Vector

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

This Section is not meant for classroom discussion. It is being given to promote self-study and self testing amongst the Resonance students.

Max. Marks : 120 **Max. Time : 1 Hr.**

Important Instructions :

1. The test is of **1 hour** duration and max. marks 120.
2. The test consists **30** questions, **4 marks** each.
3. Only one choice is correct **1 mark** will be deducted for incorrect response. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 3 above.

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1. If $\vec{p} = \hat{i} + \hat{j}$, $\vec{q} = 4\hat{k} - \hat{j}$ and $\vec{r} = \hat{i} + \hat{k}$ then the unit vector in the direction of $3\vec{p} + \vec{q} - 2\vec{r}$ is :

(1) $\frac{\hat{i} + 2\hat{j} + 2\hat{k}}{3}$ (2) $\frac{\hat{i} - 2\hat{j} - 2\hat{k}}{3}$ (3) $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$ (4) $\hat{i} + 2\hat{j} + 2\hat{k}$

2. If P is any point within a ΔABC, then $\overrightarrow{PA} + \overrightarrow{CP}$ =

(1) $\overrightarrow{AC} + \overrightarrow{CB}$ (2) $\overrightarrow{BC} + \overrightarrow{BA}$ (3) $\overrightarrow{CB} + \overrightarrow{AB}$ (4) $\overrightarrow{CB} + \overrightarrow{BA}$

3. If the position vectors of A, B, C are respectively $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ in ΔABC then $\cos^2 A$ is equal to :

(1) 0 (2) $\frac{6}{41}$ (3) $\frac{35}{41}$ (4) 1

4. $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$ and $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then α + β is equal to :

(1) 2 (2) 1 (3) 0 (4) –1

5. The value of b such that scalar product of the vectors $(\hat{i} + \hat{j} + \hat{k})$ with the unit vector parallel to the sum of the vectors $(2\hat{i} + 4\hat{j} - 5\hat{k})$ and $\left(b\hat{i} + 2\hat{j} + 3\hat{k}\right)$ is 1, is :

(1) –2 (2) –1 (3) 0 (4)1

6. A, B, C, D are any four points, then $\overrightarrow{AB}.\overrightarrow{CD} + \overrightarrow{BC}.\overrightarrow{AD} + \overrightarrow{CA}.\overrightarrow{BD}$ =

(1) $2\overrightarrow{AB}.\overrightarrow{BC} . \overrightarrow{CD}$ (2) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$ (3) $5\sqrt{3}$ (4) 0

7. Angle between the vectors $\sqrt{3}(\vec{a} \times \vec{b})$ and $\vec{b} - (\vec{a}.\vec{b})\vec{a}$

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$ (3) 0 (4) $\frac{\pi}{3}$

8. If the vector $3\hat{i}-4\hat{j}-\hat{k}$ and $2\hat{i}+3\hat{j}-6\hat{k}$ represents the diagonals of a rhombus, then the length of the side of the rhombus, is

(1) 15 (2) $15\sqrt{3}$ (3) $\frac{5\sqrt{3}}{2}$ (4) $\frac{15\sqrt{3}}{2}$

9. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}.\vec{b}=0$ and $\vec{a}\times\vec{b}=\vec{0}$ then

(1) $\vec{a}$ is parallel to $\vec{b}$ (2) $\vec{a}$ is perpendicular to $\vec{b}$

(3) Either $\vec{a}$ or $\vec{b}$ is a null vector (4) None of these

10. If $\vec{a}=2\hat{i}+3\hat{j}-5\hat{k}$, $\vec{b}=m\hat{i}+n\hat{j}+12\hat{k}$ and $\vec{a}\times\vec{b}=0$, then (m, n) =

(1) $\left(\frac{-24}{5},\frac{36}{5}\right)$ (2) $\left(\frac{24}{5},\frac{-36}{5}\right)$ (3) $\left(\frac{-24}{5},\frac{-36}{5}\right)$ (4) $\left(\frac{24}{5},\frac{36}{5}\right)$

11. If $\vec{a}=\hat{i}-\hat{j}$ and $\vec{b}=\hat{j}+\hat{k}$ then $|\vec{a}\times\vec{b}|^2+|\vec{a}.\vec{b}|^2$ is equal to :

(1) $\sqrt{2}$ (2) 2 (3) $\sqrt{6}$ (4) 4

12. If the four points with position vectors $-2\hat{i}+\hat{j}+\hat{k},\ \hat{i}+\hat{j}+\hat{k},\ \hat{j}-\hat{k},$ and $\lambda\hat{j}+\hat{k}$ are coplanar, then λ =

(1) 1 (2) 2 (3) –1 (4) 0

13. If the vectors $2\hat{i}-\hat{j}+\hat{k},\ \hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+\lambda\hat{j}+5\hat{k}$ be coplanar, then λ =

(1) –1 (2) –2 (3) –3 (4) –4

14. If three vectors $\vec{a}=12\hat{i}+4\hat{j}+3\hat{k},\ \vec{b}=8\hat{i}-12\hat{j}-9\hat{k}$ and $\vec{c}=33\hat{i}-4\hat{j}-24\hat{k}$ are edges of parallelopiped then its volume will be :

(1) 616 (2) 308 (3) 154 (4) None of these

15. If $\vec{a}=\hat{i}+\hat{j}+\hat{k},\ \vec{b}=2\hat{i}+\lambda\hat{j}+\hat{k},\ \vec{c}=\hat{i}-\hat{j}+4\hat{k}$ and $\vec{a}.(\vec{b}\times\vec{c})=10$, then λ =

(1) 6 (2) 7 (3) 9 (4) 10

16. The angle between $\vec{a}$ and $\vec{b}$ is $\frac{5\pi}{6}$ and projection of $\vec{a}$ on $\vec{b}$ is $\frac{-9}{\sqrt{3}}$, then $|\vec{a}|$ is equal to :

(1) 12 (2) 8 (3) 10 (4) 6

17. If $|\vec{a}|=3$, $|\vec{b}|=4$, then a value of λ for which $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{a}-\lambda\vec{b}$ is :

(1) $\frac{9}{16}$ (2) $\frac{3}{4}$ (3) $\frac{3}{2}$ (4) $\frac{4}{3}$

18. $\overrightarrow{AB}\times\overrightarrow{AC}=2\hat{i}-4\hat{j}+4\hat{k}$, then the area of ΔABC is :

(1) 3 (2) 4 (3) 16 (4) 9

19. If $\vec{a}$ and $\vec{b}$ are the position vectors of A and B respectively, then the position vector of a point C on AB produced such that $\overrightarrow{AC}=3\overrightarrow{AB}$ is :

(1) $3\vec{a}-\vec{b}$ (2) $3\vec{b}-\vec{a}$ (3) $3\vec{a}-2\vec{b}$ (4) $3\vec{b}-2\vec{a}$

20. If the position vectors of A and B are $\hat{i}+3\hat{j}-7\hat{k}$ and $5\hat{i}-2\hat{j}+4\hat{k}$, then the direction cosine of $\overrightarrow{AB}$ along y-axis is :

(1) $\frac{4}{\sqrt{162}}$ (2) $\frac{-5}{\sqrt{162}}$ (3) –5 (4) 11

21. Angle between the line $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and the normal to the plane $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) = 4$ is

(1) $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ (2) $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ (3) $\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ (4) $\cot^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

22. The position vector of the point in which the line joining the points $\hat{i} - 2\hat{j} + \hat{k}$ and $3\hat{k} - 2\hat{j}$ cuts the plane through the origin and the points $4\hat{j}$ and $2\hat{i} + \hat{k}$ is :

(1) $6\hat{i} - 10\hat{j} + 3\hat{k}$ (2) $\frac{1}{5}(6\hat{i} - 10\hat{j} + 3\hat{k})$ (3) $-6\hat{i} + 10\hat{j} - 3\hat{k}$ (4) None of these

23. The position vectors of two points P and Q are $3\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} - 2\hat{j} - 4\hat{k}$ respectively. The equation of the plane through Q and perpendicular to PQ is :

(1) $\vec{r}.(2\hat{i} + 3\hat{j} + 6\hat{k}) = 28$ (2) $\vec{r}.(2\hat{i} + 3\hat{j} + 6\hat{k}) = 32$

(3) $\vec{r}.(2\hat{i} + 3\hat{j} + 6\hat{k}) + 28 = 0$ (4) None of these

24. If $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{k}$ are two vectors, then the point of intersection of two lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is :

(1) $\hat{i} + \hat{j} - \hat{k}$ (2) $\hat{i} - \hat{j} + \hat{k}$ (3) $3\hat{i} + \hat{j} - \hat{k}$ (4) $3\hat{i} - \hat{j} + \hat{k}$

25. If the equation of a line through a point $\vec{a}$ and parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$ (where λ is parameter) then its perpendicular distance from the point $\vec{c}$ is :

(1) $|(\vec{c} - \vec{b}) \times \vec{a}| \div |\vec{a}|$ (2) $|(\vec{c} - \vec{a}) \times \vec{b}| \div |\vec{b}|$ (3) $|(\vec{a} - \vec{b}) \times \vec{c}| \div |\vec{c}|$ (4) $|(\vec{a} - \vec{b}) \times \vec{c}| \div |\vec{a} + \vec{c}|$

26. If $\vec{a} = 2\hat{i} - \hat{j} - m\hat{k}$ and $\vec{b} = \frac{4\hat{i}}{7} - \frac{2\hat{j}}{7} + 2\hat{k}$ are collinear, then the value of m is equal to :

(1) –7 (2) –1 (3) 2 (4) 7

27. If two vertices of a triangle are $\hat{i} - \hat{j}$ and $\hat{j} + \hat{k}$, then the third vertex can be

(1) $\hat{i} + \hat{k}$ (2) $\hat{i} - 2\hat{j} - \hat{k}$ (3) $\hat{i} - \hat{k}$ (4) All the above

28. The volume of the tetrahedron position vectors of whose vertices are given by the vectors $-\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$ with reference to the fourth vertex as origin, is :

(1) $\frac{5}{3}$ cubic unit (2) $\frac{2}{3}$ cubic unit (3) $\frac{3}{5}$ cubic unit (4) None of these

29. If vector $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and vector $\vec{b} = -2\hat{i} + 2\hat{j} - \hat{k}$, then $\frac{\text{Projection of vector } \vec{a} \text{ on vector } \vec{b}}{\text{projection of vector } \vec{b} \text{ on vector } \vec{a}}$

(1) $\frac{3}{7}$ (2) $\frac{7}{3}$ (3) 3 (4) 7

30. The moment of a force represented by $\vec{F} = \hat{i} + 2\hat{j} + 3\hat{k}$ about the point $2\hat{i} - \hat{j} + \hat{k}$ =

(1) $5\hat{i} - 5\hat{j} + 5\hat{k}$ (2) $5\hat{i} + 5\hat{j} - 5\hat{k}$ (3) $-5\hat{i} + 5\hat{j} + 5\hat{k}$ (4) $-5\hat{i} - 5\hat{j} + 5\hat{k}$

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

✍ Marked questions may have for revision questions.

1. A(3, –1, 2), B(5, 2, 4), C(–1, –1, 6) are vertices of ΔABC and D is midpoint of AC. If $2\vec{BD} = \vec{BE}$ and A_1, A_2 are areas of ΔABD and quadilatral ABCE, then $A_1 : A_2$ is equal to

(1) 12 : 1 (2) 1 : 6 (3) 1 : 24 (4) 1 : 4

2.✍ ABCD is an isosceles trapezium with $\vec{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\vec{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. If E is point on side AB such that AE : EB = 3 : 2. If A_1 and A_2 are areas of trapezium ABCD and ΔCDE respectively, then $A_1 : A_2$ is equal to

(1) $\frac{29}{15}$ (2) $\frac{74}{29}$ (3) $\frac{37}{29}$ (4) $\frac{45}{37}$

3.✍ A variable plane passes through a fixed point (a, b, c) and meets the coordinate axes in A, B, C. Locus of the point common to the planes through A, B, C and parallel to coordinate planes, is

(1) $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$ (2) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ (3) ax + by + cz = 1 (4) $\frac{a}{x} - \frac{b}{y} - \frac{c}{z} = 1$

4. If the vectors $\vec{a}, \vec{b}$ & $\vec{c}$ form the sides BC, CA & AB respectively of a triangle ABC, then :

(1) $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 0$ (2) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

(3) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a}$ (4) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$

5. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ be such that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$. Let P_1 & P_2 be planes determined by the pairs of vectors $\vec{a}, \vec{b}$ & $\vec{c}, \vec{d}$ respectively. Then the angle between P_1 and P_2 is:

(1) 0 (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

6. If $\vec{a}, \vec{b}$ & $\vec{c}$ are unit coplanar vectors, then the scalar triple product $\left[2\vec{a}-\vec{b} \quad 2\vec{b}-\vec{c} \quad 2\vec{c}-\vec{a}\right] =$

(1) 0 (2) 1 (3) $-\sqrt{3}$ (4) $\sqrt{3}$

7.✍ If, $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2$ does NOT exceed

(1) 4 (2) 9 (3) 8 (4) 6

8. Let $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$. Then $[\vec{a}\,\vec{b}\,\vec{c}]$ depends on

(1) only x (2) only y (3) Neither x nor y (4) both x and y

9. If $\vec{a}$ & $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ & $5\vec{a} - 4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ & $\vec{b}$ is:

(1) 45º (2) 60º (3) $\cos^{-1}(1/3)$ (4) $\cos^{-1}(2/7)$

10. Let $\vec{V} = 2\hat{i} + \hat{j} - \hat{k}$ & $\vec{W} = \hat{i} + 3\hat{k}$. If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $\left[\vec{U}\ \vec{V}\ \vec{W}\right]$ is:

(1) − 1 (2) $\sqrt{10} + \sqrt{6}$ (3) $\sqrt{59}$ (4) $\sqrt{60}$

11. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is

(1) $4\hat{i} - \hat{j} + 4\hat{k}$ (2) $3\hat{i} + \hat{j} - 3\hat{k}$ (3) $2\hat{i} + \hat{j} - 2\hat{k}$ (4) $4\hat{i} + \hat{j} - 4\hat{k}$

12. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i} - \hat{j}$, $4\hat{i}$, $3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$ respectively. The quadrilateral PQRS must be a

(1) parallelogram, which is neither a rhombus nor a rectangle
(2) square
(3) rectangle, but not a square
(4) rhombus, but not a square

13. Two adjacent sides of a parallelogram ABCD are given by
$\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD' . If AD' makes a right angle with the side AB, then the cosine of the angle α is given by

(1) $\frac{8}{9}$ (2) $\frac{\sqrt{17}}{9}$ (3) $\frac{1}{9}$ (4) $\frac{4\sqrt{5}}{9}$

14. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to:

(1) 0 (2) 1
(3) $\frac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$ (4) $\frac{3}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)(c_1^2 + c_2^2 + c_3^2)$

15. If the vectors $a\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ $(a \neq b \neq c \neq 1)$ are coplanar, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to

(1) 1 (2) −1 (3) 0 (4) 2

16. The vectors $\vec{a} = -4\hat{i} + 3\hat{k}$, $\vec{b} = 14\hat{i} + 2\hat{j} - 5\hat{k}$ are co-initial. The vector $\vec{d}$ which is bisecting the angle between the vectors $\vec{a}$ and $\vec{b}$ and having the magnitude $\sqrt{6}$, is

(1) $\hat{i} + \hat{j} + 2\hat{k}$ (2) $\hat{i} - \hat{j} + 2\hat{k}$ (3) $\hat{i} + \hat{j} - 2\hat{k}$ (4) $2\hat{i} - \hat{j} + 2\hat{k}$

17. Let two non-collinear unit vectors $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}\cos t + \hat{b}\sin t$. When P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$. Then,

(1) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1 + \hat{a}.\hat{b})^{1/2}$
(2) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1 + \hat{a}.\hat{b})^{1/2}$
(3) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1 + 2\hat{a}.\hat{b})^{1/2}$
(4) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1 + 2\hat{a}.\hat{b})^{1/2}$

18. Length of chord made by sphere $x^2 + y^2 + z^2 - 4x + 2y - 8z + 5 = 0$ on line $\frac{x-4}{3} = \frac{y-4}{2} = \frac{z-6}{3}$

(1) 2 (2) $2\sqrt{5}$ (3) $\sqrt{6}$ (4) $4\sqrt{3}$

19. If a, b, c are different real numbers $a\hat{i} + b\hat{j} + c\hat{k}$, $b\hat{i} + c\hat{j} + a\hat{k}$ and $c\hat{i} + a\hat{j} + b\hat{k}$, and are position vectors of three non-collinear points A, B and C, then

(1) centroid of triangle ABC is $\frac{a+b+c}{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$
(2) $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the three vectors
(3) perpendicular from the origin to the plane of triangle ABC meet at centroid
(4) all of these

20. A variable plane at a distance of 1 unit from the origin cuts the co-ordinate axes at A, B and C. If the centroid D (x, y, z) of triangle ABC satisfies the relation $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = K$, then the value of k is

(1) 9 (2) 3 (3) 1 (4) 1/3

21. If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then |d| is

(1) 3 (2) 2 (3) 5 (4) 6

22. The point P is the intersection of the straight line joining the points Q(2,3,5) and R(1, –1, 4) with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point T(2, 1,4) to QR, then the length of the line segment PS is

(1) $\frac{1}{\sqrt{2}}$ (2) $\sqrt{2}$ (3) 2 (4) $2\sqrt{2}$

23. A mirror and a source of light are situated at the origin O and at a point on OX (x-axis) respectively. A ray of light from the source strikes the mirror and is reflected. If the Drs of the normal to the plane are 1, –1, 1, then DCs of the reflected ray are

(1) $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$ (2) $-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$, (3) $-\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}$ (4) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

24. If the planes $x - cy - bz = 0$, $cx - y + az = 0$ and $bx + ay - z = 0$ pass through a straight line, then the value of $a^2 + b^2 + c^2 + 2abc$ is :

(1) 1 (2) 7 (3) $\frac{7}{2}$ (4) $\frac{8}{3}$

25. The shortest distance between the z-axis and the line, $x + y + 2z - 3 = 0$, $2x + 3y + 4z - 4 = 0$ is :

(1) 1 (2) 2 (3) 3 (4) 4

26. Three non-concurrent coplaner lines with direction ratios 1, 1, 2 ; $\sqrt{3}-1, -\sqrt{3}-1$, 4 and $-\sqrt{3}-1$, $\sqrt{3}-1$, 4 make

(1) equilateral triangle (2) isosceles triangle
(3) right angle triangle (4) obtuse angled triangle

27.* In R_3, consider the planes $P_1 : y = 0$ and $P_2 : x + z = 1$. Let P_3 be a plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0, 1, 0) from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relation is (are) true ?

(1) $2\alpha + \beta + 2\gamma + 2 = 0$ (2) $2\alpha - \beta + 2\gamma + 4 = 0$ (3) $2\alpha + \beta - 2\gamma - 10 = 0$ (4) $2\alpha - \beta + 2\gamma - 8 = 0$

28.* In R_3, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1 : x + 2y - z + 1 = 0$ and $P_2 : 2x - y + z - 1 = 0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M?

(1) $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$ (2) $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$ (3) $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$ (4) $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

29.* Let ΔPQR be a triangle. Let $\vec{a} = \overrightarrow{QR}, \vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If $|\vec{a}| = 12$, $|\vec{b}| = 4\sqrt{3}$ and $\vec{b}.\vec{c} = 24$, then which of the following is(are) true?

(1) $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$ (2) $\frac{|\vec{c}|^2}{2} + |\vec{a}| = 30$
(3) $|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 48\sqrt{3}$ (4) $\vec{a}.\vec{b} = -72$

30.* Consider a pyramid OPQRS located in the first octant ($x \ge 0, y \ge 0, z \ge 0$) with O as origin, and OP and OR along the x-axis and the y-axis, respectively. The base OPQR of the pyramid is a square with OP = 3. The point S is directly above the mid point T of diagonal OQ such that TS = 3. Then

(1) the acute angle between OQ and OS is $\frac{\pi}{3}$
(2) the equation of the plane containing the triangle OQS is $x - y = 0$
(3) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$
(4) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

31._ The equation of plane containing lines and $x = \frac{y+4}{2} = \frac{10-z}{4}$ and $\frac{4-x}{2} = -(y+2) = \frac{z}{5}$

(1) $x + y - z + 7 = 0$ (2) $4x + 2y + 2z - 7 = 0$
(3) $x + y - z + 6 = 0$ (4) $2x + y + z - 6 = 0$

COMPREHENSION

Comprehension (32 to 34)

Consider the lines

$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$

$L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$

Vector

32. The unit vector perpendicular to both L_1 and L_2 is

(1) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (2) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (3) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (4) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

33. The shortest distance between L_1 and L_2 is

(1) 0 (2) $\frac{17}{\sqrt{3}}$ (3) $\frac{41}{5\sqrt{3}}$ (4) $\frac{17}{5\sqrt{3}}$

34. The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is

(1) $\frac{2}{\sqrt{75}}$ (2) $\frac{7}{\sqrt{75}}$ (3) $\frac{13}{\sqrt{75}}$ (4) $\frac{23}{\sqrt{75}}$

DIRECTIONS (Q. NO. 35 TO 38) :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true. (2) Statement-I is true, but Statement-II is false.

(3) Statement-I is false, but Statement-II is true. (4) Both the statements are false.

35. **STATEMENT 1 :** If $\hat{u}, \hat{v}, \hat{w}$ be three non−coplanar unit vectors with angles between $\hat{u}$ & $\hat{v}$ is α, between $\hat{v}$ & $\hat{w}$ is β and between $\hat{w}$ & $\hat{u}$ is γ. If $\vec{a}, \vec{b}, \vec{c}$ are the unit vectors along angle bisectors of α, β, γ respectively, then $\left[\vec{a}\times\vec{b}.\ \vec{b}\times\vec{c}\ .\ \vec{c}\times\vec{a}\right] = \frac{1}{16}\left[\hat{u}\ \hat{v}\ \hat{w}\right]^2 \sec^2\left(\frac{\alpha}{2}\right)\sec^2\left(\frac{\beta}{2}\right)\sec^2\left(\frac{\gamma}{2}\right)$.

STATEMENT 2 : $\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$

36. **STATEMENT 1 :** If the incident ray on a surface is along the unit vector $\hat{v}$, the reflected ray is along the unit vector $\hat{w}$ and the normal is along unit vector $\hat{a}$ outwards. Then $\hat{w} = \hat{v} - 2(\hat{a}.\hat{v})\hat{a}$

STATEMENT 2 : If the incident ray on a surface is along the unit vector $\hat{v}$, the reflected ray is along the unit vector $\hat{w}$ and the normal is along unit vector $\hat{a}$ outwards. Then $\hat{w} = \hat{v} - (\hat{a}.\hat{v})\hat{a}$

37. Consider the planes 3x – 6y – 2z = 15 and 2x + y – 2z = 5

STATEMENT - 1 : The parametric equations of the line of intersection of the given planes are x = 3 + 14t, y = 1 + 2t, z = 15t.

because

STATEMENT - 2 : The vector $14\hat{i}+2\hat{j}+15\hat{k}$ is parallel to the line of interseciton of given planes.

38. Consider three planes

$P_1 : x - y + z = 1$

$P_2 : x + y - z = -1$

$P_3 : x - 3y + 3z = 2$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, P_1 and P_2, respectively.

STATEMENT -1 : At least two of the lines L_1, L_2 and L_3 are non-parallel.

and

STATEMENT-2 : The three planes do not have a common point.

APSP Answers

PART - I

1.	(1)	**2.**	(4)	**3.**	(3)	**4.**	(2)	**5.**	(4)	**6.**	(4)	**7.**	(1)
8.	(3)	**9.**	(3)	**10.**	(3)	**11.**	(4)	**12.**	(1)	**13.**	(4)	**14.**	(4)
15.	(1)	**16.**	(4)	**17.**	(2)	**18.**	(1)	**19.**	(4)	**20.**	(2)	**21.**	(2)
22.	(2)	**23.**	(3)	**24.**	(3)	**25.**	(2)	**26.**	(1)	**27.**	(4)	**28.**	(2)
29.	(2)	**30.**	(2)										

PART - II

1.	(4)	**2.**	(2)	**3.**	(1)	**4.**	(2)	**5.**	(1)	**6.**	(1)	**7.**	(2)
8.	(3)	**9.**	(2)	**10.**	(3)	**11.**	(1)	**12.**	(1)	**13.**	(2)	**14.**	(3)
15.	(1)	**16.**	(1)	**17.**	(1)	**18.**	(2)	**19.**	(4)	**20.**	(1)	**21.**	(4)
22.	(1)	**23.**	(4)	**24.**	(1)	**25.**	(2)	**26.**	(1)	**27.**	(2,4)	**28.**	(12)
29.	(1,3,4)	**30.**	(2,3,4)	**31.**	(4)	**32.**	(2)	**33.**	(4)	**34.**	(3)	**35.**	(1)
36.	(2)	**37.**	(3)	**38.**	(3)								