Exercise-3

✍ **Marked Questions may have for Revision Questions.**

*** Marked Questions may have more than one correct option.**

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. If α is a root of $25 \cos^2 \theta + 5 \cos \theta - 12 = 0$, $\frac{\pi}{2} < \alpha < \pi$, then $\sin 2\alpha$ is equal to **[AIEEE-2002,(3, –1), 225]**

(1) $\frac{24}{25}$ (2) $-\frac{24}{25}$ (3) $\frac{13}{18}$ (4) $-\frac{13}{18}$

2. The equation $a \sin x + b \cos x = c$, where $|c| > \sqrt{a^2+b^2}$ has **[AIEEE-2002,(3, –1), 225]**

(1) a unique solution (2) infinite number of solutions
(3) no solution (4) None of the above

3. If $y = \sin^2 \theta + \text{cosec}^2 \theta$, $\theta \neq 0$, then **[AIEEE-2002,(3, –1), 225]**

(1) $y = 0$ (2) $y \leq 2$ (3) $y \geq -2$ (4) $y \geq 2$

4. If $\sin(\alpha + \beta) = 1$, $\sin(\alpha - \beta) = \frac{1}{2}$ then $\tan(\alpha + 2\beta) \tan(2\alpha + \beta)$ is equal to **[AIEEE-2002,(3, –1), 225]**

(1) 1 (2) –1 (3) 0 (4) None of these

5. If $\tan\theta = -\frac{4}{3}$ then $\sin \theta$ is **[AIEEE-2002,(3, –1), 225]**

(1) $-\frac{4}{5}$ but not $\frac{4}{5}$ (2) $-\frac{4}{5}$ or $\frac{4}{5}$
(3) $\frac{4}{5}$ but not $-\frac{4}{5}$ (4) None of these

6. The value of $\frac{1-\tan^2 15^\circ}{1+\tan^2 15^\circ}$ is **[AIEEE-2002,(3, –1), 225]**

(1) 1 (2) $\sqrt{3}$ (3) $\frac{\sqrt{3}}{2}$ (4) 2

7. $\sin^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if **[AIEEE-2002,(3, –1), 225]**

(1) $x - y \neq 0$ (2) $x = y, x \neq 0$ (3) $x = y$ (4) $x \neq 0, y \neq 0$

8. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$, then the difference between the maximum and minimum values of u^2 is given by **[AIEEE-2004,(3, –1), 225]**

(1) $2(a^2+b^2)$ (2) $2\sqrt{a^2+b^2}$ (3) $(a + b)^2$ (4) $(a - b)^2$

9. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin \alpha + \sin \beta = -\frac{21}{65}$ and $\cos \alpha + \cos \beta = -\frac{27}{65}$, then the value of $\cos\left(\frac{\alpha-\beta}{2}\right)$ is **[AIEEE-2004,(3, –1), 225]**

(1) $\frac{-3}{\sqrt{130}}$ (2) $\frac{3}{\sqrt{130}}$ (3) $\frac{6}{65}$ (4) $\frac{-6}{65}$

10.✍ If the roots of the quadratic equation $x^2 + px + q = 0$ are tan 30° and tan 15° respectively, then the value of 2 + q – p is : **[AIEEE-2006 (3, – 1), 120]**
(1) 3 (2) 0 (3) 1 (4) 2

11. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then tan x is **[AIEEE-2006 (3, – 1), 120]**
(1) $\frac{4-\sqrt{7}}{3}$ (2) $-\left(\frac{4+\sqrt{7}}{3}\right)$ (3) $\frac{1+\sqrt{7}}{4}$ (4) $\frac{1-\sqrt{7}}{4}$

12. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is **[AIEEE 2006 (3, – 1), 120]**
(1) 6 (2) 1 (3) 2 (4) 4

13. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB = (a) subtends an angle of 60º at the foot of the tower and the angle of elevation of the top of the tower from A or B is 30º. The height of the tower is- **[AIEEE 2007 (3, –1), 120]**
(1) $\frac{2a}{\sqrt{3}}$ (2) $2a\sqrt{3}$ (3) $\frac{a}{\sqrt{3}}$ (4) $\sqrt{3}$

14. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60º. He moves away from the pole along the line BC to a point D such that CD = 7 m. From D the angle of elevation of the point A is 45º. Then the height of the pole is- **[AIEEE 2008 (3, –1), 105]**
(1) $\frac{7\sqrt{3}}{2}\left(\frac{1}{\sqrt{3}+1}\right)$ m (2) $\frac{7\sqrt{3}}{2}\left(\frac{1}{\sqrt{3}-1}\right)$ m (3) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)$ m (4) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)$ m

15.✍ Let A and B denote the statements **[AIEEE 2009 (4, –1), 144]**
A : $\cos\alpha + \cos\beta + \cos\gamma = 0$
B : $\sin\alpha + \sin\beta + \sin\gamma = 0$
If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$, then :
(1) A is false and B is true (2) both A and B are true
(3) both A and B are false (4) A is true and B is false

16.✍ Let $\cos(\alpha+\beta) = \frac{4}{5}$ and let $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha$ = **[AIEEE 2010 (4, –1), 144]**
(1) $\frac{56}{33}$ (2) $\frac{19}{12}$ (3) $\frac{20}{7}$ (4) $\frac{25}{16}$

17.✍ For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A **false** statement among the following is **[AIEEE 2010 (4, –1), 144]**
(1) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$. (2) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$.
(3) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$. (4) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$.

18. The possible values of $\theta \in (0,\pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are :**[AIEEE 2011 (4, –1), 120]**
(1) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$ (2) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$

(3) $\frac{2\pi}{9},\frac{\pi}{4},\frac{\pi}{2},\frac{2\pi}{3},\frac{3\pi}{4},\frac{8\pi}{9}$ (4) $\frac{2\pi}{9},\frac{\pi}{4},\frac{4\pi}{9},\frac{\pi}{2},\frac{3\pi}{4},\frac{8\pi}{9}$

19. If A = $\sin^2 x + \cos^4 x$, then for all real x : **[AIEEE 2011 (4, –1), 120]**

(1) $\frac{3}{4} \le A \le 1$ (2) $\frac{13}{16} \le A \le 1$ (3) $1 \le A \le 2$ (4) $\frac{3}{4} \le A \le \frac{13}{16}$

20.✍ The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has : **[AIEEE-2012, (4, –1)/120]**

(1) infinite number of real roots (2) no real roots
(3) exactly one real root (4) exactly four real roots

21.✍ In a ΔPQR, if 3 sin P + 4 cos Q = 6 and 4 sin Q + 3 cos P = 1, then the angle R is equal to :
[AIEEE-2012, (4, –1)/120]

(1) $\frac{5\pi}{6}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$

22.✍ ABCD is a trapezium such that AB and CD are parallel and BC ⊥ CD. If ∠ADB = θ , BC = p and CD = q, then AB is equal to : **[AIEEE - 2013, (4, – 1) 120]**

(1) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$ (2) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$ (3) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$ (4) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$

23. The expression $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$ can be written as : **[AIEEE - 2013, (4, –1),360]**

(1) sinA cosA + 1 (2) secA cosecA + 1 (3) tanA + cotA (4) secA + cosecA

24. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where x ∈ R and k ≥ 1. Then $f_4(x) - f_6(x)$ equals
[JEE(Main) 2014, (4, – 1), 120]

(1) $\frac{1}{4}$ (2) $\frac{1}{12}$ (3) $\frac{1}{6}$ (4) $\frac{1}{3}$

25. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30º, 45º and 60º respectively, then the ratio, AB : BC , is
[JEE(Main) 2015, (4, – 1), 120]

(1) $\sqrt{3}:1$ (2) $\sqrt{3}:\sqrt{2}$ (3) $1:\sqrt{3}$ (4) 2 : 3

26. If 0 ≤ x < 2π, then the number of real values of x, which satisfy the equation
cosx + cos2x + cos3x + cos4x = 0, is **[JEE(Main) 2016, (4, – 1), 120]**

(1) 5 (2) 7 (3) 9 (4) 3

27. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°, After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is :
[JEE(Main) 2016, (4, – 1), 120]

(1) 10 (2) 20 (3) 5 (4) 6

28. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of cos4x is : **[JEE(Main) 2016, (4, – 1), 120]**

(1) $\frac{-3}{5}$ (2) $\frac{1}{3}$ (3) $\frac{2}{9}$ (4) $-\frac{7}{9}$

29. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If ∠BPC = β, then tanβ is equal to

[JEE(Main) 2017, (4, – 1), 120]

(1) $\frac{6}{7}$ (2) $\frac{1}{4}$ (3) $\frac{2}{9}$ (4) $\frac{4}{9}$

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. The number of integral values of 'k' for which the equation $7 \cos x + 5 \sin x = 2k + 1$ has a solution is:
[IIT-JEE-2002, Scr., (3,–1)/90]
(A) 4 (B) 8 (C) 10 (D) 12

2. If $\sin \alpha = 1/2$ and $\cos \theta = 1/3$, then the values of $\alpha + \theta$ (if θ, α are both acute) will lie in the interval
[IIT-JEE-2004, Scr., (3,–1)/84]
(A) $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$ (B) $\left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$ (C) $\left[\frac{2\pi}{3}, \frac{5\pi}{6}\right]$ (D) $\left[\frac{5\pi}{6}, \pi\right]$

3. Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan \theta)^{\tan \theta}$, $t_2 = (\tan \theta)^{\cot \theta}$, $t_3 = (\cot \theta)^{\tan \theta}$ and $t_4 = (\cot \theta)^{\cot \theta}$, then
[IIT-JEE - 2006 , Main - (3, –1), 184]
(A) $t_1 > t_2 > t_3 > t_4$ (B) $t_2 < t_1 < t_3 < t_4$ (C) $t_3 > t_1 > t_2 > t_4$ (D) $t_2 > t_3 > t_1 > t_4$

4. If $0 < \theta < 2\pi$, then the intervals of values of θ for which $2 \sin^2\theta - 5 \sin\theta + 2 > 0$, is
[IIT-JEE-2006, Stage-1, (3,–1)/84]
(A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (B) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$
(C) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (D) $\left(\frac{41\pi}{48}, \pi\right)$

5. The number of solutions of the pair of equations $2 \sin^2\theta - \cos 2\theta = 0$, $2 \cos^2\theta - 3 \sin \theta = 0$ in the interval $[0, 2\pi]$ is **[IIT-JEE-2007, Paper-1, (3,–1)/81]**
(A) zero (B) one (C) two (D) four

6. Let $P = \{\theta : \sin \theta - \cos \theta = \sqrt{2} \cos \theta\}$ and $Q = \{\theta : \sin \theta + \cos \theta = \sqrt{2} \sin \theta\}$ be two sets. Then
(A) $P \subset Q$ and $Q - P \neq \varnothing$ (B) $Q \not\subset P$
(C) $P \not\subset Q$ (D) $P = Q$ **[IIT-JEE 2011, Paper-1, (3, –1), 80]**

7. The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right) \sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$ is equal to
[JEE (Advanced) 2016, Paper-2, (3, –1)/62]
(A) $3 - \sqrt{3}$ (B) $2\left(3 - \sqrt{3}\right)$ (C) $2\left(\sqrt{3} - 1\right)$ (D) $2\left(2 + \sqrt{3}\right)$

8. Let S = $\left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3} \sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to
[JEE (Advanced) 2016, Paper-1, (3,–1)/62]
(A) $-\frac{7\pi}{9}$ (B) $-\frac{2\pi}{9}$ (C) 0 (D) $\frac{5\pi}{9}$

9. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x \sec \theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x \tan \theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals
(A) $2(\sec\theta - \tan \theta)$ (B) $2\sec \theta$ (C) $-2\tan \theta$ (D) 0

10. Let α and β be nonzero real numbers such that $2(\cos \beta - \cos \alpha) + \cos \alpha \cos \beta = 1$. Then which of the following is/are true? **[JEE(Advanced) 2017, Paper-2,(4, –2)/61]**

(A) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$ (B) $\tan\left(\frac{\alpha}{2}\right) - \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$

(C) $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$ (D) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$

Answers

EXERCISE # 1

Section (A) :

A-1. (3) **A-2.** (3) **A-3.** (2) **A-4.** (1) **A-5.** (3) **A-6.** (2) **A-7.** (4)
A-8. (1) **A-9.** (3) **A-10.** (2) **A-11.** (4) **A-12.** (2) **A-13.** (3) **A-14.** (1)
A-15. (2) **A-16.** (1)

Section (B) :

B-1. (4) **B-2.** (1) **B-3.** (2) **B-4.** (3) **B-5.** (4) **B-6.** (3) **B-7.** (3)
B-8. (2) **B-9.** (4) **B-10.** (4) **B-11.** (3) **B-12.** (2)

Section (C) :

C-1. (1) **C-2.** (1) **C-3.** (3) **C-4.** (3) **C-5.** (2) **C-6.** (2) **C-7.** (2)
C-8. (2) **C-9.** (2)

Section (D) :

D-1. (2) **D-2.** (1) **D-3.** (2) **D-4.** (2) **D-5.** (4) **D-6.** (3)

Section (E) :

E-1. (4) **E-2.** (4) **E-3.** (2) **E-4.** (4) **E-5.** (2) **E-6.** (2) **E-7.** (1)
E-8. (1) **E-9.** (1)

Section (F) :

F-1. (2) **F-2.** (4) **F-3.** (2) **F-4.** (3) **F-5.** (4) **F-6.** (2) **F-7.** (4)
F-8. (4) **F-9.** (2) **F-10.** (4) **F-11.** (3) **F-12.** (3)

Section (G) :

G-1. (1) **G-2.** (3) **G-3.** (3) **G-4.** (3) **G-5.** (3) **G-6.** (3) **G-7.** (1)
G-8. (2)

Section (H) :

H-1. (2) **H-2.** (2) **H-3.** (4)

Section (I) :

I-1. (1) **I-2.** (2) **I-3.** (2) **I-4.** (1) **I-5.** (4)

EXERCISE # 2

PART - I

1.	(2)	**2.**	(2)	**3.**	(4)	**4.**	(2)	**5.**	(1)	**6.**	(2)	**7.**	(1)
8.	(2)	**9.**	(4)	**10.**	(1)	**11.**	(1)	**12.**	(3)	**13.**	(1)	**14.**	(4)
15.	(3)	**16.**	(4)	**17.**	(1)	**18.**	(2)	**19.**	(3)	**20.**	(4)	**21.**	(1)
22.	(3)	**23.**	(4)	**24**	(2)	**25.**	(1)	**26.**	(4)	**27.**	(2)		

PART - II

Section (A) :

A-1. (1) **A-2.** (1) **A-3.** (2) **A-4.** (3)

Section (B) :

B-1. (A) → (s), (B) → (q), (C) → (p), (D) → (r)

B-2. (A) → (q) (B) → (p), (C) → (r), (D) → (s)

Section (C) :

C-1. (2,4) **C-2.** (1,2) **C-3.** (1,2) **C-4.** (2,4) **C-5.** (1,2,3,4)

C-6. (2,4) **C-7.** (1,3,4) **C-8.** (1,2,3,4) **C-9.** (1,3,4) **C-10.** (1,2,3)

EXERCISE # 3

PART - I

1.	(2)	**2.**	(3)	**3.**	(4)	**4.**	(1)	**5.**	(2)	**6.**	(3)	**7.**	(3)
8.	(4)	**9.**	(1)	**10.**	(1)	**11.**	(2)	**12.**	(4)	**13.**	(3)	**14.**	(3)
15.	(2)	**16.**	(1)	**17.**	(2)	**18.**	(4)	**19.**	(1)	**20.**	(2)	**21.**	(2)
22.	(1)	**23.**	(2)	**24.**	(2)	**25.**	(1)	**26.**	(2)	**27.**	(3)	**28.**	(4)
29.	(3)												

PART - II

1.	(B)	**2.**	(B)	**3.**	(B)	**4.**	(A)	**5.**	(C)	**6.**	(D)	**7.**	(C)
8.	(C)	**9.**	(C)	**10.**	(BC)								