Exercise-2

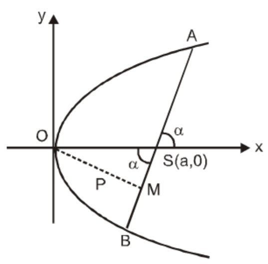
Marked Questions may have for Revision Questions.
*** Marked Questions may have more than one correct option.**

PART - I : OBJECTIVE QUESTIONS

PARABOLA

1. **Sol.** Given equation
$x^2 + 2y - 3x + 5 = 0$
$x^2 - 3x + 5 = -2y$
$x^2 - 3x + \frac{9}{4} + 5 - \frac{9}{4} = -2y$
$\left(x - \frac{3}{2}\right)^2 = -2y - \frac{11}{4}$
$\left(x - \frac{3}{2}\right)^2 = -2\left(y + \frac{11}{8}\right)$
$X^2 = -4AY$
Hence vertex $\equiv \left(\frac{3}{2}, -\frac{11}{8}\right)$
Axis $x - \frac{3}{2} = 0 \Rightarrow x = \frac{3}{2}$
Focus $x - \frac{3}{2} = 0,$ $y + \frac{11}{8} = -\frac{1}{2}$
$y = -\frac{15}{8}$
$\therefore$ Focus $\left(\frac{3}{2}, -\frac{15}{8}\right)$

2. ***Sol.*** *Distance of focal chord from (0, 0) is p*
equation of chord ; $2x - (t_1 + t_2)y + 2a\,t_1 t_2 = 0$
$2x - (t_1 + t_2)y - 2a = 0$ ...... (i)
so perpendicular length from (0, 0)



$\left|\frac{2a}{\sqrt{4 + \left(t_1 - \frac{1}{t_1}\right)^2}}\right| = p \Rightarrow \left|\frac{2a}{\left(t_1 + \frac{1}{t_1}\right)}\right| = p \Rightarrow \left(t_1 + \frac{1}{t_1}\right) = \frac{2a}{p}$

Now length of focal chord is $= a\left(t_1 + \frac{1}{t}\right)^2$
$= a\,\frac{4a^2}{p^2} = \frac{4a^3}{p^2}$

3. **Sol.** $\ell x + my = -n$
$\frac{\ell x + my}{-n} = 1$

Homogenizing, we get $y^2 = 4ax\left(\frac{\ell x + my}{-n}\right)$

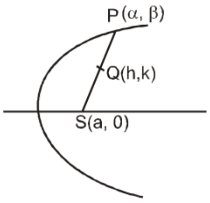
or $4a\ell x^2 + ny^2 + 4amxy = 0$

If the lines of above pair are at right angles, we must have
coeff. of x^2 + coeff. of $y^2 = 0$

$\therefore\ 4a\ell + n = 0$

4. **Sol.** $\because$ From property of parabola, tangents at the extremities of a focal chord of a parabola intersect on the directrix.

5. **Sol.**



$h = \frac{a+\alpha}{2}$, $k = \frac{\beta}{2}$

$\Rightarrow \alpha = 2h - a,\ \beta = 2k$

(α, β) satisfies the parabola

$\therefore\ \beta^2 = 4a\alpha$

$4k^2 = 4a(2h - a)$

$y^2 = a(2x - a)$

$y^2 = 2a\left(x - \frac{a}{2}\right)$

6. **Sol.** ,

a > 0, b > 0 a < 0 , b < 0

$\therefore$ ab > 0

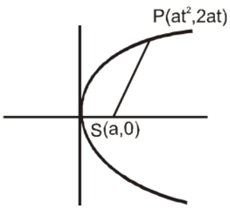
7. **Sol.** Points of intersection of $x^2 = 4y$ and $y^2 = 4x$ are (0, 0) and (4, 4)

$\Rightarrow$ Length of common chord = $4\sqrt{2}$

8. **Sol.** Point of intersection of parabola $y^2 = 4x$ and $x^2 + y^2 = 5$ are (1, 2) and (1, – 2)

$\Rightarrow$ length of common chord = 4

9. **Sol.**



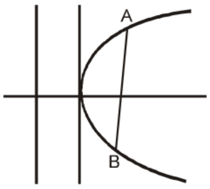
Equation of circle with PS as diameter is

$(x - a)(x - at^2) + y(y - 2at) = 0$

Since length of perpendicular from centre of circle.

$\left(\frac{a(1+t^2)}{2}, at\right)$ to the line x = 0 (tangent at vertex) is = $\frac{a(1+t^2)}{2}$
which is equal to radius of circle
so the tangent at vertex touches to circle.

10. **Sol.**



From property of parabola, a circle described on any focal chord of the parabola as its diameter will touch the directrix of the parabola.

ELLIPSE

11. **Sol.** e = $\sqrt{1-\frac{20}{36}} = \frac{2}{3}$ ⇒ $\frac{2a}{e} = \frac{2\times 6}{2}\times 3$ = 18

12. **Sol.** 2ae = 10 ⇒ a = 8

and $b^2 = 64\left(1-\frac{25}{64}\right)$ ⇒ $b^2 = 39$

∴ length of latus rectum = $\frac{2b^2}{a} = \frac{39}{4}$

13. **Sol.** $\frac{a}{e} - ae = 8$

$a\left[2-\frac{1}{2}\right] = 8$

$\frac{3}{2}a = 8$ ⇒ a = $\frac{16}{3}$

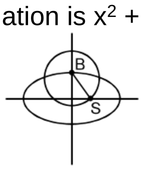
∵ $b^2 = a^2(1 - e^2)$

∴ $b^2 = \left(\frac{16}{3}\right)^2\left(1-\frac{1}{4}\right)$

⇒ $b^2 = \frac{64}{3}$ ⇒ b = $\frac{8}{\sqrt{3}}$

14. **Sol.** By definition of ellipse
∵ b > a ⇒ PS + PS' = 2b = 2 × 4 = 8

15. **Sol.** Centre ≡ (0, 3) focus S ≡ $(\sqrt{7}, 0)$
radius = BS = 4
∴its equation is $x^2 + (y - 3)^2 = 4^2$



16. **Sol.** focal distance = a ± ex_1 = 5 ± $\frac{4}{5}$ y_1

17. **Sol.** By Definition

18. **Sol.** $\frac{x^2}{r^2 - r - 6} + \frac{y^2}{r^2 - 6r + 5} = 1$

(i) $r^2 - r - 6 > 0$

$(r - 3)(r + 2) > 0$

$(-\infty, -2) \cup (3, \infty)$

(ii) $r^2 - 6r + 5 > 0$

$(r - 1)(r - 5) > 0$

$(-\infty, 1) \cup (5, \infty)$

(iii) Since in ellipse $e^2 = 1 - \frac{b^2}{a^2}$

$e < 1$

$\therefore$ $0 < 1 - \frac{r^2 - 6r + 5}{r^2 - r - 6} < 1$

$r \in (5, \infty)$

$\therefore$ for above equation to be ellipse

$\therefore$ $r \in (5, \infty)$

19. **Sol.** $\frac{x^2}{8} + \frac{y^2}{2} = 1$ ...(i)

$y^2 = 4x$ ...(ii)

let m be the slope of common tangent

equation of tangent of slope m to parabola $y^2 = 4x$

$\therefore$ $y = mx + \frac{1}{m}$ ...(iii)

since this is also tangent to ellipse

$\left(\frac{1}{m}\right)^2 = 8m^2 + 2$

$\Rightarrow 1 = 8m^4 + 2m^2 - 1 = 0$

$\Rightarrow 8m^4 + 2m^2 - 1 = 0$

$m^4 + \frac{1}{4}m^2 - \frac{1}{8} = 0$

$\left(m^2 + \frac{1}{2}\right)\left(m^2 - \frac{1}{4}\right) = 0$

$\because$ $m^2 = \frac{1}{4}$ $\therefore$ $m = \pm\frac{1}{2}$

$y = \frac{x}{2} + 2$ $\therefore$ $2y - x = 4$

$y = -\frac{x}{2} - 2$ $\therefore$ $2y + x + 4 = 0$

20. **Sol.** (3, 5) lies out side the ellipse $3x^2 + 5y^2 = 32$ and (3, 5) lies on $25x^2 + 9y^2 = 450$

number of real tangents = 0

21. **Sol.** $\frac{x^2}{16}+\frac{y^2}{25}=1$

e = $\sqrt{1-\frac{16}{25}}=\frac{3}{5}$

be = $\frac{3}{5}\times 5=3$ ⇒ $\frac{2a^2}{b}=\frac{2\times 16}{5}=\frac{32}{5}$

22. **Sol.** $\frac{x^2}{25}+\frac{y^2}{9}$ = 1 reflected ray thus will be line joining $\left(3,\frac{12}{5}\right)$ & (4, 0)

y – 0 = $\frac{-12}{5}$ (x – 4)

5y = – 12x + 48

or line joining the points $\left(3,\frac{-12}{5}\right)$ & (4, 0)

y – 0 = $\frac{12}{5}$ (x – 4) ⇒ 12x – 5y – 48 = 0

23. **Sol.** (1) Here $\frac{x^2}{9}+\frac{y^2}{5}$ = 1

∴ director circle is

$x^2 + y^2 = 9 + 5 = 14$

(2) Here b > a

∴ $SP + S_1P = 2b = 12$

(3) It is lies on tangent at vertex

24. **Sol.** Centre is intersection point of axes

$$\begin{array}{l} x+y-1=0 \\ \underline{x-y+2=0} \\ x=-\frac{1}{2}, y=\frac{3}{2} \end{array}$$

25. **Sol.** Doing partial differentiation

w.r.t .x 10x–2y+8 =0 ⇒ 5x–y+4 = 0 ....(1)

w.r.t .y 10y–2y+8 =0 ⇒ 5y–x+4 = 0 ....(2)

solving (1) & (2) centre is (–1,–1)

HYPERBOLA

26. **Sol.** Eccentricity of ellipse

e = $\sqrt{1-\frac{9}{25}}=\frac{4}{5}$

let the equation of hyperbola be

$\frac{x^2}{\ell^2}-\frac{y^2}{m^2}=1$

distance between foci

$2\ell \cdot e_h = 2\times 5\times\frac{4}{5}$

⇒ $\ell = 2$

$m^2 = \ell^2(4-1)$

$\Rightarrow m^2 = 12$
$\therefore$Equation of hyperbola is
$\frac{x^2}{4} - \frac{y^2}{12} = 1 \Rightarrow 3x^2 - y^2 - 12 = 0$

27. **Sol.** $7x + 13y - 87 = 0$
$5x - 8y + 7 = 0$
On solving we get (5,4)
Now let hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
$\because$ Passes (5,4) $\therefore \frac{25}{a^2} - \frac{16}{b^2} = 1$ ....(i)
Also, $\frac{2b^2}{a} = \frac{32\sqrt{2}}{5}$ ....(ii)
By (i), (ii) we get $a^2 = \frac{25}{2}$, $b^2 = 16$.

28. **Sol.** (1), (2), (3) are hyperbola.
While option (4) is pair of straight lines.

29. **Sol.** Hyperbola can be written in the form $\frac{(x-1)^2}{16} - \frac{(y+1)^2}{9} = 1$
Length of latus rectum = $\frac{2b^2}{a} = 2\times\frac{9}{4} = \frac{9}{2}$

30. **Sol.** Given Hyperbola
$9(x^2 + 2x + 1) - 16(y^2 - 2y + 1)$
$= 151 + 9 - 16$
$\Rightarrow \frac{(x+1)^2}{16} - \frac{(y-1)^2}{9} = 1$
foci (4, 1), (–6, 1)

31. **Sol.** We have, a = 3 and $\frac{b^2}{a} = 4 \Rightarrow b^2 = 12$
Hence, the equation of the hyperbola is $\frac{x^2}{9} - \frac{y^2}{12} = 1 \Rightarrow 4x^2 - 3y^2 = 36$

32. **Sol.** $\frac{x^2}{5} - \frac{y^2}{5\cos^2\alpha} = 1$ $\therefore$ $e^2_H = 1 + \cos^2\alpha$
& $\frac{x^2}{25\cos^2\alpha} + \frac{y^2}{25} = 1$ $\therefore$ $e_E^2 = 1 - \cos^2\alpha$
$e_H^2 = 3e_E^2$ given
$\therefore$ $1 + \cos^2\alpha = 3 - 3\cos^2\alpha$
$\therefore$ $4\cos^2\alpha = 2$
$\therefore$ $\cos^2\alpha = \frac{1}{2}$ $\therefore$ one of the value of α is $\frac{\pi}{4}$

33. **Sol.** Ends of L.R of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ be $\left(\pm\frac{a^2}{b}, be\right)$ $\therefore$ length of L.R. $\frac{2a^2}{b}$

34. **Sol.** $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$ passes through (± ae, 0)

$\therefore$ $\frac{a^2e^2}{b^2} = 1$

$\therefore$ $e^2 = \frac{b^2}{a^2}$

$1 - \frac{b^2}{a^2} = \frac{b^2}{a^2}$

$\therefore$ $\frac{b^2}{a^2} = \frac{1}{2}$

$\therefore$ $e_H = \sqrt{1+\frac{a^2}{b^2}} = \sqrt{3}$

35. **Sol.** $\frac{(x-1)^2}{9} - \frac{(y-2)^2}{16} = 1$

$e = \sqrt{1+\frac{16}{9}} = \frac{5}{3}$

$\therefore$ S (1+ 5, 2) ≡ (6, 2)

36. **Sol.** $h^2 - ab = 4 - 1 = 3 > 0$

37. **Sol.** Let , $A\left(ct_1, \frac{c}{t_1}\right)$, $B\left(ct_2, \frac{c}{t_2}\right)$ $C\left(ct_3, \frac{c}{t_3}\right)$
then orthocentre be
$H\left(\frac{-c}{t_1t_2t_3}, -ct_1t_2t_3\right)$ which lies on $xy = c^2$

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

A-1. **Ans.** **(3)**
Sol. STATEMENT-1 is false **[Parabola, E]**
since here $t^2 = 4$
$\therefore$ the normal subtends a right angle at the focus (not on the vertex)
STATEMENT-2 true (A standared result)

A-2. **Ans.** **(3)**
Sol. Standard result

A-3. **Ans.** **(3)**

Sol. **Statement-1 :** $a = 3$, $b = 2$

$a^2m^2 - b^2 = 9 \cdot \frac{4}{25} - 4 < 0$

$\therefore$ no tangent is possible

$\therefore$ **Statement-1** is false.

Statement-2 is obviously true.

Section (B) : MATCH THE COLUMN

B-1. **Sol.** (P) $y = mx \pm \sqrt{a^2m^2 - b^2}$

$y = x \pm \sqrt{5 - b^2}$

$\therefore$ $b = 0, \pm1, \pm2$

b can not be zero

$\therefore$four values are possible

(Q) We have, $a = 3$ and $\frac{b^2}{a} = 4 \Rightarrow b^2 = 12$

Hence, the equation of the hyperbola is $\frac{x^2}{9} - \frac{y^2}{12} = 1 \Rightarrow 4x^2 - 3y^2 = 36$

(R) The product of the lengths of the perpendiculars from the two focii on any tangent to the hyperbola $\frac{x^2}{25} - \frac{y^2}{3} = 1$ is 3

$\therefore 3 = , \sqrt{k}$ hence $k = 9$

(S) Equation of the hyperbola can be written as $\frac{X^2}{5^2} - \frac{Y^2}{4^2} = 1$

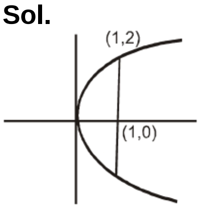
where $X = x - 3$ and $Y = y - 2$.

$\therefore$tangent $Y = X \pm \sqrt{25 - 16}$

$\Rightarrow y = x + 2$ or $y = x - 4$

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. **Sol.**



$y^2 = 4x$, the other end of focal chord will be $(1, -2)$ and this satisfy options (A) (B) & (D)

C-2. **Sol.** $3(x - 3)^2 + 4 (y + 2)^2 = C$

if $C = 0$ a point

if $C > 0$ ellipse

if $C < 0$ no locus.

C-3. **Sol.** $2ae = \frac{2b^2}{a} \Rightarrow a^2e = b^2 \Rightarrow e = \frac{b^2}{a^2} = 1 - e^2 \Rightarrow e^2 + e - 1 = 0$

$e = \frac{-1 \pm \sqrt{5}}{2}$ ($\because$ $0 < e < 1$) $\Rightarrow e = \frac{\sqrt{5} - 1}{2}$

C-4. Sol. Given Hyperbola $9(x^2 + 2x + 1) - 16(y^2 - 2y + 1)$

$= 151 + 9 - 16 \Rightarrow \frac{(x+1)^2}{16} - \frac{(y-1)^2}{9} = 1 \Rightarrow$ foci (4, 1), (–6, 1)