Exercise-1

Marked Questions may have for Revision Questions.

OBJECTIVE QUESTIONS

Section (A) : Representation of sets, Types of sets, subset and power set

A-1. **Sol.** Since, intelligency is not defined for students in a class so set of intelligent students in a class is not well defined collection.

A-2. **Sol.** (i) $x^2 - 1 = 0$ $x = \pm 1$
(ii) $x^2 + 1 = 0$ $x = \pm i$ $x \in \varphi$
(iii) $x^2 - 9 = 0$ $x = \pm 3$
(iv) $x^2 - x - 2 = 0$, $x = 2, -1$

A-3. **Sol.** $x^2 = 16 \Rightarrow$ $x = \pm 4$
$2x = 6$ $x = 3$
No common value of x

A-4. **Sol.** A = {–2, –1, 0, 1, 2}
No. of subsets = $2^n = 2^5 = 32$

A-5. **Sol.** Obvious

A-6. **Sol.** P(A) = {φ, {7}, {10}, {11}, {7, 10}, {7, 11}, {10, 11}, {7, 10, 11}}

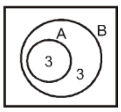
A-7. **Sol.** Collection of all beautiful women in Jalandhar is not a set as it is not a well defined collection. It is not possible to decide logically which woman is to be included in the collection and which is not to be included.

A-8. **Sol.** 2,3,5 and 7 are the only positive primes less than 10.

A-9. **Sol.** Between any two real numbers there lie infinitely many real numbers.

A-10. **Sol.** P(A) = {φ, {φ}, {{φ}}, {φ, {φ}}} = {φ, {φ}, {{φ}}, A}

Section (B) : Operations on sets, Law of Algebra of sets



B-1. **Sol.**
$n(AB) = n(A) + n(B) - n(AB)$ $\Rightarrow$ minimum value of $n(A \cup B)$
$= 3 + 6 - 3$ $= 6$

B-2. **Sol.** A = {1, 2, 3}
B = {3, 4}
C = {4, 5, 6}
$B \cap C$ = { 4 }
$A \cup (B \cap C)$ = {1, 2, 3, 4}

B-3. **Sol.** $A = [x : x \in R, -1 < x < 1]$
$B = [x : x \in R : x \leq 0 \text{ or } x \geq 2]$
$\therefore$ $A \cup B = R - D$, where $D = [x : x \in R, 1 \leq x < 2]$

B-4. **Sol.** Obvious

B-5. **Sol.** $A \cap B = \{3, 4, 10\}$
$A \cap C = \{4\}$
$(A \cap B) \cup (A \cap C) = \{3, 4, 10\}$

B-6. **Sol.** Obviously $A - (B \cup C)$

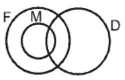
B-7. **Sol.** $B' = U - B = \{1, 2, 3, 4, 5, 8, 9, 10\}$
$A \cap B' = \{1,2,5\} = A$

B-8. **Sol.** $A = \{5, 9, 13, 17, 21\}$ and $B = \{3, 6, 9, 12, 15, 18, 21, 24\}$
$A - B = \{5, 13, 17\}$
$A - (A - B) = \{9, 21\}$

B-9. **Sol.** Let $A \cup B = A \cap B$
Now, $x \in A \Rightarrow x \in A \cup B$ $(\because A \subseteq A \cup B)$
$\Rightarrow x \in A \cap B$ $(\because A \cup B = A \cap B)$
$\Rightarrow x \in B$
Similarly, $x \in B$ implies $x \in A$ $\therefore A = B$
Conversly, let $A = B$
$\therefore A \cup B = A \cup A = A = A \cap A = A \cap B$
$\therefore A \cup B = A \cap B$

B-10. **Sol.** $bN \cap . cN$
(+ve integral multiple of b) $\cap$ (+ve integral multiple of c)
since b & c are relatively primes :
$= b\,c\,N$ $\therefore$ $d = bc$

B-11. **Sol.** 1. $(N \cup B) \cap Z = (N \cap Z) \cup (B \cap Z) = N \cup (B \cap Z)$
2. $A = \{3, 6, 9, 12, 15, 18, 21, 24\}$

B-12. **Sol.** 

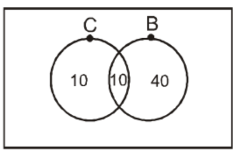
M ≡ Mother ; F ≡ Female ; D = Doctor

Section (C) : Cardinal number Problems

C-1. **Sol.** (i) $A \cup B \geq A \cup B$ (ii) $A \cap B \leq A \cup B$ (iii) $A \cap B = A \cup B$ not always

C-2. **Sol.** $n(A_c \cap B_c) = n[(\{A \cup B)_c] = n(U) - n(A \cup B)$
$= n(U) - [n(A) + n(B) - n(A \cap B)] = 700 - [200 + 300 - 100] = 300.$

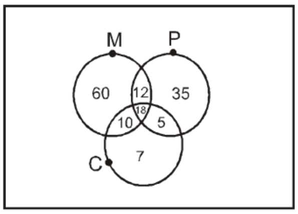
C-3. **Sol.** Let number of newspapers is x.
As every newspaper is read by 60 students
Since, every students reads 5 newspapers
$\therefore 60x = 300(5)$ $\Rightarrow$ $x = 25.$



C-4. **Sol.**
$P = 10 + 10 + 40 = 60\ \%$

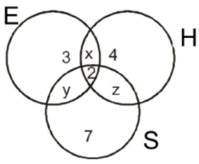
C-5. Sol. n (A) = 40% of 10,000 = 4,000
n(B) = 20% of 10,000 = 2,000
n(C) = 10% of 10,000 = 1,000
$n(A \cap B)$ = 5% of 10,000 = 500
$n(B \cap C)$ = 3% of 10,000 = 300
$n(C \cap A)$ = 4% of 10,000 = 400
$n(A \cap B \cap C)$ = 2% of 10,000 = 200
$n(A \cap B^c \cap C^c) = n[A \cap (B \cup C)^c]$
$= n(A) - n[A \cap (B \cup C)] = n(A) - n[(A \cap B) \cup (A \cap C)]$
$= n(A) - [n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)]$
$= 4000 - [500 + 400 - 200] = 4000 - 700 = 3300.$

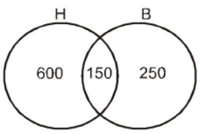
C-6.



Sol. Number of students offered maths alone = 60
n (M) = 100
n(P) = 70
n (C) = 40
$n(M \cap P) = 30$
$n(M \cap P) = 28$
$n(P \cap C) = 23$
$n(M \cap P \cap C) = 18$

C-7. Sol. $x + y = 10$; $x + z = 9$; $y + z = 11 \Rightarrow x + y + z = 15$
$x = 4, y = 6, z = 5$





C-10 Sol.
$n(H \cup B) = n(H) + n(B) - n(H \cap B)$
$1000 = 750 + 400 - n(H \cap B) = 150$
Now n(only hindi) = $n(H) - n(H \cap B) = 750 - 150 = 600$
n(only bengali) = $n(B) - n(H \cap B)$
$400 - 150 = 250$

Section (D) : Ordered pair , Cartesion product, Relation, Domain and Range of Relation

D-1. Sol. $B \cup C = \{c, d, e\}$, $B \cap C = \{d\}$
$A \cup (B \cup C) = \varphi$
$A \cup (B \cap C) = \{a, b, d\}$

$A \times (B \cup C) = \{(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)\}$
$A \times (B \cap C) = \{(a, d), (b, d)\}$

D-2. **Sol.** $n(A \times B) = n(A) \times n(B) = 3 \times 3 = 9$

D-3. **Sol.** $A = \{2, 3\}, B = \{2, 4\}, C = \{4, 5\}$
$B \cap C = \{4\}$ $\therefore$ $A \times (B \cap C) = \{(2, 4), (3, 4)\}$.

D-4. **Sol.** $n(A) = 3, n(B) = 2; n(C) = 3$
$\Rightarrow$ $n(A \times B \times C) = 3.2.3. = 18$.

D-5. **Sol.** $A \times B$ is a relation defined from set A to set B

D-6. **Sol.** Number of relation from A to B $= 2^{12}$

D-7. **Sol.** Obviously $R \subseteq A \times B$

D-8. **Sol.** $R_1 \rightarrow$ Domain = {1, 3, 5}
Range = {3, 5, 7} so R_1 is a relation

$R_2 \rightarrow$ Domain = {1,2, 3, 4, 5}
Range = {1, 3, 5} so R_2 is a relation

$R_3 \rightarrow$ Domain = {1, 3, 5}
Range = {1, 3, 5, 7} so R_3 is a relation

$R_4 \rightarrow$ Domain = $\{1, 2, 7\} \not\subset X$ so R_4 is not a relation

D-9. **Sol.** If x = 2 then y = 1, If x = 3 then y = 3,
If x = 4 then y = 5, If x = 5 then y = 7,

D-10. **Sol.** A = {1, 2, 3}
$-5 \leq a_2 - b_2 \leq 5$
R = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)}
Domain of R = {1, 2, 3}
Range of R = {1, 2, 3}

D-11. **Sol.** $-3 \leq x \leq 3$
$\Rightarrow$ $(x, y) = (x, \sqrt{(x-1)^2}) = \{(-3, 4), (-2, 3), (-1, 2), (0, 1), (1, 0), (2, 1), (3, 2)\}$

D-12. **Sol.** Domain of R
= first element of pairs (x, y)
= {– 3, – 2, – 1, 0, 1, 2, 3}

D-13. **Sol.** Range set consisting of 4, 3, 2, 1, 0, 1, 2
= {0, 1, 2, 3, 4}

D-14_ **Sol.** $(-1,2) \in A \times A$
$\Rightarrow$ $-1 \in A, 2 \in A$ and $(0,1) \in A \times A \Rightarrow 0 \in A, 1 \in A$
So , $A = \{-1,0,1,2\}$ as A has four elements
and $S = \{-1,0), (1,-1), (-1,2), (0,1), (0,2), (1,2)\}$
Hence the required element of S are given by (a)

Section (E) : Types of Relation

E-1. **Sol.** For Reflexive
$R \in \{(1,1)\ (2, 2)\ (3, 3)\}$
For symmetric $(1, 2) \in R$ but $(2, 1) \notin R$ Not symmetric
for transitive $(1, 2), (2, 3) \in R \Rightarrow (1, 3) \in R$
so transitive

E-2. **Sol.** $x < y, y < z \Rightarrow x < z\ \forall x, y, z \in N$
$\therefore$ xRy, yRz $\Rightarrow$ xRz, $\therefore$ Relation is transitive,
$\therefore$ $x < y$ does not give $y < x$.
Relation is not symmetric.
Since $x < x$ does not hold, hence relation is not reflexive.

E-3. **Sol.** For any $a \in N$, we find that a is divisible by a, therefore R is reflexive but R is not symmetric, because
aRb does not imply that bRa.

E-4. **Sol.** Since $x \nless x$, therefore R is not reflexive. Also $x < y$ does not imply that $y < x$, So R is not symmetric. Let xRy and yRz. Then $x < y$ and $y < z \Rightarrow x < z$ i.e., xRz. Hence R is transitive.

E-5. **Sol.** $xR_2y \Leftrightarrow x \geq y$ is not symmetric relation
$xR_3y \Leftrightarrow x / y$ is not symmetric relation
$xR_4y \Leftrightarrow x < y$ is not symmetric relation
$xR_1y \Leftrightarrow x_2 = y_2$ is reflexive, symmetric and transitive so equivalence relation
$xR_4y \Leftrightarrow x < y$ is not symmetric relation
$xR_1y \Leftrightarrow x_2 = y_2$ is reflexive, symmetric and transitive so equivalence relation

E-6. **Sol.** Obviously, the relation P is neither reflexive nor transitive but it is symmetric,
because $x_2 + y_2 = 1 \Rightarrow y_2 + x_2 = 1$.

E-7. **Sol.** (1) $(q, q) \notin R_1$ (2) $(p, p) \notin R_2$ (3) $(q, p) \notin R_3$
Not reflexive not reflexive not symmetric

E-8. **Sol.** For any $a \in R$, we have $a \geq a$, Therefore the relation R_1 is reflexive but it is not symmetric as $(2, 1) \in R_1$ but $(1, 2) \notin R_1$. The relation R_1 is transitive also, because $(a, b) \in R_1$, $(b, c) \in R_1$ imply that $a \geq b$ and $b \geq c$ which is turn imply that $a \geq c \Rightarrow (a, c) \in R_1$.

E-9. **Sol.** Here $\alpha R\beta \Leftrightarrow \alpha \perp \beta \Leftrightarrow \beta \perp \alpha$
Hence R is symmetric.

E-10. **Sol.** $1 + a.a = 1 + a_2 > 0, \forall a \in S$, $\therefore (a, a) \in R$
$\therefore$ R is reflexive
$(a, b) \in R \Rightarrow 1 + ab > 0 \Rightarrow 1 + ba > 0 \Rightarrow (b, a) \in R$
$\therefore$ R is symmetric.
$\because$ $(a, b) \in R$ and $(b, c) \in R$ need not imply $(a, c) \in R$
Hence, R is not transitive.

E-11. **Sol.** $2x + x = 41 \Rightarrow x = \frac{41}{3} \notin N$ $\therefore$ R is not reflexive
$2x + y = 41 \not\Rightarrow 2y + x = 41$ $\therefore$ R is not symmetric
$2x + y = 41$ and $2y + z = 41 \Rightarrow 4x - z = 41 \Rightarrow (x, z)$ R
$\therefore$ R is not transitive

E-12. **Sol.** 1. R is not symmetric so it is incorrect.

2. $S_1 \neq S_2$ so not reflexive
Let $S_1 = \{1, 2, 3\}$ & $S_2 = \{1, 2\}$
it satisfies the condition
$S_1 \not\subset S_2 \Rightarrow S_2 \not\subset S_1$
So non symmetric.
let $S_1 = \{1, 2\}, S_2 = \{4, 5\}, S_3 = \{1, 2, 3\}$
as $S_1 \not\subset S_2$ and $S_1 \not\subset S_3$ S_1 S_3
so non transitive.

E-13. Sol. $aRb \Leftrightarrow n|(a-b)$ $a, b \in Z$
$n \in I_+$
(i) $aRa \Leftrightarrow n|(a-a) \Leftrightarrow$ so R is reflexive
(ii) $aRa \Leftrightarrow n|(a-b) = n|(b-a) \Leftrightarrow$ R is symmetric
(iii) $aRb \Leftrightarrow n|(a-b)$ and $n|(b-c)$
$\Rightarrow n|(a-b)+(b-c) \Leftrightarrow n|(a-c) \Leftrightarrow$ R is transitive

E-14. Sol. We have $(a, b)R(a, b)$ for all $(a, b) \in N \times N$
Since $a + b = b + a$. Hence, R is reflexive.
R is symmetric for all $(a, b), (c, d) \in N \times N$ we have $(a, b) R (c, d)$
$\Rightarrow$ $a + d = b + c$
$\Rightarrow$ $c + b = d + a$ $\Rightarrow$ $(c, d) R (a, b)$.
$(a, b)R (c, d)$ and $(c, d)R (e, f)$
$a + d = b + c$ and $c + f = d + e$,
$\Rightarrow a + d + c + f = b + c + d + e \Rightarrow a + f = b + e$
$\Rightarrow (a, b) R (e, f) \Rightarrow$ R is transitive
Thus, $(a, b) R (c, d)$ and $(c, d) R (e, f)$ $\Rightarrow$ $(a, b) R (e, f)$

E-15. Sol. $\ell_1 \| \ell_2 \Rightarrow$ R is reflexive.
$\ell_1 \| \ell_2 \Rightarrow \ell_2 \| \ell_1$ $\therefore$ R is symmetric.
$\ell_1 \| \ell_2 \Rightarrow \ell_2 \| \ell_3 \Rightarrow \ell_1 \| \ell_3$ $\therefore$ R is transitive.

E-16. Sol. $R = \{(x, y) ; x, y \in A , x + y = 5\}$ $A = \{1, 2, 3, 4, 5\}$
$R = \{(1 ,4), (2, 3), (3, 2), (4, 1)\}$
R is symmetric but neither reflexive nor transitive.

E-17. Sol. Reflexive Relation :-
$A.A \neq 0$ for $\forall A \in S$ so Relation is not Reflexive Relation
Symmetric Relation :-
$A.B = 0 \Rightarrow BA = 0$ Not True $\forall$ $A,B \in S$
for example $A = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$ $\Rightarrow AB = 0$ but $BA \neq 0$
Transitive Relation :-
$AB = 0, BC = 0 \Rightarrow AC = 0$ Not True, $\forall$ $A,B,C \in S$
for example $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ $\Rightarrow AB = 0, BC = 0$ but $AC \neq 0$

E-18. Sol. Reflexive Relation :-
$A.A = A.A$ for $\forall A \in S$, so Relation is Reflexive Relation
Symmetric Relation :-
$A.B = BA \Rightarrow BA = AB$ $\forall$ $A,B \in S$, so Relation is Symmetric Relation
Transitive Relation :-

AB = BA, BC = CB ⇒ AC = CA Not True, $\forall$ A,B,C∈S

for example $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ ⇒ AB = BA, BC = CB but AC ≠ CA

so Relation is not Transitive Relation

Section (F) : Function & its Domain and Range.

F-1. **Sol.** $f_1 : y = 5 - x$
(1, 4)(2, 3)(3, 2)(4, 1)
f_1 is a function
$f_2 : y < x$
(2, 1)(3, 1)(3, 2)(4, 1)(4, 2)(4, 3)
f_2 is not function

F-2. **Sol.** $25 - x^2 > 0 \Rightarrow (x - 5)(x + 5) < 0 \Rightarrow x \in (-5,5)$

F-3. **Sol.** $x - 5 \neq 0 \Rightarrow x \neq 5 \Rightarrow$ Domain $R - \{5\}$

F-4. **Sol.** $y = \sqrt{25 - x^2}$
$y \geq 0$ and $x = \sqrt{25 - y^2} \Rightarrow y^2 - 25 \leq 0 \Rightarrow y \in [0,5]$

F-5. **Sol.** Obvious (according diffenation of function)

F-6. **Sol.** Obvious (according diffenation of function)

F-7. **Sol.** We know that in a function each value of x there is exactly one value of y so (2) is answer

F-8 **Sol.** Here . $f_1 = \{(2, 3), (3, 4), (4, 5)\}$
$f_2 = \{(2,5), (3,4), (3,5), (4, 3), (4,4), (4,5), (5,2), (5,3), (5,4), (5,5)\}$
$f_3 = \{(3, 2), (4, 2), (4, 3), (5, 2), (5, 3), (5, 4)\}$
and $f_4 = \{(2, 5), (3, 4), (4, 3), (5, 2)\}$
Here, only f_4 is a function from A to itself.

Exercise-2

Marked Questions may have for Revision Questions.

PART - I : OBJECTIVE QUESTIONS

Sol. $A_1 \cup A_2 \cup A_3$ is the smallest element containing subset of all we set A_1, A_2 and A_3

2. **Sol.** 1. $((A \cap B) \cup C)' \cap B')'$
$= (A \cap B) \cup C) \cup B$
$= (A \cap B) \cup B \cup C$
$= B \cup C \neq B \cap C$

2. $(A' \cap B') \cap (A \cup B \cup C')$
$= (A \cup B)' \cap ((A \cup B) \cup C')$
$= \varphi \cup ((A \cup B)' \cap C')$
$= ((A \cup B) \cup C)'$
$= (A \cup (B \cup C))'$

3. **Sol.** $n(A \cup B) = 280$

$n(A' \cup B') = 2009 - n(A \cup B)$
$= 2009 - 280 = 1729 = 12^3 + 1^3 = 10^3 + 9^3$
$n(A - B) = 1681 - 1075 = 606$
$= 4 + 2 \times 301 = 4 + 2 \times 7 \times 43$
$= (2)\ 2 + 2 \times 7 \times 43$

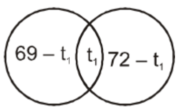
4. **Sol.** $x^3 + (x-1)^3 = 1$
$x^3 + x^3 - 3x^2 + 3x - 1 = 1$
$2x^3 - 3x^2 + 3x - 2 = 0$
$(x-1)(2x^2 - x + 2) = 0$
$x = 1$
$y = 0$ $(1, 0)$
Statement 1 is True

Statement 2 :
$X^3 + (1 - x)^3 = 1 \Rightarrow X^3 + 1 - 3x + 3x^2 - x^3 = 1$
$\Rightarrow x^2 - x = 0$
$x = 0, 1$ $(0, 1)\ (1, 0)$

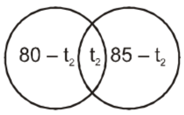
5. **Sol.** $n(M) = 23,\ n(P) = 24,\ n(C) = 19$
$n(M \cap P) = 12,\ n(M \cap C) = 9,\ n(P \cap C) = 7$
$n(M \cap P \cap C) = 4$
$n(M \cap P' \cap C') = n[M \cap (P \cup C)']$
$= n(M) - n(M \cap (P \cup C))$
$= n(M) - n[(M \cap P) \cup (M \cap C)]$
$= n(M) - n(M \cap P) - n(M \cap C) + n(M \cap P \cap C)$
$= 23 - 12 - 9 + 4 = 27 - 21 = 6$
$n(P \cap M' \cap C) = n[P \cap (M \cup C)']$
$= n(P) - n[P \cap (M \cup C) = n(P) - n[P \cap M) \cup (P \cap C)]$
$= n(P) - n(P \cap M) - n(P \cap C) + n(P \cap M \cap C)$
$= 24 - 12 - 7 + 4 = 9$
$n(C \cap M' \cap P') = n(C) - n(C \cap P) - n(C \cap M) + n(C \cap P \cap M)$
$= 19 - 7 - 9 + 4 = 23 - 16 = 7$

6.

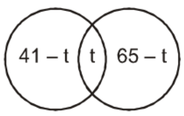
Sol.



$70 + 72 - t_1 = 100$
$t_1 = 42\%$ $\Rightarrow$ min. in $P \cap C = 42\%$



$t_2 = 85\% - 20\% = 65\%$ $\Rightarrow$ min. $M \cap E = 65\%$



$t = 42 - 35 = 7\%$
min. in $((P \cap C) \cap (M \cap E)) = 7\%$

7. **Sol.** For any $x \in R$, we have $x - x + \sqrt{2} = \sqrt{2}$ an irrational number
$\Rightarrow$ xRx for all x. So, R is reflexive.
R is not symmetric, because $\sqrt{2}R1$ but $1 \not R \sqrt{2}$, R is not transitive also because $\sqrt{2}$ R1 and

$\sqrt{2}$ 1R2 $\sqrt{2}$ but $\sqrt{2}$ $\not R_2$ $\sqrt{2}$

8. **Sol.** $((m,n), (p,q)) \in S \Rightarrow m + q = n + p$
$((p,q), (r,s)) \in S \Rightarrow p + s = r + q$
$\Rightarrow ((r,s), (m,n)) \in s$
as $r + n = m + s$
Now if we add above equation
$((m,n), (p,q)) \in s \Rightarrow m + q = n + p$
$\Rightarrow (n+p) = m + q$ & hence $((P,q), (m,n))$

9. **Sol.** $R_1 : m + 4n = 5n + (m - n)$
$R_2 : m + 9n = 10n + (m - n)$
If $5n + (m - n)$ is divisible by 5 then $10n + (m - n)$ is also divisible by 5 and vice versa.
hence $R_1 = R_2$
Also R_1 & R_2 is symmetric relation on Z.

10. **Sol.** $(x, y) \in X \Leftrightarrow x < y$ and $y = x + 5 \Rightarrow (x, x) \notin X$, Hence not reflective
If $(x, y) \in X \Rightarrow x < y$ and $y = x + 5$
$(y, x) \in X \Rightarrow$ X is not symmetric
Let $(x, y) \in X$ and $(y, z) \in X \Rightarrow x < y$; $y = x + 5$ and $y < z$; $z = y + 5$
$\Rightarrow x < z$ and $z = x + 10 \Rightarrow (x, z) \notin X$
Not transitive

PART - II : MISCELLANEOUS QUESTIONS

A-1. **Ans. (1)**
Sol. $X \cap (Y \cup X)' = X \cap (Y' \cap X') = X \cap X' \cap Y' = \varphi$
$\Rightarrow$ Statement – 1 true.
$X \Delta Y = (X \sim Y) \cup (Y \sim X) = (X \cup Y) \sim (X \cap Y)$
$\Rightarrow$ number of element in $X \Delta Y = m - n$.
$\Rightarrow$ Statement-2 is true but does explain statement-1

A-2^ **Ans. (1)**
Sol. $n(A) = 3$
$n(B) = 2$
$\therefore$ $n(A \times B) = 3 \times 2 = 6$
$\therefore$ number of subsets of $A \times B = 2_6 = 64$
$\therefore$ number of relation from A to B = 64

A-3. **Ans. (2)**
Sol. Obviously statement -1 is true and statement 2 is false

B-1. **Ans.** (1) → (q), (2) → (r), (3) → (s), (4) → (p)
Sol. (1) The set $\{3^{2n} - 8n - 1 : n \in N\}$ contains 0 and every element of this set is a multiple of 64.
(2) $2^{3n} - 1$ is always divisible by 7.
(3) $3^{2n} - 1$ is always divisible by 8.
(4) $2^{2n} - 7n - 1$ is always divisible by 49 and $2^{3n} - 7n - 1 = 0$ for n = 1.

B-2. **Ans.** (1) → (p), (2) → (r), (3) → (s), (4) → (q)
Sol. Obvious (according diffenations of Reflexive, Symmectric and Transitive relation)

C-1. **Sol.** $n(A \cup B)$ is minimum when $n(A \cap B)$ is maximum i.e. 3.
$\therefore$ minimum $n(A \cup B) = 6$
$n(A \cup B)$ is maximum when $n(A \cap B)$ is minimum i.e. 0

$\therefore$ maximum $n(A \cup B) = 9$

C-2. **Sol.** Obviously

C-3. **Sol.** $n\ R\ m \Rightarrow n$ is factor of m
(i) R is reflexive
(ii) R is not symmetric because $\frac{m}{n} = K$ but $\frac{n}{m}$ N
(iii) R is transitive.

C-4. **Sol.** $n(A) = 21$, $n(B) = 26$, $n(C) = 29$
$n(A \cap B) = 14$, $n(A \cap C) = 12$, $m(B \cap C) = 13$, $n(A \cap B \cap C) = 8$
$n(C \cap A' \cap B') = n(C \cap \overline{A \cup B}) = n(C) - n((C \cap A) \cup (C \cap B))$
$n(C) - [n(C \cap A) + n(C \cap B) - n(A \cap B \cap C)]$
$29 - [12 + 13 - 8] = 12$
$n(A \cap B \cap C') = n(A \cap B) - n(A \cap B \cap C) = 14 - 8 = 6$

Exercise-3

1. **Sol.** $(1, 1) \notin R$ $\therefore$ not reflexive
$(2, 3) \in R$ and $(3, 2) \notin R$ not symmetric
$(1, 3) \in R$ and $(3, 1) \in R$ but $(1, 1) \notin R$
$\therefore$ not transitive

2. **Sol.** $(x, x) \in R \quad \forall\ x \in W$ $\Rightarrow$ R is reflexive
Let $(x, y) \in R$, then $(y, x) \in R$ [$\because$ x, y have at least one letter in common]
$\Rightarrow$ R is symmetric.
But R is not transitive
eg. (TALL) R (LIGHT) and (LIGHT) R (HIGH) but (TALL) $\not R$ (HIGH)

3. **Sol.** ;
(1) S is not reflexive so not equivalence as $x \neq x + 1$
(2) $(x, y) \in T \Rightarrow x - y$ is an integer

(i) $x - x$ is an integer $\Rightarrow$ reflexive
(ii) $x - y$ = integer $\Rightarrow y - x$ = integer $\therefore$ T is symmetric
(iii) $x - y = m$, $y - z = n$
$\Rightarrow x - y + y - z = m + n$
$x - z = m + n$ $\Rightarrow$ Transitive
so T is equivalence relation

4. **Sol.** We have, $A \cup B = A \cup C$ $\Rightarrow$ $(A \cup B) \cap C = (A \cup C) \cap C$
$\Rightarrow (A \cap C) \cup (B \cap C) = C$ [$\because (A \cup C) \cap C = C$]
$\Rightarrow (A \cap B) \cup (B \cap C) = C$ ...(i) [$\because A \cap C = A \cap B$]
Again, $A \cup B = A \cup C$
$\Rightarrow (A \cup B) \cap B = (A \cup C) \cap B$ $\Rightarrow$ $B = (A \cap B) \cup (C \cap B)$
$\Rightarrow (A \cap B) \cup (C \cap B) = B \Rightarrow$ $(A \cap B) \cup (B \cap C) = B$ ...(ii)
From (i) and (ii), we get $B = C$

5. **Sol.** $(x, x) \in R$ for $w = 1$
$\therefore$ R is reflexive
If $x \neq 0$, then $(0, x) \in R$ for $w = 0$ but $(x, 0) \notin R$ for any w

∴ R is not symmetric
⇒ R is not equivalence relation

$\left(\frac{m}{n}, \frac{p}{q}\right) \in S \Rightarrow qm = pn \Rightarrow \frac{m}{n} = \frac{p}{q}$

(i) $\frac{m}{n} = \frac{m}{n} \Rightarrow \left(\frac{m}{n}, \frac{m}{n}\right) \in S \Rightarrow$ Reflexive

(ii) $\frac{m}{n} = \frac{p}{q} \Rightarrow \frac{p}{q} = \frac{m}{n} \Rightarrow$ symmetric

(iii) $\frac{m}{n} = \frac{p}{q}$ and $\frac{p}{q} = \frac{x}{y}$

$\Rightarrow \frac{m}{n} = \frac{x}{y} \Rightarrow$ transitive

⇒ S is equivalence relation

6. **Sol.** **Statement - 1 :**
(i) $x - x$ is an integer $\forall x \in R$ so A is reflexive relation.
(ii) $y - x \in I \Rightarrow x - y \in I$ so A is symmetric relation.
(iii) $y - x \in I$ and $z - y \in I \Rightarrow y - x + z - y \in I$
$\Rightarrow z - x \in I$ so A is transitive relation.
Therefore A is equivalence relation.
Statement - 2 :
(i) $x = \alpha x$ when $\alpha = 1 \Rightarrow$ B is reflexive relation
(ii) for $x = 0$ and $y = 2$, we have $0 = \alpha(2)$ for $\alpha = 0$
But $2 = \alpha(0)$ for no α
so B is not symmetric so not equivalence.

7. **Sol.** **for reflexive**
$(A, A) \in R \Rightarrow A = P^{-1} A P$
which is true for $P = I$
∴ reflexive
for symmetry
As $(A, B) \in R$ for matrix P
$A = P^{-1} BP \Rightarrow PA = PP^{-1} BP \Rightarrow PAP^{-1} = IBPP^{-1}$
$\Rightarrow PAP^{-1} = IBI \Rightarrow PAP^{-1} = B \Rightarrow B = PAP^{-1}$
∴ $(B, A) \in R$ for matrix P^{-1} ∴ R is symmetric
for transitivity
$A = P^{-1} BP$ and $B = P^{-1}CP \Rightarrow A = P^{-1} (P^{-1} CP)P$
$\Rightarrow A = (P^{-1})^2 CP^2 \Rightarrow A = (P^2)^{-1} C(P^2)$
∴ $(A, C) \in R$ for matrix P^2 ∴ R is transitive
so R is equivalence

8. **Sol.** xRy ⇒ $x < y$
(i) for reflexive xRx ⇒ $x < x$ Not reflexive
(ii) xRy ⇒ $x < y$
⇒ $y \le x$ ⇒ not symmetric
(iii) xRy ⇒ $x < y$
yRz ⇒ $y < z$
⇒ $x < z$
⇒ xRz transistive

9. **Sol.** $x = |y|$ & $y = |z| \Rightarrow x = |z|$

10. **Sol.** Every element has 3 options. Either set Y or set Z or none
so number of ordered pairs = 3^5

11. **Sol.** **(3)**
n(A) = 2
n(B) = 4
n(A × B) = 8
${}_8C_3 + {}_8C_4 + \ldots\ldots + {}_8C_8 = 2^8 - {}_8C_0 - {}_8C_1 - {}_8C_2$
= 256 – 1 – 8 – 28
= 219

12. **Sol.** X = {0, 9, ... $4^n - 3n - 1$}
Y = {0, 9, ..., 9(n – 1)}
Now $4^n - 3n - 1 = (3 + 1)^n - 3n - 1$
$= 3^n + n.3^{n-1} + \ldots. + {}^nC_2 .9.$
is a multiple of 9.
Also Y consists of all multiples of '9' from 0, 9,.......
Hence all values of X are subset of values of Y.
Thus X ∪ Y = Y

13.* **Sol.** Number of subsets = 2^m
also number of subsets = 2^n
$2^m - 2^n = 56$
m = 6 n = 3
(m, n) = (6, 3)

14. **Sol.** n(A) = 4
n(B) = 2
n(A × B) = 8
So atleast 3 = total – none – one – two digits
$= 2^8 - 1 - {}_8C_1 - {}_8C_2$
$= 2^8 - 1 - 8 - 28$
$= 2^8 - 37$
= 256 – 37 = 219