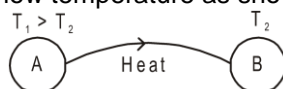


# CALORIMETRY AND THERMAL EXPANSION

## 1. HEAT

The energy that is being transferred between two bodies or between adjacent parts of a body as a result of temperature difference is called heat. Thus, heat is a form of energy. It is energy in transit whenever temperature differences exist. Once it is transferred, it becomes the internal energy of the receiving body. It should be clearly understood that the word "heat" is meaningful only as long as the energy is being transferred. Thus, expressions like "heat in a body" or "heat of a body" are meaningless. Heat transfer from a body at high temperature to low temperature as shown in figure 1.



When we say that a body is heated it means that its molecules begin to move with greater kinetic energy. So, it is energy of molecular motions.

S.I. unit of heat energy is joule (J). Another practical unit of heat energy is calorie (cal).

**1 calorie = 4.18 joules.**

**1 calorie :** The amount of heat needed to increase the temperature of 1 gm of water from 14.5 to 15.5 °C at one atmospheric pressure is 1 calorie.

### 1.1 Mechanical Equivalent of Heat

In early days heat was not recognised as a form of energy. Heat was supposed to be something needed to raise the temperature of a body or to change its phase. Calorie was defined as the unit of heat. A number of experiments were performed to show that the temperature may also be increased by doing mechanical work on the system. These experiments established that heat is equivalent to mechanical energy and measured how much mechanical energy is equivalent to a calorie. If mechanical work  $W$  produces the same temperature change as heat  $H$ , we write,

$$W = JH$$

where  $J$  is called mechanical equivalent of heat.  $J$  is expressed in joule/calorie. The value of  $J$  gives how many joules of mechanical work is needed to raise the temperature of 1 g of water by 1°C. It is conversion factor and not a physical quantity.

## Solved Examples

**Example 1.** What is the change in potential energy (in calories) of a 10 kg mass when it falls through 10 m?

**Solution :** Change in potential energy  
 $\Delta U = mgh = 10 \times 10 \times 10 = 1000 \text{ J}$   

$$= \frac{1000}{4.186} \text{ cal} \quad \text{Ans.}$$



## 2. SPECIFIC HEAT

Specific heat of substance is equal to heat gained or released by that substance to raise or fall its temperature by 1°C for a unit mass of substance.

When a body is heated, it gains heat. On the other hand, heat is lost when the body is cooled. The gain or loss of heat is directly proportional to:

- (a) the mass of the body  $\Delta Q \propto m$
- (b) rise or fall of temperature of the body  $\Delta Q \propto \Delta T$

$$\Delta Q \propto m \Delta T \quad \text{or} \quad \Delta Q = m s \Delta T$$

$$\text{or} \quad dQ = m s dT \quad \text{or} \quad Q = m \int s dT.$$

where  $s$  is a constant and is known as the specific heat of the body  $s = \frac{Q}{m \Delta T}$ . S.I. unit of  $s$  is joule/kg-kelvin and C.G.S. unit is cal./gm °C.

**Specific heat of water :**  $S = 4200 \text{ J/kg}^\circ\text{C} = 1000 \text{ cal/kg}^\circ\text{C} = 1 \text{ Kcal/kg}^\circ\text{C} = 1 \text{ cal/gm}^\circ\text{C}$

**Specific heat of steam = half of specific heat of water = specific heat of ice**

## Solved Examples

**Example 2.** Calculate the heat required to increase the temperature of 1 kg water by 20°C

**Solution :** Heat required =  $\Delta Q = ms\Delta\theta$   
 $\therefore S = 1 \text{ cal/gm}^\circ\text{C} = 1 \text{ Kcal/kg}^\circ\text{C}$   
 $= 1 \times 20 = 20 \text{ Kcal.}$



### 2.1 Heat capacity or Thermal capacity :

Heat capacity of a body is defined as the amount of heat required to raise the temperature of that body by 1°. If 'm' is the mass and 's' the specific heat of the body, then

**Heat capacity = m s.**

Units of heat capacity in: CGS system is, **cal °C<sup>-1</sup>**; SI unit is, **JK<sup>-1</sup>**

### 2.2 Important Points:

- We know,  $s = \frac{Q}{m\Delta T}$ , if the substance undergoes the change of state which occurs at constant temperature (Isothermal  $\Delta T = 0$ ), then  $s = Q/0 = \infty$ . Thus the specific heat of a substance when it melts or boils at constant temperature is infinite.
- If the temperature of the substance changes without the transfer of heat adiabatic ( $Q = 0$ ) then  $s = \frac{Q}{m\Delta T} = 0$ . Thus when liquid in the thermos flask is shaken, its temperature increases without the transfer of heat and hence the specific heat of liquid in the thermos flask is zero.
- To raise the temperature of saturated water vapours, heat (Q) is withdrawn. Hence, specific heat of saturated water vapours is negative. (This is for your information only and not in the course)
- The slight variation of specific heat of water with temperature is shown of figure 2 in the graph at 1 atmosphere pressure. Its variation is less than 1% over the interval from 0 to 100°C.

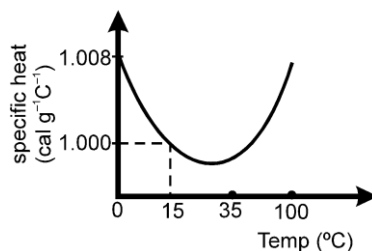


Figure : 2

### 2.3 Relation between Specific heat and Water equivalent:

It is the amount of water which requires the same amount of heat for the same temperature rise as that of the object

$$ms\Delta T = m_w s_w \Delta T \Rightarrow m_w = \frac{ms}{s_w}$$

In calorie  $s_w = 1$   
 $\therefore m_w = ms$   
 $m_w$  is also represent by W  
 So,  $W = ms$ .

### 2.4 Phase change:

Heat required for the change of phase or state,

**Q = mL**, L = latent heat.

**Latent heat (L):** The heat supplied to a substance which changes its state at constant temperature is called latent heat of the body.

**Latent heat of Fusion ( $L_f$ ) :** The heat supplied to a substance which changes it from solid to liquid state at its melting point and 1 atm. pressure is called latent heat of fusion. Latent heat of fusion of ice is 80 kcal/kg

**Latent heat of vaporisation ( $L_v$ ):** The heat supplied to a substance which changes it from liquid to vapour state at its boiling point and 1 atm. pressure is called latent heat of vaporization. Latent heat of vaporization of water is 540 kcal kg<sup>-1</sup>.

If in question latent heat of ice or steam are not mentioned and to solve the problem, it required to assume them, we should consider following values.

**Latent heat of ice :**  $L = 80 \text{ cal/gm} = 80 \text{ Kcal/kg} = 4200 \times 80 \text{ J/kg}$

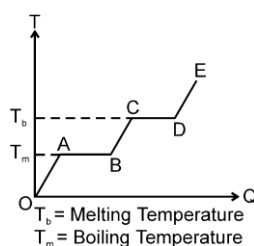
**Latent heat of steam :**  $L = 540 \text{ cal/gm} = 540 \text{ Kcal/kg} = 4200 \times 540 \text{ J/kg}$

The given figure 3, the behaviour of a solid substance when it is continuously heated is shown. Various parts show.

OA – solid state , AB – solid + liquid state (Phase change)

BC – liquid state , CD – liquid + vapour state (Phase change)

DE – vapour state



**Figure : 3**

When there is no change of state e.g. OA, BC, DE

$$\text{Slope, } \frac{\Delta T}{\Delta Q} = \frac{1}{ms} \Rightarrow \frac{\Delta T}{\Delta Q} \propto \frac{1}{s}$$

The mass (m) of substance is constant. So, slope of T – Q graph is inversely proportional to specific heat. In the given diagram.

(slope) OA > (slope) DE

then  $(s)_{OA} < (s)_{DE}$

when there is change of state e.g. AB and CD

$$\Delta Q = mL$$

If (length of AB) > (length of CD)

then (latent heat of AB) > (latent heat of CD)

## Solved Examples

### Example 3.

**Solution :**

Find amount of heat releases when 1 kg steam at 200°C is converted into –20°C ice.

Heat required  $\Delta Q$  = heat release to convert steam at 200 °C into 100°C steam + heat release to convert 100° C steam into 100°C water + heat release to convert 100° water into 0°C water + heat release to convert 0 °C water into 0°C ice + 0°C ice converted to – 20°C ice.

$$\Delta Q = 1 \times \frac{1}{2} \times 100 + 540 \times 1 + 1 \times 1 \times 100 + 1 \times 80 + 1 \times \frac{1}{2} = 780 \text{ (Kcal)}.$$



## 3. CALORIMETRY

The branch of thermodynamics which deals with the measurement of heat is called calorimetry.

A simple calorimeter is a vessel generally made of copper with a stirrer of the same material. The vessel is kept in a wooden box to isolate it thermally from the surrounding. A thermometer is used to measure the temperature of the contents of the calorimeter. Object at different temperatures are made to come in contact with each other in the calorimeter. As a result, heat is exchanged between the object as well as with the calorimeter. Neglecting any heat exchange with the surrounding, the total heat is conserved i.e. heat given by one is equals to taken by the other.

### 3.1 Law of Mixture:

## Calorimetry & Thermal Expansion

When two substances at different temperatures are mixed together, then exchange of heat continues to take place till their temperatures become equal. This temperature is then called final temperature of mixture. Here, **Heat taken by one substance = Heat given by another substance**. Referring to figure 4.

$$\Rightarrow m_1 s_1 (T_1 - T_m) = m_2 s_2 (T_m - T_2)$$

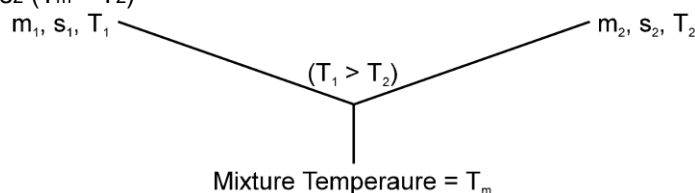


Figure : 4

### Solved Examples

**Example 4.** An iron block of mass 2 kg, falls from a height 10 m. After colliding with the ground it loses 25% energy to surroundings. Then find the temperature rise of the block. (Take sp. heat of iron 470 J/kg °C)

**Solution :**  $ms\Delta\theta = \frac{1}{4} mgh \quad \Rightarrow \quad \Delta\theta = \frac{10 \times 10}{4 \times 470}$

Note : If specific heat is given in calorie / kg°C, it is to be converted to J/kg°C by multiplying with 4.18



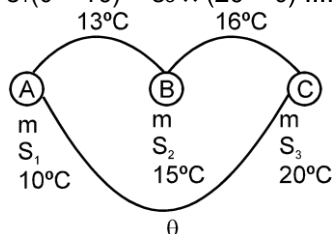
#### Zeroth law of thermodynamics :

If objects A and B are separately in thermal equilibrium with a third object C, then objects A and B are also in thermal equilibrium with each other.

### Solved Examples

**Example 5.** The temperature of equal masses of three different liquids A, B, and C are 10°C, 15°C and 20°C respectively. The temperature when A and B are mixed is 13°C and when B and C are mixed, it is 16°C. What will be the temperature when A and C are mixed?

**Solution :** when A and B are mixed  
 $ms_1 \times (13 - 10) = m \times s_2 \times (15 - 13)$   
 $3s_1 = 2s_2 \quad \dots(1)$   
 when B and C are mixed  
 $s_s \times 1 = s_s \times 4 \quad \dots(2)$   
 when C and A are mixed  
 $s_1(\theta - 10) = s_3 \times (20 - \theta) \quad \dots(3)$



by using equation (1), (2) and (3)

$$\frac{140}{11}$$

we get  $\theta = 11^\circ\text{C}$

**Example 6.** If three different liquid of different masses specific heats and temperature are mixed with each other and then what is the temperature mixture at thermal equilibrium.

$m_1, s_1, T_1 \rightarrow$  specification for liquid 1

$m_2, s_2, T_2 \rightarrow$  specification for liquid. 2

$m_3, s_3, T_3 \rightarrow$  specification for liquid. 3

**Solution :** Total heat lost or gained by all substance is equal to zero

$$\Delta Q = 0$$

$$m_1 s_1 (T - T_1) + m_2 s_2 (T - T_2) + m_3 s_3 (T - T_3) = 0$$

$$\text{So, } T = \frac{m_1 s_1 T_1 + m_2 s_2 T_2 + m_3 s_3 T_3}{m_1 s_1 + m_2 s_2 + m_3 s_3}$$

**Example 7.** In following equation calculate value of H 1 kg ice at  $-20^\circ\text{C}$  = H + 1 Kg water at  $100^\circ\text{C}$ , here H means heat required to change the state of substance.

**Solution :** Heat required to convert 1 kg ice at  $-20^\circ\text{C}$  into 1 kg water at  $100^\circ\text{C}$

$\Delta Q$  (Kcal) = 1 kg ice at  $-20^\circ\text{C}$  to 1 kg ice at  $0^\circ\text{C}$  ice + 1 kg water at  $0^\circ\text{C}$  + 1 kg water at  $0^\circ\text{C}$  to 1 kg water at  $100^\circ\text{C}$

$$= 1 \times \frac{1}{2} \times 20 + 1 \times 80 + 1 \times 100 = 190 \text{ Kcal.}$$

So H = - 190 Kcal

Negative sign indicate that 190 Kcal heat is with drawn from 1 kg water at  $100^\circ\text{C}$  to convert it into 1 kg ice at  $-20^\circ\text{C}$

**Example 8.** 1 kg ice at  $-20^\circ\text{C}$  is mixed with 1 kg steam at  $200^\circ\text{C}$ . Then find equilibrium temperature and mixture content.

**Solution :** Let equilibrium temperature is  $100^\circ\text{C}$  heat required to convert 1 kg ice at  $-20^\circ\text{C}$  to 1 kg water at  $100^\circ\text{C}$  is equal to

$$H_1 = 1 \times \frac{1}{2} \times 20 + 1 \times 80 + 1 \times 1 \times 100 = 190 \text{ Kcal}$$

Heat released by steam to convert 1 kg steam at  $200^\circ\text{C}$  to 1 kg water at  $100^\circ\text{C}$  is equal to

$$H_2 = 1 \times \frac{1}{2} \times 100 + 1 \times 540 = 590 \text{ Kcal}$$

1 kg ice at  $-20^\circ\text{C}$  =  $H_1$  + 1kg water at  $100^\circ\text{C}$  .....(1)

1 kg steam at  $200^\circ\text{C}$  =  $H_2$  + 1kg water at  $100^\circ\text{C}$  .....(2)

by adding equation (1) and (2)

1 kg ice at  $-20^\circ\text{C}$  + 1 kg steam at  $200^\circ\text{C}$  =  $H_1 + H_2$  + 2 kg water at  $100^\circ\text{C}$ .

Here heat required to ice is less than heat supplied by steam so mixture equilibrium temperature is  $100^\circ\text{C}$  then steam is not completely converted into water.

So mixture has water and steam which is possible only at  $100^\circ\text{C}$

mass of steam which converted into water is equal to

$$m = \frac{190 - 1 \times \frac{1}{2} \times 100}{540} = \frac{7}{27} \text{ kg}$$

so mixture content

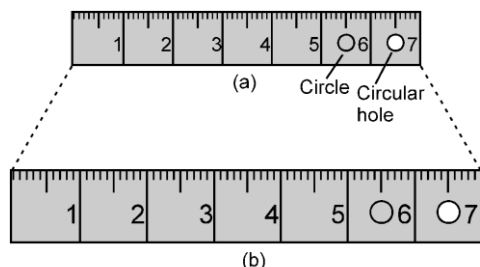
$$\text{mass of steam} = 1 - \frac{7}{27} = \frac{20}{27} \text{ kg}$$

$$\text{mass of water} = 1 + \frac{7}{27} = \frac{34}{27} \text{ kg}$$



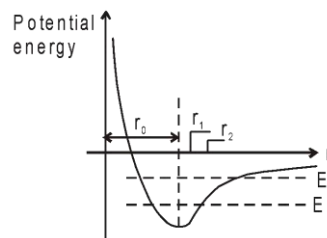
## 4. THERMAL EXPANSION

Most materials expand when their temperature is increased. Railway roads tracks, bridges all have some means of compensating for thermal expansion. When a homogeneous object expands, the distance between any two points on the object increases. Figure 5 shows a block of metal with a hole in it. **The expanded object is like a photographic enlargement.** That in the hole expands in the same proportion as the metal, it does not get smaller



The same steel ruler two different temperatures. When it expands, the scale, the numbers, the thickness, and the diameters of the circle and circular hole are all increased by the same factor. (The expansion has been exaggerated for clarity.)

**Figure : 5**



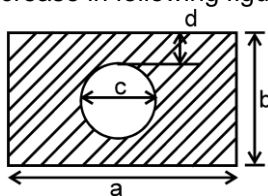
Thermal expansion arises because the well is not symmetrical about the equilibrium position  $r_0$ . As the temperature rise the energy of the atom increases. The average position when the energy is  $E_2$  is not the same as that when the energy is  $E_1$ .

**Figure : 6**

At the atomic level, thermal expansion may be understood by considering how the potential energy of the atoms varies with distance. The equilibrium position of an atom will be at the minimum of the potential energy well if the well is symmetric. At a given temperature each atom vibrates about its equilibrium position and its average remains at the minimum point. If the shape of the well is not symmetrical the average position of an atom will not be at the minimum point. When the temperature is raised the amplitude of the vibrations increases and the average position is located at a greater interatomic separation. This increased separation is manifested as expansion of the material as shown in figure 6. Almost all solids and liquids expand as their temperature increases. Gases also expand as shown in figure 6. If allowed. Solids can change in length, area or volume, while liquids change in their volumes.

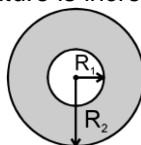
## Solved Examples

**Example 9.** A rectangular plate has a circular cavity as shown in the figure. If we increase its temperature then which dimension will increase in following figure.

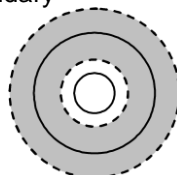


**Solution :** Distance between any two point on an object increases with increase in temperature. So, all dimension  $a$ ,  $b$ ,  $c$  and  $d$  will increase

**Example 10.** In the given figure, when temperature is increased then which of the following increases



**Solution :** (1)  $R_1$  (2)  $R_2$  (3)  $R_2 - R_1$  (4) None of these  
All of the above  
- - - - represents expanded Boundary  
- - - - - represents original Boundary



As the intermolecular distance between atoms increases on heating hence the inner and outer perimeter increases. Also if the atomic arrangement in radial direction is observed then we can say that it also increases hence all A,B,C are true.



## 5. LINEAR EXPANSION

## Calorimetry & Thermal Expansion

When the rod is heated, its increase in length  $\Delta L$  is proportional to its original length  $L_0$  and change in temperature  $\Delta T$  where  $\Delta T$  is in  $^{\circ}\text{C}$  or  $\text{K}$ . Referring figure 7.

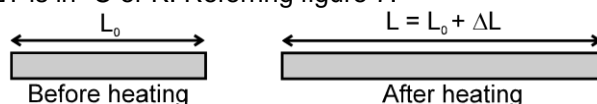


Figure : 7

$$dL = \alpha L_0 dT \Rightarrow \Delta L = \alpha L_0 \Delta T \quad \text{If } \alpha \Delta T \ll 1$$

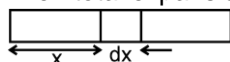
$$\alpha = \frac{\Delta L}{L_0 \Delta T} \quad \text{where } \alpha \text{ is called the coefficient of linear expansion whose unit is } ^{\circ}\text{C}^{-1} \text{ or } \text{K}^{-1}.$$

$$L = L_0 (1 + \alpha \Delta T). \quad \text{Where } L \text{ is the length after heating the rod.}$$

### Variation of $\alpha$ with temperature and distance

(a) If  $\alpha$  varies with distance,  $\alpha = ax + b$ .

$$\text{Then total expansion} = \int (ax + b) \Delta T dx.$$



(b) If  $\alpha$  varies with temperature,  $\alpha = f(T)$ . Then  $\Delta L = \int \alpha L_0 dT$

**Note :** Actually thermal expansion is always 3-D expansion. When other two dimensions of object are negligible with respect to one, then observations are significant only in one dimension and it is known as linear expansion.

### Solved Examples

**Example 11.** What is the percentage change in length of 1m iron rod if its temperature changes by  $100^{\circ}\text{C}$ .  $\alpha$  for iron is  $2 \times 10^{-5}/^{\circ}\text{C}$ .

**Solution :** percentage change in length due to temperature change

$$\% \Delta \ell = \frac{\Delta \ell}{\ell} \times 100 = \alpha \Delta \theta \times 100 = 2 \times 10^{-5} \times 100 \times 100 = 0.2\% \quad \text{Ans.}$$



### 5.1 Thermal stress of a material:

If a rod is free to expand then there will be no stress and strain. Stress and strain is produced only when an object is restricted to expand or contract according to change in temperature. When the temperature of the rod is decreased or increased under constrained condition, compressive or tensile stresses are developed in the rod. These stresses are known as thermal stresses.

$$\text{Strain} = \frac{\Delta L}{L_0} = \frac{\text{final length} - \text{original length}}{\text{original length}} = \alpha \Delta T,$$

**Note :** Original and final length should be at same temperature.

Consider a rod of length  $\ell_0$  which is fixed between two rigid supports separated at a distance  $\ell_0$  now if the temperature of the rod is increased by  $\Delta \theta$  then the strain produced in the rod will be (see figure 8):

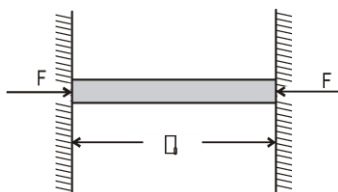


Figure : 8

$$\text{strain} = \frac{\text{length of the rod at new temperature} - \text{natural length of the rod at new temperature}}{\text{natural length of the rod at new temperature}}$$

## Calorimetry & Thermal Expansion

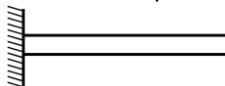
$$= \frac{\ell_0 - \ell_0(1 + \alpha\Delta\theta)}{\ell_0(1 + \alpha\Delta\theta)} = \frac{-\ell_0\alpha\Delta\theta}{\ell_0(1 + \alpha\Delta\theta)}$$

$\therefore \alpha$  is very small. So,  $\Delta\theta \ll 1$

strain =  $-\alpha \Delta\theta$  (negative sign in the answer represents that the length of the rod is less than the natural length that means is compressed at the ends.)

### Solved Examples

**Example 12.** In the given figure a rod is free at one end and other end is fixed. When we change the temperature of rod by  $\Delta\theta$ , then strain produced in the rod will be



- (1)  $\alpha\Delta\theta$  (2)  $\frac{1}{2} \alpha\Delta\theta$   
 (3) zero (4) information incomplete

**Solution :** Here rod is free to expand from one side by so by changing temperature no strain will be produced in the rod. Hence ans. is (3)

**Example 13.** An iron ring measuring 15.00 cm in diameter is to be shrunk on a pulley which is 15.05 cm in diameter. All measurements refer to the room temperature 20°C. To what minimum temperature should the ring be heated to make the job possible? Calculate the strain developed in the ring when it comes to the room temperature. Coefficient of linear expansion of iron =  $12 \times 10^{-6}/^\circ\text{C}$ .

**Solution :** The ring should be heated to increase its diameter from 15.00 cm to 15.05 cm.

$$\text{Using } \ell_2 = \ell_1 (1 + \alpha \Delta\theta), \quad \Rightarrow \quad \Delta\theta = \frac{(\ell_2 - \ell_1)}{\ell_1 \alpha}$$

$$= \frac{0.05 \text{ cm}}{15.00 \text{ cm} \times 12 \times 10^{-6}/^\circ\text{C}} = 278^\circ\text{C}$$

The temperature = 20°C + 278°C = 298°C.

$$\text{The strain developed} = \frac{\ell_2 - \ell_1}{\ell_1} = 3.33 \times 10^{-3}$$

**Example 14.** A steel rod of length 1m rests on a smooth horizontal base. If it is heated from 0°C to 100°C, what is the longitudinal strain developed?

**Solution :** in absence of external force no strain or stress will be created hear rod is free to move.

**Example 15.** A steel rod is clamped at its two ends and rests on a fixed horizontal base. The rod is in natural length at 20°C. Find the longitudinal strain developed in the rod if the temperature rises to 50°C. Coefficient of linear expansion of steel =  $1.2 \times 10^{-5}/^\circ\text{C}$ .

**Solution :** as we known that strain

$$\text{strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta\ell}{\ell_0}$$

$$\therefore \text{Strain} = \alpha\Delta\theta$$

$$= 1.2 \times 10^{-5} \times (50 - 20) = 3.6 \times 10^{-4}$$

here strain is compressive strain because final length is smaller than initial length.

**Example 16.** A steel wire of cross-sectional area 0.5 mm<sup>2</sup> is held between two fixed supports. If the wire is just taut at 20°C, determine the tension when the temperature falls to 0°C. Coefficient of linear expansion of steel is  $1.2 \times 10^{-5}/^\circ\text{C}$  and its Young's modulus is  $2.0 \times 10^{11} \text{ N/m}^2$ .

**Solution :** here final length is more than original length so that strain is tensile and tensile force is given by

$$F = AY \alpha \Delta t = 0.5 \times 10^{-6} \times 2 \times 10^{11} \times 1.2 \times 10^{-5} \times 20 = 24 \text{ N}$$



## 5.2 Variation of time period of pendulum clocks:

The time represented by the clock hands of a pendulum clock depends on the number of oscillation performed by pendulum every time it reaches to its extreme position the second hand of the clock advances by one second that means second hand moves by two seconds when one oscillation in complete

Let  $T = 2\pi \sqrt{\frac{L_0}{g}}$  at temperature  $\theta_0$  and  $T' = 2\pi \sqrt{\frac{L}{g}}$  at temperature  $\theta$ .

$$\frac{T'}{T} = \sqrt{\frac{L'}{L}} = \sqrt{\frac{L[1 + \alpha \Delta\theta]}{L}} = 1 + \frac{1}{2} \alpha \Delta\theta \text{ (Binomial approximation is used)}$$

Therefore change (loss or gain) in time per unit time lapsed is

$$\frac{T' - T}{T} = \frac{1}{2} \alpha \Delta\theta$$

gain or loss in time in duration of 't' in

$$\Delta t = \frac{1}{2} \alpha \Delta\theta t, \text{ if } T \text{ is the correct time then}$$

(a)  $\theta < \theta_0$ ,  $T' < T$  clock becomes fast and gains time

(b)  $\theta > \theta_0$ ,  $T' > T$  clock becomes slow and loses time

### Solved Examples

**Example 17.** A pendulum clock consists of an iron rod connected to a small, heavy bob. If it is designed to keep correct time at  $20^\circ\text{C}$ , how fast or slow will it go in 24 hours at  $40^\circ\text{C}$ ? Coefficient of linear expansion of iron =  $1.2 \times 10^{-6}/^\circ\text{C}$ .

**Solution :** The time difference occurred in 24 hours (86400 seconds) is given by

$$\Delta t = \frac{1}{2} \alpha \Delta\theta t$$

$$= \frac{1}{2} \times 1.2 \times 10^{-6} \times 20 \times 86400 = 1.04 \text{ sec.} \quad \text{Ans.}$$

This is loss of time as  $\theta$  is greater than  $\theta_0$ . As the temperature increases, the time period also increases. Thus, the clock goes slow.



### 5.3 Measurement of length by metallic scale:

#### Case (i)

When object expands only (figure 9a)

$$\ell_2 = \ell_1 \{1 + \alpha_0(\theta_2 - \theta_1)\}$$

$\ell_1$  = actual length of object at  $\theta_1^\circ\text{C}$  = measure length of object at  $\theta_1^\circ\text{C}$ .

$\ell_2$  = actual length of object at  $\theta_2^\circ\text{C}$  = measure length of object at  $\theta_2^\circ\text{C}$ .

$\alpha_0$  = linear expansion coefficient of object.

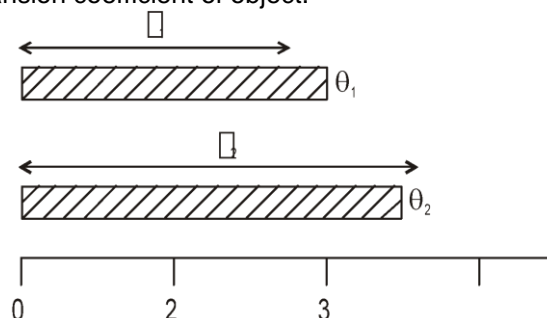


Figure : 9a

#### Case (ii)

When only measuring instrument is expanded actual length of object will not change but measured value (MV) decreases (figure 9b).

$$MV = \ell_1 \{1 - \alpha_s (\theta_2 - \theta_1)\}$$

## Calorimetry & Thermal Expansion

$\alpha_s$  = linear expansion coefficient of measuring instrument.

at  $\theta_1^\circ\text{C}$  MV = 3

at  $\theta_2^\circ\text{C}$  MV = 2.2

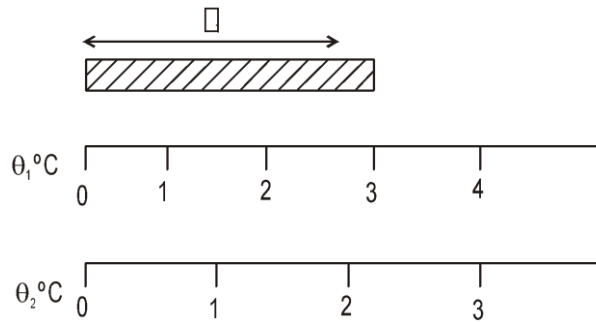


Figure : 9b

### Case (iii)

If both expanded simultaneously (figure 9c).

$$MV = \{1 + (\alpha_o - \alpha_s) (\theta_2 - \theta_1)\}$$

- (i) If  $\alpha_o > \alpha_s$ , then measured value is more than actual value at  $\theta_1^\circ\text{C}$
- (ii) If  $\alpha_o < \alpha_s$ , then measured value is less than actual value at  $\theta_1^\circ\text{C}$

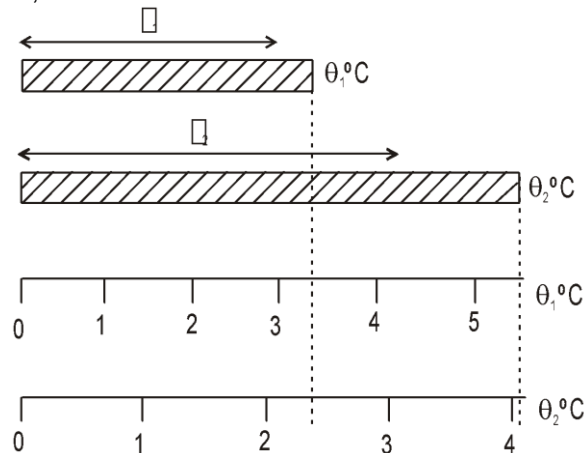


Figure : 9c

at  $\theta_1^\circ\text{C}$  MV = 3.4

$\theta_2^\circ\text{C}$  MV = 4.1

**Measured value = calibrated value  $\times \{1 + \alpha \Delta \theta\}$**

where  $\alpha = \alpha_o - \alpha_s$

$\alpha_o$  = coefficient of linear expansion of object material,  $\alpha_s$  = coefficient of linear expansion of scale material.

$$\Delta\theta = \theta - \theta_c$$

$\theta$  = temperature at the time of measurement  $\theta_c$  = temperature at the time of calibration.

For scale, **true measurement = scale reading  $[1 + \alpha (\theta - \theta_0)]$**

If  $\theta > \theta_0$  true measurement > scale reading

$\theta < \theta_0$  true measurement < scale reading

## Solved Examples

**Example 18.** A bar measured with a vernier caliper is found to be 180mm long. The temperature during the measurement is  $10^\circ\text{C}$ . The measurement error will be if the scale of the vernier caliper has been graduated at a temperature of  $20^\circ\text{C}$  : ( $\alpha = 1.1 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$ . Assume that the length of the bar does not change.)

- (1)  $1.98 \times 10^{-1} \text{ mm}$       (2\*)  $1.98 \times 10^{-2} \text{ mm}$       (3)  $1.98 \times 10^{-3} \text{ mm}$       (4)  $1.98 \times 10^{-4} \text{ mm}$

**Solution :** True measurement = scale reading  $[1 + \alpha (\theta - \theta_0)]$

$$= 180 \times \{1 + (10 - 20) \times (-1.1 \times 10^{-5})\}$$

$$\begin{aligned}\text{measurement error} &= \text{true measurement} - \text{scale reading} \\ &= 180 \times \{1 + (10 - 20) \times (-1.1 \times 10^{-5})\} - 180 \\ &= 1.98 \times 10^{-2} \text{ mm}\end{aligned}$$



### 6. SUPERFICIAL OR AREAL EXPANSION

When a solid is heated and its area increases, then the thermal expansion is called superficial or areal expansion. Consider a solid plate of area  $A_0$ . When it is heated, the change in area of the plate is directly proportional to the original area  $A_0$  and the change in temperature  $\Delta T$ .

$$dA = \beta A_0 dT \quad \text{or} \quad \Delta A = \beta A_0 \Delta T; \beta \text{ is called real expansion}$$

$$\beta = \frac{\Delta A}{A_0 \Delta T} \quad \text{Unit of } \beta \text{ is } ^\circ\text{C}^{-1} \text{ or } \text{K}^{-1}.$$

$$A = A_0 (1 + \beta \Delta T)$$

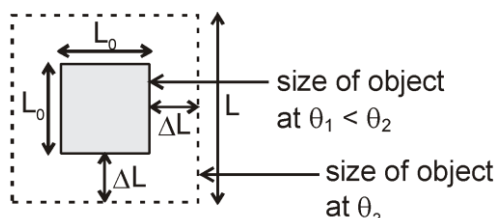


Figure : 10

Where,  $A$  is area of the plate after heating.

From figure 9, it follows that  $\beta = 2\alpha$ .

### Solved Examples

**Example 19.** A plane lamina has area  $2\text{m}^2$  at  $10^\circ\text{C}$  then what is its area at  $110^\circ\text{C}$  It's superficial expansion is  $2 \times 10^{-5}/^\circ\text{C}$

**Solution :**  $A = A_0 (1 + \beta \Delta \theta) = 2 \{1 + 2 \times 10^{-5} \times (110 - 10)\}$   
 $= 2 \times \{1 + 2 \times 10^{-3}\} \text{ m}^2 \text{ Ans.}$



### 7. VOLUME OR CUBICAL EXPANSION

When a solid is heated and its volume increases, then the expansion is called volume expansion or cubical expansion. Let us consider a solid or liquid whose original volume is  $V_0$ . When it is heated to a new volume, then the change  $\Delta V$

$$dV = \gamma V_0 dT \quad \text{or} \quad \Delta V = \gamma V_0 \Delta T$$

$$\gamma = \frac{\Delta V}{V_0 \Delta T} \quad \text{Unit of } \gamma \text{ is } ^\circ\text{C}^{-1} \text{ or } \text{K}^{-1}.$$

$$V = V_0 (1 + \gamma \Delta T)$$

where  $V$  is the volume of the body after heating.

It can be shown easily that  $\gamma = 3\alpha$  for isotropic solids.

### Solved Examples

**Example 20.** The volume of glass vessel is  $1000 \text{ cc}$  at  $20^\circ\text{C}$ . What volume of mercury should be poured into it at this temperature so that the volume of the remaining space does not change with temperature? Coefficient of cubical expansion of mercury and glass are  $1.8 \times 10^{-4}/^\circ\text{C}$  and  $9.0 \times 10^{-6}/^\circ\text{C}$  respectively.

**Solution :** Let volume of glass vessel at  $20^\circ\text{C}$  is  $V_g$  and volume of mercury at  $20^\circ\text{C}$  is  $V_m$   
 so volume of remaining space is  $= V_g - V_m$

It is given constant so that

$$V_g - V_m = V_g' - V_m'$$

where  $V_g'$  and  $V_m'$  are final volumes.

$$V_g - V_m = V_g \{1 + \gamma_g \Delta \theta\} - V_m \{1 + \gamma_{Hg} \Delta \theta\} \Rightarrow V_g \gamma_g = V_m \gamma_{Hg}$$

$$\Rightarrow V_m = \frac{100 \times 9 \times 10^{-6}}{1.8 \times 10^{-4}} \quad V_m = 50 \text{ cc.}$$



### 8. RELATION BETWEEN $\alpha$ , $\beta$ AND $\gamma$

- (i) For isotropic solids:  $\alpha : \beta : \gamma = 1 : 2 : 3$  or  $\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$   
 (ii) For non-isotropic solid  $\beta = \alpha_1 + \alpha_2$  and  $\gamma = \alpha_1 + \alpha_2 + \alpha_3$ . Here  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$  are coefficient of linear expansion in X, Y and Z direction.

### Solved Examples

**Example 21.** If percentage change in length is 1% with change in temperature of a cuboid object ( $\ell \times 2\ell \times 3\ell$ ) then what is percentage change in its area and volume.

**Solution :** percentage change in length with change in temperature = %  $\ell$

$$\frac{\Delta \ell}{\ell} \times 100 = \alpha \Delta \theta \times 100 = 1$$

$$\text{change in area} \Rightarrow \% A = \frac{\Delta A}{A} \times 100 = \beta \Delta \theta \times 100$$

$$\Rightarrow 2 (\alpha \Delta \theta \times 100) \% A = 2 \% \quad \text{Ans.}$$

Change in volume

$$\% V = \frac{\Delta V}{V} \times 100 = \gamma \Delta \theta \times 100 = 3 (\alpha \Delta \theta \times 100) \% V = 3 \% \quad \text{Ans.}$$



### 9. VARIATION OF DENSITY WITH TEMPERATURE

As we known that mass = volume  $\times$  density .

Mass of substance does not change with change in temperature so with increase of temperature, volume increases so density decreases and vice-versa.

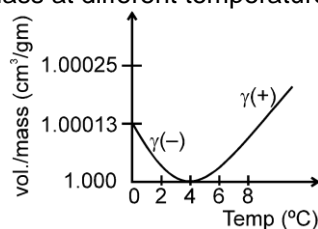
$$d = \frac{d_0}{(1 + \gamma \Delta T)}$$

For solids values of  $\gamma$  are generally small so we can write  $d = d_0 (1 - \gamma \Delta T)$  (using binomial expansion).

**Note :** (i)  $\gamma$  for liquids is of the order of  $10^{-3}$  .

(ii) **Anamolous expansion of water :**

For water density increases from 0 °C to 4 °C so  $\gamma$  is negative and for 4 °C to higher temperature  $\gamma$  is positive. At 4 °C density is maximum as shown in figure 10. This anomalous behaviour of water is due to presence of three types of molecules i.e.  $H_2O$ ,  $(H_2O)_2$  and  $(H_2O)_3$  having different volume/mass at different temperatures.



**Figure : 11**

This anomalous behaviour of water causes ice to form first at the surface of a lake in cold weather. As winter approaches, the water temperature decreases initially at the surface. The water there sinks because of its increased density. Consequently, the surface reaches 0°C first and the lake becomes covered with ice. Aquatic life is able to survive the cold winter as the lake bottom remains unfrozen at a temperature of about 4°C.

## Solved Examples

**Example 22.** The densities of wood and benzene at  $0^\circ\text{C}$  are  $880 \text{ kg/m}^3$  and  $900 \text{ kg/m}^3$  respectively. The coefficients of volume expansion are  $1.2 \times 10^{-3}/^\circ\text{C}$  for wood and  $1.5 \times 10^{-3}/^\circ\text{C}$  for benzene. At what temperature will a piece of wood just sink in benzene?

**Solution :** At just sink gravitation force = upthrust force

$$\Rightarrow mg = F_B$$

$$\Rightarrow V\rho_1 g = V\rho_2 g$$

$$\Rightarrow \rho_1 = \rho_2$$

$$\Rightarrow \frac{880}{1 + 1.2 \times 10^{-3} \theta} = \frac{900}{1 + 1.5 \times 10^{-3} \theta}$$

$$\Rightarrow \theta = 83^\circ\text{C}$$



## 10. APPARENT EXPANSION OF A LIQUID IN A CONTAINER

Initially container was full. When temperature increases by  $\Delta T$ ,

$$\text{volume of liquid} \quad V_L = V_0 (1 + \gamma_L \Delta T)$$

$$\text{volume of container} \quad V_C = V_0 (1 + \gamma_C \Delta T)$$

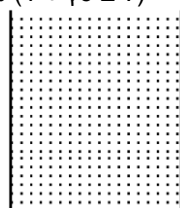


Figure : 12 (a)

So overflow volume of liquid relative to container

$$\Delta V = V_L - V_C \quad \Delta V = V_0 (\gamma_L - \gamma_C) \Delta T$$

So, coefficient of apparent expansion of liquid w.r.t. Container

$$\gamma_{\text{apparent}} = \gamma_L - \gamma_C.$$

In case of expansion of liquid + container system:

if  $\gamma_L > \gamma_C \rightarrow$  level of liquid rise

if  $\gamma_L < \gamma_C \rightarrow$  level of liquid fall

Increase in height of liquid level in tube when bulb was initially completely filled as shown in figure 11.

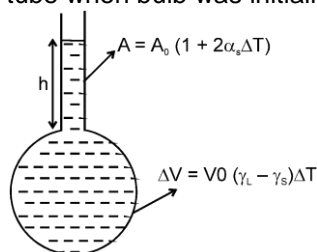


Figure : 12 (b)

$$h = \frac{\text{volume of liquid}}{\text{area of tube}} = \frac{V_0 (1 + \gamma_L \Delta T)}{A_0 (1 + 2\alpha_s \Delta T)} = h_0 \{ 1 + (\gamma_L - 2\alpha_s) \Delta T \}$$

$$h = h_0 \{ 1 + (\gamma_L - 2\alpha_s) \Delta T \}$$

where  $h_0$  = original height of liquid in container  
 $\alpha_s$  = linear coefficient of expansion of container.

## Solved Examples

**Example 23.** A glass vessel of volume  $100 \text{ cm}^3$  is filled with mercury and is heated from  $25^\circ\text{C}$  to  $75^\circ\text{C}$ . What volume of mercury will overflow? Coefficient of linear expansion of glass =  $1.8 \times 10^{-6}/^\circ\text{C}$  and coefficient of volume expansion of mercury is  $1.8 \times 10^{-4}/^\circ\text{C}$ .

**Solution :**  $\Delta V = V_0 (\gamma_L - \gamma_C) \Delta T = 100 \times \{ 1.8 \times 10^{-4} - 3 \times 1.8 \times 10^{-6} \} \times 50$

$$\Delta V = 0.87 \text{ cm}^3 \quad \text{Ans.}$$



## 11. VARIATION OF FORCE OF BUOYANCY WITH TEMPERATURE

If body is submerged completely inside the liquid

For solid, Buoyancy force

$$F_B = V_0 d_L g$$

$V_0$  = Volume of the solid inside liquid,

$d_L$  = density of liquid

Volume of body after increase its temperature  $V = V_0 [1 + \gamma_s \Delta\theta]$ ,

Density of body after increase its temperature  $d'_L = \frac{d_L}{[1 + \gamma_L \Delta\theta]}$ .

Buoyancy force of body after increase its temperature,  $F'_B = V d'_L g \Rightarrow \frac{F'_B}{F_B} = \frac{[1 + \gamma_s \Delta\theta]}{[1 + \gamma_L \Delta\theta]}$ ,

If  $\gamma_s < \gamma_L$  then  $F'_B < F_B$

(Buoyant force decreases) or apparent weight of body in liquid gets increased

$[W - F'_B > W - F_B]$ .

## Solved Examples

**Example 24.** A body is floating on the liquid if we increases temperature then what changes occur in buoyancy force. (Assume body is always in floating condition)

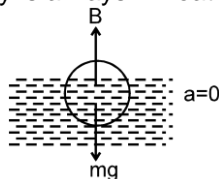


Figure : 13

**Solution :** Body is in equilibrium (see figure 13)

So,  $mg = B$  (Boyant Force)

The gravitational force does not change with change in temperature. So buoyancy force remains constant.

By increasing temperature density of liquid decreases so volume of body inside the liquid increases to kept the Buoyance force constant and equal to gravitational force)

**Example 25.** In previous question discuss the case when body move downward, upwards and remains at same position when we increases temperature.

**Solution :** Let  $f$  = fraction of volume of body submerged in liquid.

$f = \frac{\text{volume of body submerged in liquid}}{\text{total volume of body}}$

$$f = \frac{v_1}{v_0} \quad \text{at } \theta_1^\circ\text{C} \quad f_2 = \frac{v_2}{v_0(1+3\alpha_s\Delta\theta)} \quad \text{at } \theta_2^\circ\text{C}$$

$$\text{for equilibrium } mg = B = v_1 d_1 g = v_2 d_2 g.$$

$$\therefore \frac{v_1 d_1}{d_2} = \frac{v_2 d_1}{1 + \gamma_L \Delta\theta} = v_1 (1 + \gamma_L \Delta\theta) \quad \therefore f_2 = \frac{v_1 (1 + \gamma_L \Delta\theta)}{v_0 (1 + 3\alpha_s \Delta\theta)}$$

where  $\Delta\theta = \theta_2 - \theta_1$

**Case I :** Body move downward if  $f_2 > f_1$   
means  $\gamma_L > 3\alpha_s$

**Case II :** Body move upwards if  $f_2 < f_1$   
means  $\gamma_L < 3\alpha_s$

**Case III :** Body remains at same position  
if  $f_2 = f_1$   
means  $\gamma_L = 3\alpha_s$



## 12. BIMETALLIC STRIP

If two strip of different metals are welded together to form a bimetallic strip, when heated uniformly it bends in form of an arc, the metal with greater coefficient of linear expansion lies on convex side. The radius of arc thus formed by bimetal is (see figure 14) :

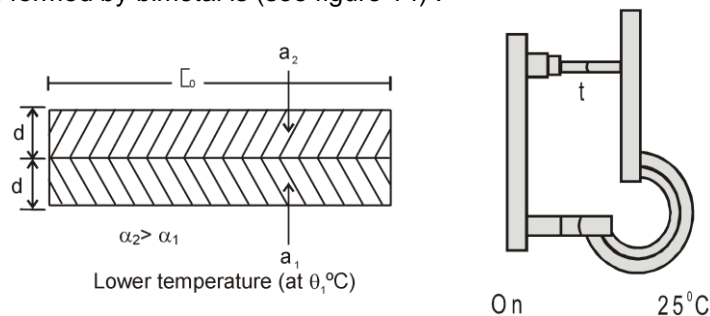


Figure : 14 (a)

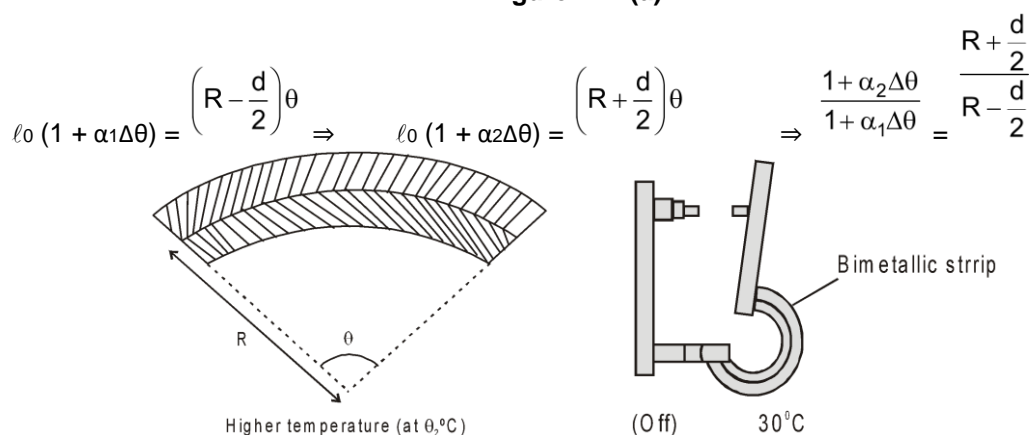


Figure : 14 (b)

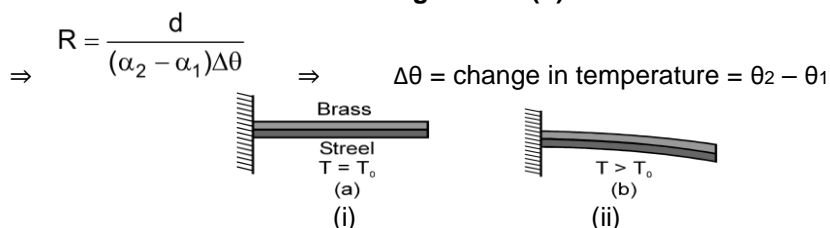


Figure : 14 (c)

A bimetallic strip, consisting of a strip of brass and a strip of steel welded together as shown in figure 14 (c). At temperature  $T_0$  in figure 14(c) (i) and figure 14 (c) (ii). The strip bends as shown at temperatures above the reference temperature. Below the reference temperature the strip bends the other way. Many thermostats operate on this principle, making and breaking an electrical constant as the temperature rises and falls as shown in figure 14 (b).

## 13. APPLICATIONS OF THERMAL EXPANSION

- A small gap is left between two iron rails of the railway.
- Iron rings are slipped on the wooden wheels by heating the iron rings
- Stopper of a glass bottle jammed in its neck can be taken out by heating the neck.
- The pendulum of a clock is made of invar [an alloy of zinc and copper].

## 14. TEMPERATURE

Temperature may be defined as the **degree of hotness or coldness** of a body. Heat energy flows from a body at higher temperature to that at lower temperature until their temperatures become equal. At this stage, the bodies are said to be in thermal equilibrium.

### 14.1 Measurement of Temperature

The branch of thermodynamics which deals with the measurement of temperature is called thermometry. A thermometer is a device used to measure the temperature of a body. The substances like liquids and gases which are used in the thermometer are called thermometric substances.

Suppose a physical quantity  $X$  varies linealy with temperature then

$$t = \frac{X_t - X_0}{X_0 \alpha}$$

$$X_t = X_0(1 + \alpha t) \Rightarrow$$

$\alpha$  is constant and  $X_0$  is value of  $X$  at reference temperature set at  $0^\circ$ . In case of absolute scale

$$\frac{X_T}{X_0} = \frac{T}{T_0} \Rightarrow T = T_0 \frac{X_T}{X_0} = 273.16 \frac{X_T}{X_0} \text{ K}$$

## 14.2 Different Scales of Temperature

A thermometer can be graduated into following scales.

- (a) The Centigrade or Celsius scale ( $^\circ\text{C}$ )
- (b) The Fahrenheit scale ( $^\circ\text{F}$ )
- (c) The Reaumur scale ( $^\circ\text{R}$ )
- (d) Kelvin scale of temperature (K)

## 14.3 Comparison Between Different Temperature Scales

	K	C	F
Water boils	373.15	100	212
body temp.	310.2	37.0	98.6
Room temp.	300	27	80.6
Triple point of water	273.16	0.01	32
Water freezes	273.15	0	32
Solid $\text{CO}_2$	195	-78	-109
Hydrogen boils	20.7	-252.5	-422.5
Absolute zero	0	-273.15	-489.67

Figure : 15

The formula for the conversion between different temperature scales is:

$$\frac{K - 273}{100} = \frac{C}{100} = \frac{F - 32}{180} = \frac{R}{80}$$

General formula for the conversion of temperature from one scale to another:

$$\frac{\text{Temp on one scale}(S_1) - \text{Lower fixed point}(S_1)}{\text{Upper fixed point}(S_2) - \text{Lower fixed point}(S_1)} = \frac{\text{Temp. on other scale}(S_2) - \text{Lower fixed point}(S_2)}{\text{Upper fixed point}(S_2) - \text{Lower fixed point}(S_2)}$$

## 14.4 Thermometers

Thermometers are device that are used to measure temperatures. All thermometers are based on the principle that some physical property of a system changes as the system temperature changes.

Required properties of good thermometric substance.

- (1) Non-sticky (absence of adhesive force)
  - (2) Low melting point (in comparison with room temperature)
  - (3) High boiling temperature
  - (4) Coefficient of volumetric expansion should be high (to increase accuracy in measurement).
  - (5) Heat capacity should be low.
  - (6) Conductivity should be high
- Mercury (Hg) suitably exhibits above properties.

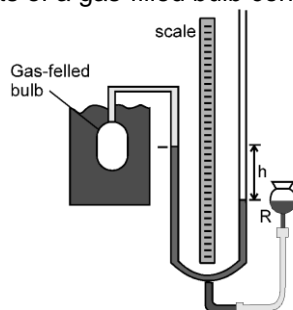
## 14.5 Types of Thermometers

## Calorimetry & Thermal Expansion

Type of thermometer and its range	Thermometric property	Advantages	Disadvantages	Particular Uses
Mercury-in-glass – 39°C to 450°C	Length of column of mercury in capillary tube	(i) Quick and easy to (direct reading) (ii) Easily portable	(i) Fragile (ii) Small size limits (iii) Limited range	(i) Every laboratory use where high accuracy is not required. (ii) Can be calibrated against constant-volume gas thermometer for more accurate work
Constant-volume gas thermometer – 270° to 1500°C	Pressure of a fixed mass of gas at constant volume	(i) Very accurate (ii) Very sensitive (iii) Wide range (iv) Easily reproducible	(i) Very large volume of bulb (ii) Slow to use and inconvenient	(i) Standard against which others calibrated (ii) He, H <sub>2</sub> or N <sub>2</sub> used depending on range (iii) can be corrected to the ideal gas scale (iv) Used as standard below -183°C
Platinum resistance –180° to 1150°C	Electrical resistance of a platinum coil	(i) Accurate (ii) Wide range	Not suitable for varying temperature (i.e., is slow to respond to changes)	(i) Best thermometer for small steady temperature differences (ii) Used as standard between 183°C and 630°C.
Thermocouple –250°C to 1150°C	Emf produced between junctions of dissimilar metals at different temperatures for measurement of emfs	(i) Fast response because of low heat capacity. (ii) wide range (iii) can be employed for remote readings using long leads.	Accuracy is lost if emf is measured using a moving-coil voltmeter (as may be necessary for rapid changes when potentiometer is unsuitable)	(i) Best thermometer for small steady temperature differences (ii) Can be made direct reading by calibrating galvanometer (iii) Used as standard between 630°C and 1063°C
Radiation pyrometer above 1000°C	Colour of radiation emitted by a hot body	Does not come into contact when temperature is measured	(i) Cumbersome (ii) Not direct reading (needs a trained observer)	(i) Only thermometer possible for very high temperatures (ii) Used as standard above 1063°C.

### 14.6 The constant-volume gas thermometer

The standard thermometer, against which all other thermometers are calibrated, is based on the pressure of a gas in a fixed volume. Figure shows such a constant volume gas thermometer; it consists of a gas-filled bulb connected by a tube to a mercury monometer.



A constant volume gas thermometer, its bulb immersed in a liquid whose temperature  $T$  is to be measured.

Figure : 16

$$T = (273.16 \text{ K}) \left( \lim_{p_3 \rightarrow 0} \frac{p}{p_3} \right)$$

$P$  = Pressure at the temperature being measured,

$P_3$  = pressure when bulb in a triple point cell.

### Solved Examples

**Example 26.** The readings of a thermometer at 0°C and 100°C are 50 cm and 75 cm of mercury column respectively. Find the temperature at which its reading is 80 cm of mercury column?

**Solution :**

By using formula

$$\frac{80 - 50}{75 - 50} = \frac{T - 0}{100 - 0} \Rightarrow T = 120^\circ\text{C}$$

### Solved Miscellaneous Problems

**Problem 1.** A bullet of mass 10 gm is moving with speed 400m/s. Find its kinetic energy in calories ?

**Solution :**

$$\Delta k = \frac{1}{2} \times \frac{10}{1000} \times 400 \times 400 = 800$$

$$\frac{800}{4.2} = 191.11 \text{ Cal.}$$

## Calorimetry & Thermal Expansion

**Problem 2.** Calculate amount of heat required to convert 1 kg steam from 100°C to 200°C steam

**Solution :** Heat req. =  $1 \times \frac{1}{2} \times 100 = 50 \text{ kcal}$

**Problem 3.** Calculate heat required to raise the temperature of 1 g of water through 1°C ?

**Solution :** heat req. =  $1 \times 10^{-3} \times 1 \times 1 = 1 \times 10^{-3} \text{ kcal}$

**Problem 4.** 420 J of energy supplied to 10 g of water will raise its temperature by

**Solution :**  $\frac{420 \times 10^{-3}}{4.20} = 10 \times 10^{-3} \times 1 \times \Delta t = 10^\circ \text{ C}$

**Problem 5.** The ratio of the densities of the two bodies is 3 : 4 and the ratio of specific heats is 4 : 3 . Find the ratio of their thermal capacities for unit volume ?

**Solution :**  $\frac{\rho_1}{\rho_2} = \frac{3}{4}, \frac{s_1}{s_2} = \frac{4}{3} \Rightarrow \theta = \frac{m \times s}{m / \rho} \Rightarrow \frac{\theta_1}{\theta_2} = \frac{s_1}{s_2} \times \frac{\rho_1}{\rho_2} = 1 : 1.$

**Problem 6.** Heat releases by 1 kg steam at 150°C if it convert into 1 kg water at 50°C.

**Solution :**  $H = 1 \times \frac{1}{2} \times 50 + 1 \times 540 + 1 \times 1 \times 50$   
 $= 540 + 75 = 615 \text{ Kcal}$   
 Heat release = 615 Kcal.

**Problem 7.** 200 gm water is filled in a calorimetry of negligible heat capacity. It is heated till its temperature is increase by 20°C. Find the heat supplied to the water.

**Solution :**  $H = 200 \times 10^{-3} \times 1 \times 20 = 4 \text{ Kcal.}$   
 Heat supplied = 4000 cal = 4 Kcal

**Problem 8.** A bullet of mass 5 gm is moving with speed 400 m/s. strike a target and energy. Then calculate rise of temperature of bullet. Assuming all the lose in kinetic energy is converted into heat energy of bullet if its specific heat is. 500J/kg°C.

**Solution :** Kinetic energy =  $\frac{1}{2} \times 5 \times 10^{-3} \times 400 \times 400 = 5 \times 10^{-3} \times 500 \times \Delta T$   
 $\Delta T = 160^\circ \text{ C}$   
 Rise in temperature is 160 °C

**Problem 9.** 1 kg ice at -10°C is mixed with 1 kg water at 100°C. Then find equilibrium temperature and mixture content.

**Solution :** Heat gained by 1 kg ice at - 10°C to convert into 0°C ice =  $1 \times \frac{1}{2} \times 10 = 5 \text{ Kcal} = 5000 \text{ cal}$   
 In the thermal equilibrium  
 $5 + 1 \times 80 + 1 \times T = 1 \times (100 - T)$   
 $85 = 100 - 2T \Rightarrow 2T = 15$   
 $T = \frac{15}{2} = 7.5^\circ \text{ C, water, entire ice melts.}$

**Problem 10.** 1 kg ice at -10° is mixed with 1kg water at 50°C. Then find equilibrium temperature and mixture content.

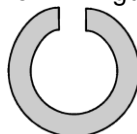
**Solution :** Heat required by ice at -10°C to convert it into 0°C water  
 $1 \times \frac{1}{2} \times 10 + 1 \times 50 = 55 \text{ Kcal}$   
 Heat released by 1 kg water to reduce it temperature from 50°C to 0°C =  $1 \times 1 \times 50 = 50 \text{ kcal}$   
 Heat required > Heat released so, ice will not complete melt. Let m g ice melt then for equilibrium.

$$1 \times \frac{1}{2} \times 10 + 80 m = 50 m \Rightarrow 80 m = 45 \Rightarrow m = \frac{45}{80}$$

## Calorimetry & Thermal Expansion

Content of mixture  $\left\{ \begin{array}{l} \text{water} \left( 1 + \frac{45}{80} \right) \text{ kg} \\ \text{ice} \left( 1 - \frac{45}{80} \right) \text{ kg} \end{array} \right\}$  and temperature is  $0^\circ\text{C}$

**Problem 11.** A small ring having small gap is shown in figure on heating what will happen to size of gap ?



**Solution :** Gap will also increase. The reason is same as in above example.

**Problem 12.** A concrete slab has a length of 10 m in a winter night when the temperature is  $0^\circ\text{C}$ . Find the length of the slab on a summer day when the temperature is  $35^\circ\text{C}$ . The coefficient of linear expansion of concrete is  $1.0 \times 10^{-5}/^\circ\text{C}$ .

**Solution :**  $\ell_t = 10(1 + 1 \times 10^{-5} \times 35) = 10.0035 \text{ m}$

**Problem 13.** A steel rod is clamped at its two ends and rests on a fixed horizontal base. The rod is unstrained at  $20^\circ\text{C}$ . Find the longitudinal strain developed in the rod if the temperature rises to  $50^\circ\text{C}$ . Coefficient of linear expansion of steel =  $1.2 \times 10^{-5}/^\circ\text{C}$ .

$$\frac{\Delta \ell}{\ell} = \frac{\ell_0 \alpha \Delta t}{\ell_0}$$

**Solution :**  $\frac{\Delta \ell}{\ell} = -3.6 \times 10^{-4}$

**Problem 14.** If rod is initially compressed by  $\Delta \ell$  length then what is the strain on the rod when the temperature

(a) is increased by  $\Delta \theta$  (b) is decreased by  $\Delta \theta$ .

**Solution:** (a) Strain =  $\frac{\Delta \ell}{\ell} + \alpha \Delta \theta$  (b) Strain =  $\left| \frac{\Delta \ell}{\ell} - \alpha \Delta \theta \right|$

**Problem 15.** A pendulum clock having copper rod keeps correct time at  $20^\circ\text{C}$ . It gains 15 seconds per day if cooled to  $0^\circ\text{C}$ . Calculate the coefficient of linear expansion of copper.

**Solution :**  $\frac{15}{24 \times 60 \times 60} = \frac{1}{2} \alpha \times 20 \Rightarrow \alpha = \frac{1}{16 \times 3600} = 1.7 \times 10^{-5}/^\circ\text{C}$

**Problem 16.** A meter scale made of steel is calibrated at  $20^\circ\text{C}$  to give correct reading. Find the distance between 50 cm mark and 51 cm mark if the scale is used at  $10^\circ\text{C}$ . Coefficient of linear expansion of steel is  $1.1 \times 10^{-5}/^\circ\text{C}$ .

**Solution :**  $\ell_t = 1 (1 - 1.1 \times 10^{-5} \times 10) = 0.99989 \text{ cm}$

**Problem 17.** A uniform solid brass sphere is rotating with angular speed  $\omega_0$  about a diameter. If its temperature is now increased by  $100^\circ\text{C}$ . What will be its new angular speed. (Given  $\alpha_B = 2.0 \times 10^{-5} \text{ per } ^\circ\text{C}$ )

$$(1) \frac{\omega_0}{1-0.002} \quad (2) \frac{\omega_0}{1+0.002} \quad (3^*) \frac{\omega_0}{1+0.004} \quad (4) \frac{\omega_0}{1-0.004}$$

**Solution :**  $I_0 \omega_0 = I_t \omega_t \Rightarrow M r_0^2 \omega_0 = M r_0^2 (1 + 2\alpha \Delta T) \omega_t \Rightarrow \omega_t = \frac{\omega_0}{1+0.004}$

**Problem 18.** The volume occupied by a thin - wall brass vessel and the volume of a solid brass sphere are the same and equal to  $1,000 \text{ cm}^3$  at  $0^\circ\text{C}$ . How much will the volume of the vessel and that of the sphere change upon heating to  $20^\circ\text{C}$ ? The coefficient of linear expansion of brass is  $\alpha = 1.9 \times 10^{-5}$ .

**Solution :**  $V = V_0 (1 + 3\alpha \Delta T) = 1.14 \text{ cm}^3 \Rightarrow 1.14 \text{ cm}^3 \text{ for both}$

**Problem 19.** A thin copper wire of length  $L$  increases in length by 1%, when heated from temperature  $T_1$  to  $T_2$ . What is the percentage change in area when a thin copper plate having dimensions  $2L \times L$  is heated from  $T_1$  to  $T_2$  ?

(1) 1% (2) 3% (3) 4% (4\*) 2%

**Solution :**  $\ell_f = L (1 + \alpha \Delta t) = \frac{L_f}{L} \times 100 = (1 + \alpha \Delta t) \times 100 = 1\%$

## Calorimetry & Thermal Expansion

$$A = 2L \times L (1 + 2\alpha\Delta t) = \frac{A_f}{2L \times L} \times 100 = (1 + 2\alpha\Delta t) \times 100 = 2\%$$

**Problem 20.** The density of water at 0°C is 0.998 g/cm<sup>3</sup> and at 4°C is 1.000 g/cm<sup>3</sup>. Calculate the average coefficient of volume expansion of water in the temperature range 0 to 4°C.

**Solution :** 
$$d_t = \frac{d_0}{1 + \gamma\Delta t} \Rightarrow 1 = \frac{0.998}{1 + \gamma \times 4} = -5 \times 10^{-4} / ^\circ\text{C}$$

**Problem 21.** A glass vessel measures exactly 10 cm × 10 cm × 10 cm at 0°C. it is filled completely with mercury at this temperature. When the temperature is raised to 10°C, 1.6 cm<sup>3</sup> of mercury overflows. Calculate the coefficient of volume expansion of mercury. Coefficient of linear expansion of glass = 6.5 × 10<sup>-6</sup>/°C

**Solution :** 
$$\Delta V = V_{Hg} - V_V$$

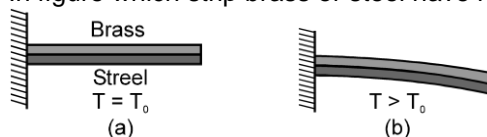
$$1.6 = 10^3 (\gamma_{\ell} \times 10 - 10^3 \times 3 \times 6.5 \times 10^{-6} \times 10)$$

$$\gamma_L = (1.6 + 0.195) \times 10^{-4} = 1.795 \times 10^{-4} / ^\circ\text{C}$$

**Problem 22.** A metal ball immersed in alcohol weighs W<sub>1</sub> at 0°C and W<sub>2</sub> at 50°C. The coefficient of cubical expansion of the metal is less than alcohol. Assuming that density of the metal is large compared to that of the alcohol, find which of W<sub>1</sub> and W<sub>2</sub> is greater?

**Solution :** 
$$\gamma_M < \gamma_{\ell} \text{ so, } \frac{F'_B}{F_B} = \frac{[1 + \gamma_S\Delta\theta]}{[1 + \gamma_{\ell}\Delta\theta]} \quad F'_B < F_B$$
 so Apparent weight increased so, W<sub>2</sub> > W<sub>1</sub>

**Problem 23.** In figure which strip brass or steel have higher coefficient of linear expansion.



**Solution :** Brass Strip ; strip of higher α is on convex side.

**Problem 24.** The upper and lower fixed points of a faulty thermometer are 5°C and 105° C. If the thermometer reads 25° C , what is the actual temperature ?

**Solution :** 
$$\frac{25 - 5}{100} = \frac{C - 0}{100} \Rightarrow C = 20^\circ \text{C}$$

**Problem 25.** At what temperature is the Fahrenheit scale reading equal to twice of Celsius ?

**Solution :** 
$$\frac{F - 32}{180} = \frac{C - 0}{100} \Rightarrow \frac{2x - 32}{180} = \frac{x - 0}{100} \Rightarrow 1x - 160 = 9x \Rightarrow x = 160^\circ \text{C}$$

**Problem 26.** Temperature of a patient is 40° C. Find the temperature on Fahrenheit scale ?

**Solution :** 
$$\frac{F - 32}{180} = \frac{40 - 0}{100} \Rightarrow F = 104^\circ \text{F}$$