Exercise-2

Marked Questions may have for Revision Questions.

PART - I : OBJECTIVE QUESTIONS

1. Sol. $I = \int \frac{\ell n\left(\frac{x-1}{x+1}\right)}{x^2-1}\,dx$ put $\ell n\left(\frac{x-1}{x+1}\right) = t \Rightarrow \frac{2}{x^2-1}\,dx = dt$

$\Rightarrow I = \int t\frac{dt}{2} = \frac{t^2}{4} + C = \log_2\left(\frac{x-1}{x+1}\right) + C = \frac{1}{4}\log_2\left(\frac{x+1}{x-1}\right) + C$

2. Sol. $I = \int \frac{\ell n(\tan x)}{\sin x\ \cos x}\,dx$ put $\ell n \tan x = t \Rightarrow \frac{1}{\sin x \cos x}dx = dt$

$I = \int t\ dt = \frac{t^2}{2} + C = \frac{1}{2}(\ell n \tan x)^2 + C = \frac{1}{2}(\ell n \cot x)^2 + C = \frac{1}{2}(\ell n^2 \cot x) + C$

$= \frac{1}{2}\ell n^2(\sin x \sec x) + C = \frac{1}{2}\ell n_2(\cos x\ \mathrm{cosec}\, x) + C$

3. Sol. $f(x) = \int \frac{2\sin x - \sin 2x}{x^3}\,dx$

$f'(x) = \frac{2\sin x - \sin 2x}{x^3} = \frac{2\sin x}{x}.\frac{1-\cos x}{x^2} = 2\left(\frac{\sin x}{x}\right).\frac{2\sin^2\frac{x}{2}}{\frac{x^2}{4}\times 4} = \frac{2\times 2}{4}\left(\frac{\sin x}{x}\right).\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2$

$\lim_{x\to 0} f'(x) = \lim_{x\to 0}\left(\frac{\sin x}{x}\right).\lim_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 = 1$

4. Sol. $I = \int x^3(x^6+x^3+1)(2x^6+3x^3+6)^{1/3}\,dx$

$= \int x^2(x^6+x^3+1)(2x^9+3x^6+6x^3)^{1/3}\,dx$

Let $2x_9 + 3x_6 + 6x_3 = t$

$18(x_8 + x_5 + x_2)\,dx = dt$

$I = \frac{1}{18}\int t^{1/3}dt = \frac{1}{18}.\frac{t^{4/3}}{4/3} + c = \frac{1}{24}(2x_9 + 3x_6 + 6x_3)_{4/3} + c$

5. Sol. $I = \int 2^{mx}.\,3^{nx}dx = \int\left(2^m.\,3^n\right)^x dx = \frac{2^{mx}.\,3^{nx}}{\ell n(2^m.\,3^n)} + c$

6. Sol. $\int \frac{dx}{\sin x.\sin(x+\alpha)}$

$= \frac{1}{\sin\alpha}\int \frac{\sin(\alpha + x - x)}{\sin x \sin(x+\alpha)} dx$

$= \text{cosec } \alpha \int \frac{\sin(x+\alpha)\cos x - \cos(x+\alpha)\sin x}{\sin x \sin(x+\alpha)}$ $\quad = \text{cosec } \alpha \left[\int \cot x\, dx - \int \cot(x+\alpha)\right] + C$

$= \text{cosec } \alpha \; [\log|\sin x| - \log|\sin(x+\alpha)|] + C$ $\quad = \text{cosec } \alpha \log \left|\frac{\sin x}{\sin(x+\alpha)}\right| + C$

7. **Sol.** $I = -\frac{1}{2}\cos 2x - \frac{\sin 2x}{2} + b = -\frac{1}{\sqrt{2}}\sin\left(2x + \frac{\pi}{4}\right) + b$

$= \frac{1}{\sqrt{2}}\sin\left(2x + \frac{5\pi}{4}\right) + b \qquad a = -\frac{5\pi}{4}, \; b \in R$

8. **Sol.** $\int (1 + \tan x \tan(x+\alpha))dx$

$= \int \frac{\sin x \sin(x+\alpha) + \cos x \cos(x+\alpha)}{\cos x \cos(x+\alpha)}$ $\quad = \int \frac{\cos(x+\alpha-x)}{\cos x \cos(x+\alpha)} dx$

$= \cot \alpha \int \frac{\sin(x+\alpha-x)}{\cos x \cos(x+\alpha)} dx$ $\quad = \cot \alpha \left[\int \frac{\sin(x+\alpha)\cos x}{\cos x \cos(x+\alpha)} - \frac{\cos(x+\alpha)\sin x}{\cos x \cos(x+\alpha)} dx\right]$

$= \cot \alpha \left[\int \tan(x+\alpha)\, dx - \int \tan x\, dx\right]$ $\quad = \cot \alpha\, [\ell n\,|\sec(x+\alpha)| - \ell n\,|\sec x|]$

$= \cot \alpha\; \ell n \left|\frac{\cos x}{\cos(x+\alpha)}\right| + C = \cot \alpha\; \ell n \left(\left|\frac{\sec(x+\alpha)}{\sec x}\right|\right) + C$

9. **Sol.** $\int 4 \sin x \cos \frac{x}{2} \cos \frac{3x}{2}\, dx$ $\quad = \int 2\sin x(\cos 2x + \cos x)dx$

$= \int 2\sin x \cos 2x\, dx + \int \sin 2x + c$ $\quad = \int (\sin 3x - \sin x + \sin 2x) + c$

$= \cos x - \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x + c$

10. **Sol.** $\int \frac{x^2 + \cos^2 x}{1 + x^2} \text{cosec}^2 x\, dx$ $\quad = \int \text{cosec}^2 x\, dx - \int \frac{1}{1+x^2} dx$

$= -\cot x - \tan^{-1} x + c$ $\quad = -\tan^{-1} x - \frac{\text{cosec } x}{\sec x} + c$

11. **Sol.** $I = \int \sqrt{\frac{x-1}{x+1}} \times \frac{1}{x^2}\, dx$ $\qquad$ Put $\frac{1}{x} = \cos 2\theta \Rightarrow -\frac{dx}{x^2} = -2\sin 2\theta\, d\theta$

$I = \int \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\; 2\sin 2\theta\, d\theta$

$= \int 4\sin^2\theta\, d\theta = 2\int (1-\cos 2\theta)\, d\theta = 2\theta - \sin 2\theta + C = \cos^{-1}\left(\frac{1}{x}\right) - \sqrt{1-\frac{1}{x^2}} + C$

12. **Sol.** $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}.\ dx = Ax + B\ell n\,|9e^{2x} - 4| + C$

put $4e^x + 6e^{-x} = P\left(9e^x - 4e^{-x}\right) + Q\,(9e^x + 4e^{-x})$

$\Rightarrow \quad 4 = 9P + 9Q \quad$ and $\quad 6 = 4Q - 4P$

comparing, $P = -\frac{19}{36}$, $Q = \frac{35}{36}$

$I = -\frac{19}{36}\int dx + \frac{35}{36}\int \frac{9e^x + 4e^{-x}}{9e^x - 4e^{-x}}\, dx = -\frac{19}{36}.x + \frac{35}{36}\ \ell n\left|\left(9e^x - 4e^{-x}\right)\right| + C$

$= -\frac{19}{36}x + \frac{35}{36}\ \ell n\left|\left(9e^{2x} - 4\right)\right| - \frac{35}{36}x + C = \frac{35}{36}\ \ell n\left|\left(9e^{2x} - 4\right)\right| - \frac{54}{36}x + C$

$= \frac{35}{36}\ \ell n\left|\left(9e^{2x} - 4\right)\right| - \frac{3}{2}x + C$

So blfy, $A = -\frac{3}{2}$, $B = \frac{35}{36}$, $C \in R$

13. **Sol.** $\int \frac{dx}{\sqrt{\sin^3 x(\sin x\cos\alpha - \cos x\sin\alpha)}}$

$= \int \frac{\operatorname{cosec}^2 x}{\sqrt{\cos\alpha - \sin\alpha\cot x}}\, dx = \frac{2}{\sin\alpha}\sqrt{\cos\alpha - \sin\alpha\cot x} + C$

14. **Sol.** $\int \frac{\cos 4x + 1}{\cot x - \tan x}\, dx = A\cos 4x + B$

$= \int \frac{2\cos^2 2x\sin x\cos x}{\cos 2x}\, dx = \int 2\cos 2x\sin x\cos x\, dx = \frac{1}{2}\int \sin 4x\, dx = \frac{1}{2}\frac{(-\cos 4x)}{4} + B$

15. **Sol.** $\int \tan^4 x\, dx = \int \tan^2 x\,(\sec^2 x - 1)dx = \int \tan^2 x\, d(\tan x) - \int (\sec^2 x - 1)\, dx = \frac{1}{3}\tan^3 x - \tan x + x + c$

16. **Sol.** $I = \int \frac{dx}{2\sin^2 x + 2\sin x\cos x}$

$= -\frac{1}{2}\int \frac{-\operatorname{cosec}^2 x.dx}{(1+\cot x)}$ Let $1 + \cot x = t$ $\therefore$ $-\operatorname{cosec}^2 x\, dx = dt$

$= -\frac{1}{2}\int \frac{(dt)}{t} = -\frac{1}{2}\ \ell n\,|t| + c = -\frac{1}{2}\ \ell n\,|(1 + \cot x)| + c$

17. **Sol.** $\int \frac{\sin^2 x}{\cos^6 x} dx = \int \tan^2 x(1+\tan^2 x)$ $\sec^2 x\, dx$

$= \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + c$

PART - II : MISCELLANEOUS QUESTIONS

A-1. **Ans.** **(2)**

Sol. STATEMENT-1 : Put sinx = t

$\Rightarrow$ cosxdx = dt

Now $\int t^5\, dt = \frac{t^6}{6} + C = \frac{\sin^6 x}{6} + C$

STATEMENT-2 : is false for n = –1

A-2. **Ans.** **(1)**

Sol. I = $\int ((\log_x e) - (\log_x e)^2)\, dx$

$= \int \left(\frac{1}{\ell nx} - \frac{1}{(\ell nx)^2} \right) dx$ put ℓn x = t $\Rightarrow$ x = e^t $\Rightarrow$ dx = e^t dt

$\Rightarrow$ I = $\int \left(\frac{1}{t} - \frac{1}{t^2} \right) e^t\, dt$ $= \frac{e^t}{t} + C = \frac{x}{\ell nx} + C =$ x $\log_x e$ + C

Section (B) : MATCH THE COLUMN

B-1. **Ans.** (A) → (s) ; (B) → (q) ; (C) → (r)

Sol. (A) v = 0 ⇒ a = b, Also a + b = u ⇒ a = $\frac{u}{2}$

Now, I = $\frac{1}{a} \int \frac{dx}{1+\cos x} = \frac{2}{u} \int \frac{dx}{2\cos^2 \frac{x}{2}} = \frac{2}{u} \int \frac{1}{2} \sec^2 \frac{x}{2} dx = \frac{2}{u} \tan \frac{x}{2} + C$

(B) v > 0 ⇒ a > b

Now, I = $\int \frac{dx}{a + b\cos x} = \int \frac{dx}{(a-b) + 2b\cos^2 \frac{x}{2}} = \int \frac{\sec^2 \frac{x}{2} dx}{(a+b) + (a-b)\tan^2 \frac{x}{2}}$

put $\tan \frac{x}{2}$ = t ⇒ $\frac{1}{2} \sec^2 \frac{x}{2}$ dx = dt

⇒ I = $\int \frac{2dt}{(a+b) + (a-b)t^2} = \frac{2}{a-b} \int \frac{dt}{t^2 + \left(\frac{a+b}{a-b} \right)}$ ...(1)

$= \frac{2}{a-b} \sqrt{\frac{a-b}{a+b}} \tan^{-1} \left(\sqrt{\frac{a-b}{a+b}}\, t \right) + C = \frac{2}{\sqrt{uv}} \tan^{-1} \left(\sqrt{\frac{v}{u}} \tan \frac{x}{2} \right) + C$

(C) v < 0 ⇒ a – b < 0 ⇒ b – a > 0

Now I = $\frac{2}{b-a}\int \frac{dt}{\frac{a+b}{b-a}-t^2}$ (using equation (1) of part (B

= $\frac{2}{b-a}\,\frac{1}{2}\sqrt{\frac{b-a}{b+a}}\ \ln\left|\frac{\sqrt{\frac{b+a}{b-a}}+t}{\sqrt{\frac{b+a}{b-a}}-t}\right|$ + C = $\frac{1}{\sqrt{-uv}}\ \ln\left|\frac{\sqrt{u}+\sqrt{-v}\tan\frac{x}{2}}{\sqrt{u}-\sqrt{-v}\tan\frac{x}{2}}\right|$ + C

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. Sol. ∵ g'(x) = f(x) here

so $\int f(x)\,dx$ = g(x) + φ(x) ⇒ $\int g'(x)dx$ = g(x) + φ(x) ⇒ g(x) + c = g(x) + φ(x)

⇒ φ(x) = c φ(0) = 1

so φ(x) = 1

∵ g'(x) = f(x) here

$\int f(x)\,dx$ = g(x) + φ(x) ⇒ $\int g'(x)dx$ = g(x) + φ(x) ⇒ g(x) + c = g(x) + φ(x)

⇒ φ(x) = c φ(0) = 1

φ(x) = 1

C-2. Sol. Since $\int (2x-1)dx$ = x^2 – x + c

so, let x^2 – x = t ⇒ (2x – 1)dx = dt

Now I = $\int \frac{(2x-1)}{(x^2-x)^2-(x^2-x)+1}dx$ = $\int \frac{1}{t^2-t+1}dt$

= $\int \frac{1}{\left(t-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dt$ = $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2(x^2-x)-1}{\sqrt{3}}\right)+C$ = $-\frac{2}{\sqrt{3}}\cot^{-1}\left(\frac{2x^2-2x-1}{\sqrt{3}}\right)+C$

C-3. Sol. I = $2\int \frac{\sin x\sec^3 x}{1+\tan^4 x}dx$ = $2\int \frac{\tan x\sec^2 x}{1+\tan^4 x}dx$

Now put $\tan^2 x$ = t

$2\tan x \sec^2 x\,dx = dt$

I = $\int \frac{1}{1+t^2}dt$ = $-\cot^{-1}(t) + C = -\cot^{-1}(\tan^2 x) + C$

a = –1, b = 1