# Exercise-1

## OBJECTIVE QUESTIONS

### Section (A) : Classical definition of probability

**A-1.** **Sol.** Max sum = 12

$$\left.\begin{array}{l} 6+6=12 \\ 6+5=11 \\ 6+4=10 \\ 5+5=10 \end{array}\right\}$$ 6 cases

P = $\frac{6}{36} = \frac{1}{6} = \frac{1}{6}$

**A-2.** **Sol.** Total number of multiple of 5=24. Total number of multiple of 15 = 8 i.e., n(A) = 24, n(B) = 8 and n(A∩B) = 8

∴ n(A∪B) = n(A) + n(B) – n(A∩B)

= 24+8–8 = 24

n(S) = 120,

∴ Reueired probability = $\frac{24}{120} = \frac{1}{5}$

**A-3.** Sol. P(A) = $\frac{1}{3}$ P(B) = $\frac{1}{4}$ and P(C) = $\frac{1}{5}$

Required probability

= $P(A\bar{B}\bar{C}$ or $\bar{A}B\bar{C}$ or $\bar{A}\bar{B}C)$

= P(A). P($\bar{B}$). P($\bar{C}$) + P($\bar{A}$) P(B) P($\bar{C}$)+ P($\bar{A}$) P($\bar{B}$) P(C)

= $\frac{1}{3}\times\frac{3}{4}\times\frac{4}{5}+\frac{2}{3}\times\frac{1}{4}\times\frac{4}{5}+\frac{2}{3}\times\frac{3}{4}\times\frac{1}{5}$ = $\frac{12+8+6}{60}$ = $\frac{13}{30}$

**A-4.** **Sol.** Total ways = 14–1! = 13!

favorable ways = 7! . 6!

**A-5.** **Sol.** 1(S) + 1(D) or 1(D) + 1(S)

$\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}=\frac{1}{2}$

**A-6.** **Sol.** ${}_4C_1$ × ${}_{13}C_9$ × ${}_{39}C_4$ = Formula card

Suit any 9 cards any 4 cards from 39 cards

${}_{52}C_{13}$ = total case

**A-7.** **Sol.** $1_{st}$ coupon can be selected in 9 ways

$2_{nd}$ coupon can be selected in 9 ways

$3_{rd}$ coupon can be selected in 9 ways

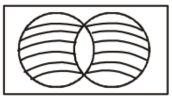
$9_7$ ways – when 9 is not take

= $9_7 – 8_7$

Total = $15_7$.

**A-8. Sol.** $\dfrac{\dfrac{2n-2!}{n-1!\,n-1!2!}\times 2!}{\dfrac{2n!}{n!\,n!2!}} = P$

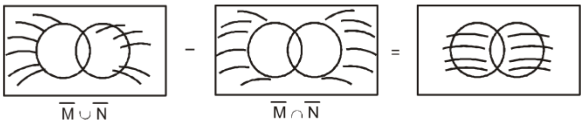
**A-9. Sol.**



P = P(M∪N) – P(M∩N)

= P(M) + P(N) – 2P(M∩N)

(c) $P(\bar{M}\cup\bar{N}) - P(\bar{M}\cap\bar{N})$




**A-10. Sol.** Favourable case = (12–1)! × 2!

Total (13 – 1)!

p = $\dfrac{11!\times 2!}{12!} = \dfrac{2}{12} = \dfrac{1}{6}$

**A-11. Sol.** Favourable no. of ways = 12

total no. of ways = 220

P = $\dfrac{12}{220} = \dfrac{3}{55}$

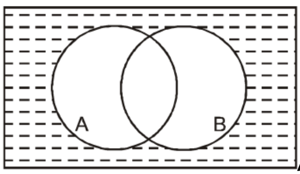
**A-12. Sol.** Let A : card is spade

B : card is an ace.

P(A) = $\dfrac{13}{52}$ P(B) = $\dfrac{4}{52}$

P(A ∩ B) = $\dfrac{1}{52}$

**A-13. Sol.**




$A_c$ – B

**A-14. Sol.** p (A) = $\dfrac{3}{6}$, p (B) = $\dfrac{2}{6}$

A ≡ {1, 3, 5 } B≡ { 3, 6 }

B $\not\subset$ A .

B – A = {6} as follows

**A-15. Sol.** $p_1 + p_2 + p_3 + p_4 = 1$ in in D obvious solution follows

**A-16. Sol.** $_2C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^0=\frac{1}{4}$

**A-17. Sol.** $P(A\cup B)=\frac{3}{4}$, $P(A\cap B)=\frac{1}{4}$, $P(A)=\frac{1}{3}$

$P(A\cap B) = P(A)+ P(B) – P(A\cup B)$

$\frac{1}{4}=\frac{1}{3}+P(B)-\frac{3}{4}\Rightarrow P(B)=\frac{2}{3}$

$P(\bar{A}\cap B)=P(B)-P(A\cap B)=\frac{2}{3}-\frac{1}{4}=\frac{5}{12}$

## Section (B) : Addition theorem

**B-1. Sol.** P(atleast one W) = $P(1W, 1M) + P(2W_1\ 0M)$

$=\frac{5\times 8}{^{13}C_2}+\frac{^5C_2}{^{13}C_2}$

**B-2. Sol.** $P(A)=\frac{3}{11}$

$P(B)=\frac{2}{7}$

$P(C) = P$

Now $P(A) + P(B) + P(C) = 1$

$\frac{3}{11}+\frac{2}{7}+P=1 \quad\Rightarrow\quad P=1-\frac{43}{77}=\frac{34}{77}$

odds against C = 43 : 34

**B-3. Sol.** Since , $0 \le P(A) \le 1$, $0 \le P(B) \le 1$, $0 \le P(C) \le 1$ and $0 \le P(A) + P(B) + P(C) \le 1$

$\therefore\ 0\le\frac{3p+1}{3}\le 1$

$\Rightarrow\quad -\frac{1}{3}\le p\le\frac{2}{3}$ ...(i)

$0\le\frac{1-p}{4}\le 1$

$\Rightarrow\quad -3\le p\le 1$ ...(ii)

$0\le\frac{1-2p}{2}\le 1$

$\Rightarrow\quad -\frac{1}{2}\le p\le\frac{1}{2}$ ...(iii)

and $0\le\frac{3p+1}{3}+\frac{1-p}{4}+\frac{1-2p}{2}\le 1$

$\Rightarrow 0 \le 13 – 3p \le 12$

$\Rightarrow\ \frac{1}{3}\le p\le\frac{13}{3}$ ...(iv)

From Eqs. (i), (ii), (iii) and (iv), we get

$\frac{1}{3}\le p\le\frac{1}{2}$.

**B-4.** **Sol.** if events are not exclusive (and are exhaustive) then p (A) + P (B) + p (C) $\geq$ 1

## Section (C) : Conditional probability, dependent and independent events

**C-1.** **Sol.** Let event A : 6 comes on Ist die
B: sum is 7

$$\left.\begin{matrix}6+1\\5+2\\4+3\end{matrix}\right\}$$

p (A) = $\frac{1}{6}$ , p (B) = $\frac{6}{36}$ = $\frac{1}{6}$

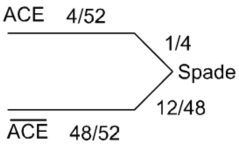
A $\cap$ B = $\frac{1}{36}$ ; p (A $\cap$ B) = p (A) × p (B)

A $\cap$ B = $\frac{1}{36}$ ; p (A $\cap$ B) = p (A) × p (B)

**C-2.** **Sol.** odd —— 1, 3, 5.

p (prime/ odd ) = $\frac{2}{3}$

**C-3.** **Sol.**




Fav., case = 1
Total case = 13

Probability = $\frac{1}{13}$

**C-4.** **Sol.** 2 + 6 = 8 p = $\frac{1}{5}$
3 + 5 = 8
4 + 4 = 8
5 + 3 = 8
6 + 2 = 8

**C-5.** **Sol.** 2W & 4B

P = $^5C_4 \times \left(\frac{2}{6}\right)^4 \left(\frac{4}{6}\right)^1 + {^5C_5}\left(\frac{2}{6}\right)^5$

**C-6.** **Sol.** 1 – P(BB)
1 – 1/2 × 1/2 = 1 – 1/4 = 3/4

**C-7.** **Sol.** $P(S_1) . P(\bar{S}_2) . P(\bar{S}_3) + P(\bar{S}_1) P(S_2) P(\bar{S}_3) + P(\bar{S}_1) P(\bar{S}_2) P(S_3)$

$= \frac{1}{3} \times \frac{5}{7} \times \frac{5}{8} + \frac{2}{3} . \frac{2}{7} . \frac{5}{8} + \frac{2}{3} . \frac{5}{7} . \frac{3}{8} = \frac{25}{56}$

**C-8.** **Sol.** p (A / B) = $\frac{p(A \cap B)}{p(B)} = \frac{0.1 + 0.1}{0.3} = \frac{2}{3}$ .
similarly evaluate others

**C-9.** **Sol.** $P(A \cap B) = \frac{1}{6}$ $\Rightarrow$ $P(A).P(B) = \frac{1}{6}$

$P(\bar{A}) = \frac{2}{3}$ $\Rightarrow$ $P(A) = \frac{1}{3}$ $\Rightarrow$ $P(B) = \frac{1}{2}$

$\therefore$ $6P(B/A) = 6P(B) = 3$

**C-11._** **Sol.** Use venn diagram

**C-12.** **Sol.** (ii) $p\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

**C-13.** **Sol.** A & B are independent

$P(A \cup B)^c = 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A \cap B)$

$= P(\bar{A}) - P(B) + P(A)\,P(B) = P(\bar{A}) - P(\bar{A})P(B)$

$= P(\bar{A})P(\bar{B})$

**C-14.** **Sol.** $P(\bar{M} \cap \bar{N}) = 1 - P(M \cup N)$ $= 1 - P(M) - P(N) + P(M)\,P(N)$

$= (1 - P(M))\,(1 - P(N))$ $= \overline{P(M)}\ \overline{P(N)}$

$P(M \cap\ ) = P(M) - P(M \cap N) = P(M) - P(M)\,P(N) = P(M)\,P()$

and $P\left(\frac{M}{N}\right) + P\left(\frac{\bar{M}}{N}\right) = \frac{P(M \cap N)}{P(N)} + \frac{P(N) - P(M \cap N)}{P(N)} = 1$

## Section (D) : Total probability theorem, Baye's theorem

**D-1.** **Sol.** p (I$^{st}$ class) = $\frac{2}{3} \times 0.28 + \frac{1}{3} \times 0.25 = 0.27$

**D-2.** **Sol.** $P(E) = P(A)\,P(E/A) + P(B)\,P(E/B) = \frac{1}{2}.\frac{2}{5} + \frac{1}{2}.\frac{4}{6} = \frac{8}{15}$

**D-3.** **Sol.** Required probability $= \frac{^5C_1}{^{12}C_1} \times \frac{^4C_1}{^{12}C_1} + \frac{^7C_1}{^{12}C_1} \times \frac{^8C_1}{^{12}C_1} = \frac{76}{144}$

**D-4.** **Sol.** $P(R) = P(P).\,P\left(\frac{V.C}{P}\right) + P(Q).P\left(\frac{V.C}{Q}\right) = \frac{1}{3} \times \frac{^2C_1.{}^3C_1}{^5C_2} + \frac{2}{3}.\frac{^1C_1.{}^4C_1}{^5C_2}$

$= \frac{1}{3} \times \frac{6}{10} + \frac{2}{3} \times \frac{4}{10} = \frac{6+8}{30} = \frac{7}{15}$

**D-5.** **Sol.** U1 – 1W + 1B U2 $\rightarrow$ 2W + 3B

U3 $\rightarrow$ 3W + 5B U4 $\rightarrow$ 4W + 7B

$P(W) = \sum_{i=1}^{4} (u_1)\,P(w/u_i) = \sum_{i=1}^{4} \frac{i^2+1}{34}$ $P(w/v_i)$

$= \frac{1^2+1}{34}\times\frac{1}{2}+\frac{2^2+1}{34}\times\frac{2}{5}+\frac{3^2+1}{34}\times\frac{3}{8}+\frac{4^2+1}{34}\times\frac{4}{11} = \frac{569}{1496}$

**D-6.** **Sol.** Number of kings left are 3.

cards are 51 $p = \frac{3}{51} = \frac{1}{17}$

**D-7.** **Sol.** $\frac{7}{12} = \frac{1}{n+1}\times 1 + \frac{n}{n+1}\times\frac{1}{2}$ solve for n of get n ds fy, gy djsaA

**D-8.** **Sol.** A : 1 ball is W & 3 black balls
$B_1$ : Urn 1 is chosen
$B_2$ : Urn 2 is chosen

$P(B_1/A) = \frac{P(A/B_1)P(B_1)}{P(A/B_1)P(B_1)+P(A/B_2)P(B_2)}$

$P(B_1/A) = \frac{\frac{1}{2}\times\frac{4}{9}\times\left(\frac{5}{9}\right)^3\times{}^4C_3}{\frac{1}{2}\times\frac{4}{9}\times\left(\frac{5}{9}\right)^3\times{}^4C_3+\frac{1}{2}\times\frac{3}{9}\times\left(\frac{6}{9}\right)^3\times{}^4C_3} = \frac{125}{287}$

**D-9.** **Sol.** A = 2 nd ball in white
$B_1$ = 1st ball in white
$B_2$ = 1st is black

$P(B_1/A) = \frac{p(A/B_1)p(B_1)}{p(A/B_1)p(B_1)+p(A/B_2)p(B_2)} = \frac{\frac{4}{7}\times\frac{3}{6}}{\frac{4}{1}\times\frac{3}{6}+\frac{3}{7}\times\frac{4}{6}}$

## Section (E) : Probability distribution and binomial probability distribution

**E-1.** **Sol.** $\frac{np}{npq} = \frac{3}{2}$ $\Rightarrow$ $q = \frac{2}{3}$ $\Rightarrow$ $p = \frac{1}{3}$

$r \le \frac{11}{1+2}$ $\Rightarrow$ $r \le \frac{1}{3}$ $\Rightarrow$ $r \le 3.66$
thus 3 succes is most probable.

**E-2.** **Sol.** $n = 3,\ p = \frac{2}{6} = \frac{1}{3}$

mean = np = $3 \times \frac{1}{3} = 1$

variance $\sigma_2$ = npq = $\sqrt{3\times\frac{1}{3}\times\frac{2}{3}} = 3\times\frac{1}{3}\times\frac{2}{3}$

$\sigma_2 = \frac{2}{3}$

**E-3.** **Sol.** ${}_3C_2\,P_2\,(1-P) = 12\,{}_3C_3\,P_3$

$1 - P = 4P \Rightarrow \frac{1}{5} = p$

**E-4.** **Sol.** P(atleast 4) = P(4) + P(5)

$= {}_5C_4 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^4 + {}^5C_5\left(\frac{9}{10}\right)^5$

**E-5.** **Sol.** $(P + q)_{99}$ $r \le \frac{99+1}{1+\left|\frac{1/2}{1/2}\right|}$ $\Rightarrow$ $r \le \frac{100}{2}$ $\Rightarrow$ $r \le 50$

Terms 50 or 51 are highest

**E-6.** **Sol.**

$$\left.\begin{array}{l} p(x=4) = {}^n c_4 \left(\frac{1}{2}\right)^n \\ p(x=5) = {}^n c_5 \left(\frac{1}{2}\right)^n \\ p(x=6) = {}^n c_6 \left(\frac{1}{2}\right)^n \end{array}\right]$$

$2\ {}_nC_5 = {}_nC_4 + {}_nC_6$

$4\ {}_nC_5 = {}_{n+1}C_5 + {}_{n+1}C_6$

$4\ {}_nC_5 = {}_{n+2}C_6$

$4 \cdot \frac{n!}{5!\,(n-5)!} = \frac{(n+2)!}{6!(n-4)!}$

$4 = \frac{(n+2)(n+1)}{6(n-4)} \Rightarrow 24\,(n-4) = (n+2)\,(n+1)$

$n = 7, 14$

**E-7.** **Sol.** We need to calculate here

Probability that out of n bombs at least two strike > 0.99

i.e. 1 – prob (none strikes) – prob (exactly one strikes) > 0.99

i.e $1 - {}_nC_0 \left(\frac{1}{2}\right)^n - {}_nC_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{n-1} > 0.99$

i.e. $0.01 \ge \frac{1+n}{2^n}$

i.e. $2_n \ge 100 + 100n$

Least values of n is 11

**E-8.** **Sol.** ${}^5C_2 \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^3 = \frac{625}{3888}$

# Exercise-2

**Marked Questions may have for Revision Questions.**

## PART - I : OBJECTIVE QUESTIONS

## ***Note : Tough Questions***

**1.** **Sol.** Since sum of 1+2+3 +......9 = $\frac{9\times10}{2}$ = 45 is disisible by 9, hance all no. will be divisible by 9.

**2.** **Sol.** p = $\frac{4}{12}.\frac{3}{11}.2$

**3.** **Sol.** $\frac{6.7.5}{{}^{18}C_3} = \frac{35}{136}$

**4.** **Sol.** Favourable case : (3,3,3,3) or (3,3,3,5) = 1+ $\frac{4!}{3!}$ = 5
total number of way → $2_4$
p = $\frac{5}{2^4}$

**5.** **Sol.** Coefficient of $x_8$ : $(x_0 + x_1 + ...x_6)_4$ = $\left(\frac{1-x^7}{1-x}\right)^4$
= $(1 - x_7)_4\ (1 - x)_{-4}$
= $(1 - x_7)_4\ (1 - x)_{-4} = (1 - 4x_7)\ (1 - x)_{-4}$
Total ways $a = {}_{4+8-1}C_8 - 4\ {}_{4+1-1}C_1 = {}_{11}C_8 - 4 \times 4$
= 165 – 16 = 149.
P = $\frac{149}{7^4}$

**6.** **Sol.**

| Unit digit in number | Unit digit in number | Unit digit in product |
|---|---|---|
| Odd | Odd | Odd |
| Odd | Even | Even |
| Even | Odd | Even |
| Even | Even | Even |

p = $\frac{3}{4}$ ⇒ q = $\frac{1}{4}$ ⇒ $\frac{p}{q}$ = 3

**7.** **Sol.** Required probability = 1 – both number are not divisible by 5 = 1 – $\frac{8}{10}\times\frac{8}{10} = \frac{9}{25}$

**8.** **Sol.** $\frac{6}{10}\times\frac{5}{9}+\frac{4}{10}\times\frac{6}{9}=\frac{54}{90}=\frac{3}{5}$

**9.** **Sol.** Even integers ends in 0, 2, 4, 6, 8. Square of an even integer ends in 4 only when the integer ends either in 2 or 8.

$\therefore$ probability = $\frac{2}{5}$

**10. Sol.** KRISHNAGIRI or DHARMAPURI
A = RI is visible
$B_1$ = its from KRISHNAGIRI
$B_2$ = its from DHARMAPURI

$$P(B_1/A) = \frac{P(A/B_1)P(B_1)}{P(A/B_1)P(B_1)+P(A/B_2)P(B_2)} = \frac{\frac{1}{2}\times\frac{2}{10}}{\frac{1}{2}\times\frac{2}{10}+\frac{1}{2}\times\frac{1}{9}} = \frac{9}{14}$$

**11. Sol.** $\left(\frac{13}{52}\right)^2\times\left(\frac{13}{52}\right)^2\times\left(\frac{13}{52}\right)^2\times\frac{6!}{2!2!2!}=P$

**12. Sol.** $E_1$ = {(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5) }
$E_2$ = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) }
$E_3$ = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}
clearly (1), (2) and (3) are correct.

**13. Sol.** P(T 1) = p
P(T 2) = q
P(T 3) = 1/2
$\frac{1}{2}$ = P(T 1, T 2) + P(T 1, T3) + P(T 1 T 2 T 3)
$\frac{1}{2}$ = pq 1/2 + p(1 – q) $\frac{1}{2}$ + pq $\frac{1}{2}$
$\frac{1}{2} = \frac{pq}{2} + \frac{p}{2}$ $\Rightarrow$ 1 = pq + p.
Now, check options

**14. Sol.** A = {1,3,5}
B = {2,4,6}
C = {4,5,6}
D = {1,2}

**15. Sol.** 10 coins 9 5 paisa 10 coins 5 paisa
1 1 Rs.

p = p (1 Rs. transfered + Back transfered) + p (1 Rs. not transfered) $\frac{{}^9C_8}{{}^{10}C_9}\times\frac{{}^{18}C_8}{{}^{19}C_9}+\frac{{}^9C_9}{{}^{10}C_9}=\frac{10}{19}$

method 2 when 1 Rs coin is in second purse and did not came back in first purse this prob. = $\frac{{}^9C_8\times{}^1C_1}{{}^{10}C_9}$ $\times\frac{{}^{18}C_9}{{}^{19}C_9}=\frac{9}{19}$ $\Rightarrow$ Required probability = $1-\frac{9}{19}=\frac{10}{19}$

**16. Sol.** p (A) = $\frac{13}{52}+\left(\frac{39}{52}\right)^3\left(\frac{13}{52}\right)+\left(\frac{39}{52}\right)^6\left(\frac{13}{52}\right)+....$

p (B) = $\left(\frac{39}{52}\right)\frac{13}{52}+\left(\frac{39}{52}\right)^4\frac{13}{52}+\left(\frac{39}{52}\right)^7\left(\frac{13}{52}\right)+...$

p (C) = $\left(\frac{39}{52}\right)^2\left(\frac{13}{52}\right) + \left(\frac{39}{52}\right)^5\left(\frac{13}{52}\right) + \ldots$

**17.** **Sol.** Required probability = p = ${}_2C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{2}$

∴ 6p = 3

**18.** **Sol.** Since line are more ${}_NC_M$ are those lines where telegrams will go

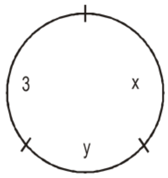
${}_NC_M$ × M ! = farvourable case

Total = $N_M$ [As first telegram can go in any ore of n lies]

[As 2nd telegram can go in any ore of n lies]

P = $\frac{{}^NC_M M!}{N^M}$

**19.** **Sol.**




x + y + z = 8 x, y, z > 0

x'+ y' + z' = 5

$\frac{{}^{5+3-1}C_3 - 1\times{}^{11}c_1}{3}$

Total = ${}_{11}C_3$

**20.** **Sol.** $\frac{2}{5}$ = (1 – P) P + $(1 – P)_3$ P + $(1\text{-}P)_5$ p + ....

$\frac{2}{5}$ = P (1 – P){1 + $(1 – P)_2$ + $(1 – P)_4$ + ....}

$\frac{2}{5}$ = P (1 – P) $\left[\frac{1}{1-(1-P)^2}\right]$

$\frac{2}{5}$ = P (1 – P) $\left[\frac{1}{P(2-P)}\right]$

3P = 1 ⇒ P = 1/3

**21.** **Sol.** $625p_2 – 175p + 12 < 0$ gives p ∈ $\left(\frac{3}{25}, \frac{4}{25}\right)$

$\left(\frac{4}{5}\right)^{n-1}.\frac{1}{5} = p$

∴ $\frac{3}{25} < \left(\frac{4}{5}\right)^{n-1} . \frac{1}{5} < \frac{4}{25}$

i.e. $\frac{3}{5} < \left(\frac{4}{5}\right)^{n-1} < \frac{4}{5}$

value of n is 3

**22. Sol.** $1 - \left(\frac{3}{5}\right)^{0} . \left(\frac{2}{5}\right)^{3} = \frac{117}{125}$

## PART - II : MISCELLANEOUS QUESTIONS

### Section (A) : ASSERTION/REASONING

**A-1. Ans. (3)**

**Sol. Statement - 2 :** True (By definition)

**Statement -1 :** False because the sample points are not equally likely.

**A-2. Ans. (1)**

**Sol.** Statement-2 $P(A/B) = P(A) \Leftrightarrow P(A) = \frac{P(A \cap B)}{P(B)} \Leftrightarrow P(A)\,P(B) = P(A \cap B)$

A and B are mutually exclusive $\Rightarrow P(A \cap B) = 0$

$\Rightarrow P(B)\,P(A/B) = 0 \Rightarrow P(A/B) = 0 \Rightarrow P(A/B) \neq P(A)$

$\therefore$ statement-2 is true

Statement-1 Suppose A and B are mutually exclusive, then by statement-2 $P(A/B) \neq P(A)$ which is a contradiction.

$\therefore$ statement-1 is true.

$P(A/B) = P(A) \Leftrightarrow P(A) = \frac{P(A \cap B)}{P(B)} \Leftrightarrow P(A)\,P(B) = P(A \cap B)$

$\Rightarrow P(B)\,P(A/B) = 0 \Rightarrow P(A/B) = 0 \Rightarrow P(A/B) \neq P(A)$

**A-3. Ans. (1)**

**Sol. (2)**

(2) We must have

$0 \leq \frac{1+4P}{4} \leq 1,\ 0 \leq \frac{1-P}{4} \leq 1$ and $0 \leq \frac{1-2P}{4} \leq 1$

$\Rightarrow -\frac{1}{4} \leq P \leq \frac{3}{4},\ -3 \leq P \leq 1,\ -\frac{3}{2} \leq P \leq \frac{1}{2}$

Again the events are pair-wise mutually exclusive so

$0 \leq \frac{1+4P}{4} + \frac{1-P}{4} + \frac{1-2P}{4} \leq 1$

$\Rightarrow -3 \leq P \leq 1$

Taking intersection of all four intervals of 'P'

We get $-\frac{1}{4} \leq P \leq \frac{1}{2}$

**A-4. Ans. (1)**

**Sol.** $ax + by = 0$ $\quad a, b, c, d \in \{0, 1\}$

$cx + dy = 0$

system has unique solution if and only if $ad - bc \neq 0$

For which a = d = 1 and bc = 0 $\Rightarrow$ 3 combination
Similarly if bc = 1, ad = 0 $\Rightarrow$ 3 combination
Total choice for a, b, c, d is $2^4$

Hence probability of unique solution is $\frac{6}{16} = \frac{3}{8}$
Statement-2 is also true since (0, 0) is a solution

**Aliter :** ad – bc ≠ 0
If (i) ad = 1, bc = 0
(ii) ad = 0, bc = 1

$P(ad = 1) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$P(bc = 0) = 1 - P(bc = 1) = 1 - \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}$

$\therefore$ $P(ad = 1 \text{ and } bc = 0) = \frac{3}{16}$

$P(ad = 0 \text{ and } bc = 1) = \frac{3}{16}$

$\therefore$ required probability $= \frac{3}{8}$

## Section (B) : MATCH THE COLUMN

**B-1.** **Sol.** Total cases = 5! = 120
(A) Favourable cases = ${}_5C_2 \times 2$
required probability = $\frac{20}{120} = \frac{1}{6}$
(B) Favourable cases = ${}_5C_3 \times 1 = 10$
required probability = $\frac{10}{120} = \frac{1}{1/12}$
(C) Favourable cases $5! - (44 + {}_5C_1 \times 9)$
= 31
required probability = $\frac{31}{120}$
(D) Favourable cases = $3!\left(\frac{1}{2!} - \frac{1}{3!}\right) + 4!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right) = 2 + 9 = 11$
required probability =

## Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

**C-1.** **Sol.** $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
$= 0.4 + 0.6 + 0.5 + 0.75 - (0.4 + 0.6) - P(B \cap C) - 0.35 + 0.2$
$= 1.1 - P(B \cap C)$ [Since $P(A \cup B) = P(A) + P(B) - P(A \cap B)$]
But , $0.75 \le P(A \cup B \cup C) \le 1$
$\Rightarrow 0.75 \le 1.1 - P(B \cap C) \le 1$
$\Rightarrow 0.1 \le P(B \cap C) \le 0.35$, but from when diagram $P(B \cap C) \ne .1$

**C-2.** **Sol.** $P(E_1) = \frac{2}{4} = \frac{1}{2}$

$P(E_2) = \frac{2}{4} = \frac{1}{2}$

$P(E_3) = \frac{2}{4} = \frac{1}{2}$

$P(E_1 \cap E_2) = \frac{1}{4} = P(E_1)\,P(E_2)$

$\therefore$ $E_1$ and $E_2$ are independent

$P(E_2 \cap E_3) = \frac{1}{4} = P(E_2)\,P(E_3)$

$\therefore$ $E_2$ and $E_3$ are independent

$P(E_3 \cap E_1) = \frac{1}{4} = P(E_3)\,P(E_1)$

$\therefore$ $E_3$ and $E_1$ are indendpent

$P(E_1 \cap E_2 \cap E_3) = \frac{1}{4} \neq P(E_1)\,P(E_2)\,P(E_3)$

$\therefore$ $E_1, E_2, E_3$ are not independent.

**C-3.** **Sol.** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{5}{8} = \frac{3}{8} + \frac{4}{8} - P(A \cap B) \Rightarrow P(A \cap B) = \frac{2}{8} = \frac{1}{4}$

Now $P(A^c/B) = \frac{P(A^C \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$

$= 1 - \frac{2}{8} \cdot \frac{8}{4} = \frac{1}{2}$

$2P(A/B^c) = \frac{2P(A \cap B^c)}{P(B^c)} = \frac{2(P(A) - P(A \cap B))}{1 - P(B)} = 4\left(\frac{3}{8} - \frac{2}{8}\right) = \frac{1}{2} \Rightarrow$ (1) is correct

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{4} \cdot \frac{2}{1} = \frac{1}{2} = P(B) \Rightarrow$ (2) is correct

again $P(A^c/B^c) = \frac{P\left(A^c \cap B^c\right)}{P(B^c)} = \frac{1 - P(A \cup B)}{1 - P(B)} = 2\left(1 - \frac{5}{8}\right) = \frac{3}{4} \neq P(A \cup B)$. So (3) is incorrect.

again $2P(A/B^c) = \frac{1}{2}$ from (1) $\Rightarrow$ $P(A/B^c) = \frac{1}{4} = P(A \cap B)$

hence (4) is correct

**C-4.** **Sol.** P(A wins the game)

= P (H or TH or TTTH or TTTTH or TTTTTTH or ...........

$= \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^7} + \ldots\ldots\ldots\ldots$

$= \left(\frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \ldots\ldots\ldots\right) + \left(\frac{1}{2^2} + \frac{1}{2^5} + \frac{1}{2^8} + \ldots\ldots\ldots\right) = \frac{\frac{1}{2}}{1 - \frac{1}{8}} + \frac{\frac{1}{4}}{1 - \frac{1}{8}} = \frac{4}{7} + \frac{2}{7} = \frac{6}{7}.$

$\beta = 1 - x = 1 - \frac{6}{7} = \frac{1}{7}$

**C-5.** **Sol.** $P(E_1) = 1 - P$(unit's place in both is 1, 2, 3, 4, 6, 7, 8, 9)

P($E_1$ = 0 or 5) = 1 – $\frac{8}{10}\cdot\frac{8}{10} = \frac{9}{25}$

P($E_2$ : 5) = P (1 3 5 7 9) – P (1 3 7 9) for both numbers

$= \frac{5}{10}\cdot\frac{5}{10} - \frac{4}{10}\cdot\frac{4}{10} = \frac{25-16}{100} = \frac{9}{100}$

$\frac{P(E_2)}{P(E_1)} = \frac{9}{100}\cdot\frac{25}{9} = \frac{1}{4}$

P($E_1$) = 4 P($E_2$) $\Rightarrow$ (1) is not correct

P($E_2$/$E_1$) = $\frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{P(E_2)}{P(E_1)} = \frac{9}{100}\cdot\frac{25}{9} = \frac{1}{4}$

P($E_1$/$E_2$) = $\frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{P(E_2)}{P(E_2)} = 1$

# Exercise-3

* **Marked Questions may have more than one correct option.**

## PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

**1.** **Sol.** Since, the probabilities of solving the problem by A, B and C are $\frac{1}{2}, \frac{1}{3}$, and $\frac{1}{4}$ respectively.

$\therefore$ Probability that the problem is not solved

$= P(\bar{A})P(\bar{B})P(\bar{C})$

$= \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)$

$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$

Hence, the probability that the problem is solved

$= 1 - \frac{1}{4} = \frac{3}{4}$

**2.** **Sol.** The total number of ways in which numbers can be choosed = 25 × 25 = 625

The number of ways in which either players can choose same numbers = 25

$\therefore$ Probability that they with a prize = $\frac{25}{625} = \frac{1}{25}$

Thus, the probability that they will not win a prize in a single trial = $1 - \frac{1}{25} = \frac{24}{25}$

**3.** **Sol.** Since, A and B are two mutually exclusive events.

$\therefore$ $A \cap B = \varphi$

$\Rightarrow$ either $A \subseteq \bar{B}$ or $B \subseteq \bar{A}$

$\Rightarrow P(A) \le P(\bar{B})$

or $P(B) \le P(\bar{A})$.

**4.** **Sol.** Let $A_1$, $A_2$ and $A_3$ be the events of match winning in first, second and third match respectively. And whose probabilities are

$P(A_1) = P(A_2) = P(A_3) = \frac{1}{2}$

$\therefore$ Required probability

$= P(A_1 A'_2, A_3) + P(A'_1, A_2 A_3)$

$= P(A_1) P(A'_2) P(A_3) + P(A'_1) P(A_2) P(A_3)$

$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$.

**5.** **Sol.** Let the probability of getting a head is p and not getting a head is q.

Since, head appears first time in an even throw 2 or 4 or 6....

$\therefore$ $\frac{2}{5} = qp + q_3 p + q_5 p + ....$

$\Rightarrow \frac{2}{5} = \frac{qp}{1-q^2}$

$\Rightarrow \frac{2}{5} = \frac{(1-p)p}{1-(1-p)^2}$ $(\because\ q = 1 - p)$

$\Rightarrow \frac{2}{5} = \frac{1-p}{2-p}$

$\Rightarrow 4 - 2p = 5 - 5p$

$\Rightarrow p = \frac{1}{3}$

**6.** **Sol.** Probability of getting success, $p = \frac{1}{2}$ and probability of failure, $q = \frac{1}{2}$

$\therefore$ Required probability = ${}^7C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^5 \times \frac{1}{6} = \frac{{}^7C_2 \times 5^5}{6^8}$

**7.** **Sol.** The probability that Mr. A selected the loosing horse = $\frac{4}{5} \times \frac{3}{4} = \frac{3}{5}$

The probability that Mr. A selected the winning horse = $1 - \frac{3}{5} = \frac{2}{5}$.

**8.** **Sol.** Since , $0 \le P(A) \le 1$, $0 \le P(B) \le 1$, $0 \le P(C) \le 1$ and $0 \le P(A) + P(B) + P(C) \le 1$

$\therefore 0 \le \frac{3x+1}{3} \le 1$

$\Rightarrow -\frac{1}{3} \le x \le \frac{2}{3}$ ...(i)

$0 \le \frac{1-x}{4} \le 1$

$\Rightarrow -3 \le x \le 1$ ...(ii)

$0 \le \frac{1-2x}{2} \le 1$

$\Rightarrow -\frac{1}{2} \le x \le \frac{1}{2}$ ...(iii)

and $0 \le \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$

$\Rightarrow 0 \le 13 - 3x \le 12$

$\Rightarrow \frac{1}{3} \le x \le \frac{13}{3}$ ………...(iv)

From Eqs. (i), (ii), (iii) and (iv), we get

$\frac{1}{3} \le x \le \frac{1}{2}$

**9.** **Sol.** Given that, for binomial distribution mean, $np = 4$ and variance, $npq = 2$

$\therefore q = \frac{1}{2}$, but $p + q = 1 \Rightarrow p = \frac{1}{2}$

and $n \times \frac{1}{2} = 4 \Rightarrow n = 8$

We know, $P(X = r) = {}^nC_r\, p^r q^{n-r}$

$\therefore$ $P(X = 1) = {}^8C_1 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^1 = 8 \times \frac{1}{2^8} = \frac{1}{2^5} = \frac{1}{32}$

**10.** **Sol.** Given probabilities of speaking truth are

$P(A) = \frac{4}{5}$ and $P(B) = \frac{3}{4}$

And their corresponding probabilities of not speaking truth are

$P(\bar{A}) = \frac{1}{5}$ and $P(\bar{B}) = \frac{1}{4}$

The probability that they contradict each other

$= P(A) \times P(\bar{B}) + P(\bar{A}) \times P(B)$

$= \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4}$

$= \frac{1}{5} + \frac{3}{20} = \frac{7}{20}$

**11.** **Sol.** Given, $E = \{X \text{ is a prime number}\}$

$= \{2, 3, 5, 7\}$

$P(E) = P(X = 2) + P(X = 3) + P(X = 5) + P(X = 7)$

$\Rightarrow P(E) = 0.23 + 0.12 + 0.20 + 0.07 = 0.62$

and $F = \{X < 4\} = \{1, 2, 3\}$

$\Rightarrow P(F) = P(X = 1) + P(X = 2) + P(X = 3)$

$\Rightarrow P(F) = 0.15 + 0.23 + 0.12 = 0.5$

and $E \cap F = \{X \text{ is prime number as well as } < 4\}$

$= \{2, 3\}$

$P(E\cap F) = P(X = 2) + P(X = 3)$

$= 0.23 + 0.12 = 0.35$

$\therefore$ Required probability

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\Rightarrow P(E \cup F) = 0.62 + 0.5 - 0.35$

$\Rightarrow P(E \cup F) = 0.77$

**12.** **Sol.** Given that, mean = 4

$\Rightarrow np = 4$

and variance = 2

$\Rightarrow npq = 2 \Rightarrow 4q = 2$

$\Rightarrow q = \frac{1}{2}$

$\therefore$ $p = 1 - q = 1 - \frac{1}{2} = \frac{1}{2}$

Also, $n = 8$

Probability of 2 successes = $P(X = 2)$

$= {}^8C_2\, p^2 q^6$

$= \frac{8!}{2! \times 6!} \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^6$

**13. Sol.** Given that,

$P(A \cap B) = \frac{1}{4}$, $P(\bar{A}) = \frac{1}{4}$ and $P(\overline{A \cup B}) = \frac{1}{6}$

$\Rightarrow 1 - P(A \cup B) = \frac{1}{6}$ $\Rightarrow 1 - P(A) - P(B) + P(A \cup B) = \frac{1}{6}$ $\Rightarrow P(\bar{A}) - P(B) + \frac{1}{4} = \frac{1}{6}$

$\Rightarrow P(B) = \frac{1}{4} + \frac{1}{4} - \frac{1}{6}$ $\Rightarrow P(B) = \frac{1}{3}$ and $P(A) = \frac{3}{4}$

Now, $P(A \cap B) = \frac{1}{4} = \frac{3}{4} \times \frac{1}{3} = P(A)\,P(B)$.

Hence, the events A and B are independent events but not equally likely.

**14. Sol.** All the three persons has three options to apply a house.

$\therefore$ Total number of cases = $3_3$

Now, favourable cases = 3 (An either all has applied for house 1 or 2 or 3)

$\therefore$ Required probability = $\frac{3}{3^3} = \frac{1}{9}$.

**15. Sol.** Probability of getting score 9 in a single throw

$= \frac{4}{36} = \frac{1}{9}$

Probability of getting score 9 exactly in double throw

$= {}^3C_2 \times \left(\frac{1}{9}\right)^2 \times \frac{8}{9} = \frac{8}{243}$.

**16. Sol.** Let the events,

A = Ist aeroplane hit the target

B = IInd aeroplane hit the target

And their corresponding probabilites are

$P(A) = 0.3$ and $P(B) = 0.2$

$\Rightarrow$ $P(\bar{A}) = 0.7$ and $P(\bar{B}) = 0.8$

$\therefore$ Required probability

$= P(\bar{A})\,P(B)$

$= (0.7)(0.2) = 0.14$

**17. Sol.** Given that , $P(A) = \frac{1}{4}$, $P\left(\frac{A}{B}\right) = \frac{1}{2}$ and $P\left(\frac{B}{A}\right) = \frac{2}{3}$

we know, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ ...(i)

and $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$ ...(ii)

$\therefore P(B) = \frac{P\left(\frac{B}{A}\right).P(A)}{P\left(\frac{A}{B}\right)} = \frac{\left(\frac{2}{3}\right)\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)} = \frac{1}{3}$

**18.** **Sol.** $\because$ A = {4, 5, 6} and B = {1, 2, 3, 4}

$\therefore$ $A \cap B = \{4\}$

$\because$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{3}{6} + \frac{4}{6} - \frac{1}{6} = 1.$

**19.** **Sol.** By binomial distribution, $\left(\frac{1}{4} + \frac{3}{4}\right)^n$

$\therefore$ probability of at least one success = 1 – no. of success = $1 - {}_nC_n \left(\frac{3}{4}\right)^n = 1 - \left(\frac{3}{4}\right)^n \geq \frac{9}{10}$

$\Rightarrow \left(\frac{3}{4}\right)^n \leq \frac{1}{10}$

Taking $\log_{10}$ on both sides

$n(\log_{10}3 - \log_{10}4) \leq -1$ or $n \geq \frac{-1}{\log_{10}3 - \log_{10}4}$ $\Rightarrow$ $n \geq \frac{1}{\log_{10}4 - \log_{10}3}$

**20.** **Sol.** Case in which sum of digits in 8 are 08, 17, 26, 35, 44

Total cases : 00, 01, 02, ...., 09, 10, 20, 30, 40

Required probability $= \frac{1}{14}$

**21.** **Ans.** **(2)**

**Sol.** Statement-1 Total ways = ${}_{20}C_4$

number of AP's of common difference 1 is = 17
number of AP's of common difference 2 is = 14
number of AP's of common difference 3 is = 11
number of AP's of common difference 4 is = 8
number of AP's of common difference 5 is = 5
number of AP's of common difference 6 is = 2

total = 57

probability $= \frac{57}{{}^{20}C_4} = \frac{1}{85}$

Statement-2 common difference can be ± 6 , so statement -2 is false

Hence correct option is (2)

vr% lgh fodYi (2) gSA

**22.** **Ans.** **(1)**

**Sol.** $= \frac{{}^3C_1\ {}^4C_1\ {}^2C_1}{{}^9C_3} = \frac{3.4.2}{\frac{9.8.7}{3.2.1}} = \frac{2}{7}$

Hence correct option is (1)

**23.** **Sol.** **(3)**

$1 - P_5 \geq \frac{31}{32}$

$P_5 \leq \frac{1}{32}$

$P \le \frac{1}{2}$

$P \in \left[0, \frac{1}{2}\right]$

**24. Sol. (2)**

$P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)} \Rightarrow \frac{1}{P(D)} \ge 1$

$\frac{P(C)}{P(D)} \ge P(C) \Rightarrow P(C) \le P\left(\frac{C}{D}\right)$

**25. Sol. (4)**

$P(A^c \cap B^c/C) = \frac{P((A^c \cap B^c) \cap C)}{P(C)} = \frac{P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}{P(C)}$

$= \frac{P(C) - P(A).P(C) - P(B)P(C) + 0}{P(C)} = 1 - P(A) - P(B) = P(A^c) - P(B)$

**26. Sol.** Let Event (Given : {1, 2, 3,.........8})

A : Maximum of three numbers is 6.

B : Minimum of three numbers is 3

$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{{}^2C_1}{{}^5C_2} = \frac{2}{10} = \frac{1}{5}$

**27. Sol.** (3)

$p = \frac{1}{3}$, $q = \frac{2}{3}$

${}_5C_4 \left(\frac{1}{3}\right)^4 . \frac{2}{3} + {}_5C_5 \left(\frac{1}{3}\right)^5 = 5.\frac{2}{3^5} + \frac{1}{3^5} = \frac{11}{3^5}$

**28. Sol. Ans. (1)**

Given $P\left(\overline{AUB}\right) = \frac{1}{6}$, P = , P

$1 - P(A ∪ B) = \frac{1}{6}$

$\Rightarrow 1 - P(A) - P(B) + P(A ∩ B) = \frac{1}{6}$

$\Rightarrow 1 - \frac{3}{4} - P(B) + \frac{1}{4} = \frac{1}{6}$ ( $P(A) = 1 - P(\overline{A})$ ))

$\Rightarrow P(B) = \frac{1}{2} - \frac{1}{6} = \frac{3-1}{6} = \frac{1}{3}$

A and B are not equally likely.

Further $P(A).P(B) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4} = P(A ∩ B)$

A and B are independent events

**29. Ans. (1)**

**Sol.** There seems to be ambiguity in the question. It should be mentained that boxs are different and one particular box has 3 balls :
then

number of ways = $\frac{^{12}C_3 \times 2^9}{3^{12}} = \frac{55}{3}\left(\frac{2}{3}\right)^{11}$

**Alter**

${}_3C_1\ {}_{12}C_3\ ({}_9C_0 + {}_9C_1 + {}_9C_2 + {}_9C_4 + {}_9C_5 + {}_9C_5 + {}_9C_7 + {}_9C_8 + {}_9C_9)$

$+ \frac{\lfloor 12 \times \lfloor 3}{\lfloor 3 \lfloor 3 \lfloor 6 \lfloor 3}$

$= {}_3C_1\ {}_{12}C_3\ (2_9 - 2_9C_3) + \frac{\lfloor 12}{\lfloor 3 \lfloor 2 \lfloor 6}$

correct answer should have been $\frac{^3C_1{}^{12}C_3(2^9 - 2.{}^9C_3) + \frac{\lfloor 12}{\lfloor 3 \lfloor 2 \lfloor 6}}{3^{12}}$

**30.** **Ans. (3)**

**Sol.** $E_1$ : {(4, 1) ,............... (4,6,)} 6 cases
$E_2$ : {(1,2), ............... (6,2)} 6 cases
$E_3$ : 18 cases (sum of both are odd)}

$P(E_1) = \frac{6}{36} = \frac{1}{6} = P(E_2)$

$P(E_3) = \frac{18}{36} = \frac{1}{2}$

$P(E \cap E_2) = \frac{1}{36}$

$P(E \cap {}_2E_3) = \frac{1}{12}$

$P(E \cap {}_3E_1) = \frac{1}{12}$

$P(E \cap {}_1E \cap {}_2E_3) = 0$

$\therefore E_1, E_2, E_3$ are not independent

$\therefore E_1, E_2, E_3$ Loar=k ugha gS

**31.** **Ans. (2)**

**Sol.** $P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$

$P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$

$P(A) + P(C) - 2P(A \cap C) = \frac{1}{4}$

$P(A \cap B \cap C) = \frac{1}{16}$

$\therefore P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{1}{16} = \frac{6+1}{16} = \frac{7}{16}$

**32.** **Ans. (1)**

**Sol.** $P = \frac{6}{^{11}C_2} = \frac{6}{55}$

$x_1 - x_2 = \pm 4\lambda$

$x_1 + x_2 = 4\alpha$

---

$2x_1 = 4(\lambda \pm \alpha)$

$x_1 = 2(\lambda \pm \alpha)$

| $x_1$ | $x_2$ |
|---|---|
| 0 | 4, 8 |
| 2 | 6, 10 |
| 4 | 0, 8 |
| 6 | 2, 10 |
| 8 | 0, 4 |
| 10 | 2, 6 |

**33.** **Ans. (1)**

**Sol.** 15 green + 10 yellow = 25 balls

P(green ) = $\frac{3}{5} = p_1$

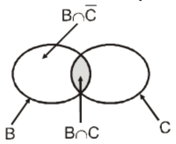
P(yellow ) = $\frac{2}{5} = q$

n =10

$\therefore$ Variance = npq $= 10\,\frac{3}{5}\cdot\frac{2}{5} = \frac{60}{25} = \frac{12}{5}$

## PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

**1.** **Sol.** We have $P(B \cap \bar{C}) = P[(A \cup \bar{A}) \cap (B \cap \bar{C})] = P(A \cap B \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$




$P(B \cap C) = P(B) - P(B \cap \bar{C}) = \frac{3}{4} - \frac{2}{3} = \frac{1}{12}$

**2.** **Sol.** Let E denote the event that minimum of two numbers is less than 4, then E' denote the event that minimum number ≥ 4.

$P(E') = \frac{{}^3C_2}{{}^6C_2} = \frac{3}{15} = \frac{1}{5}$

⇒ P(E) = 4/5.

**3.** **Sol.** Such numbers are 6, 12, 18, ...... 96

i.e. 16 such numbers. Hence required probability = $\frac{{}^{16}C_3}{{}^{100}C_3} = \frac{16\times15\times14}{100\times99\times98} = \frac{4}{1155}$

Hence (D)

**4.** **Sol.** $P\left(\frac{E^c \cap F^c}{G}\right) = \frac{P(E^c \cap F^c \cap G)}{P(G)} = \frac{P(G) - P(E \cap G) - P(G \cap F)}{P(G)}$

$= \frac{P(G)[1 - P(E) - P(F)}{P(G)}$ [ $\therefore P(G) \neq 0$ ] $= 1 - P(E) - P(F) = P(E_c) - P(F)$.

**5.** **Sol.** If A : Indian Men sit with their wife

B : American men sit with their wife

$P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)} = \frac{n(A\cap B)}{n(B)} = \frac{4!(2!)^5}{5!(2!)^4} = \frac{2}{5}$

**6.** **Sol.** n(A) = 4

Let n(B) = x > 0

$\because$ A & B are independent events

$\Rightarrow$ P(A) . P(B) = P(A ∩ B)

$\frac{4}{10} . \frac{x}{10} = \frac{y}{10}$ where y = n (A ∩ B) ≤ min (4, x)

$x = \frac{5y}{2}$

| y | x |
|---|---|
| 0 | 0 |
| 1 | not an integer |
| 2 | 5 |
| 4 | 10 |

So x = 5 or 10

**7.** **Sol.** $P(X = 3) = \frac{5}{6} . \frac{5}{6} . \frac{1}{6} = \mathbf{\frac{25}{216}}$

**8.** **Sol.** $P(X \geq 3) = \frac{5}{6} \frac{5}{6} 1 = \mathbf{\frac{25}{36}}$

**9.** **Sol.** $P((X \geq 6) / (X > 3)) = \frac{P((X>3)/(X\geq 6)) . P(X \geq 6)}{P(X > 3)} = \frac{1 . \left[\left(\frac{5}{6}\right)^5 . \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^6 . \left(\frac{1}{6}\right) + .....\infty\right]}{\left[\left(\frac{5}{6}\right)^3 . \frac{1}{6} + \left(\frac{5}{6}\right)^4 . \frac{1}{6} + ......\infty\right]} = \mathbf{\frac{25}{36}}$

**10.**

**Sol.** Probability i (P) = $\frac{P(GGG) + P(GRG)}{P(GGG) + P(GRG) + P(RGG) + P(RRG)}$

$\Rightarrow$ $P = \frac{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}}{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{5}\times\frac{3}{4}\times\frac{1}{4}}$ $\Rightarrow$ $P = \frac{36+4}{36+4+3+3} = \frac{40}{46} = \frac{20}{23}$

**11.** **Sol.** $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ ; $r_1, r_2, r_3$ are to be selected from {1, 2, 3, 4, 5, 6}

As we know that $1 + \omega + \omega^2 = 0$

$\therefore$ from $r_1, r_2, r_3$, one has remainder 1, other has remainder 2 and third has remainder 0 when divided by 3.

$\therefore$we have to select $r_1, r_2, r_3$ from (1, 4) or (2, 5) or (3, 6) which can be done in ${}_2C_1 \times {}_2C_1 \times {}_2C_1$ ways

value of $r_1, r_2, r_3$ can be interchanged in 3! ways.

$\therefore$ required probability = $\frac{({}^2C_1 \times {}^2C_1 \times {}^2C_1) \times 3!}{6 \times 6 \times 6} = \frac{2}{9}$

**12*.** **Sol.** P(E ∩ F) = P(E) . P(F) ....(1)

$P(E \cap \bar{F}) + P(\bar{E} \cap F) = \frac{11}{25}$ ....(2)

$P(\bar{E} \cap \bar{F}) = \frac{2}{25}$ ....(3)

by (2)

$P(F) + P(E) - 2P(E \cap F) = \frac{11}{25}$ ....(4)

by (3)

$1 - [P(E) + P(F) - P(E \cap F)] = \frac{2}{25}$

$[P(E) + P(F) - P(E \cap F)] = \frac{23}{25}$ ....(5)

by (4) & (5) $P(E)\,P(F) = \frac{12}{25}$ ....(6)

and $P(E) + P(F) = \frac{7}{5}$ ....(7)

By (6) and (7) $P(E) = \frac{4}{5}$, $P(F) = \frac{3}{5}$ or $P(E) = \frac{3}{5}$, $P(F) = \frac{4}{5}$

**13*.** **Sol.** $P(X/Y) = \frac{1}{2}$

$\frac{P(X \cap Y)}{P(Y)} = \frac{1}{2} \Rightarrow P(Y) = \frac{1}{3}$

$P(Y/X) = \frac{1}{3}$

$\frac{P(X \cap Y)}{P(X)} = \frac{1}{3} \Rightarrow P(X) = \frac{1}{2}$

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = \frac{2}{3}$ A is correct

$P(X \cap Y) = P(X) \cdot P(X) \Rightarrow$ X and Y are independent B is correct

$= P(Y) - P(X \cap Y)$

$= \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$ D is not correct

**14.** **Sol.** Favourable : $D_4$ shows a number and
only 1 of $D_1D_2D_3$ shows same number
or only 2 of $D_1D_2D_3$ shows same number
or all 3 of $D_1D_2D_3$ shows same number

Required Probability = $\frac{{}^6C_1({}^3C_1 \times 5 \times 5 + {}^3C_2 \times 5 + {}^3C_3)}{216 \times 6}$

$= \frac{6 \times (75 + 15 + 1)}{216 \times 6}$

$= \frac{6 \times 91}{216 \times 6}$

$= \frac{6 \times 91}{216 \times 6}$

**15.** **Sol.** **(A)**

P (problem solved by at least one) = 1 – P(problem is not solved by by all)

$= 1 - P(\bar{A})\,P(\bar{B})\,P(\bar{C})\,P(\bar{D})$

$= 1 - \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{7}{8}\right) = 1 - \frac{21}{256} = \frac{235}{256}$

**16. Sol. (A)**

| Bag 1 | Bag 2 | Bag 3 |
|---|---|---|
| 1 W<br>3 R<br>2 B | 2 W<br>3 R<br>4 B | 3 W<br>4 R<br>5 B |

P(W W W) + P(R R R) + P(B B B)

$$\left(\frac{1}{6}\times\frac{2}{9}\times\frac{3}{12}\right) + \left(\frac{3}{6}\times\frac{3}{9}\times\frac{4}{12}\right) + \left(\frac{2}{6}\times\frac{4}{9}\times\frac{5}{12}\right) \Rightarrow \frac{6+36+40}{6\times9\times12} \Rightarrow \frac{82}{648}$$

**17. Sol. (D)**

P (Ball drawn from box 2 / one is W one is R) = $\frac{P(A\cap B)}{P(B)}$

$$= \frac{\frac{1}{3}\times\frac{2\times3}{{}^9C_2}}{\frac{1}{3}\left[\frac{1\times3}{{}^6C_2}+\frac{2\times3}{{}^9C_2}+\frac{3\times4}{{}^{12}C_2}\right]} = \frac{\frac{2\times3\times2}{9\times8}}{\frac{3\times2}{6\times5}+\frac{2\times6\times2}{9\times8}+\frac{3\times4\times2}{12\times11}}$$

$$= \frac{\frac{1}{6}}{\frac{1}{5}+\frac{1}{6}+\frac{2}{11}} = \frac{\frac{1}{6}}{\frac{66+55+60}{55\times60}} = \frac{55}{181}$$

**18. Ans. (A)**

**Sol.** 3 Boys & 2 Girls..................

(1) B (2) B (3) B (4)

Girl can't occupy $4_{th}$ position. Either girls can occupy 2 of 1, 2, 3 position or they can both be a position (1) or (2).

Hence total number of ways in which girls can be seated is ${}_3C_2 \times 2! \times 3! + {}_2C_1 \times 2! \times 3! = 36 + 24 = 60$.

Number of ways in which 3 B & 2 A can be seated = 5 !

Hence required prob. = $\frac{60}{5!} = \frac{1}{2}$ .

**19. Ans. (A,B)**

**Sol.** $\text{Box}-\text{I} < \begin{matrix}\text{Red}\to n_1\\ \text{Black}\to n_2\end{matrix}$ $\qquad$ $\text{Box}-\text{II} < \begin{matrix}\text{Red}\to n_3\\ \text{Black}\to n_4\end{matrix}$

P(R) = $\frac{1}{2}.\frac{n_1}{n_1+n_2}+\frac{1}{2}.\frac{n_3}{n_3+n_4}$

R(II/R) = $\frac{\frac{1}{2}.\frac{n_3}{n_3+n_4}}{\frac{1}{2}.\frac{n_1}{n_1+n_2}+\frac{1}{2}.\frac{n_3}{n_3+n_4}} = \frac{\frac{n_3}{n_3+n_4}}{\frac{n_1}{n_1+n_2}+\frac{n_3}{n_3+n_4}}$

by option $\qquad$ $n_1 = 3$, $n_2 = 3$, $n_3 = 5$, $n_4 = 15$

P(II/R) = $\frac{\frac{5}{20}}{\frac{3}{6}+\frac{5}{20}} = \frac{n_4}{\frac{1}{2}+\frac{1}{4}} = \frac{1}{4}\times\frac{4}{2+1} = \frac{1}{3}$

**20. Ans. (C,D)**

**Sol.** Given $\frac{n_1}{n_1+n_2}.\frac{n_1-1}{n_1+n_2-1}+\frac{n_2}{n_1+n_2}.\frac{n_1}{n_1+n_2-1}=\frac{1}{3}$

$3(n_1^2 - n_1 + n_1n_2) = (n_1 + n_2)(n_1 + n_2 - 1)$

$3n_1(n_1 + n_2 - 1) = n_1 + n_2(n_1 + n_2 - 1)$

$2n_1 = n_2$

**21.** **Ans.** **(C)**

**Sol.** Let x = P(computer turns out to be defective given that it is produced in Plant $T_2$),

$\frac{7}{100}=\frac{1}{5}\times(10x)+\frac{4}{5}x \Rightarrow \quad 7 = 200x + 80x \quad \Rightarrow \quad x = \frac{7}{280}$

P(produced in $T_2$/ not defective) = $\frac{P(A\cap B)}{P(B)}$

$\frac{4/5(1-x)}{\frac{1}{5}(1-10x)+\frac{4}{5}(1-x)} = \frac{\frac{4}{5}\left(\frac{273}{280}\right)}{\frac{1}{5}\left(\frac{280-70}{280}\right)+\frac{4}{5}\left(\frac{273}{280}\right)} = \frac{4\times 273}{210+4\times 273} = \frac{2\times 273}{105+2\times 273} = \frac{546}{651} = \frac{78}{93}$

**22.** **Ans.** **(B)**

**Sol.** $P(X > Y) = T_1T_1 + DT_1 + T_1D$ (Where $T_1$ represents wins and D represents draw)

$\frac{1}{2}.\frac{1}{2}+\frac{1}{6}.\frac{1}{2}+\frac{1}{2}.\frac{1}{6} = \frac{5}{12} =$ ⇒ (B) is correct

**23.** **Ans.** **(C)**

**Sol.** $P(X = Y) = DD + T_1T_2 + T_2T_1 = \frac{1}{6}.\frac{1}{6}+\frac{1}{2}.\frac{1}{3}+\frac{1}{3}.\frac{1}{2} = \frac{1}{36}+\frac{1}{3} = \frac{39}{36\times 3} = \frac{13}{36}$ ⇒ (C) is correct lgh gSA

**24.** **Ans.** **(A,B)**

**Sol.** $\frac{P(X\cap Y)}{P(Y)} = \frac{1}{2}$

$\frac{P(Y\cap X)}{P(X)} = \frac{2}{5}$

$P(X \cap Y) = \frac{P(Y)}{2} = \frac{2}{5} P(X) = \frac{2}{5}.\frac{1}{3} = \frac{2}{15} \Rightarrow P(Y) = \frac{4}{15}$

$\frac{P(\bar{X}\cap Y)}{P(Y)} = \frac{P(Y)-P(X\cap Y)}{P(Y)} = \frac{\frac{4}{15}-\frac{2}{15}}{\frac{4}{15}} = \frac{2}{4}=\frac{1}{2}$

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = \frac{1}{3}+\frac{4}{15}-\frac{2}{15}=\frac{7}{15}$

**25.** **Ans.** **(C)**

**Sol.** $x + y + z = 10$

Total number of non-negative solutions = ${}^{10+3-1}C_{3-1} = {}^{12}C_2 = 66$

Now Let $z = 2n$.

$x + y + 2n = 10$ ; $n \geq 0$

Total number of non-negative solutions = 11 + 9 + 7 + 5 + 3 + 1 = 36

Required probability = $\frac{36}{66} = \frac{6}{11}$