \frac{420}{21} = 20$

∴ $V = \sqrt{\frac{T}{\mu}} = 20$ ⇒ $T = (20)^2 \mu = (20)^2 \times 0.2 = 80N$

6. Sol. $\frac{\omega}{K} = V_\omega$ for either component waves

7. Sol. $\frac{n}{2\ell}\sqrt{\frac{T}{\mu}} = 384$ $\frac{n-1}{2\ell}\sqrt{\frac{T}{\mu}} = 288$

∴ $\frac{n}{n-1} = \frac{4}{3}$ ∴ n = 4 Now ; $4\left(\frac{V}{2L}\right) = 384$

Put L = 75 cm ∴ V = 144 m/sec.

11. Sol. By defination

12. Sol. As $\langle P \rangle = 2\pi^2 f^2 A^2 \mu v$ put values

13. Sol. $\mu = \frac{m}{\ell} = \rho A$

∴ $m_1 = \rho\pi r^2$

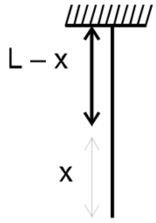
$m_2 = \rho 4 \pi r^2$

∴ $\frac{v_1}{v_2} = \frac{\sqrt{T/\mu_1}}{\sqrt{T/\mu_2}}$

Let P loops and q loops are formed respectively 1st and 2nd ωire.

∴ $\frac{p}{2\ell} V_1 = \frac{q}{2\ell} V_2$ ⇒ $\frac{p}{q} = \frac{1}{2}$

14. Sol.



For the pulse;

$V = \sqrt{\frac{\mu x g}{\mu}} = \sqrt{xg} = \frac{dx}{dt}$

$\frac{dx}{dt} = \sqrt{xg} \Rightarrow \int_o^x \frac{dx}{\sqrt{x}} = \sqrt{g}\int_o^t dt$

$t = 2\sqrt{\frac{x}{g}}$ ———(1)

for the particle ,

$L - x = \frac{1}{2} gt^2$

$t = \sqrt{\frac{2(L-x)}{g}}$ ———(2)

1 = 2 $\Rightarrow \therefore x = \frac{L}{3}$ from the bottom

15.

Sol. As $f_1 = f$, $f_2 = \frac{f}{2}$, $f_3 = f$

$\therefore \omega_1 = 2\pi f \quad \Rightarrow \omega_3 = 2\pi f \quad$ and $\omega_2 = \pi f$

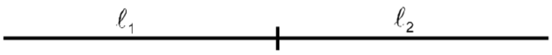
16. **Sol.** $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\times 1}\sqrt{\frac{\frac{YA}{\ell}.\Delta\ell}{\rho A}} = \frac{1}{2}\sqrt{\frac{9\times 10^{10}\times 5\times 10^{-14}}{9\times 10^3}}$ = 35 Hz

17. **Sol.** $y = a \sin \omega t \cos Kx$

$y = \frac{1}{2}(2a \sin \omega t \cos Kx)$

$\therefore$ Amplitude of component wave is $a/2$

18. **ol.**



and $n = \frac{v}{2\ell}$ (for complete length of wire)

$n_1 = \frac{v}{2\ell_1} \qquad n_2 = \frac{v}{2\ell_2}$

As $\ell = \ell_1 + \ell_2 + ...$

$\frac{V}{2n} = \frac{V}{2n_1} + \frac{V}{2n_2} + ...$

$\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + ...$

19. **Sol.** Given equation is

$y = 0.3 \sin (314\, t - 1.57\, x)$

Standard equation is

$y = A \sin(\omega t - kx)$

Comparing (1) and (2), we get

$\omega = 314$ rad/s

$k = 1.57\ m^{-1}$ ∴ Velocity of wave, $\upsilon = \frac{\omega}{k} = \frac{314}{1.57} = 200$ m/s

20. **Sol.** $y = \frac{1}{1+x^2}$ (t = 0)

$y = \frac{1}{1+(x-2v)^2}$ (t = 2)

Now comparing

$x - 2v = x - 1$

$v = 0.5\ \frac{m}{sec}$

$y = \frac{1}{1+x^2}$ (t = 0) ⇒ $y = \frac{1}{1+(x-2v)^2}$ (t = 2)

$x - 2v = x - 1$

$v = 0.5\ \frac{m}{sec}$

PART - II : MISCELLANEOUS QUESTIONS

A-1. **Sol.** **(4)** Every small segment is acted upon by forces from both sides of it hence energy is not conserved, rather it is transmitted by the element.

A-2. **Sol.** **(4)** Two waves moving in uniform string with uniform tension shall have same speed and may be moving in opposite directions. Hence both waves may have velocities in opposite direction. Hence statement-1 is false

Section (B) : Match the column

B-1. **Ans.** (1 → R); (2 → P); (3 → S); (4 → Q).

Sol. Use x = 0; t = 0 for y and particle velocity $\frac{\partial y}{\partial t}$. Like for (a), y = 0 at x = 0 and t = 0. $\frac{\partial y}{\partial t} > 0$ i.e. positive therefore it matches with (R).

Section (C) : One or More Than One Options Correct

C-1*. **Sol.** Standard equation

C-2*. **Sol.** Compare with $y = a \sin(\omega t + Kx)$

C-3*. **Sol.** Compare with y = A sin (ωt – Kx

C-4*. **Sol.** Satisfy the standard equation of wave

C-5*. **Sol.** Comparing with $y = A \cos(\omega t - kx)$

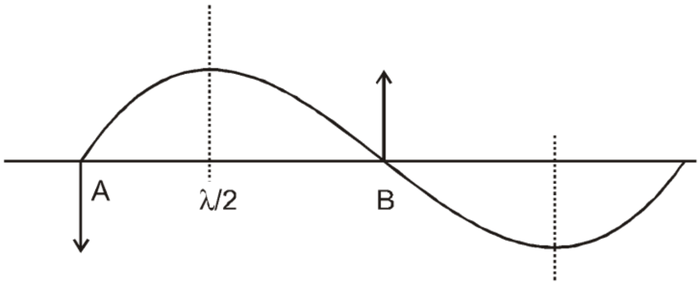
$\omega = 500\ s^{-1}$, $k = 0.025\ m^{-1}$,

$v = \frac{500}{0.025} = 2 \times 10^4$ m/s

$\lambda = \frac{2\pi}{0.025} = 80\pi$ m

$y = A \cos(\omega t - kx)$

C-6*. **Sol.**



A and B point will move in opposite direction displacement level of A and B will be equal

C-7*. **Sol.** $\ell = 1$ m $\quad \varepsilon = 10 \sin(80\pi t - 4\pi x)$

For superimposed second wave is

$\varepsilon_2 = 10\sin(80\pi t + 4\pi x)$

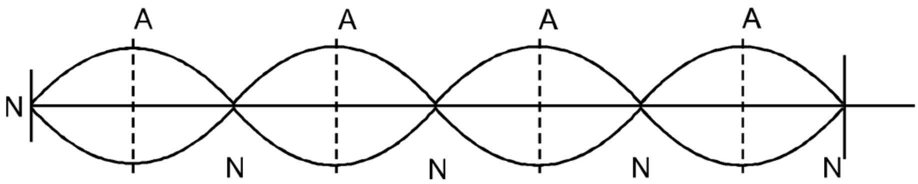
Amplitude of stationary wave = 2A = 2 × 10 = 20m

$K = 4\pi = \frac{2\pi}{\lambda}$

$\lambda = \frac{2\pi}{4\lambda} = \frac{1}{2} = 0.5$ m

$\ell = 1$m

($\ell = 2\lambda$)



Total (N = 5)

Exercise-3

Marked Questions can be Revision Questions.

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. **Sol.** $y_1 + y_2 = a \sin(\omega t - kx) - a \sin(\omega t + s\, kx)$

$= -2a \cos \omega t \times \sin kx$

$\Rightarrow \quad y_1 + y_2 = 0$ at $x = 0$

2. **Sol.** $\frac{\lambda}{2} = \ell \qquad \lambda = 80$ cm

3. **Sol.** $V = \frac{\omega}{k} = \frac{600}{2} = 300$ m/sec

4. **Sol.** $\eta = \frac{2}{2\ell}\sqrt{\frac{T}{\mu}} = 100$ Hz

5. **Sol.** $V = \frac{\omega}{k} = \frac{100}{20} = 5$ m/sec

6. **Sol.** $(n+1)\frac{v}{2\ell} = 420$ ......(1) $\frac{nv}{2\ell} = 315$ ......(2)

(1) – (2) $\frac{V}{\mu}$ = 105 Hz f_{min} = = 105 Hz

7. **Sol.** $\frac{2\pi}{\lambda} = \alpha$ $\alpha = \frac{2\pi}{0.05} = 25\pi$ $\frac{2\pi}{T} = \beta = 2$ $T = \pi$

8. **Sol.** By equation

$f = \frac{1}{0.04}$ and $\lambda = 0.5$

$\Rightarrow$ $V = \frac{1}{0.04} \times 0.5 = \frac{25}{2}$

by $V = \sqrt{\frac{T}{\mu}}$ $\Rightarrow$ $\left(\frac{25}{2}\right)^2 = \frac{T}{0.04} \Rightarrow T = \frac{625}{4} \times 0.04$

T = 6.25 N

9. **Ans.** **(2)**

Sol. $y(x,t) = e^{-[\sqrt{a}x+\sqrt{b}t]^2}$

It is transverse type $y(x,t) = e^{-(ax+bt)^2}$

Speed $v = \frac{\sqrt{b}}{\sqrt{a}}$

and wave is moving along –x direction.

$y(x,t) = e^{-[\sqrt{a}x+\sqrt{b}t]^2}$

10. **Ans.** **(4)**

Sol. Y = A sin(ωt – kx) + A sin(ωt + kx)

Y = 2A sinωt coskx standing wave

For nodes, coskx = 0

$\frac{2\pi}{\lambda}.x = (2n+1)\frac{\pi}{2}$ $\therefore$ $x = \frac{(2n+1)\lambda}{4}$, n = 0,1,2,3,........

11. **Ans.** **(1)**

Sol. Since, $I \propto A^2 \omega^2$

$I_1 \propto (2a)^2 \omega^2$

$I_2 \propto a^2 (2\omega)^2$

$I_1 = I_2$

Intensity depends on frequency also.

12. **Sol.** $f = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{Ad}}$

Also $Y = \frac{T\ell}{A\Delta\ell}$ $\Rightarrow$ $\frac{T}{A} = \frac{Y\Delta\ell}{\ell}$ $\Rightarrow$ $f = \frac{1}{2\ell}\sqrt{\frac{y\Delta\ell}{\ell d}}$

$\ell = 1.5$m, $\frac{\Delta\ell}{\ell} = 0.01$, $d = 7.7 \times 10^3$ kg/m^3

$y = 2.2 \times 10^{11}$ N/m^2

After solving

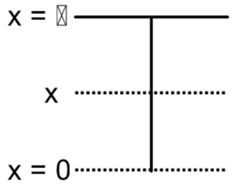
$f = \sqrt{\frac{2}{7} \times \frac{10^3}{3}}$ Hz

$f \approx 178.2$ Hz

Ans. (2)

13. Ans. (2)

Sol. Let mass per unit length be λ.



$T = \lambda g x$ $v = \sqrt{\frac{T}{\lambda}} = \sqrt{gx}$

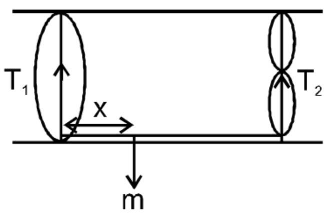
$v^2 = gx,$

$a = \frac{vdv}{dx} = \frac{g}{2}$

$\ell = \frac{1}{2}\frac{g}{2}t^2 \Rightarrow t = \sqrt{\frac{4\ell}{g}} = 2\sqrt{2}$ sec

14.

Sol.



$\frac{\lambda_1}{2} = \ell \Rightarrow \lambda_1 = 2\ell$

$\lambda_2 = \ell \quad \Rightarrow \therefore \frac{\lambda_1}{\lambda_2} = 2$

$\frac{v_1/f}{v_2/f} = 2$

$\frac{v_1}{v_2} = 2 = \sqrt{\frac{T_1/\mu}{T_2/\mu}} \Rightarrow \frac{T_1}{T_2} = 4$ ——(1)

Now moment about P : $T_1 x = T_2(\ell - x)$

$\ell - x = 4x$ $\quad x = \ell/5$

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. Sol. $f = \frac{1}{2\ell}\sqrt{\frac{T}{\rho A}}$ $\quad \therefore \frac{f_1}{f_2} = \frac{\frac{1}{2L}\sqrt{\frac{T}{\rho}\pi 4r^2}}{\frac{1}{4L}\sqrt{\frac{T}{\rho\pi r^2}}} = \frac{1}{1}$

2. **Sol.** Energy $\propto A^2\,\omega^2$

$\therefore \frac{E_1}{E_2} = \frac{A^2\omega^2}{A^2(2\omega)^2}$ $\therefore$ $E_2 = 4E_1$

3. **Sol.** After 2 sec.
$D = D_1 + D_2 = (2 \times 2) - (2 \times 2) = 0$
As their amplitude is same
$\therefore$ P.E. = 0
$\therefore$ purely kinetic

4. **Sol.** $f = \frac{5}{2L}\sqrt{\frac{9g}{\mu}}$ noω $f^1 = \frac{3}{2L}\sqrt{\frac{mg}{\mu}}$
as f $f = f^1$ $\Rightarrow$ m = 25 Kg

5. **Sol.** $f = \frac{5}{2L}\sqrt{\frac{9g}{\mu}}$ now $f' = \frac{3}{2L}\sqrt{\frac{Mg}{\mu}}$
as f $f = f'$ $\Rightarrow$ M = 25 Kg

6. **Sol.** Maximum particle velocity $v_{max} = \omega A = 3$ m/s

Maximum particle acceleration $a_{max} = \omega^2 A = 90$ m/s^2

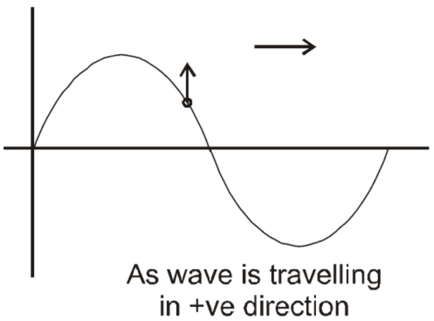
$a_{max} = v_{max}\,\omega = \omega \times 3 = 90$ m/s^2 $\Rightarrow$ $\omega = 30$ s^{-1} $\Rightarrow$ $A = \frac{3}{\omega} = \frac{3}{30} = 0.1$ m

$k = \frac{\omega}{v} = \frac{30}{20} = \frac{3}{2}$ [where v is velocity of wave]

Equation of wave in string $y = 0.1 \sin\left(30 \quad t \quad \pm \frac{3}{2}x + \phi\right)$ [where ϕ is initial phase]

Ans. Equation of wave in string $y = 0.1 \sin\left(30 \quad t \quad \pm \frac{3}{2}x + \phi\right)$ [where ϕ is initial phase]

7 **Sol.**



$V = \omega\sqrt{A^2 - y^2}$

$V_P = 2\pi f\sqrt{A^2 - y^2}$

$V_P = 2\pi\left(\frac{V}{\lambda}\right)\sqrt{A^2 - y^2} = \frac{2\pi}{0.5} \times 0.1\sqrt{(0.1)^2 - (0.05)^2}$

$V_P = \frac{\sqrt{3}\pi}{50}$ j m/s

Ans. (1)

8. **Sol.** $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{0.5}{10^{-3}/0.2}} = 10$ m/sec.

$v = f\lambda$

$10 = (100)\lambda \quad \Rightarrow \quad \lambda = 0.1$ m = 10 cm

distance between two successive nodes = $\frac{\lambda}{2} = 5$ cm

Ans. 5

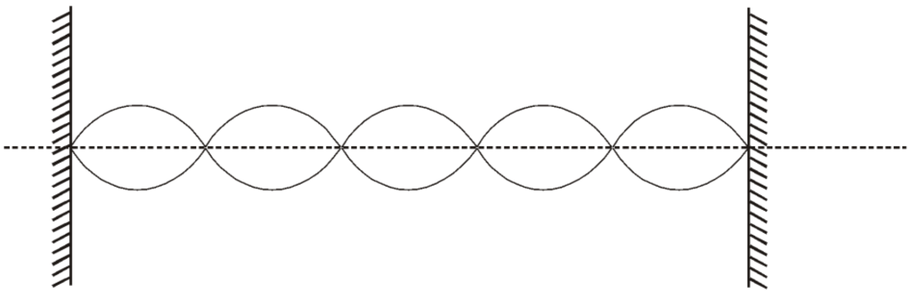
9. **Ans.** **5**

Sol. $A_{eq} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$A_{eq} = \sqrt{4^2 + 3^2 + 2(4)(3)\cos\frac{\pi}{2}}$

$A_{eq} = 5$.

10.* **Sol.**



(1) There are 5 complete loops.

Total number of nodes = 6

(2) $\omega = 628\ \text{sec}^{-1}$

$k = 62.8\ \text{m}^{-1} = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{1}{10}$

$v_w = \frac{\omega}{k} = \frac{628}{62.8} = 10\ \text{ms}^{-1}$

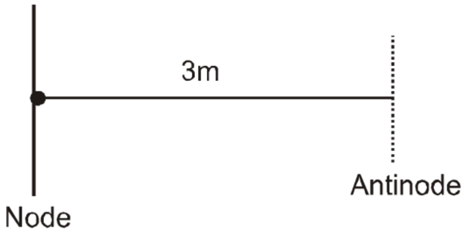
$L = \frac{5\lambda}{2} = 0.25$

(3) 2A = 0.01 = maximum amplitude of antinode

(4) $f = \frac{v}{2\ell} = \frac{10}{2 \times 0.25} = 20$ Hz.

11.* **Ans.** **(A), (C), (D)**

Sol. V = 100 m/s



Possible modes of vibration

$\ell = (2n+1)\frac{\lambda}{4}$

$\lambda = \frac{12}{(2n+1)}$ m

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{12/(2n+1)} = \frac{(2n+1)\pi}{6}$

$\omega = vk = 100\ (2n + 1)\ \frac{\pi}{6} = \frac{(2n+1)\ \ 50\pi}{3}$

if n = 0 $k = \frac{\pi}{6}$ $\omega = \frac{50\pi}{3}$

n = 1 $k = \frac{5\pi}{6}$ $\omega = \frac{250\pi}{3}$

n = 7 $k = \frac{5\pi}{2}$ $\omega = 250\ \pi$

12. **Ans.** (ABD or BD)

Sol. $V = \sqrt{\frac{T}{\mu}}$, so speed at any position will be same for both pulses, therefore time taken by both pulses will be same.

$\lambda f = v = \Rightarrow \lambda = \lambda \propto V$, since when pulse 1 reaches at A, speed decreases therefore λ decreases.

Note : If we refer velocity by magnitude only, then option (A, B, D) will be correct, else only (B, D) will be correct.

Additional Problems For Self Practice (APSP)

PART-I : PRACTICE TEST PAPER

1. **Sol.** $V = \sqrt{\frac{T}{\mu}} = \frac{YA\alpha\Delta T}{\mu}$ Put values

2. **Sol.** Distance between boat = $\frac{\lambda}{2}$ = 10 m

$\Rightarrow$ λ = 20m

time penod T = 4 sec .

$\therefore$ $V = \lambda / T$ = 20 m / 4sec.

= 5m/s.

3. **Sol.** Second string is denser so speed will decrease hence wavelength also $\lambda^1 < \lambda$

4. **Sol.** At $x_1 = \frac{\pi}{3K}$ and $x_2 = \frac{3\pi}{2K}$

Nodes are not formed because neither x_1 nor x_2 gives sin kx = 0

$\therefore \Delta x = x_2 - x_1 = \frac{7\pi}{6K}$

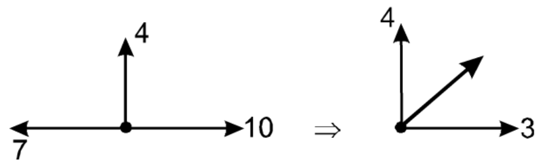
As this Δx is between λ and $\frac{\lambda}{2}$

$\therefore \phi_1 = \pi$

and $\phi_2 = K\Delta x = \frac{7\pi}{6}$ $\therefore \frac{\phi_1}{\phi_2} = \frac{6}{7}$

5. **Sol.** $v = f\lambda = \frac{54}{60} \times 10 = 9$ m/sec.

6. **Sol.**



Resultant Amplitude $= \sqrt{3^2 + 4^2} = 5\mu m$

7. **Sol.** $n = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$ $n = \frac{1}{2\ell}\sqrt{\frac{YA\alpha T}{\mu}}$

Put values $\therefore n = 11$ Hz

8. **Sol.** As, $T = \frac{YA}{20} 4 \Rightarrow$ $v_0 = \frac{1}{2 \times 24}\sqrt{\frac{YA}{20}.\frac{4}{\mu}}$ ....(1)

and $T' = \frac{YA}{20} 6 \Rightarrow$ $v' = \frac{1}{2 \times 26}\sqrt{\frac{YA}{20}.\frac{6}{\mu}}$ ....(2)

$\therefore \frac{v'}{v_0} = \frac{24}{26}\sqrt{\frac{6}{4}}$ $\therefore v' > v_0$

9. **Sol.** $K = 0.025\,\pi = \frac{2\pi}{\lambda}$ $\lambda = \frac{2cm}{0.025}$

Required length $= \frac{\lambda}{2} = \frac{1}{0.025} = 40$ cm

10. **Sol.** $2\lambda = \ell \Rightarrow \lambda = \frac{\ell}{2}$

As x = 0 is a node

$\therefore A_{s.\omega.} = A \sin \frac{2\pi}{\lambda} x$

$= a \sin\left(\frac{\frac{2\pi}{\lambda}\frac{1}{3}}{2}\right) = a\frac{\sqrt{3}}{2}$

11. **Sol.** $V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{4.5 \times 10^7}{0.05}}$

$\frac{n}{2\ell}\sqrt{\frac{4.5 \times 10^7}{0.05}} = 420$ ———(1)

$\frac{n+1}{2\ell}\sqrt{\frac{4.5 \times 10^7}{0.05}} = 490$ ———(2)

$1 \div 2 \Rightarrow \frac{n}{n+1} = \frac{6}{7} \Rightarrow n = 6$

Put n in (1) $\therefore \frac{6}{2\ell}\ 3 \times 10^2 = 420$

$\ell = \frac{30000}{140}$ $\ell = \frac{1500}{7}$ = 214 cm

12. **Sol.** By difination

13. **Sol.** $p_0 = A_0^2\ \omega_0^2\ \mu v$

$\frac{p_0}{2} = A^2\ \omega^2\ \mu v$

$\therefore 2 = \frac{A_0^{\ 2}\omega_0^{\ 2}}{A^2\omega^2}$

$\therefore$ As, $\omega = \omega_0$ (frequency remains the same)

$\therefore A = \frac{A_0}{\sqrt{2}}$

15. **Sol.** As x = 0 is node $\Rightarrow$ standing wave should be $y = 2a \sin kx \sin \omega t$

17. **Sol.** **Key Ideal :** The expression of travelling wave is sine or cosine function of $\omega t \pm kx$.

The general expression of travelling wave can be written as

$y = A \sin (\omega t \pm kx)$ ...(i)

For travelling wave along positive x-axis we should use minus (–) sign only

$\therefore$ $y = A \sin (\omega t - kx)$

but $\omega = \frac{2\pi v}{\lambda}$ and $k = \frac{2\pi}{\lambda}$

So, $y = A \sin \frac{2\pi}{\lambda} (vt - x)$ ...(ii)

Given, A = 0.2 m , v = 360 m/s , λ = 60 m

Substituting in Eq. (ii), we have

$y = 0.2 \sin \frac{2\pi}{60} (360t - x)$

or $y = 0.2 \sin 2\pi \left(6t - \frac{x}{60}\right)$

18. **Sol.** **Key Idea :** The standard transverse wave propagating along x -axis can be written as

$y = a \sin(kx - \omega t + \phi)$

The given equation is

$y(x, t) = 8.0 \sin \left(0.5\pi x - 4\pi t - \frac{\pi}{4}\right)$ ...(i)

The standard wave equation can be written as ,

$y = a \sin(kx - \omega t + \phi)$ ...(ii)

where a is amplitude, k the propagation constant and ω the angular frequency, comparing the Eqs. (i) and (ii), we have

$k = 0.5\pi$, $\omega = 4\pi$

$\therefore$ Speed of transverse wave

$v = \frac{\omega}{k} = \frac{4\pi}{0.5\pi}$ = 8 m/s

19. **Sol.** The given wave euqation is

$y = A\sin(\omega t - kx)$

Wave velocity, $v = \frac{\omega}{k}$ ...(i)

Particle velocity, $v_p = \frac{dy}{dt} = A\omega\cos(\omega t - kx)$

Maximum particle velocity, $(v_p)_{max} = A\omega$ ...(ii)

According to the given question

$v = (v_p)_{max}$

$\frac{\omega}{k} = A\omega$ (Using (i) and (ii)

$\frac{1}{k} = A$ or $\frac{\lambda}{2\pi} = A$ $\left(\because k = \frac{2\pi}{\lambda}\right)$ $\lambda = 2\pi A$

20. **Sol.** $60 = \sqrt{\frac{mg}{\mu}}$ ....(1)

$62 = \sqrt{\frac{m(g^2 + a^2)^{1/2}}{\mu}}$ ....(2)

Solve (1) and (2)

$\therefore a = 3.7 \frac{m}{sec^2}$

21. **Sol.** Given $\omega = 3\pi$

$\therefore$ $f = \frac{\omega}{2\pi} = 1.5$,

Also $\Delta x = 1.0$ cm

Given, $\phi = \frac{2\pi}{\lambda}\Delta x$ $\Rightarrow$ $\frac{\pi}{8} = \frac{2\pi}{\lambda} \times 1$

$\Rightarrow$ $\lambda = 16$ cm $\Rightarrow$ $v = f\lambda = 16 \times 1.5 = 24$ cm/sec

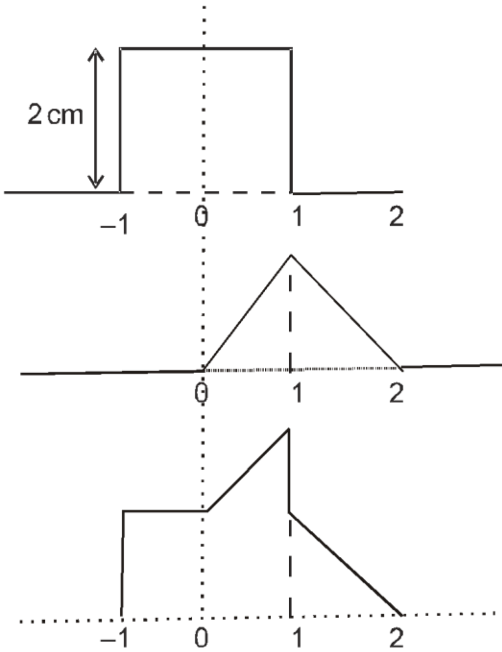
22. **Sol.** $y = a \sin \frac{2\pi}{\lambda}\ vt - x)$

$y = 2 \sin \frac{2\pi}{\lambda}\ (24t - x)$

for t = 1, x = 4 cm

$y = 2\sin\left(\frac{2\pi}{16} \times 20\right) = 2 \sin\left(2\pi + \frac{\pi}{2}\right) =$ $y = 2$ cm. **Ans.**

23. **Sol.** At t = 2 second, the position of both pulses are separately given by fig.(a) and fig. (b); the superposition of both pulses is given by fig. (c)



24. **Sol.** y (x, t) = 2 sin (0.1 πx) cos (100 πt)
compare with
$y = A \sin (Kx) \cos \omega t$
$K = 0.1\,\pi = \frac{2\pi}{\lambda}$
$\lambda = 20$ cm
$\frac{\pi}{4} = \frac{20}{4} = 5$ cm

25. **Sol.** $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$
If radius is doubled and length is doubled, mass per unit length will become four times. Hence
$f' = \frac{1}{2 \times 2\ell}\sqrt{\frac{2T}{4\mu}} = \frac{f}{2\sqrt{2}}$

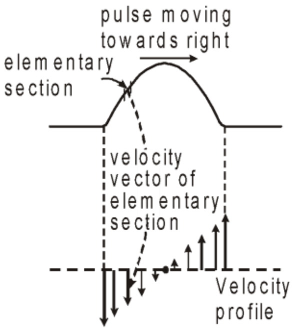
26. **Sol.** Wavelength and velocity are medium dependent. Final amplitude is decided by the superposition of individual amplitudes.

27. **Sol.** As wave has been reflected from a rarer medium, therefore there is no change in phase. Hence equation for the opposite direction can be written as
$y = 0.5A \sin (-kx - \omega t + \theta)$
$= -0.5A \sin (kx + \omega t - \theta)$

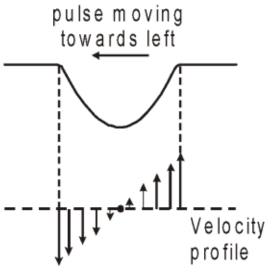
PART - II : PRACTICE QUESTIONS

1. **Ans.** The waves can be added using a phasor diagram.

2. **Sol.** The velocity of profile of each elementary section of the pulse is shown in figure 1 and figure 2.



(Figure-1)



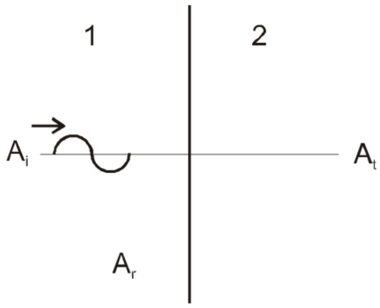
(Figure-2)

When both the pulses completely overlaps, the velocity profiles of both the pulses in overlap region are identical. By superposition, velocity of each elementary section doubles. Therefore K.E. of each section becomes four times. Hence the K.E. in the complete width of overlap becomes four times, i.e., 4k.

3. **Sol.** $A_r = 0.02 \times 0.75 = 0.015$; $y_r = +0.015 \sin 8\pi\left[t + \frac{x}{20}\right]$; their is no phase difference of π produced as the reflection is from rarer medium.

$A_r = 0.02 \times 0.75 = 0.015$; $y_r = +0.015 \sin 8\pi\left[t + \frac{x}{20}\right]$;

4. **Sol.**



$V_2 = \frac{V_1}{4}$

$$\left|\frac{A_t}{A_r}\right| = \frac{\frac{2V_2}{V_1+V_2}A;}{\frac{V_2-V_1}{V_1+V_2}A;}$$

$$= \left(\frac{2V_2}{V_2-V_1}\right)$$

$$= \left(\frac{2.\frac{V_1}{4}}{\frac{V_1}{4}-V_1}\right) = \frac{2.\frac{V_1}{4}}{\frac{3V_1}{4}} = \frac{2}{3}$$

5. **Sol.** $\frac{\lambda_n}{2} = 6$ cm $\Rightarrow$ $\frac{2L}{2n} = 6$ $\Rightarrow$ $n = \frac{L}{6}$

$\frac{\lambda_{n+1}}{2} = 4$ cm $\Rightarrow$ $\frac{2\ L}{2(n+1)} = 4$ $\Rightarrow$ $n+1 = \frac{L}{4}$

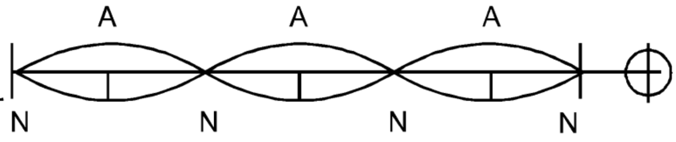
$\Rightarrow$ $\frac{L}{4} - \frac{L}{6} = 1$ $\Rightarrow$ L = 12 cm

6.

Sol. At point C node and at B antinode $\frac{\lambda}{4} = 20$ cm

7. **Sol.** Fundamental mode $\longrightarrow L = \frac{\lambda}{2} \Rightarrow \lambda = 2L$

$f_0 = \frac{V}{\lambda} = \frac{V}{2L}$



$L = \frac{3\lambda'}{2} \Rightarrow \lambda' = \frac{2L}{3} = \frac{\lambda}{3}$ and $f' = \frac{V}{\lambda'} \Rightarrow f' = 3f_0$

II^{nd} overtone

So it contain 4 node and –3 antinode frequency is 3 time at fundamental frequency wavelength is $\frac{1}{3}$ time at fundamental wavelength.

8.

Sol. Length of BC to AC is $\frac{20}{80} = \frac{1}{4}$

So, value of loops in BC to AC will also be 1 : 4

So, of vibration will be

$f_1 = \frac{1}{4(0.2)}\sqrt{\frac{T}{\mu}} = 125$ Hz

$f_2 = \frac{3}{4(0.2)}\sqrt{\frac{T}{\mu}} = 275$ Hz.

9.

Sol. Initially,

$f_0 = \frac{3}{2\ell}\sqrt{\frac{T}{\mu}}$

$f_0 = \frac{3}{2 \times 1}\sqrt{\frac{100}{0.01}} = 150$ Hz

In the second case suppose the tuning fork is in resonance with n^{th} harmonic

$f_0 = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$

$150 = \frac{n}{2\ell}\sqrt{\frac{400}{0.01}}$ $\qquad \ell = \frac{n}{1.5}$

n = 1 $\quad \ell_1 = \frac{1}{1.5} = 0.67$

n = 2 $\quad \ell_2 = \frac{2}{1.5} = 1.33$

n = 3 $\quad \ell_3 = \frac{3}{1.5} = 2$m.

10.

Sol. $\frac{T_1}{T_2} = \frac{(3)^2}{(2)^2} = \frac{9}{4}$, $T_2 = T_1 - B$ $\Rightarrow$ $B = T_1 - T_2$

specific gravity of block = $\frac{\text{wt. in air}}{\text{loss in wt. in water}} = \frac{T_1}{T_1 - T_2}$

$= \frac{1}{1 - \frac{T_2}{T_1}} = \frac{1}{1 - \frac{4}{9}} = \frac{9}{5} = 1.8$

11. **Sol.** $420 = \frac{P}{2L}\sqrt{\frac{T}{m}}$

$490 = \frac{P+1}{2L}\sqrt{\frac{T}{m}}$

$\frac{490}{420} = \frac{P+1}{P}$

P = 6

substituting P = 6 in above equation

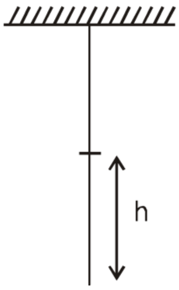
$420 = \frac{P}{2L}\sqrt{\frac{T}{m}}$

$L = \frac{10}{7}$ m

12. **Sol.** $v = f\lambda$ $\Rightarrow$ $\lambda = \frac{300}{(1/0.04)} = 12$

$\Delta\phi = \frac{2\pi}{\lambda}(x_2 - x_1) = \frac{2\pi}{12}(6) = \frac{\pi}{1} = \pi$.

13. **Sol.**



$T_h = \mu hg$

$V_h = \sqrt{\frac{T_h}{\mu}}$

$V_h = \sqrt{gh}$

$v_h^2 = gh$

14. **Sol.** $y = A\sin(-\omega t + kx + \phi)$

$\omega = 2 \times \frac{314}{100} \times 125 = 785$

$k = \frac{2\pi}{x} = 2 \times \frac{3.14}{15.6} \times 100 = 40.0$

15. **Sol.** $y = A \sin(\omega t - kx + \phi)$

$\phi = \pi$ for $t = 0$

$y = A \sin(kx - \omega t)$ for positive direction

$y = A \sin(kx + \omega t)$ for negative direction

Ans. (2)

16.

Sol. $v_y = \sqrt{\frac{T_y}{\mu_y}}$

$T_y = \left\{\int_0^y \mu_0 e^y dy\right\} g$

$T_y = \mu_0(e^y - 1).g$

$v_y = \sqrt{g - \frac{g}{e^y}}$ $\quad v_y^2 = g(1 - e^{-y})$.