Exercise-1

Marked questions may have for revision questions.

OBJECTIVE QUESTIONS

Section (A) : Equation of circle, Intercepts on axes

A-1. **Sol.** $r = \sqrt{4+9-4} = 3 \Rightarrow 2r = 6$

A-2. **Sol.** In fourth option coefficient of x^2 = coefficient of y^2 and coefficient of $xy = 0$ & $\Delta \neq 0$

A-3. **Sol.** Midpoint of (1, 0) & (0, 1) is circumcentre so radius = $\sqrt{\frac{1}{4}+\frac{1}{4}} = \frac{1}{\sqrt{2}}$

A-4. **Sol.** $\frac{x+3}{2} = 2 \Rightarrow x = 1$

$\frac{3+5}{2} = y \Rightarrow y = 4$

A-5. **Sol.** $p = 3,\ 2 - q = 0 \Rightarrow q = 2$

A-6. **Sol.** (3,4) & (2,5) are ends of diameter of circle

So, Equation $(x-3)(x-2) + (y-4)(y-5) = 0$

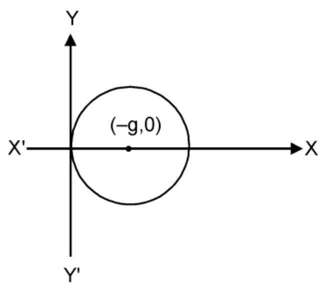
$x^2 + y^2 - 5x - 9y + 26 = 0$

A-7. **Sol.** Centre (2, 2), r = 2 so touches both axes

A-8. **Sol.** We have the equation of circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

But it passes through (0,0)



and (2,1) , then

$c = 0$ ....(i)

$5 + 4g + 2f = 0$ ....(ii)

As it touches y-axis

$\therefore 2\sqrt{f^2 - c} = 0$ $[\because c = 0]$

or f = 0, from (ii) $g = -\frac{5}{4}$

A-9. Sol. Centre (1, –1)

equation $(x - 1)^2 + (y + 1)^2 = 1$

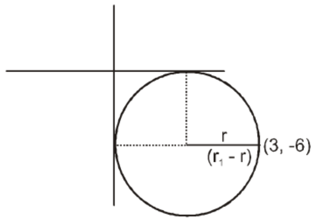
A-10. Sol. Now

$(r - 3)^2 + (-r + 6)^2 = r^2$

$r^2 - 18r + 45 = 0$

$\Rightarrow r = 3, 15$

Hence circle



$(x - 3)^2 + (y + 3)^2 = 3^2$

$x^2 + y^2 - 6x + 6y + 9 = 0$

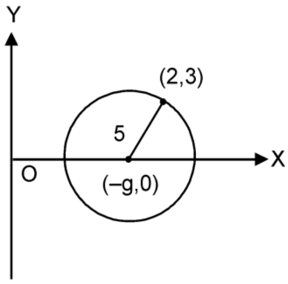
$(x - 15)^2 + (y + 15)^2 = (15)^2$

$\Rightarrow x^2 + y^2 - 30x + 30y + 225 = 0$

A-11. Sol. Centre (–g,0) and passes through (2,3)

$\therefore (2 + g)^2 + (3 - 0)^2 = (5)^2$

$\therefore g = -6, 2$



Hence the equation are $x^2 + y^2 - 12x + 11 = 0$ and $x^2 + y^2 + 4x - 21 = 0$

A-12. Sol. Centre (2, 3)

midpoint of intercept on x-axis is (2, 0) and on y-axis (0, 3)

equation of line

$\frac{x}{2} + \frac{y}{3} = 1$

A-13. Sol. Intercept on x-axis $= 2\sqrt{\frac{25}{4} + 14} = 9$,

Intercept on y-axis $= 2\sqrt{\frac{169}{4} + 14} = 15$,

A-14. **Sol.** Equation of circle (x – 0) (x – a) + (y – 1)(y – b) = 0
it cuts x-axis put y = 0 ⇒ $x^2 - ax + b = 0$

A-15. **Sol.** $x^2 + y^2 = \frac{25}{4}$ parametric equation

$C \equiv (0, 0)$ $r = \frac{5}{2}$

$x = 0 + \frac{5}{2}\cos\theta$, $y = 0 + \frac{5}{2}\sin\theta$

Section (B) : Power and position of a point and line, Tangents and normal

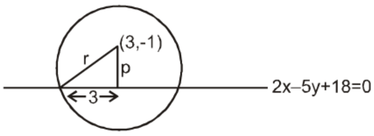
B-1. **Sol.** We have $\lambda^2 + (\lambda + 1)^2 - 1 < 0$
$\Rightarrow 2\lambda^2 + 2\lambda < 0 \Rightarrow \lambda \in (-1, 0)$

B-2. **Sol.** From centre (2, –3), length of perpendicular on line 3x + 5y + 9 = 0 is
$p = \frac{6-15+9}{\sqrt{25+9}} = 0$; line is diameter.

B-3. **Sol.** Centre (2, 4), r = 5
so
$\left|\frac{6-16-m}{5}\right| < 5$
$|m + 10| < 25$
$-25 < m + 10 < 25$
$-35 < m < 15$

B-4. **Sol.** Point on the line x + y + 13 = 0 nearest to the circle $x^2 + y^2 + 4x + 6y - 5 = 0$ is foot of ⊥ from centre
$\frac{x+2}{1} = \frac{y+3}{1} = -\left(\frac{-2-3+13}{1^2+1^2}\right) = -4$
x = –6 y = –7

B-5. **Sol.** Centre (6, 2), $r = \sqrt{10}$
$OC = \sqrt{36+4} = 2\sqrt{10}$
slope line OC = 1/3
Let point be (x, y)
so $= \frac{\frac{x-6}{3}}{\sqrt{10}} = \frac{\frac{y-2}{1}}{\sqrt{10}} = \sqrt{10}$
$\Rightarrow x = 6 + 3 = 9$
$y = 2 + 1 = 3$



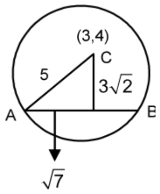
B-6. **Sol.** $p = \dfrac{6+5+18}{\sqrt{29}} = \sqrt{29}$

$r^2 = p^2 + 3^2 = 38 \Rightarrow r = \sqrt{38}$

B-7. **Sol.** Foot of ⊥ from (0, 0) to this line is required point of contact

$= \dfrac{\alpha - 0}{3} = \dfrac{\beta - 0}{4} = -\left(\dfrac{0+0-25}{25}\right)$

B-8. **Sol.** AB $= 2\sqrt{AC^2 - CM^2}$



$= 2\sqrt{(5)^2 - (3\sqrt{2})^2} = 2\sqrt{25-18} = 2\sqrt{7}$

B-9. **Sol.** $4x - y - 15(x + 4) + 3(y - 1) - 109 = 0$

$11x - 2y - 46 = 0$

B-10. **Sol.** The equation of circle in third quadrant touching the coordinate axes with centre (–a,–a) and radius 'a' is $x^2 + y^2 + 2ax + 2ay + a^2 = 0$ and we known

$\left|\dfrac{3(-a) - 4(-a) + 8}{\sqrt{9+16}}\right| = a \Rightarrow a = 2$

Hence the required equation is

$x^2 + y^2 + 4x + 4y + 4 = 0$

B-11. **Sol.** $p = \left|\dfrac{(-g+g)\cos\theta + (-f+f)\sin\theta - k}{\sqrt{\cos^2\theta + \sin^2\theta}}\right|$

$= \sqrt{g^2 + f^2 - c} \Rightarrow g^2 + f^2 = c + k^2$

B-12. **Sol.** Equation of line parallel to the given line $4x + 3y + \lambda = 0$

so $5 = \left|\dfrac{12 - 6 + \lambda}{5}\right| \Rightarrow \lambda = -6 \pm 25$

$\lambda = -31, 19$

B-13. **Sol.** Equation of tangent $x - 2y = 5$

Let required point be (α, β)

$\alpha x + \beta y - 4(x + \alpha) + 3(y + \beta) + 20 = 0$

$x(\alpha - 4) + y(\beta + 3) - 4\alpha + 3\beta + 20 = 0$

Comparing

$$\frac{\alpha-4}{1} = \frac{\beta+3}{-2} = \frac{4\alpha-3\beta-20}{5}$$

Similarly (α,β) $(3, -1)$

B-14. Sol. Equation of line parallel to $x + 2y - 3 = 0$ is

$$x + 2y + k = 0$$

This is normal of $x^2 + y^2 - 2x = 0$

Hence centre of circle satisfies it

$1 + 0 + k = 0 \Rightarrow k = -1$

$x + 2y - 1 = 0$

B- 15. Sol. Centre $(2, -2)$

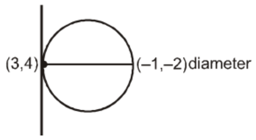
so equation of normal

$y + 2 = \frac{1+2}{1-2}(x - 2)$

$y + 2 = -3x + 6$

$\Rightarrow 3x + y = 4$

B-16. Sol.



$(x-3)(x+1) + (y-4)(y+2) = 0$

Equation $x^2 + y^2 - 2x - 2y - 11 = 0$

Section (C) : Pair of tangents (Joint equation and length of tangent) Director circle, chord of contact, chord with given middle point, and chord joining two point

C-1. Sol. Point (8,6) lies on circle ; $S_1 = 0$ one tangent.

C-2. Sol. Length of tangent = $\sqrt{PA.PB}$ = 12

C-3. Sol. The equation of tangents is $SS_1 = T^2$

$\Rightarrow h^2(x^2 + y^2 - 2x - 2hy + h^2) = (rx + hy - h^2)^2$

$\Rightarrow (h^2 - r^2)x^2 - 2rhxy = 0 \Rightarrow x(h^2 - r^2)\, x - 2rhy\} = 0$

$\Rightarrow x = 0,\ (h^2 - r^2)\, x - 2rhy = 0$

C-4. Sol. Length of tangent $= \sqrt{S_1} = \sqrt{16+1-\frac{1}{2}} = \sqrt{17-\frac{1}{2}} = \sqrt{\frac{33}{2}}$

C-5. Sol. $\because\ 7 = \sqrt{25+9+10+3k+17}$

$\Rightarrow 3k = 49 - 61$

k = –4

C-6. Sol. Let any point on the circle $x^2 + y^2 + 2gx + 2fy + p = 0$ (α, β)

This point satisfies $\alpha^2 + \beta^2 + 2g\alpha + 2f\beta + p = 0$

Length of tangent from this point to circle $x^2 + y^2 + 2gx + 2fy + q = 0$

length = $\sqrt{S_1} = \sqrt{\alpha^2 + \beta^2 + 2g\alpha + 2f\beta + q} = \sqrt{q - p}$

C-7. Sol. Locus in director circle

C-8. Sol. Let tangent be y = mx

$$\left|\frac{7m+1}{\sqrt{1+m^2}}\right| = 5$$

$\Rightarrow$ $49m^2 + 1 + 14m = 25(1+m^2)$

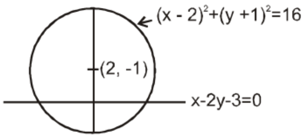
$24m^2 + 14m - 24 = 0$

$m_1m_2 = -1$ angle = 90º

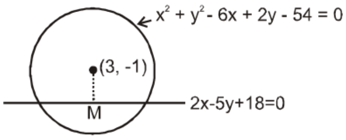
C-9. Sol. Required diameter is $\perp$ to given line.

Hence y + 1 = –2(x – 2)

$\Rightarrow$ 2x + y – 3 = 0



C-10. Sol. Required point is foot of $\perp$



$$\frac{x-3}{2} = \frac{y+1}{-5} = -\left(\frac{6+5+8}{4+25}\right) = -1$$

x = 1, y = 4

C-11. Sol. Equation of chords of contact from (0, 0) & (g, f)

gx + fy + c = 0

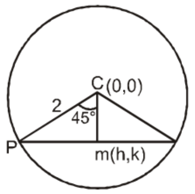
gx + fy + g(x + g) + f(y + f) + c = 0

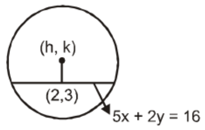
$$gx + fy + \frac{(g^2 + f^2 + c)}{2} = 0$$

Distance between these parallel lines = $\left|\frac{g^2 + f^2 - c}{2\sqrt{g^2 + f^2}}\right|$

C-12. Sol. $\cos 45° = \frac{cm}{cp} = \frac{\sqrt{h^2 + k^2}}{2}$

Hence locus $x^2 + y^2 = 2$



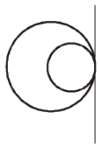
C-13. **Sol.** 

$\frac{k-3}{h-2} = \frac{2}{5} \Rightarrow$ $2x - 5y + 11 = 0$

Section (D) : Common tangents, common chord and orthogonality

D-1. **Sol.** $C_1(0, 1)$, $r_1 = 3$
$C_2(1, 0)$, $r_2 = 5$
$C_1C_2 = \sqrt{2}$
$|r_1 - r_2| = 2$
$\Rightarrow c_1c_2 < |r_1 - r_2|$

D-2. **Sol.** $C_1C_2 = 5$, $r_1 = 7$, $r_2 = 2$



$C_1C_2 = |r_1 - r_2| \Rightarrow$ one common tangent

D-3. **Sol.** Equation of common tangent at point of contact is $S_1 - S_2 = 0$
$\Rightarrow 10x + 24y + 38 = 0$
$\Rightarrow 5x + 12y + 19 = 0$

D-4. **Sol.** $S_1 : x^2 + y^2 - 2x - 6y + 9 = 0$ $C_1(1, 3)$, $r_1 = 1$
$S_2 : x^2 + y^2 + 6x - 2y + 1 = 0$ $C_2(-3, 1)$, $r_2 = 3$
$C_1C_2 = \sqrt{16+4} = \sqrt{20} \Rightarrow n + r_2 = 4$
Hence $C_1C_2 > r_1 + r_2$ Both circles are non-intersecting.
Hence there are four common tangents.
Transverse common tangents :
coordinate of P $\left(\frac{3-3}{1+3}, \frac{1+9}{1+3}\right) \equiv \left(0, \frac{5}{2}\right)$
Let slope of these tangents is m
$y - \frac{5}{2} = m(x - 0) \Rightarrow mx - y + \frac{5}{2} = 0$

Now $\left|\dfrac{m-3+\frac{5}{2}}{\sqrt{1+m^2}}\right| = 1 \Rightarrow \left|m-\frac{1}{2}\right| = \sqrt{1+m^2}$

$\Rightarrow m^2 + \frac{1}{4} - m = 1 + m^2 \Rightarrow m = -\frac{3}{4}$, other tangents is vertical

Equation of tangents x = 0

$-\frac{3}{4}x - y + \frac{5}{2} = 0 \Rightarrow -3x - 4y + 10 = 0 \Rightarrow 3x + 3y = 10$

Direct common tangents

coordinate of Q $\left(\dfrac{-3-3}{1-3}, \dfrac{1-9}{1-3}\right) \equiv Q(3, 4)$

Hence equations $y - 4 = m(x - 3) \Rightarrow mx - y + (4 - 3m) = 0$

$\Rightarrow \left|\dfrac{m-3+4-3m}{\sqrt{1+m^2}}\right| = 1 \Rightarrow |1 - 2m| = \sqrt{1+m^2} \Rightarrow 1 + 4m^2 - 4m = 1 + m^2 \Rightarrow 3m^2 - 4m = 0 \Rightarrow m = 0, \frac{4}{3}$

Hence equation $y - 4 = 0(x - 3) \Rightarrow y = 4 \Rightarrow y - 4 = \frac{4}{3}(x - 3) \Rightarrow 4x - 3y = 0$

D-5. **Sol.** They touch each other

D-6. **Sol.** Standard result = $\dfrac{a(h^2+k^2-a^2)^{3/2}}{h^2+k^2} = \dfrac{3(25-9)^{3/2}}{25}$

$= \dfrac{3\times16\times4}{25} = \dfrac{192}{25}$

D-7 **Sol.** $r_1 = 2$ $r_2 = 1$

$c_1(1,2)$ $c_2(5,2)$

$c_1c_2 = 4 = d$

Length of direct common tangent = $\sqrt{d^2-(r_1-r_2)^2} = \sqrt{4^2-(2-1)^2} = \sqrt{16-1} = \sqrt{15}$

D-8. **Sol.** as we know $L_{int} = \sqrt{d^2-(r_1+r_2)^2} = 7 \Rightarrow L_{ext} = \sqrt{d^2-(r_1-r_2)^2} = 11$

squaring & subtact $r_1r_2 = 18$

D-9. **Sol.** If two circles touch each other, then

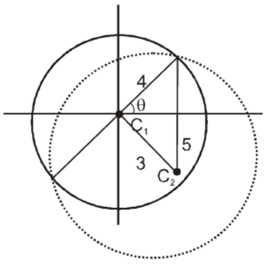
$C_1C_2 = r_1 + r_2$

$\sqrt{(-g_1+g_2)^2+(-f_1+f_2)^2} = \sqrt{g_1^2+f_1^2} + \sqrt{g_2^2+f_2^2}$ squaring both sides

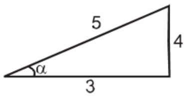
$-2g_1g_2 - 2f_1f_2 = 2\sqrt{\left(g_1^2+f_1^2\right)\left(g_2^2+f_2^2\right)}$

$\Rightarrow (g_1f_2)^2 + (g_2f_1)^2 - 2g_1g_2f_1f_2 = 0 \Rightarrow \dfrac{g_1}{g_2} = \dfrac{f_1}{f_2}$

D-10. **Sol.**



slope of C_1C_2 is $\tan\alpha = -\frac{4}{3}$



By using parametric coordinates C_2 ($\pm$ 3 cos α , $\pm$ 3 sin α)

C_2 ($\pm$ 3 (–3/5) , $\pm$ 3 (4/5))

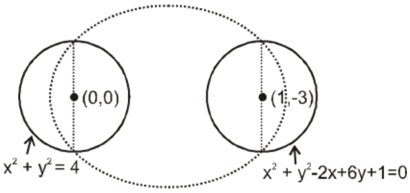
C_2 ($\pm$ 9/5 , 12/5)

D-11. Sol. Let required circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

Hence common chord with $x^2 + y^2 - 4 = 0$

is $2gx + 2fy + c + 4 = 0$

This is diameter of circle $x^2 + y^2 = 4$ hence $c = -4$.



Now again common chord with other circle

$2x(g + 1) + 2y(f - 3) + (c - 1) = 0$

This is diameter of $x^2 + y^2 - 2x + 6y + 1 = 0$

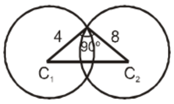
$2(g + 1) - 6(f - 3) - 5 = 0$

$2g - 6f + 15 = 0$

locus $2x - 6y - 15 = 0$ which is st. line.

D-12. Sol. $C_1C_2 = \sqrt{80}$

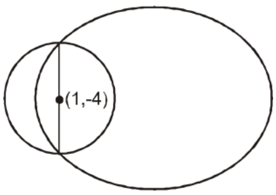
Area = $\frac{1}{2} \times 4 \times 8 = \frac{1}{2} \times \sqrt{80} \times \frac{\ell}{2}$



$$\ell = \frac{64}{\sqrt{80}} = \frac{16}{\sqrt{5}}$$

D-13. Sol. Common chord of given circle

$6x + 4y + (p + q) = 0$



This is diameter of $x^2 + y^2 - 2x + 8y - q = 0$

centre (1, –4)

$6 - 16 + (p + q) = 0 \Rightarrow p + q = 10$

D-14 **Sol.** $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ (Formula for orthogonal intersection)

$2(1)(0) + 2(k)(k) = 6 + k$

$\Rightarrow \quad 2k^2 - k - 6 = 0$

$\Rightarrow \quad k = -\frac{3}{2}, 2$

Therefore,(1) is the answer.

D-15. **Sol.** $S_1 - S_2 = 0 \quad \Rightarrow \quad 7x - 8y + 16 = 0$

$S_2 - S_3 = 0 \quad \Rightarrow \quad 2x - 4y + 20 = 0$

$S_3 - S_1 = 0 \quad \Rightarrow \quad 9x - 12y + 36 = 0$

On solving centre (8,9)

Length of tangent

$= \sqrt{S_1} = \sqrt{64 + 81 - 16 + 27 - 7} = \sqrt{149}$

$= (x - 8)^2 + (y - 9)^2 = 149$

$= x^2 + y^2 - 16x - 18y - 4 = 0$

D-16. **Sol.** Let equation of circle be $x^2 + y^2 + 2gx + 2y = 0$ ($\because$ C = 0)

$-g - f = 4 \qquad$ ....(1)

$2(g)(-2) + 2(f)(1) = 4$

$-2g + f = 2 \qquad$ ...(2)

from (1) & (2)

$g = -2, f = -2$

so $x^2 + y^2 - 4x - 4y = 0$

Section (E) : Radical axis and family of circle

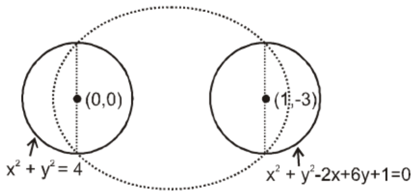
E-1. **Sol.** Let required circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

Hence common chord with $x^2 + y^2 - 4 = 0$

is $2gx + 2fy + c + y = 0$

This is diameter of circle $x^2 + y^2 = 4$ hence $c = -4$.

Now again common chord with other circle



$2x(g + 1) + 2y(f - 3) + (c - 1) = 0$

This is diameter of $x^2 + y^2 - 2x + 6y + 1 = 0$

$2(g + 1) - 6(f - 3) + 5 = 0$

$2g - 6f + 15 = 0$

locus $2x - 3y - 15 = 0$ which is st. line.

E-2. **Sol.** Common chord of given circle

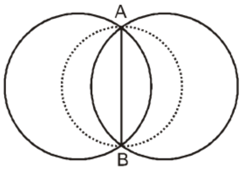
$2x + 3y - 1 = 0$

family of circle passing through point of intersection of given circle

$(x^2 + y^2 + 2x + 3y - 5) + \lambda(x^2 + y^2 - 4) = 0$

$(\lambda + 1)x^2 + (\lambda + 1)y^2 + 2x + 3y - (4\lambda + 5) = 0$

$x^2 + y^2 + \frac{2x}{\lambda+1} + \frac{3}{\lambda+1}y - \frac{(4\lambda+5)}{\lambda+1} = 0$



centre $\left(-\frac{1}{\lambda+1}, \frac{-3}{2(\lambda+1)}\right)$

This centre lies on

$2\left(-\frac{1}{\lambda+1}\right) + 3\left(\frac{-3}{2(\lambda+1)}\right) - 1 = 0$

$-4 - 9 - 2\lambda - 2 = 0$

$\Rightarrow 2\lambda = -15$

$\Rightarrow \lambda = -15/2$

$\left(-\frac{15}{2}+1\right)x^2 + \left(-\frac{15}{2}+1\right)y^2 + 2x + 3y - \left(-4\times\frac{15}{2}+5\right) = 0$

$\Rightarrow -\frac{13x^2}{2} - \frac{13y^2}{2} + 2x + 3y + 25 = 0$

$\Rightarrow 13(x^2 + y^2) - 4x - 6y - 50 = 0$

E-3. **Sol.** Requried equation is

$(x^2 + y^2 + 13x - 3y) + \lambda(2x^2 + 2y^2 + 4x - 7y - 25) = 0$

which passes through (1,1) , so $\lambda = \frac{1}{2}$

Hence required equation is

$4x^2 + 4y^2 + 30x - 13y - 25 = 0$

E-4. **Sol.** $S + \lambda L = 0$

$(x - 1)^2 + (y - 1)^2 + \lambda (x + y - 2) = 0$

$9 + 9 + \lambda (4 + 4 + 2) = 0$

$\lambda = -\frac{18}{6} = -3$

$\therefore (x - 1)^2 + (y - 1)^2 - 3 (x + y - 2) = 0$

$x^2 + y^2 - 5x - 5y + 8 = 0$

E-5. **Sol.** $S + \lambda L = 0$

$(x - 1)(x - 3) + (y - 1)(y - 3) + \lambda (x - y) = 0$

$x^2 + y^2 - x(4 - \lambda) - y(4 + \lambda) + 6 = 0$

Centre $\left(\frac{4-\lambda}{2}, \frac{4+\lambda}{2}\right)$

$\therefore \frac{4+\lambda}{2} = 0$

$\lambda = -4$

$\therefore x^2 + y^2 - 8x + 6 = 0$

Exercise-2

Marked Questions may have for Revision Questions.

PART - I : OBJECTIVE QUESTIONS

1. **Sol.** Let centre be (α, α) & α > 0

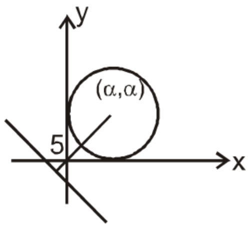
$\Rightarrow 5 = \left|\dfrac{3\alpha + 4\alpha + 11}{5}\right| \quad \Rightarrow 7\alpha + 11 = \pm 25$

$7\alpha = 14,\ 7\alpha = -36$

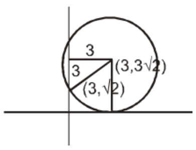
$\alpha = 2,\ \alpha = \dfrac{-36}{7}$

so circle

$(x - 2)^2 + (y - 2)^2 = 4$



2. **Sol.** $(x \pm 3)^2 + \left(y \pm 3\sqrt{2}\right)^2 = \left(3\sqrt{2}\right)^2$



3. **Sol.** $x^2 + y^2 - 10x + \lambda(2x - y) = 0$ ....(i)

$x^2 + y^2 + 2x(\lambda - 5) - \lambda y = 0$

Centre $(-(\lambda - 5), \lambda/2)$

Using on $y = 2x$

$\dfrac{\lambda}{2} = -2(\lambda - 5)$

$\dfrac{5\lambda}{2} = 10$

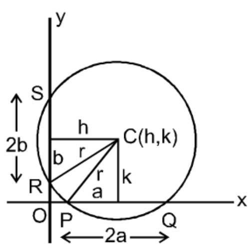
Putting $\lambda = 4$

$x^2 + y^2 - 2x - 4y = 0$

4. **Sol.** $h^2 + b^2 = r^2$

$k^2 + a^2 = r^2$

$\Rightarrow \quad h^2 - k^2 = a^2 - b^2$



$\therefore$ locus is $x^2 - y^2 = a^2 - b^2$

5. **Sol.** $\left|\frac{4c+3c-12}{5}\right| = c \Rightarrow c = 1, 6$

6. **Sol.** Let equation be $x^2 + y^2 + 2gx + 2fy + g^2 = 0$

Which passes through (1, –2) & (3, –4)

$\Rightarrow g = -3, 5$ and $f = 2, 10$

7. **Sol.** Point $\left(t, \frac{1}{t}\right)$ lies on $x^2 + y^2 = 16$

$t^2 + \frac{1}{t^2} = 16$

$\Rightarrow \quad t^4 - 16t^2 + 1 = 0$ ....(i)

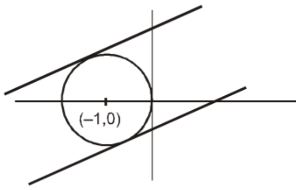
If roots are t_1, t_2, t_3, t_4 then

$t_1t_2t_3t_4 = 1$ ....(ii)

8. **Sol.** $\left|\frac{-1-0+c}{\sqrt{2}}\right| = \sqrt{2} \Rightarrow c - 1 = \pm 2 \Rightarrow C = 3, -1$

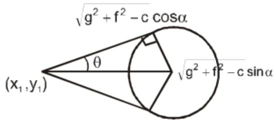
One common point is obtained

For inequalities to satisfy and for only one common point



C = – 1

9. **Sol.**



$\tan\theta = \tan\alpha \Rightarrow \theta = a$

angle = 2α

10. **Sol.** (0, 0) & (8, 6) lie on the director circle of $x^2 + y^2 - 14x + 2y + 25 = 0$

so $\alpha - \beta = 0$

11. **Sol.** $x^2 + y^2 - 8x - 12y + p = 0$

Power of (2, 5) is $S_1 = 4 + 25 - 16 - 60 + P = P - 47 < 0 \Rightarrow P < 47$

Circle neither touches nor cuts coordinate axes

$g^2 - c < 0 \Rightarrow 16 - p < 0 \Rightarrow p > 16$

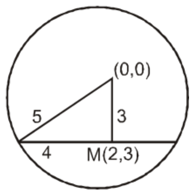
$f^2 - c < 0 \Rightarrow 36 - p < 0 \Rightarrow p > 36$

taking intersection $P \in (36, 47)$

12. **Sol.** Let slope of required line is m

$y - 3 = m(x - 2)$

$\Rightarrow mx - y + (3 - 2m) = 0$



length of $\perp$ from origin

$= 3$

$\Rightarrow 9 + 4m^2 - 12m = 9 + 9m^2$

$\Rightarrow 5m^2 + 12m = 0 \Rightarrow m = 0, -\frac{12}{5}$

Hence lines are $y - 3 = 0 \Rightarrow y = 3$

and $\quad y - 3 = -\frac{12}{5}(x - 2) \Rightarrow 5y - 15 = -12x + 24$

$\Rightarrow 12x + 5y = 39.$

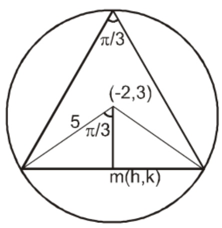
13. **Sol.** Let point on line be

$(h, 4 - 2h)$ (chord of contact)

$hx + y(4 - 2h) = 1$

$h(x - 2y) + 4y - 1 = 0$ $\qquad$ Point $\left(\frac{1}{2}, \frac{1}{4}\right)$

14. **Sol.** $\cos \pi/3 = \frac{\sqrt{(h+2)^2 + (k-3)^2}}{5}$



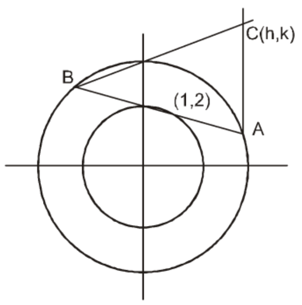
Locus $(x + 2)^2 + (y - 3)^2 = 6.25$

15. **Sol.** Tangent at (1, 2) to the circle $x^2 + y^2 = 5$

$x + 2y - 5 = 0$

chord of contact from C(h, k) to $x^2 + y^2 = 9$

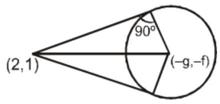
$hx + ky - 9 = 0$



compare both equations $\frac{h}{1} = \frac{k}{2} = \frac{9}{5}$

$(h, k) \equiv \left(\frac{9}{5}, \frac{18}{5}\right)$

16. **Sol.**



$(x + g)(x - 2) + (y + f)(y - 1) = 0$

17. **Sol.** Let any point $P(x_1, y_1)$ to the circle $x^2 + y^2 - \frac{16x}{5} + \frac{64y}{15} = 0$

$\Rightarrow x_1^2 + y_1^2 - \frac{16x_1}{5} + \frac{64y_1}{15} = 0$

Length of tangent from $P(x_1, y_1)$ to the circle are in ratio

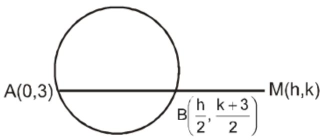
$$\frac{\sqrt{S_1}}{\sqrt{S_2}} = \frac{\sqrt{x_1^2 + y_1^2 - \frac{24}{5}x_1 + \frac{32}{5}y_1 + 15}}{\sqrt{x_1^2 + y_1^2 - \frac{48}{5}x_1 + \frac{64}{5}y_1 + 60}}$$

$$= \sqrt{\frac{\frac{16}{5}x_1 - \frac{64}{15}y_1 - \frac{24}{5}x_1 + \frac{32}{5}y_1 + 15}{\frac{16}{5}x_1 - \frac{64}{15}y_1 - \frac{48}{5}x_1 + \frac{64}{5}y_1 + 60}}$$

$$= \sqrt{\frac{-24x_1 + 32y_1 + 225}{-96x_1 + 128y_1 + 900}}$$

$$= \sqrt{\frac{-24x_1 + 32y_1 + 225}{4(-24x_1 + 32y_1 + 225)}} = \frac{1}{2}$$

18. **Sol.**



B lies on circle

$\left(\frac{h}{2}\right)^2 + 4\left(\frac{h}{2}\right) + \left(\frac{k+3}{2} - 3\right)^2 = 0$

$\Rightarrow \frac{h^2}{4} + 2h + \frac{(k-3)^2}{4} = 0$

Hence locus of (h, k) $x^2 + 8x + (y - 3)^2 = 0$

(h, k) dk $x^2 + 8x + (y - 3)^2 = 0$

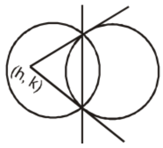
19. **Sol.** Let point be (h,k)

Equation of chord of contact $hx + ky - 12 = 0$

Equation of common chord

$S_1 - S_2 = 0 \Rightarrow 5x - 3y - 10 = 0$

Comparing $\frac{g}{5} = \frac{k}{-3} = \frac{6}{5}$



$(h,k) \equiv \left(6, -\frac{18}{5}\right)$

20. **Sol.** as we know $L_{int} = \sqrt{d^2 - (r_1 + r_2)^2} = 7$

$L_{ext} = \sqrt{d^2 - (r_1 - r_2)^2} = 11$

squaring & subtact $r_1 r_2 = 18$

21. **Sol.** $S_1 : x^2 + y^2 - 2x - 6y + 9 = 0$ $C_1(1, 3), r_1 = 1$

$S_2 : x^2 + y^2 + 6x - 2y + 1 = 0$ $C_2(-3, 1), r_2 = 3$

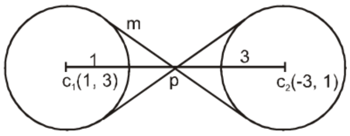
$C_1C_2 = \sqrt{16+4} = \sqrt{20}$

$r_1 + r_2 = 4$

Hence $C_1C_2 > r_1 + r_2$ Both circles are completely outside each other.

Hence there are four common tangents.

Transverse common tangents :



coordinate of P $\left(\frac{3-3}{1+3}, \frac{1+9}{1+3}\right) \equiv \left(0, \frac{5}{2}\right)$

Let slope of these tangents is m

$y - \frac{5}{2} = m(x - 0) \Rightarrow mx - y + \frac{5}{2} = 0$

Now $\left|\frac{m - 3 + \frac{5}{2}}{\sqrt{1+m^2}}\right| = 1 \Rightarrow \left|m - \frac{1}{2}\right| = \sqrt{1+m^2}$

$\Rightarrow m^2 + \frac{1}{4} - m = 1 + m^2 \Rightarrow m = -\frac{3}{4}$, other tangents is vertical

Equation of tangents x = 0

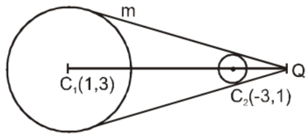
$-\frac{3}{4}x - y + \frac{5}{2} = 0 \Rightarrow -3x - 4y + 10 = 0 \Rightarrow 3x + 4y = 10$

Direct common tangents

coordinate of Q $\left(\frac{-3-3}{1-3}, \frac{1-9}{1-3}\right) \equiv Q(3, 4)$

Hence equations $y - 4 = m(x - 3) \Rightarrow mx - y + (4 - 3m) = 0$

$$\Rightarrow \left|\frac{m-3+4-3m}{\sqrt{1+m^2}}\right| = 1$$



$\Rightarrow |1 - 2m| = \sqrt{1+m^2}$

$\Rightarrow 1 + 4m^2 - 4m = 1 + m^2 \Rightarrow 3m^2 - 4m = 0 \Rightarrow m = 0, \frac{4}{3}$

Hence equation $y - 4 = 0(x - 3) \Rightarrow y = 4$

$y - 4 = \frac{4}{3}(x - 3) \Rightarrow 4x - 3y = 0$

22. **Sol.** Two fixed pts. are point of intersection of

$x^2 + y^2 - 2x - 2 = 0$ & $y = 0$

$x^2 - 2x - 2 = 0$

$(x - 1)^2 - 3 = 0$

$x - 1 = -\sqrt{3}$, $x - 1 = -\sqrt{3}$

$(1+\sqrt{3},\ 0)$, $(1-\sqrt{3},\ 0)$

PART - II : MISCELLANEOUS QUESTIONS

1. **Sol.** Statement-1 is false as 3 points are collinear statement-2 is true.

2. **Sol.** Statement-1 : There is exactly one circle whose centre is the radical centre and the radius equal to the length of tangent drawn from the radical centre to any of the given circles.

Statement-2 is True But does not explain Statement-1.

Section (B) : MATCH THE COLUMN

B-1. Ans. (A) → s ; (B) → p ; (C) → q ; (D) → r

Sol. (A) C_1 (1, 0), $r_1 = 1$ and C_2 (–3, 3), $r_2 = 4$

distance between centres C_1 and C_2 = d = 5

$d = r_1 + r_2 = 5$ ⇒ 3 common tangents

(B) C_1 (2, 5), $r_1 = 5$ and C_2 (3, 6), $r_2 = 10$

distance between centres C_1 and C_2 =

$d = \sqrt{2}$

$d < |r_1 - r_2|$

⇒no common tangent

(C) C_1 (1, 2), $r_1 = \sqrt{5}$ and C_2 (0, 4), $r_2 = 2\sqrt{5}$

distance between centres C_1 and C_2 = $d = \sqrt{5}$

$|r_1 - r_2| = d$

number of common tangents is 1

(D) $C_1(-1, 4)$, $r_1 = 2$ and $C_2(3, 1)$, $r_2 = 2$

distance between centres C_1 and C_2 = d = 5

$d > r_1 + r_2$

$\Rightarrow$ number of direct common tangents is 2

B-2. **Ans.** A → q ; B → s ; C → p ; D → r

Sol. (A) Let length of common chord be 2a, then

$\sqrt{9-a^2} + \sqrt{16-a^2} = 5$

$\sqrt{16-a^2} = 5 - \sqrt{9-a^2}$

$16 - a^2 = 25 + 9 - a^2 - 10\sqrt{9-a^2}$

$10\sqrt{9-a^2} = 18 \Rightarrow (9-a^2) = 324$

i.e , $100a^2 = 576$

$\therefore a = \sqrt{\frac{576}{100}} = \frac{24}{10}$

$\therefore 2a = \frac{24}{10} = \frac{k}{5} \Rightarrow k = 24$

(B) Equation of common chord is 6x + 4y + p + q = 0

common chord pass through centre

$\therefore p + q = 36$

(C) Equation of the circle is $2x^2 + 2y^2 - 2\sqrt{2}x - y = 0$

Let (α,0) be mid point of a chord. Then equation of the chord is

$2\alpha x - \sqrt{2}(x + \alpha) - \frac{1}{2}(y + 0) = 2\alpha^2 - 2\sqrt{2}\alpha$

Since it passes through the point $\left(\sqrt{2}, \frac{1}{2}\right)$

$\therefore 2\sqrt{2}\alpha - \sqrt{2}(\sqrt{2} + \alpha) + 9 = 0$ i.e. $(2\sqrt{2}\alpha - 3)^2 = 0$

i.e., $\alpha = \frac{3}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}$ $\therefore$ Number of chords is 1.

(D) Mid point of AB = (1,4)

$\therefore$ Equation perpendicular bisector of AB is x = 1

A diameter is 4y = x + 7

$\therefore$ Centre of the circle is (1,2)

$\therefore$ Sides of the rectangle are 8 and 4

$\therefore$ area = 32

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. **Sol.** Let the circle $(x - h)^2 + (y - k)^2 = k^2$

$x^2 + y^2 - 2hx - 2ky + h^2 = 0$

passes through (1, –2) and (3, –4), so

$-2h + 4k + h^2 + 5 = 0$ ....(i)

and $-6h + 8k + h^2 + 25 = 0$ ....(ii)

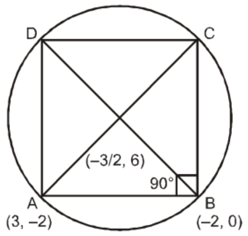
by (i) and (ii)

at $h = -5, k = -10$

and $h = 3, \quad k = -2$

so the circles are $x^2 + y^2 + 10x + 20y + 25 = 0$, $x^2 + y^2 - 6x + 4y + 9 = 0$

C-2. **Sol.** The centre $\left(-\frac{3}{2}, -6\right)$ is mid-poins of diameter AC and BD



$\therefore$ C is (p, q) = (–6, –10)

$\therefore$ D is (r, s) = (–1, –12)

C-3. **Sol.** If (h, k) be the point of intersection of tangents then A, B is chord of contact of (h, k).

$\therefore$ AB is $hx + ky - a^2 = 0$. It touches the second circle (a, 0), a. Hence apply condition of tangency p=r

$\therefore$ $\frac{ha - a^2}{\sqrt{h^2 + k^2}} = a$ or $x^2 + y^2 = (x - a)^2$ or $y^2 = a(a - 2x)$

C-4. **Sol.** The equation of the circle passing through the intersection of the given lines and axes xy = 0 is

$(\lambda x - y + 1)(x - 2y + 3) + kxy = 0$

Condition for circle a = b, h = 0

$\lambda = 2$ and $1 - 2\lambda + k = 0$

$\therefore$ $k = 5$ as $\lambda = 2$

$\therefore$ $(2x - y + 1)(x - 2y + 3) + 5xy = 0$ or $2x^2 + 2y^2 + 7x - 5y + 3 = 0$ ....(1)

It passes through all the four points of intersection of the given lines with co-ordinate axes, one of which is $\left(-\frac{1}{\lambda}, 0\right)$, it will satisfy (1).

or $\frac{2}{\lambda^2} - \frac{7}{\lambda} + 3 = 0$ or $3\lambda^2 - 7\lambda + 2 = 0$

or $(\lambda - 2)(3\lambda - 1) = 0$ $\therefore$ $\lambda = 2, . \frac{1}{3}$