Exercise-3

1. Sol. $\because$ α is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$.

$\therefore$ $25\cos^2\alpha + 5\cos\alpha - 12 = 0$

$\Rightarrow$ $\cos\alpha = -\frac{4}{5}, \frac{3}{5}$ But $\frac{\pi}{2} < \alpha < \pi$ (IInd quadrant)

$\therefore$ $\cos\alpha = \frac{-4}{5}$ and $\sin\alpha = \frac{3}{5}$

$\Rightarrow$ $\sin 2\alpha = 2\sin\alpha\cos\alpha = -\frac{24}{25}$

2. Sol. $\because$ $|c| > \sqrt{a^2+b^2}$

$\Rightarrow$ $c < -\sqrt{a^2+b^2}$ & $c > \sqrt{a^2+b^2}$

But $-\sqrt{a^2+b^2} \le a\sin x + b\cos x \le \sqrt{a^2+b^2}$ .... (i)

& $a\sin x + b\cos x = c$ .... (ii)

$\therefore$ from (i) & (ii)

no slution

3. Sol. $y = \sin^2\theta + \operatorname{cosec}^2\theta$

$= (\sin\theta - \operatorname{cosec}\theta)^2 + 2$

$\Rightarrow$ $y \ge 2$, $\theta \ne 0$

4. Sol. $\sin(\alpha+\beta) = 1$, $\Rightarrow$ $\alpha + \beta = \frac{\pi}{2}$ ..... (i)

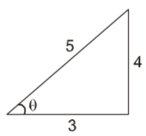
$\sin(\alpha-\beta) = \frac{1}{2}$, $\Rightarrow$ $\alpha - \beta = \frac{\pi}{6}$ ..... (ii)

on solving (i) & (ii)

$\alpha = \frac{\pi}{3}$, $\beta = \frac{\pi}{6}$

$\therefore$ $\tan(\alpha+2\beta).\tan(2\alpha+\beta) = \tan\left(\frac{2\pi}{3}\right)\tan\left(\frac{5\pi}{6}\right)$

5. Sol. $\tan\theta = -\frac{4}{5}$ $\Rightarrow$ $\theta \in$ IInd or IVth quadrant



$\therefore$ $\sin\theta = \frac{4}{5}$ or $-\frac{4}{5}$

6. Sol. $\frac{1-\tan^2 15^\circ}{1+\tan^2 15^\circ} = \cos 30^\circ = \frac{\sqrt{3}}{2}$

7. **Sol.** $\because$ $\sin^2\theta \le 1$

$\therefore$ $\frac{4xy}{(x+y)^2} \le 1$ $\Rightarrow$ $x^2 + y^2 + 2xy - 4xy \ge 0$

$\Rightarrow$ $(x - y)^2 \ge 0$

which is true for all real values of x & y

provided $x + y \ne 0$, otherwise $\frac{4xy}{(x+y)^2}$ will be meanigless.

8. **Sol.** $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$

$\Rightarrow$ $u^2 = a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta + 2\sqrt{a^2\cos^2\theta + b^2\sin^2\theta} \times \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$

$\Rightarrow$ $u^2 = (a^2 + b^2) + 2\sqrt{\{a^2 + (b^2 - a^2)\sin^2\theta\} \times \{a^2 + (b^2 - a^2)\cos^2\theta\}}$

$\Rightarrow$ $u^2 = (a^2 + b^2) + 2\sqrt{a^4 + a^2(b^2 - a^2) + (b^2 - a^2)^2 \sin^2\theta\cos^2\theta}$

$\Rightarrow$ $u^2 = (a^2 + b^2) + 2\sqrt{a^2b^2 + \left(\frac{b^2 - a^2}{2}\right)^2 \sin^2 2\theta}$.

$\therefore$ $\min(u^2) = a^2 + b^2 + 2ab = (a+b)^2$

and $\max(u^2) = a^2 + b^2 + (a^2 + b^2) = 2(a^2 + b^2)$

Now, $\max(u^2) - \min(u^2) = (a - b)^2$

9. **Sol.** $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$

squaring and adding, we get

$\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \cos^2\alpha + \cos^2\beta + 2\cos\alpha \cdot \cos\beta$

$= \left(-\frac{21}{65}\right)^2 + \left(-\frac{27}{65}\right)^2$

$\Rightarrow$ $2 + 2\cos(\alpha - \beta) = \frac{1170}{4225}$

$\Rightarrow$ $\cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1170}{4 \times 4225} = \frac{9}{130}$

$\Rightarrow$ $\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{-3}{\sqrt{130}}$ $\left(\because \pi < \alpha - \beta < 3\pi \Rightarrow \frac{\pi}{2} < \left(\frac{\alpha-\beta}{2}\right) < \frac{3\pi}{2}\right)$

10. **Sol.** Since, tan 30º and tan 15º are the roots of equation $x^2 + px + q = 0$.

$\therefore$ tan 30º + tan 15º = – p and tan 30º tan 15º = q

Therefore, 2 + q – p = 2 + tan 30º tan 15º + (tan 30º + tan 15º)

$\Rightarrow$ 2 + q – p = 2 + tan 30º tan 15º + 1 – tan 30º . tan 15º $\left(\because \tan 45^\circ = \frac{\tan 30^\circ + \tan 15^\circ}{1 - \tan 30^\circ \tan 15^\circ}\right)$

$\Rightarrow$ 2 + q – p = 3

11. **Sol.** $\cos x + \sin x = \frac{1}{2}$

$\therefore \frac{1-\tan^2 x/2}{1+\tan^2 x/2} + \frac{2\tan x/2}{1+\tan^2 x/2} = \frac{1}{2}$, Let $\tan\frac{x}{2} = t$

$\frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2} = \frac{1}{2} \Rightarrow 3t^2 - 4t - 1 = 0$

$\therefore\ t = \frac{2 \pm \sqrt{7}}{3}$

as $0 < x < \pi \Rightarrow 0 < \frac{x}{2} < \frac{\pi}{2}$

$\therefore \tan\frac{x}{2}$ is positive

$\therefore t = \tan\frac{x}{2} = \frac{2+\sqrt{7}}{3}$

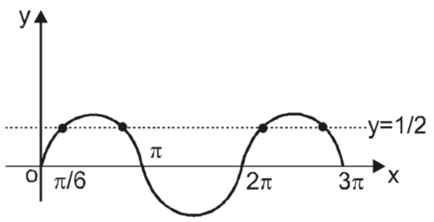
Now $\tan x = \frac{2\tan x/2}{1-\tan^2 x/2} = \frac{2t}{1-t^2}$

$\Rightarrow\ \tan x = \frac{2\left(\frac{2+\sqrt{7}}{3}\right)}{1-\left(\frac{2+\sqrt{7}}{3}\right)^2} = -\left(\frac{4+\sqrt{7}}{3}\right)$

12. **Sol.** Given equation is $2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow (2\sin x - 1)(\sin x + 3) = 0$

$\Rightarrow \sin x = \frac{1}{2}$ $(\because\ \sin x \neq -3)$



It is clear from figure that the curve intersect the line at four points in the given interval.
Hence, number of solutions are 4.

13. **Sol.** Let h be the height of a tower,

$\angle AOB = 60°$

$\therefore$ ΔOAB is equilateral

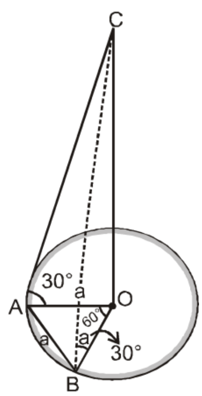
$\therefore$ $OA = OB = AB = a$

Now in ΔOAC,

$\tan 30° = \frac{h}{a}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{a}$

$\Rightarrow h = \frac{a}{\sqrt{3}}$



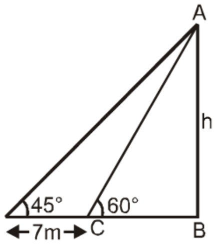
14. **Sol.** In Δ ABC, BC = h cot 60° and in Δ ABD, BD = h cot 45°

Since, BD – BC = DC

⇒ h cot 45° – h cot 60° = 7

∴ h = $\frac{7}{\cos 45° - \cot 60°}$ = $\frac{7}{\left(1-\frac{1}{\sqrt{3}}\right)}$

= $\frac{7\sqrt{3}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$ = $\frac{7\sqrt{3}}{2}\left(\sqrt{3}+1\right)$m



15.

Sol. 2{cos (β – γ) + cos (γ – α) + cos (α – β)} + 3 = 0

(cos α + cos β + cos γ)^2 + (sin α + sin β + sin γ)^2 = 0

∑ cos α = 0 = ∑ sin α

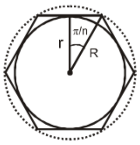
16. Ans. (1)

Sol. tan 2α = tan ((α + β) + (α – β)) = $\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$ = $\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}}$ = $\frac{(9+5)4}{48-15}$ = $\frac{14\times 4}{33}$ = $\frac{56}{33}$

Hence correct option is (1)

17. Sol. $\frac{r}{R}$ = cos $\left(\frac{\pi}{n}\right)$

Let cos $\frac{\pi}{n}$ = $\frac{2}{3}$ for some n ≥ 3, n ∈ N



As $\frac{1}{2} < \frac{2}{3} < \frac{1}{\sqrt{2}}$ ⇒ cos $\frac{\pi}{3}$ < cos $\frac{\pi}{n}$ < cos $\frac{\pi}{4}$ ⇒ $\frac{\pi}{3} > \frac{\pi}{n} > \frac{\pi}{4}$

⇒ 3 < n < 4, which is not possible

so option (2) is the false statement

so it will be the right choice

Hence correct option is (2)

18. Sol. sin 4θ+ 2sin 4 θ cos 3θ = 0 ∵ θ , ∈ (0, π)

sin 4θ (1 + 2 cos 3 θ) = 0

sin 4θ = 0 or cos 3θ = – $\frac{1}{2}$

4θ = nπ ; n ∈ I or 3θ = 2nπ ± $\frac{2\pi}{3}$, n ∈ I

$\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ or $\theta = \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{4\pi}{9}$

19. Sol. (1)

$A = \sin^2 x + \cos^4 x$

$= \sin^2 x + (1 - \sin^2 x)^2$

$= \sin^4 x - \sin^2 x + 1$

$= \left(\sin^2 x - \frac{1}{2}\right)^2 + \frac{3}{4}$

$= \frac{3}{4} \le A \le 1$

20. Sol. Ans. (2)

Let $e^{\sin x} = t$

$\Rightarrow t^2 - 4t - 1 = 0 \Rightarrow t = \frac{4 \pm \sqrt{16+4}}{2}$

$\Rightarrow t = e^{\sin x} = 2 \pm \sqrt{5} \Rightarrow e^{\sin x} = 2 - \sqrt{5}$,

$e^{\sin x} = 2 + \sqrt{5}$

$e^{\sin x} = 2 - \sqrt{5} < 0$, $\Rightarrow \sin x = ln(2 + \sqrt{5}) > 1$

so rejected so rejected

hence no solution

21. Sol. Ans. (2)

$3\sin P + 4\cos Q = 6$ ...(i)

$4\sin Q + 3\cos P = 1$ ...(ii)

Squaring and adding (i) & (ii) we get $\sin(P + Q) = \frac{1}{2}$

$\Rightarrow P + Q = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$\Rightarrow R = \frac{5\pi}{6}$ or $\frac{\pi}{6}$

If $R = \frac{5\pi}{6}$ then $0 < P, Q < \frac{\pi}{6}$

$\Rightarrow \cos Q < 1$ and $\sin P < \frac{1}{2}$

$\Rightarrow 3\sin P + 4\cos Q < \frac{11}{2}$

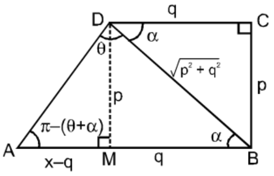
So $R = \frac{\pi}{6}$

Hindi $3\sin P + 4\cos Q = 6$ ...(i)

22. Sol. (1)

Let AB = x

$\tan(\pi - \theta - \alpha) = \frac{p}{x - q} \Rightarrow \tan(\theta + \alpha) = \frac{p}{q - x}$



⇒ $q - x = p\cot(\theta + \alpha)$

⇒ $x = q - p\cot(\theta + \alpha)$

$= q - p\left(\dfrac{\cot\theta\ \cot\alpha - 1}{\cot\alpha + \cot\theta}\right)$

$= q - p\left(\dfrac{\frac{q}{p}\ \cot\theta\ \ -1}{\frac{q}{p}\ \ +\cot\theta}\right) = q - p\left(\dfrac{q\cot\theta\ \ -p}{q + p\cot\theta}\right) = q - p\left(\dfrac{q\cos\theta\ \ -p\sin\theta}{q\sin\theta + p\cos\theta}\right)$

⇒ $x = \dfrac{q^2\ \ \sin\theta + pq\cos\theta - pq\cos\theta + p^2\sin\theta}{p\cos\theta + q\sin\theta}$ ⇒ $AB = \dfrac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$.

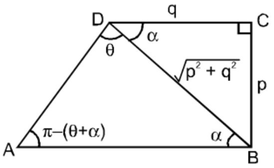
Alternative

From Sine Rule

$\dfrac{AB}{\sin\theta} = \dfrac{\sqrt{p^2 + q^2}}{\sin(\pi - (\theta + \alpha))}$

$AB = \dfrac{\sqrt{p^2 + q^2}\ \ \sin\theta}{\sin\theta\ \ \cos\alpha + \cos\theta\ \ \sin\alpha}$

$= \dfrac{(p^2 + q^2)\ \ \sin\theta}{q\sin\ \ \theta + p\cos\theta}$ $\left(\because\ \cos\alpha = \dfrac{q}{\sqrt{p^2 + q^2}}\right)$



$= \dfrac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$.

23. Sol. (2)

Given expression

$= \dfrac{\sin A}{\cos A} \times \dfrac{\sin A}{\sin A - \cos A} + \dfrac{\cos A}{\sin A} \times \dfrac{\cos A}{\cos A - \sin A}$

$= \dfrac{1}{\sin A - \cos A}\left\{\dfrac{\sin^3 A - \cos^3 A}{\cos A\ \ \sin A}\right\} = \dfrac{\sin^2 A + \sin A\cos A + \cos^2 A}{\sin A\ \ \cos A}$ $= 1 + \sec A \operatorname{cosec} A$

24. Sol. Ans. (2)

$f_k(x) = \dfrac{1}{k}\ (\sin^k x + \cos^k x)$

$f_4 - f_6 = \dfrac{1}{4}(\sin^4 x + \cos^4 x) - \dfrac{1}{6}(\sin^6 x + \cos^6 x) = \dfrac{1}{4}\ (1 - 2\sin^2 x\cos^2 x) - \dfrac{1}{6}\ (1 - 3\sin^2 x\cos^2 x)$

$\dfrac{1}{4} - \dfrac{1}{6} = \dfrac{1}{12}$

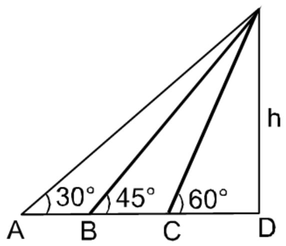
25. **Ans.** **(1)**

Sol. $\tan 30° = \frac{h}{AD} \Rightarrow AD = h\sqrt{3}$

BD = h ; CD = $\frac{h}{\sqrt{3}}$

$\frac{AB}{BC} = \frac{AD - BD}{BD - CD}$

$= \frac{\sqrt{3}-1}{1-\frac{1}{\sqrt{3}}} = \frac{3-\sqrt{3}}{\sqrt{3}-1} = \sqrt{3}$



26. **Ans.** **(2)**

Sol. $0 \le x < 2\pi$

$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$(\cos x + \cos 4x) + (\cos 2x + \cos 3x) = 0$

$2\cos\frac{5x}{2}\cos\frac{3x}{2} + 2\cos\frac{5x}{2}\cos\frac{x}{2} = 0$

$2\cos\frac{5x}{2}\left[2\cos x\cos\frac{x}{2}\right] = 0$

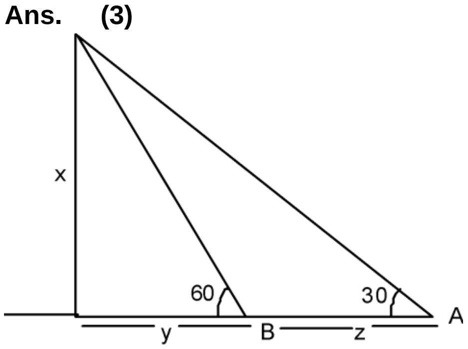
$\cos\frac{5x}{2} = 0$ or $\cos x = 0$ or $\cos\frac{x}{2} = 0$

$x = \frac{(2n+1)\pi}{5}$ or $x = (2n+1)\frac{\pi}{2}$ or $x = (2n+1)\pi$

$x = \left\{\frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$

Number of solution is 7

27. **Ans.** **(3)**



Sol.

$\tan 30° = \frac{x}{y+z} = \frac{1}{\sqrt{3}} \Rightarrow \sqrt{3}x = y + z \Rightarrow$ $\tan 60° = \frac{x}{y} = \sqrt{3} \Rightarrow x = \sqrt{3}y = y + z$

$3y = y + z \Rightarrow 2y = z$

for 2y distance time = 10 min.
so for y dist time = 5 min.

28. **Ans.** **(4)**

Sol. $5(\tan^2x - \cos^2x) = 2\cos2x + 9$

$5\left(\tan^2 x - \frac{1}{1+\tan^2 x}\right) = 2\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right) + 9$

$5(\tan^4x + \tan^2x - 1) = 2 - 2\tan^2x + 9 + 9\tan^2x$

$5\tan^4x - 2\tan^2x - 16 = 0$

$5\tan^4x - 10\tan^2x + 8\tan^2x - 16 = 0$

$5\tan^2x(\tan^2x - 2) + 8(\tan^2x - 2) = 0$

$(5\tan^2x + 8)(\tan^2x - 2) = 0$

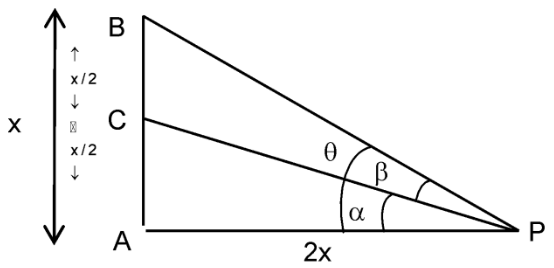
$\tan^2x = 2$

$\cos2x = \frac{1-2}{1+2} = -\frac{1}{3}$

$\cos4x = 2\cos^2 2x - 1 = -\frac{7}{9}$

29. **Ans.** **(3)**

Sol.



$\tan\theta = \frac{1}{2}$, $\tan\alpha = \frac{1}{4}$, $\tan\beta = y$

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

IIT-JEE - 2002

1. **Sol.** The given equation is $7\cos x + 5\sin x = 2k + 1$
Now, let $z = 7\cos x + 5\sin x$

$= \sqrt{74}\left(\frac{7}{\sqrt{74}}\cos x + \frac{5}{\sqrt{74}}\sin x\right)$

$= \sqrt{74}\cos(x-a)$, where a = $\tan^{-1}\left(\frac{5}{7}\right)$

$\Rightarrow -\sqrt{74} \le 2k+1 \le \sqrt{74}$

$\Rightarrow$ – 8 < 2k + 1 < 8 (since k is an integer)

$\Rightarrow -9 < 2k < 7$

Number of possible integral values of k = 8.

IIT-JEE - 2004

2. **Solution**

Clearly θ = 30º and φ∈ (60º, 90º)

Hence θ + φ lies in (90º, 120º).

3. **Sol.** $\theta \in \left(0, \frac{\pi}{4}\right)$

∵ tan θ ↑ in $\theta \in \left(0, \frac{\pi}{4}\right)$ and 0 < tan θ < 1

cot θ ↓ in $\theta \in \left(0, \frac{\pi}{4}\right)$ and cot θ > 1

Let tan θ = 1 – λ_1 and cot θ = 1 + λ_2 where λ_1 and λ_2 are very small and positive, then

$t_1 = (1-\lambda_1)^{1-\lambda_1}$, $t_2 = (1-\lambda_1)^{1+\lambda_2}$, $t_3 = (1+\lambda_2)^{1-\lambda_1}$, $t_4 = (1+\lambda_2)^{(1+\lambda_2)}$

∴ $t_4 > t_3 > t_1 > t_2$

OR

∵ tan θ ↑ in $\theta \in \left(0, \frac{\pi}{4}\right)$ and 0 < tan θ < 1

cot θ ↓ in $\theta \in \left(0, \frac{\pi}{4}\right)$ and cot θ > 1 think only above and conculude result.

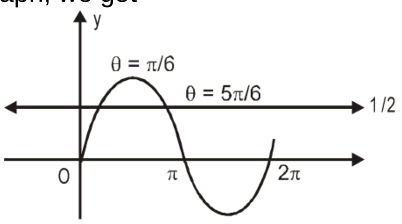
IIT-JEE - 2006

4. **Sol.** $2\sin^2\theta - 5\sin\theta + 2 > 0$

$\Rightarrow$ (sinθ – 2)(2sinθ –1) > 0

$\Rightarrow \sin\theta < \frac{1}{2}$ [∵ –1≤ sin θ ≤1]

From graph, we get $\theta \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$



IIT-JEE - 2007

5. **So.**

$2\sin^2\theta - \cos 2\theta = 0$ ....(i)

$\Rightarrow \sin\theta = \pm\frac{1}{2}$

$2\cos^2\theta - 3\sin\theta = 0$ ....(ii)

$-2\sin^2\theta - 3\sin\theta + 2 = 0$

$\sin\theta = \frac{1}{2}, \ -2$

So $\sin\theta = \frac{1}{2}$ is the only solution

at $\theta = \frac{\pi}{6}, \ \frac{5\pi}{6}$

6. Ans. (D)

Sol. $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$

$\sin\theta = (\sqrt{2} + 1)\cos\theta \Rightarrow \tan\theta = \sqrt{2} + 1 \Rightarrow \theta = n\pi + \frac{3\pi}{8}$; $n \in I$

$Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$

$\therefore \cos\theta = (\sqrt{2} - 1)\sin\theta \Rightarrow \tan\theta = \frac{1}{\sqrt{2}-1} = \sqrt{2} + 1 \Rightarrow \theta = n\pi + \frac{3\pi}{8}$; $n \in I$

$\therefore P = Q$

7. Ans. (C)

Sol. $\sum_{k=1}^{13} \frac{\sin\left[\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)-\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)\right]}{\sin\frac{\pi}{6}\left(\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)\sin\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)\right)} = 2\sum_{k=1}^{13}\left(\cot\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)\right)$

$= 2\left(\cot\frac{\pi}{4}-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)\right) = 2\left(1-\cot\left(\frac{29\pi}{12}\right)\right) = 2\left(1-\cot\left(\frac{5\pi}{12}\right)\right) = 2(1-(2-\sqrt{3})) = 2(-1+\sqrt{3})$

$= 2(\sqrt{3}-1)$

8. Ans. (C)

Sol. $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0 \Rightarrow \sqrt{3}\sin x + \cos x + 2(\sin^2 x - \cos^2 x) = 0$

$\sqrt{3}\sin x + \cos x - 2\cos 2x = 0 \Rightarrow \sin\left(x+\frac{\pi}{3}\right) = \cos 2x$

$\cos(\pi/3 - x) = \cos 2x \Rightarrow 2x = 2n\pi \pm (\pi/3 - x)$

$x = \frac{2n\pi}{3} + \frac{\pi}{9}$ or $x = 2n\pi - \frac{\pi}{3}$.

$-100^\circ - 60^\circ + 20^\circ + 140^\circ = 0$

9. Ans. (C)

Sol. $x^2 - 2x\sec\theta + 1 = 0$

$\Rightarrow x = \frac{2\sec\theta \pm \sqrt{4\sec^2\theta - 4}}{2} \Rightarrow x = \sec\theta + \tan\theta, \ \sec\theta - \tan\theta \Rightarrow \alpha_1 = \sec\theta - \tan\theta$

now $x^2 + 2x\tan\theta - 1 = 0 \Rightarrow x = \frac{-2\tan\theta \pm \sqrt{4\tan^2\theta + 4}}{2}$

$\Rightarrow x = -\tan\theta \pm \sec\theta \quad \Rightarrow \quad \alpha_2 = (\sec\theta - \tan\theta) \quad \Rightarrow \quad \beta_2 = -(\sec\theta + \tan\theta)$

$\therefore \quad \alpha_1 + \beta_2 = -2\tan\theta$

Alt : (i) $x^2 - 2x\sec\theta + 1 = 0$

$x = \dfrac{2\sec\theta \pm \sqrt{4\sec^2\theta - 4}}{2} = \sec\theta \pm \tan\theta$

$\alpha_1 = \sec\theta - \tan\theta \qquad \beta_1 = \sec\theta + \tan\theta$

(ii) $x^2 + 2x\tan\theta - 1 = 0 \Rightarrow x = \dfrac{-\tan\theta \pm \sqrt{4\tan^2\theta + 4}}{2}$

$x = -\tan\theta \pm \sec\theta$

$\alpha_2 = -\tan\theta + \sin\theta \qquad \beta_2 = -\tan\theta - \sec\theta$

$\alpha_1 + \beta_2 = -2\tan\theta$

10. Ans. (BC) Trigonometric Ratio & identities

Sol. $\cos\alpha = \left(\dfrac{1-a}{1+a}\right) \quad ; \quad a = \tan^2\dfrac{\alpha}{2}$

$\cos\beta = \left(\dfrac{1-b}{1+b}\right) \quad ; \quad b = \tan^2\dfrac{\beta}{2}$

$2\left(\left(\dfrac{1-b}{1+b}\right) - \left(\dfrac{1-a}{1+a}\right)\right) + \left(\left(\dfrac{1-a}{1+a}\right)\left(\dfrac{1-b}{1+b}\right)\right) = 1$

$\Rightarrow \quad 2((1-b)(1+a) - (1-a)(1+b)) + (1-a)(1-b) = (1+a)(1+b)$

$\Rightarrow \quad 2(1 + a - b - ab - (1 + b - a - ab)) + 1 - a - b + ab = 1 + a + b + ab$

$\Rightarrow \quad 4(a - b) = 2(a + b)$

$\Rightarrow \quad 2a - 2b = a + b$

$\Rightarrow \quad a = 3b$

$\tan^2\dfrac{\alpha}{2} = 3\tan^2\dfrac{\beta}{2}$

$\tan\dfrac{\alpha}{2} = \pm\sqrt{3}\tan\left(\dfrac{\beta}{2}\right)$

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

1. Sol. $\sin\theta + \cos\theta = m$ ....(1)

$\sin\theta\cos\theta = \frac{m}{n}$ ...(2)

squaring (1)

$1 + 2\sin\theta\cos\theta = m^2$

$\sin\theta\cos\theta = \frac{m^2-1}{2}$ ...(3)

by (2) & (3) $n(m^2 - 1) = 2m$

2. Sol. $E = \frac{13}{4} - (\cos x + \frac{1}{2})^2 \Rightarrow$ maximum value $= \frac{13}{4}$

minimum value = 1

3. Sol. 1 radian = 57°

2 radian = 114°

This lies in 2nd quadrant

$\tan 1 > 0, \tan 2 < 0 \Rightarrow \tan 1 > \tan 2$

4. Sol. $\sin^2\theta - 2\cos\theta + \frac{1}{4} = 0 \Rightarrow \cos\theta = \frac{1}{2}$ or $\cos\theta = \frac{-5}{2} \Rightarrow \theta = 60°$

5. Sol. $\sin\theta - \cos\theta = 1$

$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{4} + \frac{\pi}{4}$.

6. Sol. $\cos x = 0, \cos x = \frac{-2}{3}, \cos x = \frac{1}{2}$

Hence either x = 90° or x = 60° so required difference = $\frac{\pi}{2} - \frac{\pi}{3} = \pi/6$

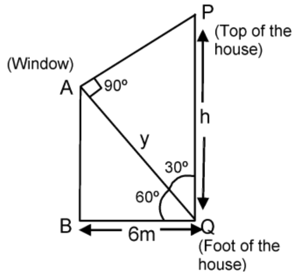
7. Sol. $\tan(A + B) = \frac{\frac{a}{a+1} + \frac{1}{2a+1}}{1 - \frac{a}{(a+1)(2a+1)}} = \frac{2a^2 + a + a + 1}{2a^2 + 3a + 1 - a}$

$= \frac{2a^2 + 2a + 1}{2a^2 + 2a + 1} \Rightarrow A + B = \frac{\pi}{4}$

8. Sol. $\sin 15° \cos 15° = \frac{1}{2} \sin 30° = \frac{1}{4}$

9. Sol. Checking option A = 60°, B = 30°, C = 0°

10. Sol.



In ΔABQ

$\frac{6}{y} = \cos 60^\circ = \frac{1}{2}$

y = 12

in ΔPAQ

∠AQP = 30º

$\frac{y}{h} = \cos 30^\circ = \frac{\sqrt{3}}{2}$

h = $8\sqrt{3}$

11. Sol. cos2θ = sin α

cos2θ = cos (π/2 – α)

2θ = 2nπ± (π/2 – α)

⇒ θ = nπ± $\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$

12. Sol. $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\left(2\pi - \frac{8\pi}{7}\right)$

$\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = \frac{\sin 3\left(\frac{2\pi}{2\times 7}\right)}{\sin\frac{\pi}{7}} \cos\left[\frac{8\pi}{2\times 7}\right] = \frac{\sin\frac{3\pi}{7}\cos\frac{4\pi}{7}}{\sin\frac{\pi}{7}} = \frac{-\sin\frac{3\pi}{7}\cos\frac{3\pi}{7}}{\sin\pi/7} = -\frac{1}{2}$

13. Sol. cos (x – π/4) = cos A ⇒ x – π/4 = 2nπ± A ⇒ x = 2nπ+ π/4± A

14. Sol. cosx = $\frac{1}{2}$

x = $\frac{\pi}{3}$, $\frac{5\pi}{3}$ and cosx = $\frac{-3}{2}$ rejected

15. Sol. $\frac{\sin x\cos y + \cos x\sin y}{\sin x\cos y - \cos x\sin y} = \frac{a+b}{a-b}$

$\frac{2\sin x\cos y}{2\cos x\sin y} = \frac{a}{b}$ ⇒ $\frac{\tan x}{\tan y} = \frac{a}{b}$

16. Sol. $\frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ} = \frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sin 20^\circ\cos 20^\circ} = \frac{2\sin(60^\circ - 20^\circ)}{\sin 20^\circ\cos 20^\circ} = 4$

17. **Sol.** $\frac{\sin(A+2B)}{\sin A}=\frac{1}{3}$ $\Rightarrow$ $\frac{2\sin(A+B)\cos B}{2\cos(A+B)\sin B}=\frac{1+3}{1-3}=-2$

tan (A + B) = – 2 tan B

18. **Sol.** θ is in 3rd or 4th quadrant

$\Rightarrow \sin\theta = \pm\frac{4}{5}$

19. **Sol.** $\frac{\cot 20^\circ-\tan 20^\circ}{\cot 40^\circ} = \frac{2(1-\tan^2 20^\circ)}{2\tan 20^\circ}\times\tan 40^\circ = \frac{2}{\tan 40^\circ}\times\tan 40^\circ$

20. **Sol.** $\frac{3\cos\theta+4\cos^3\theta-3\cos\theta}{3\sin\theta-(3\sin\theta-4\sin^3\theta)}$

$= \frac{4\cos^3\theta}{4\sin^3\theta} = \cot^3\theta$

21.
Sol. sin10° + sin20° + sin30° .....+ sin170° + sin180° + sin190° ..... sin300° + sin360°
= sin10 + sin20 + sin30 + ...+ sin170° + sin180°
– sin170° ....– sin10° + sin360°
= 0

22. **Sol.** sinθ = 1

⇒ $\sin^n\theta + \text{cosec}^n\theta = 2$

23. **Sol.** sum of roots = $\frac{1+\tan^2\frac{\theta}{2}}{\tan\frac{\theta}{2}} = \frac{2}{\sin\theta}=6$

$x^2 - 6x + 1 = 0$

24. **Sol.** tanA + tanB + tanC = tanA tanB tanC

6 = tanA + tanB tanC ⇒ cotA cotB cotC = $\frac{1}{6}$

25. **Sol.** sin5θ sin2θ = cos2θ cos5θ

cos (7θ) = 0

$7\theta = n\pi+\frac{\pi}{2}$

$\theta = \frac{n\pi}{7}+\frac{\pi}{14}$

26. **Sol.** $\frac{\sin\frac{2^5\pi}{33}}{2^5\sin\frac{\pi}{33}} = \frac{\sin\frac{32\pi}{33}}{32\sin\frac{\pi}{33}} = \frac{\sin\left(\pi-\frac{\pi}{33}\right)}{32\sin\frac{\pi}{33}} = \frac{\sin\frac{\pi}{33}}{32\sin\frac{\pi}{33}} = \frac{1}{32}$

27. **Sol.** $t + \frac{16}{t} = 10 \Rightarrow t = 2$ or 8

$16^{\sin^2 x} = 2$ or 8

$4\sin^2 x = 1$ or $4\sin^2 x = 3$

$\sin x = \pm 1/2$ or $\sin x = \pm \frac{\sqrt{3}}{2}$

28. **Sol.** $-13 \le 12\sin\frac{11\theta}{2} + 5\cos\frac{11\theta}{2} \le 13$

$0 \le 13 + 12\sin\frac{11\theta}{2} + 5\cos\frac{11\theta}{2} \le 26$

range = [0,26]

29. **Sol.** $\frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2} = \pi$

$\tan\left(\frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2}\right) = 0$

30. **Sol.** $\sec\theta + \tan\theta = e^x$

$\sec\theta - \tan\theta = e^{-x}$

$\sec\theta = \frac{e^x + e^{-x}}{2}$

$\cos\theta = \frac{2}{e^x + e^{-x}}$

PART - II : PRACTICE QUESTIONS

1. **Sol.** $\sin\alpha\sin\beta - \cos\alpha\cos\beta + 1 = 0$

$\Rightarrow$ $\cos(\alpha + \beta) = 1$

$\alpha + \beta = 2n\pi$

$\Rightarrow$ $1 + \cot\alpha\tan\beta = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta} = \frac{\sin(\alpha+\beta)}{\sin\alpha\cos\beta} = 0$

2. **Sol.** $3\sin x - 4\sin^3 x = k$ $0 < k < 1$

$\sin 3x = k$

Also $\sin 3A = k$

$\sin 3B = k$

$\Rightarrow$ $0 < 3A < \pi$ $0 < 3B < \pi$ ....(i)

Also $\sin 3A - \sin 3B = 0$

$\Rightarrow$ $2\cos\frac{3}{2}(A + B)\sin\frac{3}{2}(A - B) = 0$

$\cos\frac{3}{2}(A + B) = 0$ $\sin\frac{3}{2}(A - B) = 0$ ....(ii)

Given $A > B$ ......(iii)

$\Rightarrow \quad \sin \frac{3}{2}(A - B) \neq 0 \quad \Rightarrow \quad \cos\left(3 \frac{(A+B)}{2}\right) = 0$

$A + B = \frac{\pi}{3} \qquad C = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

3. **Sol.** $\frac{1}{\sin\frac{\pi}{n}} - \frac{1}{\sin\frac{3\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}}$

$\frac{2\cos\frac{2\pi}{n}\sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}}$

$\sin\frac{4\pi}{n} = \sin\frac{3\pi}{n}$

$\frac{4\pi}{n} = (-1)^k \frac{3\pi}{n} + k\pi,\ k \in I$

If $k = 2m \quad \Rightarrow \quad \frac{\pi}{n} = 2m\pi$

$\frac{1}{n} = 2m$, not possible

If $k = 2m + 1 \quad \Rightarrow \quad \frac{7\pi}{n} = (2m + 1)\pi$

$\Rightarrow \quad n = 7,\ m = 0$

Ans. $n = 7$

4*. **Sol.** $\because 0 < \theta < \frac{\pi}{2}$ and

$\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\ \pi}{4}\right) \operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$

$\Rightarrow \sum_{m=1}^{6} \frac{1}{\sin\left(\theta + \frac{(m-1)\pi}{4}\right)\sin\left(\theta + \frac{m\pi}{4}\right)} = 4\sqrt{2}$

$\Rightarrow \sum_{m=1}^{6} \frac{\sin\left[\theta + \frac{m\pi}{4} - \left(\theta + \frac{(m-1)\pi}{4}\right)\right]}{\sin\frac{\pi}{4}\left\{\sin\left(\theta + \frac{(m-1)\pi}{4}\right)\sin\left(\theta + \frac{m\pi}{4}\right)\right\}} = 4\sqrt{2}$

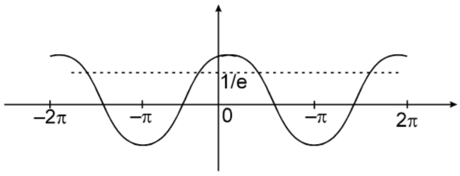
$\Rightarrow \sum_{m=1}^{6} \frac{\cot\left(\theta + \frac{(m-1)\pi}{4}\right) - \cot\left(\theta + \frac{m\pi}{4}\right)}{\frac{1}{\sqrt{2}}} = 4\sqrt{2}$

$\Rightarrow \sum_{m=1}^{6} \left(\cot\left(\theta + \frac{(m-1)\pi}{4}\right) - \cot\left(\theta + \frac{m\pi}{4}\right)\right) = 4$

$\Rightarrow \cot(\theta) - \cot\left(\theta + \frac{\pi}{4}\right) + \cot\left(\theta + \frac{\pi}{4}\right) - \cot\left(\theta + \frac{2\pi}{4}\right) + \ldots + \cot\left(\theta + \frac{5\pi}{4}\right) - \cot\left(\theta + \frac{6\pi}{4}\right) = 4$

$\Rightarrow$ $\cot\theta - \cot\left(\frac{3\pi}{2}+\theta\right) = 4$
$\Rightarrow$ $\cot\theta + \tan\theta = 4$
$\Rightarrow$ $\tan^2\theta - 4\tan\theta + 1 = 0$
$\Rightarrow$ $(\tan\theta - 2)^2 - 3 = 0$
$\Rightarrow$ $(\tan\theta - 2 + \sqrt{3})(\tan\theta - 2 - \sqrt{3}) = 0$
$\Rightarrow$ $\tan\theta = 2 - \sqrt{3}$ or $\tan\theta = 2 + \sqrt{3}$
$\Rightarrow$ $\theta = \frac{\pi}{12}$ or $\theta = \frac{5\pi}{12}$ $\because$ $\theta \in \left(0, \frac{\pi}{2}\right)$

5. **Sol.**



$\alpha - \beta = 0, -2\pi$ or 2π

$\alpha - \beta = 0$ $\Rightarrow$ $\alpha = \beta$ $\Rightarrow$ $\cos 2\beta = \frac{1}{e}$
This is true for '4' values of 'α', 'β'
If $\alpha - \beta = -2\pi$ $\Rightarrow$ $\alpha = -\pi$ and $\beta = \pi$ and $\cos(\alpha + \beta) = 1$ $\Rightarrow$ (No solution)
similarly if $\alpha - \beta = 2\pi$ $\Rightarrow$ $\alpha = \pi$ and $\beta = -\pi$ again no solution results

6. **Sol.** As $\tan(2\pi - \theta) > 0$, $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$, $\theta \in [0, 2\pi]$
$\Rightarrow \frac{3\pi}{2} < \theta < \frac{5\pi}{3}$
Now $2\cos\theta(1 - \sin\varphi) = \sin^2\theta(\tan\theta/2 + \cot\theta/2)\cos\varphi - 1$
$\Rightarrow 2\cos\theta(1 - \sin\varphi) = 2\sin\theta\cos\varphi - 1$
$\Rightarrow 2\cos\theta + 1 = 2\sin(\theta + \varphi)$
As $\theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right) \Rightarrow 2\cos\theta + 1 \in (1, 2)$
$\Rightarrow 1 < 2\sin(\theta + \varphi) < 2$
$\Rightarrow \frac{1}{2} < \sin(\theta + \varphi) < 1$
As $\theta + \varphi \in [0, 4\pi]$
$\Rightarrow \theta + \varphi \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ or $\theta + \varphi \in \left(\frac{13\pi}{6}, \frac{17\pi}{6}\right)$
$\Rightarrow \frac{\pi}{6} - \theta < \varphi < \frac{5\pi}{6} - \theta$ or $\frac{13\pi}{6} - \theta < \varphi < \frac{17\pi}{6} - \theta$
$\Rightarrow \varphi \in \left(-\frac{3\pi}{2}, \frac{-2\pi}{3}\right) \cup \left(\frac{2\pi}{3}, \frac{7\pi}{6}\right)$ $\left(\because \theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)\right)$

7. **Sol.** $a_1 + a_2\cos 2x + a_3\sin^2 x = 0$
$\Rightarrow$ $a_1 + a_2(1 - 2\sin^2 x) + a_3\sin^2 x = 0$
$\Rightarrow$ $(a_1 + a_2) + \sin^2 x(a_3 - 2a_2) = 0$ is an identity

$\Rightarrow$ $a_1 + a_2 = 0$ & $a_3 - 2a_2 = 0$

$\frac{a_1}{-1} = \frac{a_2}{1} = \frac{a_3}{2}$

infinite triplets of (a_1, a_2, a_3) are possible

8. **Ans.** **(D)**

Sol. $\sin x + 2\sin 2x - \sin 3x = 3.$

$\sin x\,(1 + 2\cos x - 3 + 4\sin^2 x) = 3.$

$(4\sin^2 x + 2\cos x - 2) = \frac{3}{\sin x}$

$2 - 4\cos^2 x + 2\cos x = \frac{3}{\sin x}$

$\frac{9}{4} - \left(2\cos x - \frac{1}{2}\right)^2 = \frac{3}{\sin x}.$

L.H.S. $\le \frac{9}{4}$ R.H.S. ≥ 3.

No solution.

9. **Sol.** $\sec^2 (a + 2)\, x = 1 - a^2$

as $\sec^2\theta \ge 1$

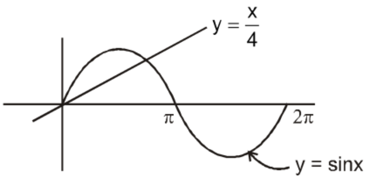
$\therefore$ $a = 0$ and $\sec^2 2x = 1$

$\Rightarrow$ $\cos^2 2x = 1$ $\Rightarrow$ $2x = -\pi, 0, \pi$ ($\because$ $2x \in (-2\pi, 2\pi)$)

$\therefore$ $a = 0$ and $x \in \left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

$\therefore$ Three ordered pairs are possible for (a, x)

10. **Sol.** Two solution of sin x



11. **Sol.** $\cos 2x + a\sin x = 2a - 7$

$\Rightarrow$ $1 - 2\sin^2 x + a\sin x = 2a - 7$

$\Rightarrow$ $2\sin^2 x - a\sin x + 2(a - 4) = 0$ ....(1)

$\Rightarrow$ $\sin x = \frac{a \pm \sqrt{a^2 - 16(a-4)}}{4}$

$\Rightarrow = \frac{a \pm \sqrt{(a-8)^2}}{4} = \frac{a \pm (a-8)}{4} = \frac{2a-8}{4}, \frac{8}{4}$

$\Rightarrow$ $\sin x = \frac{a-4}{2}$ $\Rightarrow$ $-1 \le \frac{a-4}{2} \le 1$

$\Rightarrow$ $2 \le a \le 6$ $\Rightarrow$ $a = 2,3,4,5,6$

$\therefore$ No. of solutions = 5

12. **Sol.** $\sin^6 x + \cos^6 x = a^2$

$\Rightarrow$ $(\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x\cos^2 x) = a^2$

$\Rightarrow$ $(\sin^2 x + \cos^2 x)^2 - 3\sin^2 x\cos^2 x = a^2$ $\Rightarrow$ $1 - 3\sin^2 x\cos^2 x = a^2$

$\Rightarrow \quad 1 - \frac{3}{4}\sin^2 2x = a^2 \qquad \Rightarrow \quad \frac{4\;(1-a^2)}{3} = \sin^2 2x$

$\Rightarrow \quad 0 \le \frac{4}{3}(1 - a^2) \le 1$

$1 - a^2 \ge 0 \qquad$ and $4 - 4a^2 \le 3$

$a^2 \le 1 \qquad$ and $\qquad \frac{1}{4} \le a^2$

$-1 \le a \le 1 \qquad$ So $\quad a \ge \frac{1}{2}$ or $a \le -\frac{1}{2}$

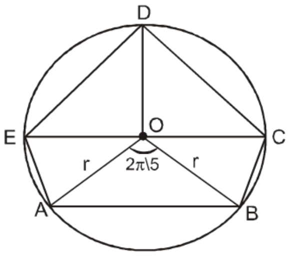
$a \in \left[-1, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, 1\right]$

13. Sol. $A_1 = \pi r^2$

$A_2 = 5 \times \frac{1}{2}\text{ r.r. } \sin\frac{2\pi}{5}$

$\frac{A_1}{A_2} = \frac{2\pi}{5\sin\frac{2\pi}{5}} = \frac{2\pi}{5}\operatorname{cosec}\frac{2\pi}{5}$

$= \frac{2\pi}{5}\sec\left(\frac{\pi}{2} - \frac{2\pi}{5}\right) = \frac{2\pi}{5}\sec\frac{\pi}{10}$



14. Sol. Area of pentagon $= 5 \times \frac{1}{2} \times r \times r.\sin\frac{4\pi}{10} = \frac{5}{8}\,\frac{a^2\sin\frac{4\pi}{10}}{\sin^2\frac{2\pi}{10}}$

$A_1 = \frac{5}{4}a^2\cot\frac{2\pi}{10}$

$\cos\frac{4\pi}{10} = \frac{r^2 + r^2 - a^2}{2r^2} \Rightarrow \cos\frac{4\pi}{10} = 1 - \frac{a^2}{2r^2} \Rightarrow \frac{a^2}{2r^2} = 2\sin^2\frac{2\pi}{10} \Rightarrow r^2 = \frac{a^2}{4\sin^2\frac{2\pi}{10}}$

For decagon, $\cos\frac{2\pi}{10} = \frac{r_1^2 + r_1^2 - \left(\frac{a}{2}\right)^2}{2r_1^2} \Rightarrow \frac{a^2}{8r_1^2} = 2\sin^2\frac{\pi}{10} \Rightarrow r_1^2 = \frac{a^2}{16\sin^2\frac{\pi}{10}}$

Area of decagon, $A_2 = 10 \times \frac{1}{2}r_1^2\sin\frac{2\pi}{10} = 5.\frac{a^2}{16\sin^2\frac{\pi}{10}}.\sin\frac{2\pi}{10} \Rightarrow A_2 = \frac{5}{8}a^2\cot\frac{\pi}{10}$

$A_1 : A_2 = 2\cot\frac{2\pi}{10} : \cot\frac{\pi}{10} = 2\cot\frac{\pi}{5} : \cot\frac{\pi}{10}$

$= \frac{2\cos\frac{\pi}{5}}{\sin\frac{\pi}{5}}.\frac{\sin\frac{\pi}{10}}{\cos\frac{\pi}{10}} = \frac{\sin\frac{2\pi}{5}\sin\frac{\pi}{10}}{\sin^2\frac{\pi}{5}\cos\frac{\pi}{10}} = \frac{\cos\frac{\pi}{10}\sin\frac{\pi}{10}}{\sin^2\frac{\pi}{5}\cos\frac{\pi}{10}} = \frac{\frac{\sqrt{5}-1}{4}}{1 - \left(\frac{\sqrt{5}+1}{4}\right)^2} = 2 : \sqrt{5}$

15. Sol. $1 + 2\operatorname{cosec} x = \dfrac{-\sec^2\left(\frac{x}{2}\right)}{2}$ $\Rightarrow 1 + \dfrac{2}{\sin x} = \dfrac{-1}{1+\cos x}$

$\Rightarrow (2 + \sin x)(1 + \cos x) = -\sin x \Rightarrow 2 + 2\cos x + \sin x + \sin x\cos x = -\sin x$

$\Rightarrow 2(\sin x + \cos x) + \sin x\cos x + 2 = 0$

Put $\sin x + \cos x = t$

$\Rightarrow 1 + 2\sin x\cos x = t^2$ $\therefore\ 2t + \dfrac{t^2-1}{2} + 2 = 0 \Rightarrow t^2 + 4t + 3 = 0$

$\Rightarrow t = -1, -3 \Rightarrow \sin x + \cos x = -1 \Rightarrow \cos\left(x - \dfrac{\pi}{4}\right) = \dfrac{-1}{\sqrt{2}} = \cos\dfrac{3\pi}{4}$

$\Rightarrow x - \dfrac{\pi}{4} = 2n\pi \pm \dfrac{3\pi}{4} \Rightarrow x = 2n\pi + \pi,\ 2n\pi - \dfrac{\pi}{2}$

$\Rightarrow x = 2n\pi + \pi$ at which cosec x is not defined

$\therefore\ x = 2n\pi - \dfrac{\pi}{2}$.

16. Sol. $\dfrac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$

$\Rightarrow\ \dfrac{5}{4}\cos^2 2x + 1 - \dfrac{1}{2}\sin^2 2x + 1 - \dfrac{3}{4}\sin^2 2x = 2$

$\Rightarrow\ \cos^2 2x = \sin^2 2x$

$\Rightarrow\ \tan^2 2x = 1$

Now $2x \in [0, 4\pi]$ $\Rightarrow$ $x = \dfrac{\pi}{8}, \dfrac{3\pi}{8}, \dfrac{5\pi}{8}, \dfrac{7\pi}{8}, \dfrac{9\pi}{8}, \dfrac{11\pi}{8}, \dfrac{13\pi}{8}, \dfrac{15\pi}{8}$

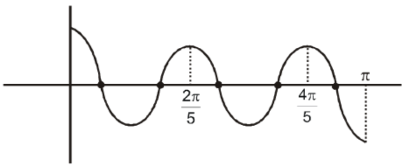
so number of solution = 8

17. Sol. $2\sin\theta + \tan\theta = 0 \Rightarrow \sin\theta = 0$ or () $2 + \dfrac{1}{\cos\theta} = 0 \Rightarrow \theta = n\pi$ or() $2 = -\dfrac{1}{\cos\theta} \Rightarrow \cos\theta = -\dfrac{1}{2}$

$\Rightarrow \theta = 2n\pi \pm \dfrac{2\pi}{3}$.

18. Sol. $\sin x \,.\, \tan 4x = \cos x \Rightarrow \sin x\sin 4x = \cos x\cos 4x$

$\Rightarrow \cos 5x = 0 \Rightarrow$ five solutions.



19*. Sol. We have, $2\cos 2\theta + \sqrt{2\sin\theta} = 2$

$\Rightarrow\ \sqrt{2\sin\theta} = 2(1 - \cos 2\theta) \Rightarrow \sqrt{2\sin\theta} = 4\sin^2\theta$ $\Rightarrow \sqrt{2\sin\theta} - 4\sin^2\theta = 0$

$\Rightarrow\ \sqrt{2\sin\theta}\,[1 - 2\sqrt{2}\sin^{3/2}\theta] = 0$

If $\sqrt{2\sin\theta} = 0,\ \sin\theta = 0$ $[\because \sqrt{z} = 0 \Rightarrow z = 0]$

$\Rightarrow\ \theta = n\pi,\ n \in I.$

If $1 - 2\sqrt{2}\sin^{3/2}\theta = 0,\ \sin^{3/2}\theta = \dfrac{1}{2\sqrt{2}}$ $\Rightarrow$ $(\sin\theta)^{3/2} = \left(\dfrac{1}{2}\right)^{\frac{3}{2}}$

$\Rightarrow \quad \sin\theta = \frac{1}{2} = \sin\frac{\pi}{6} \quad \Rightarrow \quad \theta = n\pi + (-1)^n\frac{\pi}{6}$, n ∈ I.

$\therefore \quad \theta = n\pi,\ n\pi + (-1)^n\frac{\pi}{6}$, n ∈ I.

20*. **Sol.** We have, $\frac{\sqrt{3}}{2}\sin x = \cos x + \cos^2 x$.

Squaring both sides, we get

$3(1 - \cos^2 x) = 4(\cos^2 x + 2\cos^3 x + \cos^4 x)$

$\Rightarrow \quad 4\cos^4 x + 8\cos^3 x + 7\cos^2 x - 3 = 0 \Rightarrow \quad (\cos x + 1)(2\cos x - 1)(2\cos^2 x + 3\cos x + 3) = 0$

When $\cos x = -1 = \cos\pi$, $x = 2n\pi + \pi = (2n + 1)\pi$.

When $\cos x = \frac{1}{2} = \cos\frac{\pi}{3}$, $x = 2n\pi \pm \frac{\pi}{3}$.

When $2\cos^2 x + 3\cos x + 3 = 0$,

the discriminant = $9 - 4 \cdot 2 \cdot 3 < 0$

This factor does not given any real values of cosx.

Hence, $x = (2n + 1)\pi,\ 2n\pi \pm \frac{\pi}{3}$, n ∈ I