High Level Problems (HLP)

PART - I : DIFFERENTIATION

Find the derivative of functions using quotient rule.

1. $g(x) = \dfrac{x^2 - 4}{x + 0.5}$

2. Suppose u and v are differentiable functions of x and that
$u(1) = 2, \quad u'(1) = 0 \quad v(1) = 5 \quad v'(1) = -1.$
Find the values of the following derivatives at x = 1.

(a) $\dfrac{d}{dx}(uv)$ (b) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)$ (c) $\dfrac{d}{dx}\left(\dfrac{v}{u}\right)$ (d) $\dfrac{d}{dx}(7v - 2u)$.

3. Find $\dfrac{ds}{dt}$. $s = \dfrac{1+\operatorname{cosec} t}{1-\operatorname{cosec} t}$

4. $s = \dfrac{\sin t}{1-\cos t}$

Find $\dfrac{dy}{dx}$ as a function of x.

5. $y = \sin^3 x$

6. $y = 5\cos^{-4} x$.

Find the derivatives of the functions

7. $r = (\operatorname{cosec}\theta + \cot\theta)^{-1}$

8. $r = -(\sec\theta + \tan\theta)^{-1}$

PART - II : INTEGRATION

Find an antiderivative for each function. Do as many as you can mentally.Check your answer by differentiation.

1. (a) csc x cot x (b) – csc 5x cot 5x (c) $-\pi \csc\dfrac{\pi x}{2}\cot\dfrac{\pi x}{2}$

2. $(1 + 2\cos x)^2$

Evaluating Integrals. Check your answers by differentiation.

3. $\int\left(\dfrac{\sqrt{x}}{2} + \dfrac{2}{\sqrt{x}}\right)dx$ **4.** $\int\left(8y - \dfrac{2}{y^{1/4}}\right)dy$ **5.** $\int 2x(1 - x^{-3})\,dx$

6. $\int(-3\csc^2 x)\,dx$ **7.** $\int\left(-\dfrac{\sec^2 x}{3}\right)dx$ **8.** $\int\dfrac{\csc\theta\cot\theta}{2}\,d\theta$

9. $\int\dfrac{2}{5}\sec\theta\tan\theta\,d\theta$ **10.** $\int(4\sec x\tan x - 2\sec^2 x)\,dx$

11. $\int\dfrac{1}{2}(\csc^2 x - \csc x\cot x)\,dx$ **12.** $\int(\sin 2x - \csc^2 x)\,dx$

13. $\int(2\cos 2x - 3\sin 3x)\,dx$ **14.** $\int 4\sin^2 y\,dy$

15. $\int\dfrac{\csc\theta}{\csc\theta - \sin\theta}\,d\theta$

Evaluate Integrals by substitution method.

16. $\int\dfrac{1}{\sqrt{5s+4}}\,ds$ **17.** $\int 3y\sqrt{7-3y^2}\,dy$ **18.** $\int\sin^5\dfrac{x}{3}\cos\dfrac{x}{3}\,dx$.

19. $\int \tan^7 \frac{x}{2} \sec^2 \frac{x}{2}\, dx$ **20.** $\int r^2 \left(\frac{r^3}{18} - 1\right)^5 dr$ **21.** $\int r^4 \left(7 - \frac{r^5}{10}\right)^3 dr$

22. $\int x^{1/3} \sin(x^{4/3} - 8)\, dx$ **23.** $\int \csc\left(\frac{\upsilon - \pi}{2}\right) \cot\left(\frac{\upsilon - \pi}{2}\right) d\upsilon$

24. $\int \sqrt{\cot y} \csc^2 y\, dy$ **25.** $\int \frac{\sec z \tan z}{\sqrt{\sec z}}\, dz$ **26.** $\int \frac{1}{t^2} \cos\left(\frac{1}{t} - 1\right) dt$

Find the definite integrals of following Functions

27. $\int_{1/2}^{3/2} (-2x + 4)\, dx$ **28.** $\int_{-2}^{1} |x|\, dx$

Evaluate definite integrals of following Functions

29. $\int_{0}^{\pi/2} \theta^2\, d\theta$ **30.** $\int_{0}^{3b} x^2\, dx$

PART - III : VECTOR

1. Four forces of magnitudes P, 2P, 3P and 4P act along the four sides of a square ABCD in cyclic order. Use the vector method to find the resultant force .

2. A sail boat sails 2km due East, 5km 37^0 South of East and finally has an unknown displacement. If the final displacement of the boat from the starting point is 6km due East, the third displacement is _____.

3. The resultant of two vectors **u** and **v** is perpendicular to the vector u and its magnitude is equal to half of the magnitude of vector **v**. Find the angle between **u** and **v**.

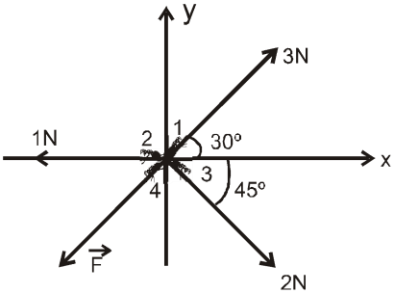
4. Let the resultant of three forces of magnitude 5N, 12N & 13N acting on a body be zero. If sin 23° = (5/13), find the angle between the 5N force & 13N force.

5. Two vectors $\vec{A}$ & $\vec{B}$ have the same magnitude. Under what circumstances does the vector $\vec{A} + \vec{B}$ have the same magnitude as $|\vec{A}|$ or $|\vec{B}|$. When does the vector difference $\vec{A} - \vec{B}$ have this magnitude.

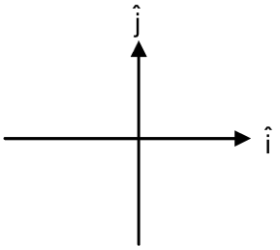
6. The resultant of $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If magnitude of $\vec{Q}$ is doubled, magnitude of resultant is also doubled, when direction of $\vec{Q}$ is reversed from initial condition then magnitude of resultant is again doubled, find P : Q : R.

7. If five consecutive sides of a regular hexagon represent five unit vectors acting in the same sense, find their resultant vector. (taking first side on x-axis)

8. Four ants 1, 2, 3 and 4 are pulling a grain with force of magnitudes 3 N, 1N, 2N and $|\vec{F}|$ N as shown in the figure. Find force $\vec{F}$ if the grain remains in equilibrium under the action of the above forces.



9. An insect moves in a circular path of radius R. If it rotates through an angle θ, find its displacement means $\vec{s} = \Delta\vec{r}$, where $\Delta\vec{r}$ is "the change in position vector".

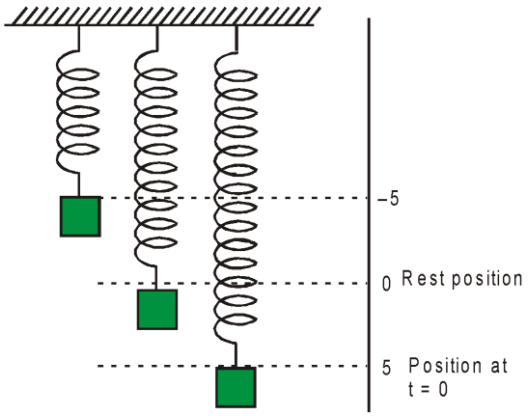


10. Find the vector equation of a line which is parallel to a given vector $\vec{A}$ and passes through a given point P having position vector $\vec{r_0}$.

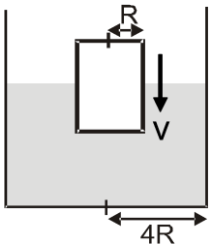
11. By using the concept of scalar product prove that $|\vec{A}+\vec{B}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta}$.

12. The vector $\vec{A}$ varies with time as $\vec{A} = t\hat{i} - \sin\pi t\hat{j} + t^2\hat{k}$. Find the derivative of the vector at t = 1.

13. A body hanging from a spring (fig.) is stretched 5 units beyond its rest position and released at time t = 0 to oscillate up and down. Its position at any later time t is s = 5 cos t. What are its velocity and acceleration at time t ?



14._ A cylinder of radius R is moving down with speed v into water placed in a cylinder of radius 4R as shown. Find the rate by which



(a) Water surface is rising
(b) Wet surface area of cylinder of radius R is increasing.

15._ Find the maximum area of the rectangle that can be inscribed in a circle of radius r ?

HLP Answers

PART – I

1. $g'(x) = \frac{x^2+x+4}{(x+0.5)^2}$ **2.** (a) – 2 (b) $\frac{2}{25}$ (c) $-\frac{1}{2}$ (d) –7 **3.** $\frac{-2\ \text{cosec}\ t \cot t}{(1-\text{cosec}\ t)^2}$

4. $\frac{ds}{dt} = \frac{1}{\cos t - 1}$ **5.** $3\sin^2 x (\cos x)$ **6.** $\frac{dy}{dx} = 20\sin x \cos^{-5} x$

7. $\frac{\text{cosec}\theta}{\cot\theta + \text{cosec}\theta}$ **8.** $\frac{\sec\theta}{\sec\theta + \tan\theta}$

PART – II

1. (a) – csc x + C (b) $\frac{1}{5}\csc(5x) + C$ (c) $2\csc\left(\frac{\pi x}{2}\right) + C$ **2.** $3x + \sin 2x + 4\sin x + C$

3. $\frac{x^{\frac{3}{2}}}{3} + 4x^{1/2} + C$ **4.** $4y^2 - \frac{8}{3}y^{3/4} + C$ **5.** $x^2 + \frac{2}{x} + C$

6. $3\cot x + C$ **7.** $\frac{-\tan x}{3} + C$ **8.** $-\frac{1}{2}\csc\theta + C$

9. $\frac{2}{5}\sec\theta + C$ **10.** $4\sec x - 2\tan x + C$ **11.** $-\frac{1}{2}\cot x + \frac{1}{2}\csc x + C$

12. $-\frac{1}{2}\cos 2x + \cot x + C$ **13.** $\sin 2x + \cos 3x + C$ **14.** $2y - \sin 2y + C$

15. $\tan\theta + C$ **16.** $\frac{2}{5}(5s+4)^{1/2} + C$ **17.** $-\frac{1}{3}(7 - 3y^2)^{3/2} + C$

18. $\frac{1}{2}\sin^6\left(\frac{x}{3}\right) + C$ **19.** $\frac{1}{4}\tan^8\frac{x}{2} + C$ **20.** $\left(\frac{r^3}{18} - 1\right)^6 + C$

21. $-\frac{1}{2}\left(7 - \frac{r^5}{10}\right)^4 + C$ **22.** $-\frac{3}{4}\cos(x^{4/3} - 8) + C$ **23.** $-2\csc\left(\frac{\upsilon - \pi}{2}\right) + C$

24. $-\frac{2}{3}(\cot^3 y)^{1/2} + C$ **25.** $2\sqrt{\sec z} + C$ **26.** $-\sin\left(\frac{1}{t} - 1\right) + C$

27. Area = 2 square units **28.** Area = 2.5 square units **29.** $\frac{\pi^3}{24}$

30. $9b^3$

PART – III

1. $2\sqrt{2}$ P **2.** 3 km in North **3.** 150°

4. 113° **5.** when the angle between A & B is 120° ; when it is 60°

6. P:Q:R.= $\sqrt{2} : \sqrt{3} : \sqrt{2}$ **7.** $\vec{A}_3 = \frac{1}{2}(-\hat{i} + \sqrt{3}\hat{j})$

8. $F_4 = \sqrt{x^2 + y^2} = \sqrt{\left(\frac{3\sqrt{3}}{2} + \sqrt{2} - 1\right)^2 + \left(\frac{3}{2} - \sqrt{2}\right)^2}$ N, $\varphi = \tan^{-1}\left[\frac{3 - 2\sqrt{2}}{3\sqrt{3} + 2\sqrt{2} - 2}\right]$

9. The magnitude of displacement = $|\Delta\vec{r}| = 2R\sin\frac{\theta}{2}$. The direction of $\Delta\vec{r}$ is given as $\beta = \frac{\pi}{2} + \frac{\theta}{2}$

10. $\vec{r} = \vec{r}_0 + n\hat{a}$ **12.** $\hat{i} + \pi\hat{j} + 2\hat{k}$

13. Velocity : – 5 sin t, Acceleration : – 5 cos t

14. (a) $\frac{dh}{dt} = \frac{v}{15}$ (b) $\frac{32\pi R\nu}{15}$

15. $A_{max} = 2r^2$.