Exercise-1

Marked Questions can be uesed as Revision Questions.

PART - I : OBJECTIVE QUESTION

Section (A) : Flux And Faraday's laws of electromagnetic induction

A-1 **Sol.** $\phi = NBA = 5 \times 3 \times 10^{-5} \times 1 = 15 \times 10^{-5}$ Wb

A-3 **Sol.** $e = -\frac{d\phi}{dt} = -\frac{d(\vec{B}.\vec{A})}{dt}$

A-5 **Sol.** Flux emerging out from side EACF

So $\phi = B . A = Ba^2$

A-6 **Sol.** Since $\Delta\phi = 0$ hence EMF induced is zero.

A-7 **Sol.** The direction of current in the loop such that it opposes the the change in magnetic flux in it.

A-8 **Sol.** The direction of current in the loop such that it opposes the the change in magnetic flux in it.

A-9 **Sol.** Since the magnetic flux in the loop is zero hence the current induced in it is zero.

Section (B) : Lenz's Law

B-2 **Sol.** When metal ring approaches the ring then induced current in ring has direction. So that it repels the bar magnet. After it crosses the ring, current has direction so that it attracts the bar magnet. Hence the acceleration of bar magnet will be less than g.

B-3 **Sol.** When A is moved towards B. Magnetic flux through B increases. So current induced in B should be negative to decrease the flux.

B-4 **Sol.** When A is moved towards B. Magnetic flux through B increases. So current induced in B should be opposite to that of A to decrease the flux.

B-5 **Sol.** By moving away from solenoid the ring will resist the changing flux in it.

B-6 **Sol.** The repulsion is to resist the increasing magnetic flux in coil B.

B-7 **Sol.** Q will move towards P to resist the increasing magnetic flux in the loop formed due to rails R,S and conductors P,Q.

B-8 **Sol.** The decrease in current is to oppose the increasing magnetic flux in the circular loops.

B-9 **Sol.** When the coil is entering and coming out of the field the magnetic flux in it is changing but when it is within the field the magnetic flux in it is constant.

B-10 **Sol.** When the magnet goes away from the ring the flux in the ring decreases hence the induced current will be such that it opposes the decreasing flux in it hence ring will behave like a magnet having face A as north pole and face B as south pole.

Section (C) : Induced EmF in a moving rod in uniform magnetic field

C-1 **Sol.** $\varepsilon = VB\ell = 1 \times 0.5 \times 2 = 1V$

C-2 **Sol.** $\varepsilon = VB\ell = 200 \times 2 \times 10^{-4} \times 50 = 2V$

C-3 **Sol.** $\varepsilon_{max} = VB\ell = 7 \times 0.9 \times 0.4 = 2.52$ Volt

C-4 **Sol.** We have $\varepsilon = (\vec{V} \times \vec{B}).\vec{\ell}$ $\Rightarrow$ when any two vectos are parallel then $\varepsilon = 0$

C-5 **Sol.** When the loop enters the magnetic field the magnetic flux in it changes till it covers a distance 'a'. Hence the EMF induced in the surface afer that flux in it remains constant till its back portion has not entered in magnetic field. No emf is induced during this time.when it is out of magnetic field the magnetic flux in it decreases. EMF is again induced in the circuit hence total time for which emf is induced is $\frac{2a}{v}$.

C-6 **Sol.** $\varepsilon = |(\vec{V}\times\vec{B}).\vec{\ell}| = \left|\left[1\hat{i}\times\left(3\hat{i}+4\hat{j}+5\hat{k}\right)\right].5\hat{j}\right|$

$= \left|\left(4\hat{k}-5\hat{j}\right).5\hat{j}\right| = 25$ V

C-7

Sol. For constant v $\frac{d\phi}{dt}$ will be same in both cases hence the induced emf thus inudced current will remain same.

Section (D) : Circuit Problems & Mechanics

D-1 **Sol.** It the magnitude of I_A is very large such that force due to magnetic field on PQ exceeds its weight then it will move upwards otherwise it will move downwards.

D-2 **Sol.** Force acting on the rod because of the induced current due to change in magnetic flux will try to oppose the motion of rod. Hence the accelereation of the rod will decrease with time $\frac{dp}{dt} = F\frac{dv}{dt}$ = f × a. Thus, rate of power delivered by external force will be decreasing continuously.

D-3 **Sol.** W = (0.5)F

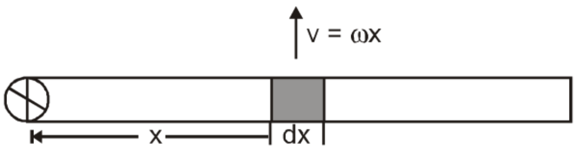
= 0.5 × ILB or H = I^2 RT

$= 0.5 \times \frac{L^2B^2V}{R}$

$= \frac{0.5\times(0.5)^2\times(1)^2\times\left(\frac{0.5}{2}\right)}{10} = 3.125 \times 10^{-3}$ J.

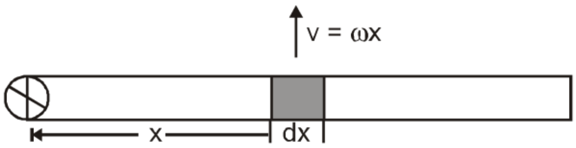
Section (E) : Induced emf in a rod, Ring, Disc rotating in a uniform magnetic field

E-1 **Sol.**

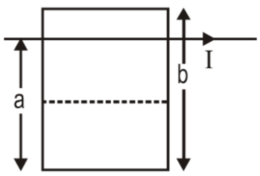


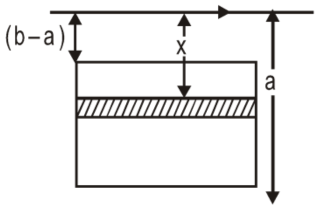
$\varepsilon = \int_0^L B(\omega x)dx = B\frac{\omega L^2}{2}$

E-2 **Sol.**



$$\varepsilon = \int_0^L B(\omega x)dx = B\frac{\omega L^2}{2} = \frac{B(2\pi f)L^2}{2} = B\pi f L^2$$

E-3 **Sol.** 

⇒ 

$$\int d\phi = \int \frac{\mu_0 I}{2\pi x}(bdx)$$

$$\phi = \frac{\mu_0 I b}{2\pi}\int_{(b-a)}^{a}\frac{dx}{x}$$

$$\phi = \frac{\mu_0 I b}{2\pi}\ell n\left(\frac{a}{b-a}\right).$$

E-4 **Sol.** Since magnetic field lines around the wire AB are circular, therefore magnetic flux through the circular loop will be zero, hence induced emf in the loop will be zero.

Section (F) : Fixed loop in a time varying magnetic field & Induced electric field

F-1 **Sol.** $I = I_0 \sin(\omega t + \phi)$ ⇒ $\frac{dI}{dt} = I_0\omega\cos(\omega t + \phi)$

we have $\varepsilon = -\frac{d\phi}{dt} = -\frac{d(B.A)}{dt} = \frac{-A.d\left(\frac{\mu_0 I}{2r}\right)}{dt} = -\frac{\mu_0 A}{2r}\times\frac{dI}{dt} = -\frac{\mu_0 A}{2r}\times I_0\omega\cos(\omega t+\phi)$

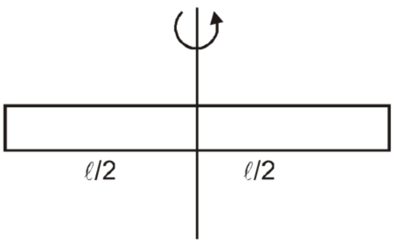
So , $\varepsilon \propto \omega$ ⇒ $\varepsilon \propto n$ $\varepsilon_1 : \varepsilon_2 = n_1 : n_2$

F-2 **Sol.** $\phi = LI$ ⇒ $\varepsilon = \frac{d\phi}{dt} = L\frac{dI}{dt}$ ⇒ $45 = L \times 1.5$

⇒ $L = 30H$

F-3 **Sol.** $I = \frac{1}{R}\frac{d\phi}{dt}$ ⇒ $\frac{d\phi}{dt} = IR = 0.2 \times 5 = 1$ Wb/s

F-4 **Sol.**



both ends of rod will have same potential $v = \frac{1}{2}B\omega\left(\frac{\ell}{2}\right)^2 = \frac{B\omega\ell^2}{8}$.

F-5 **Sol.** $\varepsilon = \int_{7cm}^{10cm} B(\omega x)dx$

$= B\omega\left[\frac{x^2}{2}\right]_{7cm}^{10cm}$

$= \frac{2\times10}{2} \times (100 - 40) \times 10^{-4}$

$= 10^{-3} \times 51 = 0.051$ volt.

F-6 **Ans.** $\mathbf{2B\omega R^2}$

Sol. Here effective length is 2R

$\varepsilon = \frac{1}{2} B\omega(2R)^2 = 2B\omega R^2$

Section (G) : Self induction, self inductance self induced emF & Magnetic energy density

G-1 **Sol.** $\phi = BA = \left(N\mu_0\left(\frac{N}{\ell}\right)\times I\right)A = LI \Rightarrow L = \frac{\mu_0 N^2 \times A}{\ell}$

$\Rightarrow L \propto N^2$

G-2 **Sol.** $\varepsilon = L\frac{dI}{dt} \Rightarrow E \,.\, d = L\frac{dI}{dt} \Rightarrow \frac{F}{q}d = L\frac{dI}{dt}$

$\Rightarrow \frac{MLT^{-2}\times L}{A\times T} = LAT^{-1} \Rightarrow [L] = ML^2T^{-2}A^{-2}$

G-3 **Sol.** $\phi = BA = \left(N\mu_0\left(\frac{N}{\ell}\right)\times I\right)A = LI \Rightarrow L = \frac{\mu_0 N^2 \times A}{\ell}$

G-5 **Sol.** $\varepsilon = -L\frac{dI}{dt} \Rightarrow 5 = -L\times\frac{2-3}{10^{-3}} \Rightarrow L = 5 \times 10^{-3} = 5$ mH

G-6

Sol. $\int E.d\ell = A\frac{dB}{dt}$

$\Rightarrow E \,.\, 2\pi R = \pi R^2 \frac{dB}{dt}$

$\Rightarrow E = \frac{R}{2}\frac{dB}{dt}$

$\Rightarrow a = \frac{qE}{m} = \frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$

G-7

Sol. Since $P_2 = P_2$ or $i_1v_1 = i_2v_2$ & $\frac{v_1}{v_2} = \frac{L_1\frac{di_1}{dt}}{L_2\frac{di_2}{dt}}$

or $\frac{v_1}{v_2} = 4$ so $\frac{i_1}{i_2} = \frac{1}{4}$ $\Rightarrow$ $\frac{w_2}{w_1} = \frac{\frac{1}{2}L_2i_2^2}{\frac{1}{2}L_1i_1^2} = 4$

Section (H) : Circuit containing inductance, Resistance & battery, Growth and decay Of Current in a circuit containing inductor

H-1 **Sol.** R has dimensions $ML^2T^{-3}A^2$, C has dimensions $M^{-1}L^{-2}T^4A^{-2}$, L has dimensions $ML^2T^{-2}A^2$, frequency has dimensions T^{-1}.

$$\sqrt{LC} = \sqrt{ML^2T^{-2}A^2} \times \sqrt{M^{-1}L^{-2}T^4A^{-2}}$$

H-2

Sol. $L_1\frac{di_1}{dt} = L_2\frac{di_2}{dt}$

or $L_1di_1 = L_2di_2$ or $L_1i_1 = L_2i_2$

$\therefore \frac{i_1}{i_2} = \frac{L_2}{L_1}$

H-3 **Sol.** EMF induced = $-L\frac{di}{dt}$.

H-4 **Sol.** $i = i_0\left(e^{\frac{-t}{\tau}}\right)$ or $\tau = \frac{2}{\ell n\left(\frac{10}{9}\right)}$.

H-5 **Sol.** Initially the inductor offers infinite resistance hence i_1 is 1A. Finally, at steady state inductor offers zero resistance and current i_2 is 1.25 A in the battery.

H-6 **Sol.** Initially inductor will offer infinite resistance and capacitors zero resistor and finally capacitor will offer infinite resistance and inductor will offer zero resistance. So initial and final current will be $I = \frac{E}{2R}$

Section (I) : Mutual Induction & Mutual inductance

I-1 **Sol.** EMF = $\left|-M\frac{dI}{dt}\right|$ $25 \times 10^{-3} = M \times 15$

or $M = \frac{5}{3} \times 10^{-3}$ H

$\phi = MI = \frac{5}{3} \times 10^{-3} \times 3.6 = 6.00$ mWb.

I-2 **Sol.** As the flux in the ring due to wire will be zero hence mutual inductance will be zero.

I-4 **Sol.** $M = K\sqrt{L_1L_2} = \sqrt{L_1L_2}$ (since K = 1)

$= \sqrt{0.1 \times 0.1} = 0.1$ H

Section (J) : L C oscillations

J-1 **Sol.** $f = \frac{1}{2\pi}\frac{1}{\sqrt{L_{eff} \times c_{eff}}} = \frac{1}{2\pi\sqrt{3L \times 3C}} = \frac{1}{6\pi\sqrt{LC}}$.

J-2 **Sol.** $C_{eq} = 3C$

$Q_{eq} = 3Q$

$E = \frac{1}{2}\frac{Q_{eq}}{C_{eq}} = \frac{3Q^2}{2C}$.

Exercise-2

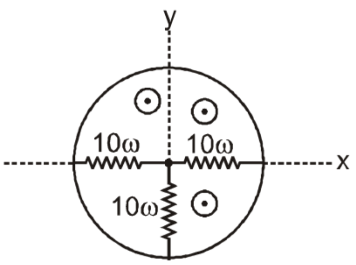
Marked Questions can be used as Revision Questions.

PART - I : OBJECTIVE QUESTION

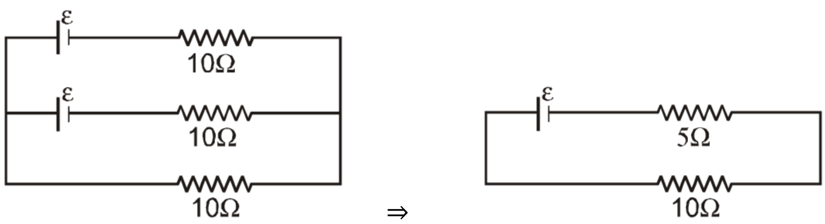
1. **Sol.** $U = \frac{1}{2}LI^2$

$P = I^2R$ or $\frac{2U}{P} = \frac{L}{R} = \tau$.

2. **Sol.**

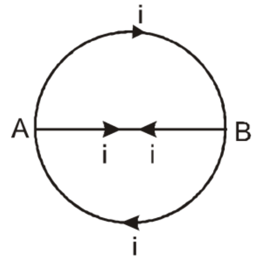


Equivalent circuit



So current $I = \frac{\varepsilon}{R} = \frac{\frac{1}{2}B\omega r^2}{15} = \frac{\frac{1}{2} \times 50 \times 20 \times (0.1)^2}{15} = \frac{1}{3}$ Amp.

3. **Sol.** The induced current in upper semicircular and lower semicircular will cancel each other in diameter (AB).



4. **Sol.** $V_{ab} = L\frac{dI}{dt} + IR$

$8 = L \times 1 + 2 \times R$

$4 = -L \times 1 + 2 \times R$

Solving the above equations

we get $R = 3\Omega$

$L = 2H.$

5. **Sol.** $I = I_1 + I_2$

$I_1 = \varepsilon/R$

$L\frac{dI_2}{dt} = \varepsilon.$

$I_2 = \frac{\varepsilon t}{L}$

$I = \varepsilon/R + \frac{\varepsilon t}{L}$

$I = 12A.$

6. **Sol.** Magnetic force on electron acts towards B. So electrons accumulate at B and A becomes positively charged.

7. **Sol.** Since the magnitude flux in the ring due to motion of charge particle is zero hence the induced emf will be zero. So current is also zero.

8. **Sol.** $q = \frac{\Delta\phi}{R}$

$\therefore$ $\Delta\phi = qR$ = (area of It graph) × R.

$= \frac{1}{2} \times 0.1 \times 4 \times 10 = 2$

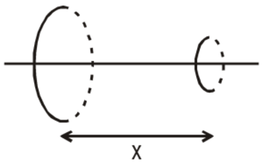
9. **Sol.** Since the tube is very long the force on magnet due to induced current will continue to oppose its motion till it acquires a constant speed.

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : Assertion/Reasoning

A-1. **Sol:** Magnetic field cannot do work, hence statement-1 is false.

A-2. **Sol.** **(Moderate)**



It is obvious that flux linkage in one ring due to current in other coaxial ring is maximum when x = 0 (as shown) or the rings are also coplanar. Hence under this condition their mutual induction is maximum.

A-3. **Sol.** Obiviously statement 2 is correct explanation of statement-1

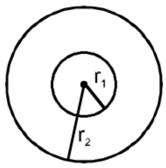
Section (B) : Match the column

B-1. **Ans.** (1 → q); (2 → p); (3 → t); (4 → t)

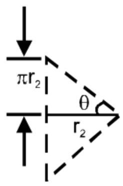
Sol. **(Moderate)** When both S_1 and S_2 are either open or closed; current through ad is zero. With S_1 closed, current 2×10^{-7} A flows from a to d. With S_2 closed, current 2×10^{-7} A flows from d to a.

B-2. **Ans.** (1 → r); (2 → s); (3 → s); (4 → r)

Sol. B at the smaller coil due to 'i' in larger loop = $\frac{\mu_0 i}{2r_2}$



∴ ϕ linked = $\frac{\mu_0 i}{2r_2}\pi r_1^2$



∴ $M_1 = \frac{\frac{\mu_0 i}{2r_2}\pi r_1^2}{} = \frac{\mu_0 A}{2.\sqrt{\frac{nA}{\pi}}} = \frac{\mu_0\sqrt{\pi}}{2}\sqrt{\frac{A}{n}}$

M_2 and M_3 are zero

($\because$ plane is parallel to B, no flux)

where $M_4 = \frac{\mu_0 \sin\theta}{2\pi r_2}\pi r_1^2 = \frac{\mu_0 \sin\theta}{2\sqrt{\pi}}\sqrt{\frac{A}{n}}$.

Section (C) : One or More Than One Options Correct

C-1.*# **Sol:** Magnetic field cannot do work

C-2.*# **Sol.** Rate of work done by external agent is :

$\frac{dw}{dt} = \frac{BIL.dx}{dt}$ = BILv & thermal power dissipated in the resistor = eI = (BvL) I

clearly both are equal, hence (1) .

If applied external force is doubled, the rod will experience a net force and hence acceleration. As a result velocity increases, hence (2)

Since ; $I = \frac{e}{R}$

On doubling 'R', current and hence required power becomes half.

Since, P = BILv

Hence (4).

C-3*. **Sol.** EMF is induced in the ring if there is change in flux which occurs either due to rotation about a diameter or due to its deformation.

C-4.*

Sol. $e = -\frac{d\phi}{dt}$, $e = -\frac{dBA\sin\omega t}{dt}$ = –BAw cosωt.

C-5*. **Sol.** Magnetic lines of force do noy pass inside a super conducting loop

hence $\varepsilon = 0$

$\frac{d\phi}{dt} = 0$

or ϕ = constant.

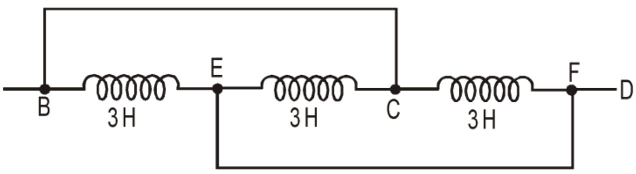
C-6.* **Sol.** EMF induced = $-L\frac{di}{dt} \neq 0$, rest quantities are zero.

Exercise-3

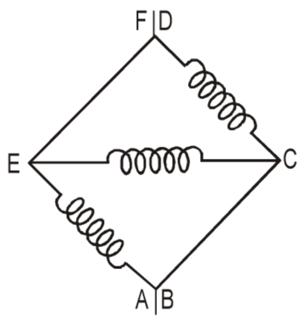
Marked Questions can be used as Revision Questions.

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Sol.



Here, inductors are in parallel



$\therefore \quad \frac{1}{L} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

L = 1

2. **Sol.** Mutual inductance of the pair of coils depends on distance between two coils and geometry of two coils

3. **Sol.** $e = -L\frac{di}{dt} = L\frac{(-2-2)}{0.05}$

$8 = L\frac{(4)}{0.05}$

$L = \frac{8\times 0.05}{4}$ =0.1H

4. **Sol.** $\frac{1}{2}Li^2 = \frac{1}{2}\frac{q^2}{C}$

⇒ $q^2 = Li^2C$ .....(i)

and $U_{E(max)} = U_{B(max)}$ (given) .....(ii)

Since, $U_B = \frac{1}{2}Li^2$

where $i = \frac{\varepsilon}{R}(1-e^{Rt/L})$

at $t = \infty$; = $i_\infty = \frac{\varepsilon}{R}$

$U_{B(max)} = \frac{1}{2}Li_\infty^2$ .....(iii)

and $U_{E(max)} = \frac{Q^2}{2C}$ .....(iv)

Where Q is maximum charge on capacitor. The energy is equally divided between electric and magnetic fields. Therefore

∴ $U_B = \frac{U_{B(max)}}{2}$

⇒ $\frac{1}{2}Li^2 = \frac{1}{2}\left(\frac{1}{2}Li_\infty^2\right)$

⇒ $i_\infty^2 = 2i^2$ ......(v)

From equation (ii), (iii), (iv) and (v), we get

∴ $\frac{1}{2}L.2i^2 = \frac{1}{2}\frac{Q^2}{C}$

$LCi^2 = \frac{Q^2}{2}$

∴ $q^2 = \frac{Q^2}{2}$ [from equation (i)]

$q = \frac{Q}{\sqrt{2}}$

Method II

$\frac{q^2}{2C} = \frac{1}{2}\left(\frac{Q^2}{2C}\right)$ ⇒ $q = \frac{Q}{\sqrt{2}}$ **Ans.**

5. **Sol.** The rate of change of flux or emf induced in the coil is

$\varepsilon = \frac{-\Delta\phi}{\Delta t}$

∴ Induced current

$i = \frac{\varepsilon}{R_{eq}} = -\frac{1}{R}\frac{\Delta\phi}{\Delta t}$ ...(i)

Given: $R_{eq.} = R + 4R = 5R$, $\Delta\phi = n(w_2 - w_1)\,A$, $\Delta t = t$. (Here W_1 and W_2 are associated with one turn.)

Putting the given values in eq. (i), we get

∴ $i = -\frac{n}{5R}\,\frac{(W_2 - W_1)A}{t}$

6. **Sol.** The flux associated with coil of area A and magnetic induction B is

$\phi = BA\cos\theta$

$= \frac{1}{2}B\pi r^2\cos\omega t$ $\left[\because A = \frac{1}{2}\pi r^2\right]$

∴ $e_{induced} = -\frac{d\phi}{dt}$

$= -\frac{d}{dt}\left(\frac{1}{2}B\pi r^2\cos\omega t\right)$

$= \frac{1}{2}B\pi r^2\omega\sin\omega t$

∴ Power $P = \frac{e_{induced}^2}{R}$

$= \frac{B^2\pi^2r^4\omega^2\sin^2\omega t}{4R}$

Hence, $P_{mean} = \langle P \rangle$

$= \frac{B^2\pi^2r^4\omega^2}{4R}\cdot\frac{1}{2}$ $\left[\because \langle \sin\omega t \rangle = \frac{1}{2}\right]$

$= \frac{(B\pi r^2\omega)^2}{8R}$

7. **Sol.** The emf induced between ends of conductor

$e = \frac{1}{2}B\omega L^2$

$= \frac{1}{2}\times 0.2\times 10^{-4}\times 5\times (1)^2$

$= 0.5\times 10^{-4}\,V = 5\times 10^{-5}\,V = 50\,\mu V$

8. **Sol.** Relative velocity $= \upsilon - (-\upsilon) = 2\upsilon = \frac{dl}{dt}$

Now, $e = \frac{d\phi}{dt}$

$e = \frac{Bdl}{dt}$ $\left(\frac{dl}{dt} = 2\upsilon\right)$

Induced emf

$e = 2Bl\upsilon$

9. **Sol.** The current at any instant is given by

$I = I_0(1 - e^{-Rt/L})$

$\frac{I_0}{2} = I_0(1 - e^{-Rt/L})$

$\frac{1}{2} = (1 - e^{-Rt/L})$

$e^{-Rt/L} = \frac{1}{2}$

$\frac{Rt}{L} = ln\,2$

$\therefore$ $t = \frac{L}{R}\ln 2$

$= \frac{300 \times 10^{-3}}{2} \times 0.693 = 150 \times 0.693 \times 10^{-3}$

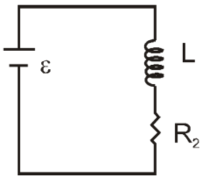
$= 0.10395$ sec $= 0.1$ sec.

10. **Sol.** $I = I_0 e^{-Rt/L} = \frac{1}{e}$ A.

11. **Sol.** $M = \mu_0 n_1 A N_2 = \left(4\pi \times 10^{-7}\right)\left(\frac{300}{0.20}\right)(10 \times 10^{-4})(400)$

$= 2.4\pi \times 10^{-4}$ H

12. **Sol.**



$V_L = \varepsilon e^{-\frac{R_2 t}{L}} = 12.\, e^{-\frac{2t}{400 \times 10^{-3}}} = 12\, e^{-5t}$.

13. **Ans.** **(2)**

Sol. At t = 0, current does not flow through inductor.

$\therefore i = \frac{V}{R_2}$

At $t = \infty$ inductor behaves as wire $\Rightarrow$ $R_{eq} = \frac{R_1R_2}{R_1 + R_2}$

$\therefore i = \frac{V(R_1 + R_2)}{R_1R_2}$

14. **Ans.** **(2)**

Sol. In LC oscillation energy is transfered C toL

or L to C maximum energy in L is = $\frac{1}{2}LI^2_{max}$

Maximum energy in C is = $\frac{q^2_{max}}{2C}$

Equal energy will be when

$\frac{1}{2}LI^2 = \frac{1}{2}\frac{1}{2}LI^2_{max}$

$I = \frac{1}{\sqrt{2}}I_{max}$

$I = I_{max}\sin\omega t = \frac{1}{\sqrt{2}}I_{max}$

$\omega t = \frac{\pi}{4}$

or $\frac{2\pi}{T}t = \frac{\pi}{4}$ or $t = \frac{T}{8}$

$t = \frac{1}{8}2\pi\sqrt{LC} = \frac{\pi}{4}\sqrt{LC}$ **Ans.**

15. **Ans.** **(4)**

Sol. $E_{ind} = B \times v \times \ell$

$= 5.0 \times 10^{-5} \times 1.50 \times 2$

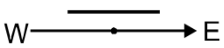
$= 10.0 \times 10^{-5} \times 1.5$

$= 15 \times 10^{-5}$ vot.

= 0.15 mv

16. **Ans.** **(1)**

Sol.



$\varepsilon_{ind} = Bv\ell$

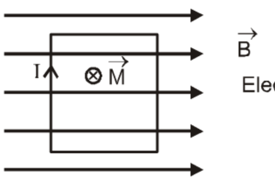
$= 0.3 \times 10^{-4} \times 5 \times 20$

$= 3 \times 10^{-3}$ v

= 3 mv.

17. **Ans.** (4)

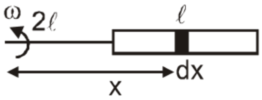
Sol.



Electomagnetic damping

18.

Sol. $e = \int_{2\ell}^{3\ell} (\omega x)Bdx = B\omega \frac{[(3\ell)^2 - (2\ell)^2]}{2}$



$= \frac{5B\ell^2\omega}{2}$

Ans. (4)

19. **Sol.** $\frac{\mu_0(2)(20\times10^{-2})^2}{2\left[(0.2)^2+(0.15)^2\right]} \times \pi\,(0.3\times10^{-2})^2$

on solving

$= 9.216 \times 10^{-11}$

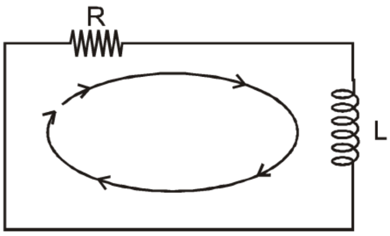
$\approx 9.2 \times 10^{-11}$ weber

Ans (1)

20. **Ans. (3)**

Sol. After changing the switch, the circuit will act like an L–R discharging circuit.

Applying Kirchoff loop eqation.



$V_R + V_L = 0$

$\Rightarrow V_R = -V_L$

So $\frac{V_R}{V_L} = -1$

21. **Ans. (4)**

Sol. Current at t = 0 $I_0 = \frac{E_0}{R}$

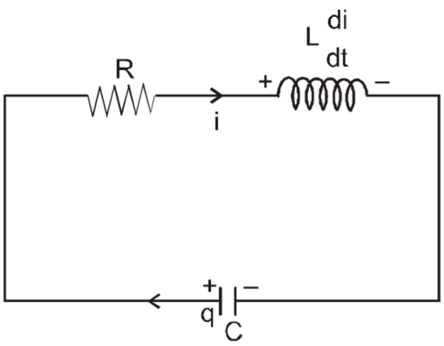
t = 0 $I_0 = \frac{E_0}{R}$

For decay circuit $I = I_0 e^{-\frac{tR}{L}}$

$I = \frac{E_0}{R} e^{-\frac{tR}{L}}$ $\Rightarrow$ I = 0.67 mA

22. **Ans. (1)**

Sol.



$\frac{q}{C} - iR - L\frac{di}{dt} = 0$

at any time 't' apply KVL

$i = -\frac{dq}{dt}$ $\Rightarrow$ $\frac{q}{C} + \frac{dq}{dt}R + \frac{Ld^2q}{dt^2} = 0$

$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = 0$

from damped harmonic oscillator, the amplitude is given by

$A = A_0 e^{-\frac{dt}{2m}}$, for general equation of double differential equation $\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$

$A = A_0 e^{-\frac{dt}{2m}}$, $\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$

$\Rightarrow$ $Q_{max}^{(t)} = Q_0 e^{-\frac{Rt}{2L}}$ $\Rightarrow$ $Q_{max}^2 = Q_0^2 e^{-\frac{Rt}{L}}$

lesser the self inductance, faster will be damping hence

23. **Ans.** **(4)**

Sol. $\Delta Q = \frac{\Delta\phi}{r} = \text{Area under } i - t \text{ graph}$

$= \frac{\Delta\phi}{100} = \frac{1}{2} \times 10 \times .5$

$\Rightarrow \Delta\phi = 2.5 \times 100 = 250$

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. **Ans.** D

Sol. When switch S is closed magnetic field lines passing through Q increases in the direction from right to left. So, according to Lenz's law induced current in Q will flow in such a direction so that the magnetic flux through Q decreases. This is possible when current in Q flows in anticlockwise direction as seen by E. Opposite is the case when switch S is opened, induced current in Q will be clockwise as seen by E.

2. **Ans.** B

Sol. Power P = $\frac{e^2}{R}$

here e = induced emf = – $\left(\frac{d\phi}{dt}\right)$ Where ϕ = NBA

$\therefore$ e = – NA $\left(\frac{dB}{dt}\right)$

also R $\propto \frac{1}{r^2}$

where R = resistance, r = radius, ℓ = length

$\therefore$ P $\propto \frac{N^2r^2}{\ell}$ $\therefore$ $\frac{P_1}{P_2}$ = 1

3. **Sol.** **(1)** zero, as there is no flux change.

4. Sol. (B)

$Q = Q_0(1 - e^{-t/\tau})$

$Q = CV(1 - e^{-t/\tau})$ after time interval 2τ.

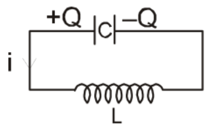
5. Sol. (D)

$q = Q_0 \cos \omega t$

$i = -\dfrac{dq}{dt} = Q_0\, \omega \sin \omega t \quad \Rightarrow \quad i_{max} = C\omega V = V\sqrt{\dfrac{C}{L}}$

6. Ans. C

Sol.



$L\dfrac{di}{dt} - \dfrac{Q}{C} = 0, \quad -L\dfrac{d^2Q}{dt^2} - \dfrac{Q}{C} = 0, \quad Q = -LC\dfrac{d^2Q}{dt^2}$

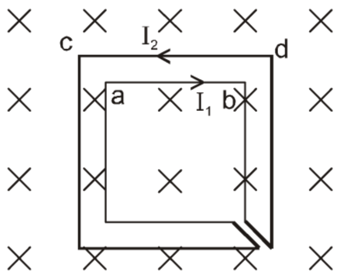
7.# Ans. (A)

Sol. Due to induce current in coil, force between two coil is generated.

8. Solution : Current $I_1 = I_2$,

Since magnetic field increases with time

So induced net flux should be outward (opposite to external field) i.e. current will flow in loop in anticlockwise. I_1 from a to b and I_2 from d to c



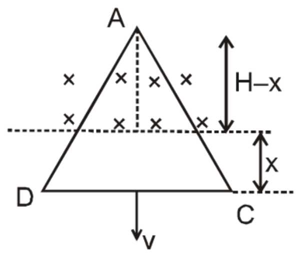
9. Ans. (C)

Sol. True for induced electric field and magnetic field.

Additional Problems For Self Practice (APSP)

PART-I : PRACTICE TEST PAPER

1. **Sol.** $\phi = -\frac{1}{2}(2)\frac{H-I}{\sqrt{3}}(H-X)$



$|-d\phi/dt| = \varepsilon = \frac{2(H-x)}{\sqrt{3}}$

$i = \frac{2}{\sqrt{3}R}(H–x)$

Hence answer is (B)

2. **Sol.** Induced motional emf in MNQ is equivalent to the motional emf in an imaginary wire MQ i.e.,

$e_{MNQ} = e_{MQ} = Bv\ell = Bv\,(2R)$

$[\ell = MQ = 2R]$

Therefore , potential difference developed across the ring is 2RBv with Q at higher potential.

3. **Sol.** $I = \frac{\frac{1}{2}B\omega L^2}{R} = \frac{\frac{1}{2}\times 0.10\times 40\times(5\times 10^{-2})^2}{1} = 5\text{ mA}$

4. **Sol.** EMF $= -\frac{d\phi}{dt} = -\frac{dB\pi r^2}{dt} = -\pi r^2\frac{dB}{dt}$ or $E = \left(\frac{EMF}{r}\right) = \left(\frac{\pi dB}{dt}\right)r$

or $E \propto r$ for $r \le R$.

$E \propto \frac{1}{r}$ for $r > R$.

5. **Sol.** If the circuit Q C P containing rod PQ is completed then the direction of induced current will be from Q to C to P hence Q will be at higher potential than P.

6. **Sol.** $R = \frac{V}{I}$

$\tau = \frac{L}{R} = 1$ ms.

7. **Sol.** $\phi = M \times I$

$\frac{\int_d^{d+b} B.ds}{I} = M$

$M = \frac{\mu_0 a}{2\pi}\ell n\frac{b+d}{d}$

Hence $M \propto a$.

8. Sol. $M_{max} = \sqrt{L_1L_2} = \sqrt{100 \times 400}$ mH = 200 mH.

9. Sol. $L_{eff} = 2H$

Energy stored in inductor = $\frac{1}{2}LI^2$

$= \frac{1}{2} \times (2) \times (1)^2 = 1J.$

Energy developed in resistance = $I^2RT = 1^2 \times 10 \times 10 = 100$ J

Hence the required ratio is $\frac{1}{100}$.

10. Sol. $\oint E.d\ell = -\frac{d\phi}{dt}$

$E \times 2\pi R = \pi R^2 \frac{dB}{dt}$.

$E = \frac{R}{2} \times 8t = R8$

$(qE)R = (\mu mg) R$

$\mu = \frac{8qR}{mg}$

11. Sol. In the loop containing wire AB the flow of current will be from B to A because emf generated in that loop is less than the emf generated in the loop containing CD.

12. Sol. $q = \int Idt = \int -\frac{1}{r}\frac{d\phi}{dt} dt = -\frac{\Delta\phi}{r} = \frac{\mu_0 Ia}{\pi r}\ell n2$.

13. Sol. $P = F.V \quad = Bi\ell V = B\left(\frac{Bv\ell}{R}\right)\ell V$, $P \propto V^2$

14. Sol. $\int Ed\ell = \varepsilon$, $E = \frac{r}{2}\frac{dB}{dt}$,

$E\cos\theta = \frac{r\cos\theta}{2} B_0 = B_0$

$V_Q - V_P = 2\ell \quad = B_0 \ell$

15. Sol. $U = \frac{1}{2}LI^2$

$\frac{dU}{dt} = LI\frac{dI}{dt} = \quad RI_0^2(1 - e^{-t/\tau})e^{-t/\tau}$

$\frac{dU}{dt}$ is maximum when $e^{-t/\tau} = \frac{1}{2}$ or $\left(\frac{dU}{dt}\right)_{max} = \frac{E^2}{4R} = 1$ W.

16. Sol. $E = \frac{1}{2}LI^2 \qquad E = \frac{1}{2}L\frac{V^2}{R^2}$

$= \frac{1}{2} \times 5 \times 10^{-3} \times (1)^2$

$= 2.5$ mJ.

17. **Sol.** Since the tube is very long the force on magnet due to induced current will continue to oppose its motion till it acquires a constant speed.

18. **Sol.** $\langle i \rangle = \frac{\langle \varepsilon \rangle}{R} = \frac{BA}{Rt} = \frac{(\mu_0 n I)\, AN}{Rt}$

$= \frac{4\pi \times 10^{-7} \times 400 \times 0.40 \times 6 \times 10^{-4} \times 10}{1.5 \times 0.050}$.

19. **Sol.** $B = \frac{n_0 \mu_0 I_0}{2R_0}$, $\phi = B \times \pi r_i^2 \times n_i$, $EMF = -\frac{\Delta\phi}{\Delta t}$

$EMF = -\frac{n_0 \mu_0 \times \pi r_i^2 n_i}{2R_0} \times \frac{\Delta B}{\Delta t} = 493\mu V$.

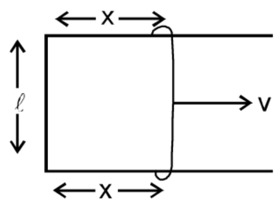
20. **Sol.** $\varepsilon = BVL = 0.2 \times 2 \times 10^{-2}$ volt

21. **Sol.** $\varepsilon = BV(L \sin\theta)$

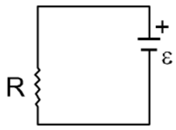
$= 0.1 \times 0.2 \times 1 \sin 60^{o}$

$= \sqrt{3} \times 10^{-2} V$

22. **Sol.** $\varepsilon = BV\ell$

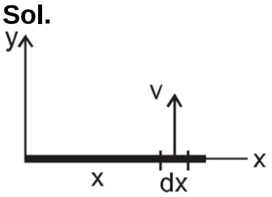


Net resistance $R = (2\ell + 2x)\, r$



$i = \frac{\varepsilon}{R} = \frac{BV\ell}{(2\ell + 2x)r} = \frac{BV\ell}{2r(\ell + Vt)}$

23. **Sol.**



$\varepsilon = \int_0^L \frac{B_0 x}{L} v_0\, dx = \frac{B_0 v_0 L}{2}$

24. **Ans.** C

Sol. For understanding, let us assume that the two loops are lying in the plane of paper as shown. The current in loop 1 will produce ⊙ magnetic field in loop 2. Therefore, increase in current in loop 1 will produces ⊗ magnetic field passing through it i.e. induced current in loop 2 will also be clockwise as shown alongside.

25. **Ans.** D

Sol. The current-time (i- t) equation in L-R circuit is given by [Growth of current in L -R circuit]

$i = i_0 (1 - e^{-t/\tau})$

where $i_0 = \frac{V}{R} = \frac{12}{6} = 2A$

and $\tau_L = \frac{L}{R} = \frac{8.4 \times 10^{-3}}{6} = 1.4 \times 10^{-3}$ s

and $i = 1A$ (given)

$t = ?$

Substituting these values in equation (1), we get -

$t = 0.97 \times 10^{-3}$ s

or $t = 0.97$ ms

$t \approx 1$ ms

26. **Ans.** B

Sol. **(2)**

$\oint \vec{E} \cdot \vec{dl} = \left|\frac{d\phi}{dt}\right|$

$= S\left|\frac{dB}{dt}\right|$

or $E(2\pi r) = \pi a^2 \left|\frac{dB}{dt}\right|$ for $r \geq a$

$\therefore$ Induced electric field $\propto \frac{1}{r}$

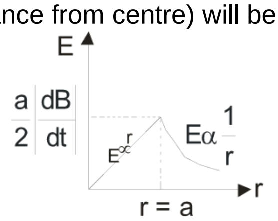
for $r \leq a$

$E(2\pi r) = \pi r^2 \left|\frac{dB}{dt}\right|$

or $E = \frac{r}{2}\left|\frac{dB}{dt}\right|$ or $E \propto r$

At $r = a$, $E = \frac{a}{2}\left|\frac{dB}{dt}\right|$

Therefore, variation of E with r (distance from centre) will be as follows :



27. **Ans.** D

Sol. (4) Electric field will be induced in both AD and BC. (
because of induced emf.

28. **Sol.** $\frac{dB}{dt} = 2T/s$

$E = \frac{AdB}{dt} = 800 \times 10^{-4} m^2 \times 2 = 0.16$ V

$I = \frac{E}{R} = \frac{0.16}{1} = 0.16$ A, clockwise

29. **Sol.** At t = 2s B = 4T; $\frac{dB}{dt} = 2T/s$

t = 2 s B = 4T; $\frac{dB}{dt}$ = 2T/s
A = 20 × 30 cm^2
= 600 × 10^{-4} m^2; $\frac{dA}{dt}$ = –(5 × 20) cm^2/s
= – 100 × 10^{-4} m^2/s

$E = -\frac{d\phi}{dt} = -\left[\frac{d(BA)}{dt}\right] = -\left[\frac{BdA}{dt} + \frac{AdB}{dt}\right]$

= – [4×(–100 × 10^{-4}) + 600 × 10^{-4} × 2]
= – [–0.04 + 0.120] = – 0.08 v

Alternative :
ϕ = BA = 2t x 0.2 (0.4 – vt)
= 0.16t – 0.4 vt^2
$E = -\frac{d\phi}{dt}$ = 0.8 vt – 0.16
at t = 2s
t = 2s
E = – 0.08 V

30. **Sol.** At t = 2s, length of the wire = (2 × 30 cm) + 20 cm = 0.8 m
Resistance of the wire = 0.8 Ω

Current through the rod = $\frac{\varepsilon}{R} = \frac{0.08}{0.8} = \frac{1}{10}$ A

Force on the wire = $i\ell B$ = $\frac{1}{10}$ × (0.2) × 4 = 0.08 N
Same force is applied on the rod in opposite direction to make net force zero.

PART - II : PRACTICE QUESTIONS

1. **Ans.** A
Sol. (1) When current flows in any of the coils, the flux linked with the other coil will be maximum in the first case. Therefore, mutual inductance will be maximum in case (1) .

2.# **Ans.** D
Sol. When switch S is closed magnetic field lines passing through Q increases in the direction from right to left. So, according to Lenz's law induced current in Q i.e. I_{Q1} will flow in such a direction so that the magnetic field lines due to $]_{Q2}$ passes from left to right through Q. This is possible when I_{Q1} flows in anticlockwise direction as seen by E. Opposite is the case when switch S is opened i.e. I_{Q2} will be clockwise as seen by E.

3. **Ans.** B
Sol. Power $P = \frac{e^2}{R}$

here e = induced emf = $-\left(\frac{d\phi}{dt}\right)$ Where ϕ = NBA

∴ $e = -NA\left(\frac{dB}{dt}\right)$

also $R \propto \frac{1}{r^2}$

where R = resistance, r = radius, ℓ = length

$\therefore \quad P \propto \frac{N^2r^2}{\ell} \qquad \therefore \quad \frac{P_1}{P_2} = 1$

4.# **Sol.** $\phi_A = \frac{\mu_0 i\pi R^2}{2\pi(R^2+x^2)^{3/2}}.\pi r^2$

$E_A = -\frac{d\phi}{dt} = \frac{\mu_0 i\pi}{2} R^2 r^2 \ (-3/2)\ (R^2 + x^2)^{-5/2} \ . \ 2x\ (v)$

E_A is maximum when $\frac{dE_A}{dx} = 0$

$\Rightarrow \quad \frac{d}{dx}\frac{x}{(R^2+x^2)^{5/2}} = 0$

or $\quad (R^2 + x^2)^{5/2} - \frac{5x}{2}(R^2 + x^2)^{3/2}\ 2x = 0$

or, $\quad R^2 + x^2 - 5x^2 = 0$

or, $\quad x = \frac{R}{2}$ **Ans.**

5.#

Sol. $E = \frac{d\phi}{dt} = \frac{Bd(b\ell)}{dt}$

$= Bbv = B \times 2 \times 10^{-2} \times 20 \qquad = 0.40\ B$

$\Delta t = \frac{1\times10^{-2}}{20} \qquad = 5 \times 10^{-4}$ sec $= 500\ \mu$ sec

$t = \frac{6\times10^{-2}}{20} \qquad = 3 \times 10^{-3}$ sec $= 3000\ \mu$ sec

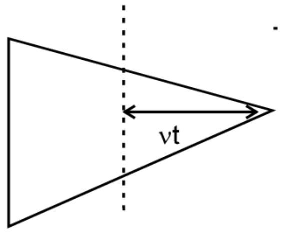
Ans. (A)

6.# **Sol.** $e = \frac{BdA}{dt}$

$= \frac{Bd}{dt}(\pi r^2) \qquad = B2\pi r\frac{dr}{dt}$

7.# **Sol.** $A = \frac{1}{2} \times 6 \times 4 - \frac{1}{2} \times 2\ vt \tan 37^o \times vt$

$\Rightarrow \phi = B\ A$



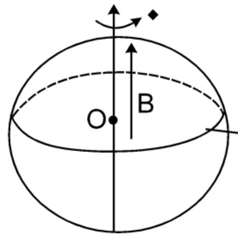
$\Rightarrow e = \frac{-d\phi}{dt} = +Bv^2\ \frac{3}{4} \times 2t = iR'$

$i \propto t$

$p \propto t^2$

$\Rightarrow P \propto t^2$ (parabolic variation)

8. **Sol.**



the equator can be seen as a conducting ring of radius R_e revolving with angular velocity ω in a perpendicular magnetic field B.

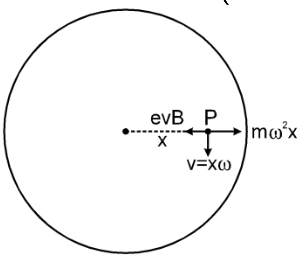
∴ Potential difference. across its center and periphery = $\frac{B\omega R_e^2}{2}$

Potential at pole = potential of the axis of earth i.e. potential at point O

∴ $V_{eqvator} - V_{pole} = \frac{B\omega {R_e}^2}{2}$.

9.# **Sol.** Consider a free electron in the disc at point P distant x from centre of disc.

The magnetic force on free electron is = evB = eωxB (towards left)



Centrifugal force = $m\omega^2x$. (towards right)

For net force on the electron at P to be zero

i.e., $e\omega x B = m\omega^2 x$

or $\omega = \frac{eB}{m}$

There shall be no flow of free electrons radially outwards and hence no electric field shall develop within the disc

Ans. $\frac{eB}{m}$

10. 

Using ; $V_A - V_B = RI + L\frac{dI}{dt}$

140 = 5R + 10 L

60 = 5R – 10 L

⇒ L = 4H. **Ans.**

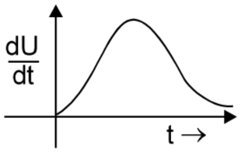
13. **Sol.** Rate of increment of energy in inductor = $\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt}$

Current in the inductor at time t is:

$i = i_0\,(1 - e^{-\frac{t}{\tau}})$ and $\frac{di}{dt} = \frac{i_0}{\tau}e^{-\frac{t}{\tau}}$

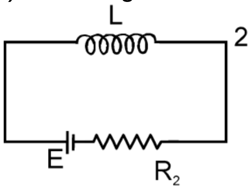
∴ $\frac{dU}{dt} = \frac{Li_0}{\tau}e^{-\frac{t}{\tau}}(1 - e^{-\frac{t}{\tau}})$

$\frac{dU}{dt} = 0$ at t = 0 and t = ∞



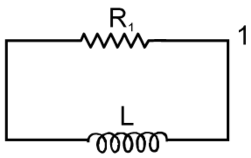
Hence E is best represented by :

15.# **Sol.** When the key is at position (2) for a long time ; the energy stored in the inductor is :



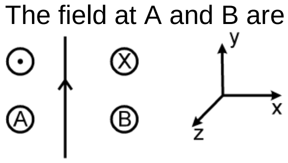
$U_B = \frac{1}{2} L i_0^2 = \frac{1}{2} . L . \left(\frac{E}{R_2}\right)^2 = \frac{L.E^2}{2R_2^2}$

This whole energy will be dissipated in the form of heat when the inductor is connected to R_1 and no source is connected.



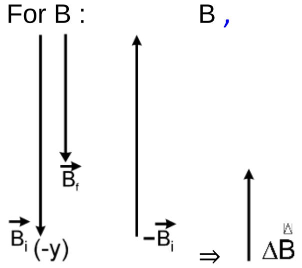
Hence (A).

28. **Sol.** The field at A and B are out of the paper and inside the paper respectively.



As the current in the straight wire decreases the field also decreases.

For B : B ,



The change in the magnetic field which causes induced current $(\Delta\vec{B})$ is along (+)z direction.

Hence, induced emf and hence current should be such as to oppose this change $\Delta\vec{B}$.

Hence, induced emf should be along – z direction which results in a clockwise current in 'B'. Similarly, there will be anticlockwise current in 'A'. Hence (B).

29. **Sol.** Inductance and potential difference across terminals will not change with time.

30. **Sol.** Even after insertion of the rod the current in circuit will increase with time till steady state is reached.

31. **Sol.** At steady state inductor will offer zero resistance and hence $I = \frac{\varepsilon}{R}$.