Exercise-1

Marked Questions may have for Revision Questions.

OBJECTIVE QUESTIONS

Section (A) : Arithmetic Progression and Arithmetic Mean

A-1. Sol. $a = 4, d = -3 \Rightarrow t_{15} = a + 14d = 4 - 42 = -38$

A-2. Sol. First tem $a = 7$, last term $\ell = 102$

common difference $\Rightarrow$ $d = 5$

$\therefore \quad \ell = a + (n-1)d$

$\Rightarrow \quad 102 = 7 + (n-1) 5 \quad \Rightarrow \quad n = 20$

A-3. Sol. First term in the given series is 2 and common difference is 4.

$\therefore a = 2, d = 4$

$T_n = a + (n-1) d$

$T_{19} = 2 + 18 \times 4 = 74$

A-4. Sol. Given A.P. is 4, 9, 14, ..., 104

Here $a = 4$ and $d = 5$

$4 + (n-1) 5 = 104 \quad \Rightarrow \quad n = 21$

so middle term is T_{11}

$T_{11} = 4 + 10 \times 5 = 54$

A-5. Sol. first term $a = 10$, common difference $d = 2$

$S_{20} = \frac{20}{2} [-20 + (20-1)(2)] = 180$

A-6. Sol. We have to find the sum of numbers lying between 10 and 200 which are divisible by 7.

Here first number is 14 and last number = 196

common difference $d = 7$

$\therefore \ 196 = 14 + (n-1)7$

$\Rightarrow \quad n = 27$

$\therefore$sum of numbers $= \frac{27}{2} (14 + 196) = 2835$

A-7. Sol. All positive even integers less than 200 are 2, 4, ..., 198

Here $a = 2, d = 2, T_n = 2 + (n-1) 2 = 198$

$\Rightarrow \quad n - 1 = \frac{196}{2} = 98$

$\therefore \quad n = 99$

positive even integers which are divisible by 6 are 6, 12, 18, ..., 198

Here no of integers is 33

So sum of even positive integers, less than 200, which are not divisible by 6 is

$(2 + 4 + 6 + ... + 198) - (6 + 12 + ... + 198)$

$= \frac{99}{2} (198 + 2) - \frac{33}{2} (6 + 198)$

$= 100 \times 99 - 102 \times 33 = 6534$

A-8. Sol. Equation is $\frac{x-1}{x} + \frac{x-2}{x} + \frac{x-3}{x} + ,....... + \frac{1}{x} = 3$

There are $(x - 1)$ terms

So sum = $\frac{(x-1)}{2}\left[\frac{x-1}{x}+\frac{1}{x}\right] = 3$

$\Rightarrow \quad \frac{(x-1)}{2} = 3$

$\Rightarrow \quad x - 1 = 6 \quad \Rightarrow \quad x = 7$

A-9. Sol. Sum of n terms of A.P. is $An^2 + Bn$ and common difference is two times coefficient of n^2 i.e. $2A$

$S_n = n^2 a + \frac{n}{4}(n-1)d$

$= n^2\left(a+\frac{d}{4}\right) - \frac{nd}{4}$

$\therefore$ common difference $2A = 2a + \frac{d}{2}$

$\therefore$ lkoZvUrj $2A = 2a + \frac{d}{2}$

A-10. Sol. Numbers a, b, c, d, e are in A.P.
we have to find $a - 4b + 6c - 4d + e$
here by properties of A.P.

$a + e = b + d$

$2c = b + d$

so given expression is $(a + e) - 4(b + d) + 6c = (b + d) - 4(b + d) + 3(b + d) = 0$

A-11. Sol. $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

$\Rightarrow \quad 3(a_1 + a_{24}) = 225 \qquad (\because$ sum of terms equidistant from beginning and end are equal)

$\Rightarrow \quad a_1 + a_{24} = 75$

Now $\quad a_1 + a_2 + \ldots\ldots.. + a_{23} + a_{24}$

$= \frac{24}{2}[a_1 + a_{24}] = 12 \times 75 = 900$

A-12. Sol. $\frac{1}{q+r}, \frac{1}{r+p}, \frac{1}{p+q}$ are in A.P

$\frac{2}{r+p} = \frac{1}{q+r} + \frac{1}{p+q} = \frac{p+2q+r}{(q+r)(p+q)}$

$\therefore \quad 2(pq + q^2 + rp + rq) = rp + 2qr + r^2 + p^2 + 2pq + pr$

$\Rightarrow \quad 2q^2 = r^2 + p^2$

$\therefore \quad p^2, q^2, r^2$ are in A.P.

A-13. Sol. Let T_n of first A.P. be identical with T_m of second

$\therefore \quad 3n-1 = 2m + 1$ or $3n - 1 - 5 = 2m + 1 - 5$

or $\quad 3(n - 2) = 2(m - 2)$

or $\quad \frac{n-2}{2} = \frac{m-2}{3} = \lambda$ (say)

$\therefore \quad n = 2\lambda + 2, m = 3\lambda + 2$

where $1 \le n \le 60$ and $1 \le m \le 50$

or $\quad 1 \le 2\lambda + 2 \le 60$ and $1 \le 3\lambda + 2 \le 50$

or $\quad -\frac{1}{2} \le \lambda \le 29$ and $-\frac{1}{3} \le \lambda \le 16$

Clearly $\lambda = 0, 1, 2, 3, \ldots., 16$ satisfy both the above inequalities.

Thus there are 16 + 1 = 17 identical terms

A-14. Sol. Given A.P. are 17, 21, 25, ... and 16, 21, 26 ......

Let m_{th} term of first A.P. is common to n_{th} term of second A.P.

$\therefore \quad 17 + 4(m - 1) = 16 + 5(n - 1)$

$\Rightarrow \quad 13 + 4m = 11 + 5n \quad \Rightarrow \quad 12 + 4m = 10 + 5n$

$\therefore \quad 4(m + 3) = 5(n + 2)$

$\therefore \quad \frac{m+3}{5} = \frac{n+2}{4} = \lambda(\text{say})$

$\therefore \quad m = 5\lambda - 3,\ n = 4\lambda - 2$

For λ = 1, 2, 3, 4 ....

m = 2, 7, 12, 17 ....

$\therefore$ 2nd, 7th, 12th, 17th .... terms of first A.P. is common to second A.P.

and these terms are 21, 41, 61, ..., 4001

so sum of first 200 common terms = $\frac{200}{2}$ [4001 + 21] = 402200

A-15. Sol. Let the four numbers which are in A.P. be a – 3d, a – d, a + d, a + 3d

Their sum is 48

$\therefore \quad 4a = 48$

a = 12

and $\frac{a^2 - 9d^2}{a^2 - d^2} = \frac{27}{35}$ (Given condition)

$\therefore \quad \frac{144 - 9d^2}{144 - d^2} = \frac{27}{35} \quad \Rightarrow \quad d = 2$

so numbers are

12 – 6, 12 – 2, 12 + 2, 12 + 6

or 6, 10, 14, 18

A-16. Sol. Let A_1, A_2, A_3, ... A_{11} are 11 AM's between 28 and 10

$\Rightarrow \quad d = \frac{10 - 28}{12} = -\frac{3}{2}$

Hence vr % $A_6 = 28 + 6d = 28 - 9 = 19$

A-17. Sol. Sum of n AM's between a and b is $\frac{n}{2}$ (a + b)

Hence sum of 6 AM's between 3 and 97 is = $\frac{6}{2}$ (3 + 97) = 300

A-18. Sol. In AP $t_{26} = a + 25d = 51$

$S_{51} = \frac{51}{2}(2a + 50d) = 51(a + 25d) = 51.51 = 2601$

A-19. Sol. $d = \frac{31 - 1}{m + 1} = \frac{30}{m + 1}$

$$\frac{A_7}{A_{m-1}} = \frac{1 + 7\frac{30}{m+1}}{1 + (m-1)\frac{30}{m+1}} = \frac{5}{9}$$

$\Rightarrow \quad \dfrac{m+211}{31m-29} = \dfrac{5}{9} \Rightarrow 146\,m = 2044 \quad \Rightarrow \; m = 14 \qquad \therefore \; \dfrac{m}{7} = 2$

Section (B) : Geometric progression and Geometric mean

B-1. **Sol.** $a = \sqrt{3}$, $r = 1/3 \Rightarrow t_6 = ar^5 = \dfrac{\sqrt{3}}{243}$

B-2. **Sol.** $a = 1/2$ and $r = 1/2$

$\Rightarrow S_{10} = \dfrac{\frac{1}{2}\left(1-\frac{1}{2}\right)^{10}}{1-\frac{1}{2}} = \dfrac{2^{10}-1}{2^{10}}$

B-3. **Sol.** If the first term of G.P. is a and common ratio is r, then $T_5 = ar^4 = 81$

$T_8 = ar^7 = 2187$

$\Rightarrow \quad r^3 = 27, r = 3$

$\therefore \quad a(3)^4 = 81 \quad \Rightarrow \quad a = 1$

$\therefore \quad T_3 = ar^2 = 1.3^2 = 9$

B-4. **Sol.** Here first term of G.P. a = 18

and common ratio $r = -\dfrac{12}{18} = -\dfrac{2}{3}$

$T_n = \dfrac{512}{729}$

$T_n = ar^{n-1} = 18\left(\dfrac{-2}{3}\right)^{n-1} = \dfrac{512}{729}$

$\Rightarrow \quad \left(-\dfrac{2}{3}\right)^{n-1} = \left(\dfrac{2}{3}\right)^{8}$

$\therefore \quad n - 1 = 8, n = 9$

B-5. **Sol.** Given G.P. is $x, x^2 + 2, x^3 + 10$

$\therefore \quad (x^2 + 2)^2 = x(x^3 + 10)$

$\Rightarrow \quad 4x^2 - 10x + 4 = 0$

$\Rightarrow \quad 2x^2 - 5x + 2 = 0$

$\Rightarrow \quad (2x - 1)(x - 2) = 0$

$\therefore \quad x = 2, \dfrac{1}{2}$

For $\quad x = 2$

G.P. is 2, 6, 18, 54

so next term of G.P. is 54

B-6. **Sol.** $3 + 3a + 3a^2 + ... \infty = \dfrac{45}{8}$ (given)

Here first term is 3 and common ratio is a

$\therefore \quad \dfrac{3}{1-a} = \dfrac{45}{8} \quad \Rightarrow \quad a = \dfrac{7}{15}$

B-7. **Sol.** We have to find value of $9^{1/3} . 9^{1/9} . 9^{1/27}$ .... $\infty = x$ (say)

We can see that powers are in G.P. and $|r| < 1$

$\therefore \quad x = 9^{\frac{\frac{1}{3}}{1-\frac{1}{3}}} = 9^{1/2} = 3$

B-8. Sol. Let first term is a and common ratio is r then

$$\frac{a}{1-r} = \frac{4}{3} \qquad .... (1)$$

and $\dfrac{a^2}{1-r^2} = \dfrac{16}{27}$ ...(2)

by $(1)^2$ (2) ls

$\Rightarrow \quad \dfrac{1+r}{1-r} = 3 \quad \Rightarrow \quad r = 1/2$

B-9. Sol. $T_n = ar^{n-1} = 128$ ....(1)

$S_n = \dfrac{a(r^n - 1)}{r-1} = 225$ ....(2)

$\Rightarrow \quad \dfrac{128r - a}{r-1} = 255$

Put r = 2 $\Rightarrow$ a = 1

B-10. Sol. a, b, c are in A.P.

$\therefore \quad b = \dfrac{a+c}{2}$

a, b, c are also in G.P.

$\therefore \quad b^2 = ac \quad \Rightarrow \quad \left(\dfrac{a+c}{2}\right)^2 = ac$

$\therefore \quad (a + c)^2 = 4ac \quad \Rightarrow \quad (a - c)^2 = 0 \quad \Rightarrow \quad a = c$

$b = \dfrac{a+c}{2} = \dfrac{a+a}{2} = a$

$\therefore \quad a = b = c$

B-11. Sol. First term of infinite G.P. = 1 and sum = 5

Let common ratio is r

So $S = \dfrac{1}{1-r}$

Let sum of squares of its terms is S'.

$\therefore \quad S' = 1 + r^2 + r^4$ .... ∞

$\therefore \quad S' = \dfrac{1}{1-r^2} = \dfrac{1}{(1+r)(1-r)} = \dfrac{S}{\left(1+\dfrac{S-1}{S}\right)} \qquad \left[\because \quad S = \dfrac{1}{1-r}\right]$

$\therefore \quad S' = \dfrac{S}{\left(\dfrac{2S-1}{S}\right)} = \dfrac{S^2}{2S-1}$

B-12. Sol. Three distinct real numbers a, b, c are in G.P. such that a + b + c = xb

If common ratio is r.
a, b, c will be a, ar, ar^2 respectively

$\therefore$ $a + ar + ar^2 = x . ar$

or $r^2 + r(1 - x) + 1 = 0$

$\because$ r is real

$\therefore$ $D > 0$

$\Rightarrow$ $(1 - x)^2 - 4 > 0$

$\Rightarrow$ $x^2 - 2x - 3 > 0$

$\Rightarrow$ $(x + 1)(x - 3) > 0$

$\Rightarrow$ $x < -1$ or $x > 3$

B-13. **Sol.** $S = \frac{a}{1-r}$ $\Rightarrow$ $r = \frac{S-a}{S}$

$S' = \frac{a[1-r^n]}{1-r} = S\left[1-\left(\frac{S-a}{S}\right)^n\right]$

B-14. **Sol.** $y = 2.\overline{357}$

y = 2.357357357..................(1)
1000 y = 2357.357357...........(2)
so 999y = 2355

$y = \frac{2355}{999}$

B-15. **Sol.** The sum of 10 terms of series .7 + .77 + .777 + ...

$S_{10} = + \frac{7}{10} + \frac{77}{100}$ ....to 10 terms

$= \frac{7}{9}\left[\frac{9}{10}+\frac{99}{100}+\text{.........to 10 terms}\right]$

$= \frac{7}{9}\left[\frac{10-1}{10}+\frac{100-1}{100}+\frac{1000-1}{1000}+\text{.........to 10 terms}\right]$

$= \frac{7}{9}$ [(1 + 1 + 1........10 terms) $-\left(\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+\text{.........to 10 terms}\right)$]

$= \frac{7}{9}\left[10-\frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^{10}\right)}{1-\frac{1}{10}}\right] = \frac{7}{9}\left[10-\frac{\left(10^{10}-1\right)}{9.10^{10}}\right]$

$= \frac{7}{81}\left[90-1+\frac{1}{10^{10}}\right] = \frac{7}{81}\left[89+\frac{1}{10^{10}}\right]$

B-16. **Sol.** The sum of the first ten terms of G.P. is S_1

$S_1 = \frac{a(1-r^{10})}{1-r}$

where a is first term and r is common ratio
S_2 is sum from 11th to 20th term.

$S_2 = \dfrac{ar^{10}(1-r^{10})}{1-r}$

$\therefore \dfrac{S_2}{S_1} = \dfrac{ar^{10}(1-r^{10})}{1-r} \cdot \dfrac{(1-r)}{a.(1-r^{10})}$

$\therefore r^{10} = \dfrac{S_2}{S_1}$ $\qquad r = \pm \sqrt[10]{\dfrac{S_2}{S_1}}$

B-17. **Sol.** a, b, c, d are in G.P.

Let common ratio is r

$\therefore$ $b = ar,\ c = ar^2,\ d = ar^3$

$\therefore$ $(a-c)^2 + (b-c)^2 + (b-d)^2 - (a-d)^2 = (a-ar^2)^2 + (ar-ar^2)^2 + (ar-ar^3)^2 - (a-ar^3)^2$

$= a^2[1 + r^4 - 2r^2 + r^2 + r^4 - 2r^3 + r^2 + r^6 - 2r^4 - 1 - r^6 + 2r^3]$

$= a^2 \times 0 = 0$

B-18. **Sol.** Let G_1, G_2, G_3, G_4, are 4 GM's between 3 and 729

$\Rightarrow$ $r = \left(\dfrac{729}{3}\right)^{1/5} = 3$

$\Rightarrow$ $G_3 = 3\ (r)^3 = 81$

B-19. **Sol.** Let the numbers are a and b.

A.M. of a and b = $\dfrac{a+b}{2} = 34$ (Given)

$\therefore$ $a + b = 68$

G.M. of a and b = $\sqrt{ab} = 16$ (Given)

$\therefore$ $ab = 256$

$a + b = 68$ and $ab = 256$

$\therefore$ $a = 64,\ b = 4$ Numbers are 64 and 4.

B-20. **Sol.** There are 6 G.M.s between 8 and $\dfrac{1}{16}$

$\therefore$ $8, G_1, G_2, \ldots\ldots, G_6, \dfrac{1}{16}$ are in G.P.

we know that product of n G.M. between two numbers is n^{th} power of single G.M. between two numbers.

G.M. of 8 and $\dfrac{1}{16}$ is $\sqrt{8.\dfrac{1}{16}} = \dfrac{1}{\sqrt{2}}$

$\therefore$ product of 6 G.Ms = $\left(\dfrac{1}{\sqrt{2}}\right)^6 = \dfrac{1}{8}$

B-21. **Sol.** Given numbers are p and q

A.M. = $\dfrac{p+q}{2}$, GM = $\sqrt{pq}$

= 2 (Given)

$\therefore \dfrac{p+q}{2\sqrt{pq}} = 2$

$\therefore \dfrac{p+q+2\sqrt{pq}}{p+q-2\sqrt{pq}} = \dfrac{3}{1}$ (componendo dividendo method)

$$\frac{(\sqrt{p}+\sqrt{q})^2}{(\sqrt{p}-\sqrt{q})^2} = \frac{3}{1} \Rightarrow \frac{\sqrt{p}+\sqrt{q}}{\sqrt{p}-\sqrt{q}} = \sqrt{3}$$

$$\therefore \frac{2\sqrt{p}}{2\sqrt{q}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\therefore \frac{p}{q} = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2 = \frac{4+2\sqrt{3}}{4-2\sqrt{3}} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

B-22. **Sol.** Let numbers are a, b

a, G_1, G_2, b or ;k $A = \frac{a+b}{2}$

$$r = \left(\frac{b}{a}\right)^{\frac{1}{3}}$$

$$G_1 = a\left(\frac{b}{a}\right)^{\frac{1}{3}}, \quad G_2 = a.\left(\frac{b}{a}\right)^{\frac{2}{3}} \qquad \therefore \frac{G_1^3+G_2^3}{G_1G_2} = \frac{a^3.\frac{b}{a}+a^3.\frac{b^2}{a^2}}{a^2.\frac{b}{a}} = \frac{a^2b+ab^2}{ab} = a + b = 2A$$

Section (C) : Harmonic and Arithmetic Geometric Progression

C-1. **Sol.** m_{th} term of H.P. is n

so m_{th} term of A.P. is $\frac{1}{n}$

n_{th} term of H.P. is m

n_{th} term of A.P. is $\frac{1}{m}$

Let first term of A.P. is a and common difference is d.

$\therefore$ $t_m = a + (m-1)\,d = \frac{1}{n}$

$t_n = a + (n-1)\,d = \frac{1}{m}$

$\therefore$ $(m-n)\,d = \frac{m-n}{mn}$ $\Rightarrow$ $d = \frac{1}{mn}$

$a = \frac{1}{n} - \frac{m-1}{mn} = \frac{1}{mn}$

$\therefore$ r_{th} term of A.P. is $T_r = \frac{1}{mn} + \frac{r-1}{mn} = \frac{r}{mn}$

$\therefore$ rth term of H.P.= $\frac{mn}{r}$

C-2. **Sol.** a and b are in H.P.

$\therefore$ $\frac{1}{a}, \frac{1}{b}$ are in A.P.

common difference d for A.P. = $\frac{1}{b} - \frac{1}{a}$

n_{th} term of A.P. = $\frac{1}{a} + (n-1)\left(\frac{1}{b}-\frac{1}{a}\right) = \frac{b+(n-1)(a-b)}{ab}$

$\therefore$ n_{th} term of H.P. = $\frac{ab}{b+(n-1)(a-b)}$

C-3. Sol. Put b = $\frac{2ac}{a+c}$ $\Rightarrow$ $\frac{b+a}{b-a} + \frac{b+c}{b-c} = \frac{\frac{2ac}{a+c}+a}{\frac{2ac}{a+c}-a} + \frac{\frac{2ac}{a+c}+c}{\frac{2ac}{a+c}-c}$

$= \frac{a+3c}{c-a} + \frac{3a+c}{a-c} = \frac{a+3c-3a-c}{c-a} = \frac{2(c-a)}{c-a} = 2$

C-4. Sol. Let H.P. is $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}$, ...

$\Rightarrow$ $\frac{1}{a} = \frac{2}{5}$ & $\frac{1}{a+d} = \frac{12}{13}$ $\Rightarrow$ a = $\frac{5}{2}$ & a + d = $\frac{13}{12}$ $\Rightarrow$ d = $-\frac{17}{12}$

$t_n = \frac{1}{a+(n-1)d} = \frac{1}{\frac{5}{2}-\frac{(n-1)17}{12}} = \frac{12}{47-17n}$

Clearly t_n is largest only if n = 2 $\Rightarrow$ $t_2 = \frac{12}{13}$

C-5. Sol. loga, logb, logc are in AP

$\Rightarrow$1 + loga, 1 + logb, 1 + logc are in AP

$\Rightarrow$ $\frac{1}{1+\log a}, \frac{1}{1+\log b}, \frac{1}{1+\log c}$ are in H.P.

C-6. **Sol.** a, b, c, d are in A.P.

$\frac{a}{abcd}, \frac{b}{abcd}, \frac{c}{abcd}, \frac{d}{abcd}$ are in A.P.

$\frac{1}{bcd}, \frac{1}{acd}, \frac{1}{abd}, \frac{1}{abc}$ are in A.P.

$\frac{1}{abc}, \frac{1}{abd}, \frac{1}{acd}, \frac{1}{bcd}$ are in A.P.

abc, abd, acd, bcd are in H.P.

C-7. **Sol.** $\because$ equal numbers are always in A.P.

C-8. **Sol.** $\left(\frac{1}{x}\right)^a, \left(\frac{1}{x}\right)^b, \left(\frac{1}{x}\right)^c$ are in G.P.

$\therefore$ $\left(\frac{1}{x}\right)^{2b} = \left(\frac{1}{x}\right)^a . \left(\frac{1}{x}\right)^c$

$\Rightarrow$ $\left(\frac{1}{x}\right)^{2b} = \left(\frac{1}{x}\right)^{a+c}$ $\Rightarrow$ 2b = a + c

$\therefore$ a, b, c are in A.P.

C-9. **Sol.** $\dfrac{1}{\sqrt{b}+\sqrt{c}} - \dfrac{1}{\sqrt{c}+\sqrt{a}} = \dfrac{1}{\sqrt{c}+\sqrt{a}} - \dfrac{1}{\sqrt{a}+\sqrt{b}}$

$\dfrac{\sqrt{c}+\sqrt{a}-\sqrt{b}-\sqrt{c}}{(\sqrt{b}+\sqrt{c})(\sqrt{c}+\sqrt{a})} = \dfrac{\sqrt{a}+\sqrt{b}-\sqrt{c}-\sqrt{a}}{(\sqrt{a}+\sqrt{b})(\sqrt{a}+\sqrt{c})}$

$(a - b) = b - c \quad \Rightarrow \quad a + c = 2b$

$a, b, c \rightarrow$ A.P. $\quad \Rightarrow \quad ax, bx, cx \rightarrow$ A.P.

$ax + 1, bx + 1, cx + 1 \rightarrow$ A.P.

$9_{ax+1}, 9_{bx+1}, 9_{cx+1} \rightarrow$ G.P.

C-10. **Sol.** let three numbers in HP are $\dfrac{1}{a-d}, \dfrac{1}{a}, \dfrac{1}{a+d} \quad \Rightarrow \quad \dfrac{1}{a-d} + \dfrac{1}{a} + \dfrac{1}{a+d} = 37$.........(i)

and $a - d + a + a + d = \dfrac{1}{4} \quad \Rightarrow \quad a = \dfrac{1}{12}$

put the value of a in (i) $\Rightarrow \dfrac{12}{1-12d} + \dfrac{12}{1+12d} = 25 \quad \Rightarrow \dfrac{24}{25} = 1 - 144d_2 \quad \Rightarrow d = \pm \dfrac{1}{60}$

Hence number are $\dfrac{1}{\frac{1}{12}-\frac{1}{60}}, 12, \dfrac{1}{\frac{1}{12}+\frac{1}{60}} = 15, 12, 10$

C-11. Sol. $a = \dfrac{1}{3}$, $b = \dfrac{1}{13}$

Hence common diff. $d = \dfrac{13-3}{5} = 2$

$H_1 = \dfrac{1}{3+2} = \dfrac{1}{5}$

$H_2 = \dfrac{1}{3+2.2} = \dfrac{1}{7}$

$H_1 = \dfrac{1}{3+2.2} = \dfrac{1}{9}$

$H_1 = \dfrac{1}{3+2.4} = \dfrac{1}{11} \quad \Rightarrow \quad = 5 + 7 + 9 + 11 = 32$

C-12. **Sol.** Let number are a and b then

$A = \dfrac{a+b}{2}$, $\quad G = \sqrt{ab} \quad$ and $\quad H = \dfrac{2ab}{a+b}$

$xA = yG \quad \Rightarrow \quad \dfrac{x}{y} = \dfrac{2\sqrt{ab}}{a+b}$

also , $yG = zH \quad \Rightarrow \dfrac{y}{z} = \dfrac{2\sqrt{ab}}{a+b} \quad \Rightarrow \dfrac{x}{y} = \dfrac{y}{z}$ Hence x,y, z are in GP

C-13. **Sol.** let $H_1, H_2, H_3, \ldots, H_{18}$ are ae 18 HM's between $\dfrac{1}{3}$ & $\dfrac{1}{41}$ then

$\dfrac{1}{H_1}, \dfrac{1}{H_2}, \dfrac{1}{H_3}$ ....., $\dfrac{1}{H_{18}}$ are AM's between 3 and 41 $\Rightarrow \dfrac{1}{H_4} = 3 + 4\left(\dfrac{41-3}{19}\right) = 3 + 8 = 11 \Rightarrow H_4 = 1/11$

C-14. **Sol.** $S_n = 1+\frac{2}{3}+\frac{3}{3^2}+\frac{4}{3^3}+$ ....+ $\frac{n}{3^{n-1}}$ ....(i)

$\Rightarrow \frac{1}{3}S_n = \frac{1}{3}+\frac{2}{3^2}+\frac{3}{3^3}$ +......+ $\frac{n}{3^n}$ ....(ii)

by (i) and (ii)

$\frac{2}{3}S_n = 1+\frac{1}{3}+\frac{1}{3^2}$ + ...nterms – $\frac{n}{3^n}$ $\Rightarrow$ $S_n = \frac{3}{2}\left(\frac{3}{2}\left(1-\frac{1}{3^n}\right)-\frac{n}{3^n}\right)$

C-15. **Sol.** Clearly the given sequence in AGP where

t_n = (1+ (n–1)3) $\frac{1}{5^{n-1}} = \frac{3n-2}{5^{n-1}}$

C-16. **Sol.** $S = 5-\frac{7}{3}+\frac{9}{3^2}-\frac{11}{3^3}$ +...............∞ ....(i)

.............(ii)

by (i) and (ii) $\frac{4}{3}S = 5 - \frac{2}{3}+\frac{2}{3^2}-\frac{2}{3^3}$ +........

$= 5 - \frac{\frac{2}{3}}{1+\frac{1}{3}}$ $\Rightarrow s = \frac{27}{8}$

C-17. **Sol.** $S = 4\ \frac{8}{3} + \frac{12}{3^2} + \frac{16}{3^3}$ +..........+ $\frac{4n}{3^{n-1}}$ ...(i)

$\frac{S}{3} = \frac{4}{3} + \frac{8}{3^2} + \frac{12}{3^3}$ + ...+ $\frac{4n}{3^n}$ ...(ii)

(i) – (ii) we get

$\frac{2}{3}S = 4 + \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3}$ +..........+ $\frac{4}{3^{n-1}} - \frac{4n}{3^n}$

$= 4\left(\frac{1-\left(\frac{1}{3}\right)^n}{1-\frac{1}{3}}\right) - \frac{4n}{3^n}$

$S = \frac{4\cdot 3}{2} . \frac{3}{2}\left(1-\left(\frac{1}{3}\right)^n\right) - \frac{4n}{3^n}\ \frac{3}{2}$ $\therefore S_\infty = 9$

C-18. **Sol.** $S_\infty = 2^{1/4} . 4^{1/8} . 8^{1/16}$...............∞

$S_\infty = 2^{1/4} . 2^{2/8} . 2^{3/16}$...............∞

$S_\infty = 2^{1/4 + 2/8 + 3/16 ....\infty} = 2^{S'_\infty}$

Let $S'_\infty = \frac{1}{4} + \frac{2}{8} + \frac{3}{16}$ ....

$S'_n = \frac{1}{4} + \frac{2}{8} + \frac{3}{16}$ .... $\frac{n}{2^{n+1}}$ ...(i)

$\frac{S'_n}{2} = \frac{1}{8} + \frac{2}{16} + \frac{3}{16}$ .... $\frac{n-1}{2^{n+1}} + \frac{n}{2^{n+2}}$ ...(ii)

(i) – (ii) we get

$\frac{S'_n}{2} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... \frac{1}{2^{n+1}} - \frac{n}{2^{n+2}}$

$\frac{S'_n}{2} = \frac{1}{4}\left(\frac{1-(1/2)^n}{1-1/2}\right) - \frac{n}{2^{n+2}}$

$S'_n = \frac{2.2}{4}\left(1-\left(\frac{1}{2}\right)^n\right) - \frac{2n}{2^{n+2}}$

$S'_\infty = 1$

$\therefore S_\infty = 2^{S'_\infty} = 2$

Section (D) : Relation between A.M. , G.M. and H.M.

D-1. Sol. Sum of first 15 odd nutural numbers = 1 + 3 + 5 +.......+ 29

D-2. Sol. HM = $\frac{50}{\frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_{50}}} = \frac{50}{\sum_{r=1}^{50}(r+1)} = \frac{50}{\frac{50}{2}(2+51)} = \frac{2}{53}$

D-3. Sol. HM between a and c = b

and GM = $\sqrt{ac}$

Also HM between b and d = c

and GM = $\sqrt{bd}$

But GM > HM

$\therefore$ $\sqrt{ac}$ > b and $\sqrt{bd}$ > c

$\Rightarrow$ $\sqrt{ac}\sqrt{bd}$ > bc $\Rightarrow$ ad > bc

D-4. Sol. Let numbers are a_1 , a_2 , a_3 .... a_n

$\therefore$ a_1a_2 .... a_n = 1 (Given)

AM ≥ GM

$\therefore$ $\frac{a_1 + a_2 + a_3 ..... + a_n}{n} \geq (a_1a_2 ..... a_n)^{1/n}$

$a_1 + a_2 + ... + a_n \geq n(1)^{1/n}$

$\therefore$ $a_1 + a_2 + ... + a_n \geq n$

so sum of positive numbers will not be less than n.

D-5. Sol. $\frac{x^{100}}{1+x+x^2+x^3+.....+x^{200}}$

AM ≥ GM

$\frac{1+x+x^2+x^3+.....+x^{200}}{201} \geq \left(1.x.x^2.....x^{200}\right)^{\frac{1}{201}}$

$\frac{1+x+x^2+x^3+.....+x^{200}}{201} \geq \left(x^{\frac{201}{2}.200}\right)^{\frac{1}{201}}$

$\frac{1+x+x^2+x^3+.....+x^{200}}{201} \geq x^{100}$

$$\frac{x^{100}}{1+x+x^2+x^3+.....+x^{200}} \le \frac{1}{201}$$

D-6. **Sol.** (1) $\frac{a}{b}+\frac{b}{c}+\frac{c}{a} \ge \left(\frac{a}{b}.\frac{b}{c}.\frac{c}{a}\right)^{1/3} \Rightarrow \frac{a}{b}+\frac{b}{c}+\frac{c}{a} \ge 3$

(2) AM ≥ HM ⇒ $\frac{a+b+c}{3} \ge \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

(3) $\frac{a+b+c}{3} \ge (abc)^{1/3}$

(4) $\frac{a^2b+b^2c+c^2a}{3} \ge (a^3b^3c^3)^{1/3} \Rightarrow a^2b+b^2c+c^2a \ge 3abc$

D-7. **Sol.** Let the two positive numbers are a and b
By given condition
A.M. = GM. + 5 or A = G + 5 ....(i)
G.M. = HM. + 4 or G = H + 4 ....(ii)
we know G_2 = AH
or G_2 = (G + 5) (G – 4) [By (i) and (ii)]
⇒ G = 20
∴ A = 25 and H = 16
Again A = $\frac{a+b}{2}$ ∴ $\frac{a+b}{2}$ = 25 ⇒ a + b = 50
G = $\sqrt{ab}$
∴ $\sqrt{ab}$ = 20 ⇒ ab = 400
solving the equation we get a = 10, b = 40

D-8. **Sol.** Let A = 5x & G = 3x

Hence $\frac{a}{b} = \frac{A+\sqrt{A^2-G^2}}{A-\sqrt{A^2-G^2}} = \frac{5x+\sqrt{25x^2-9x^2}}{5x-\sqrt{25x^2-9x^2}} \Rightarrow \frac{a}{b} = \frac{9}{1}$

Section (E) : Σn, Σn_2, Σn_3, Method of difference and V_n method

E-1. **Sol.** $\sum_{r=1}^{n} t_r = 2\sum_{r=1}^{n} t^2 + \sum_{r=1}^{n} 3 = \frac{2n(n+1)(2n+1)}{6} + 3n$
Put n = 10

E-2. **Sol.** = r (r+1)

⇒ $S_n = \sum_{r=1}^{n} t_r = \sum_{r=1}^{n} r(r+1) = \frac{n(n+1)(n+2)}{3}$

$S_{20} = \frac{20.21.22}{3} = 3080$

E-3. **Sol.** $\sum_{r=1}^{n} t_r = 2\sum_{r=1}^{n} r + \sum_{r=1}^{n} 2^r$

$= \frac{2n(n+1)}{2} + \frac{2(2^n-1)}{2-1}$

Put n = 6 $\Rightarrow \sum_{r=1}^{6} t_r$ = 42 + 126 = 168

E-4. **Sol.** The sum of the series $1.3^2 + 2.5^2 + 3.7^2 +$........to 20 terms
T_n of A.P. 1,2,3.....is n.
T_n of A.P. 3,5,7,9.....is 3 + (n –1)2 = 2n + 1.
$\therefore$ T_n of the given series is $n(2n + 1)^2$ or $T_n = 4n^3 + 4n^2 + n$
$S_n = 4 \sum n^3 + 4\sum n^2 + \sum n$
$= 4\left[\frac{n(n+1)}{2}\right]^2 + 4\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{n(n+1)}{2}$
Now putting n = 20
we get S_{20} = 188090

E-5. **Sol.** The sum of the series 1.3.5 + 3.5.7 + 5.7.9 + ... up to n terms
here $T_n = (2n – 1)(2n + 1)(2n + 3)$
$T_n = 8n^3 + 12n^2 – 2n – 3$
$S_n = 8 \sum n^3 + 12 \sum n^2 – 2 \sum n – 3n$
$= 8\left[\frac{n(n+1)}{2}\right]^2 + \frac{12n(n+1)(2n+1)}{6} - \frac{2n(n+1)}{2} - 3n$
$= 2n^2 (n + 1)^2 + 2n (n + 1)(2n + 1) – n(n + 1) – 3n$
$= n (n + 1) [2n (n + 1) + 2 (2n + 1) – 1] – 3n$
$= n (n + 1) [2n^2 + 6n + 1] – 3n$
$= 2n^4 + 8n^3 + 7n^2 – 2n$
$= n(2n^3 + 8n^2 + 7n – 2)$

E-6. **Sol.** $S = 1^2 + 2^2 x + 3^2x^2 + \ldots\ldots$
$Sx = 1^2x + 2^2x^2 + \ldots\ldots$

$S(1 – x) = 1 + 3x + 5x^2 + \ldots\ldots\ldots$
$Sx(1 – x) = x + 3x^2 + \ldots\ldots\ldots$
$\therefore$ Subtracting
$S(1 – x)^2 = 1 + 2x + 2x^2$
$S(1 – x)^2 = 1 + \frac{2x}{1-x}$
$\therefore$ $S(1 – x)^2 = \frac{1+x}{1-x}$
$S = \frac{1+x}{(1-x)^3}$

E-7. **Sol.** $(1^2 – t_1) + (2^2 – t_2) + \ldots\ldots + (n^2 – t_n) = \frac{1}{3} n (n^2 – 1)$
Let () $S_n = t_1 + t_2 + \ldots\ldots + t_n$
$\therefore$ $\sum n^2 – S_n = \frac{1}{3} n(n^2 – 1)$
$S_n = \frac{1}{6} n(n + 1) (2n + 1) – \frac{1}{3} n (n + 1) (n – 1)$
$\Rightarrow$ $S_n = \frac{1}{6} n (n + 1) . 3 = \frac{n}{2} (n + 1)$
$\Rightarrow$ $S_n = \sum n = 1 + 2 + 3 + \ldots\ldots + n$
$\Rightarrow$ $t_n = n$

E-8. **Sol.** Given sequence is 1, 3, 6, 10, 15, 21, ..., 5050

First order difference is 2, 3, 4, 5, 6

second order difference is 1, 1, 1, 1 ....

so $T_n = an^2 + bn + c$

$\therefore$ $T_1 = a + b + c = 1$

$T_2 = 4a + 2b + c = 3$

$T_3 = 9a + 3b + c = 6$

Solving those three equations we get

$a = \frac{1}{2}$, $b = \frac{1}{2}$ and $c = 0$

$\therefore$ $T_n = \frac{n^2}{2} + \frac{n}{2}$

Last term of the sequence is 5050

Let $T_n = 5050$

$\therefore$ $\frac{n^2}{2} + \frac{n}{2} = 5050$

$\Rightarrow$ $n^2 + n - 10100 = 0$ $\Rightarrow$ $(n + 101)(n - 100) = 0$

$n = -101, 100$

n cannot be negative

So n = 100

The number of terms in the given sequence = 100

E-9. **Sol.** $S = 2 + 3 + 6 + 11 + 18 + ...+ t_n$ ...(i)

$S = \quad 2 + 3 + 6 + 11 + 18 + ...+ t_n$ ...(ii)

(i) – (ii) we get ls

$0 = 2 + 1 + 3 + 5 + 7 + ...(n - 1)$ terms in $- t_n$

$t_n = 2 + (1 + 3 + 5 + 7 + ...(n - 1)$ term in

$t_n = 2 + (n - 1)^2$

$t_{50} = 2 + 49^2$

E-10. **Sol.** $S = \sum_{r=2}^{\infty}\left(\frac{1}{r^2 - 1}\right)$

$S = \frac{1}{2}\sum_{r=2}^{\infty}\left(\frac{1}{r-1} - \frac{1}{r+1}\right)$

$S = \frac{1}{2}\left[\left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{4} - \frac{1}{6}\right) + ...\infty\right]$

$= \frac{1}{2}\left[1 + \frac{1}{2}\right] = \frac{3}{4}$ **Ans**

E-11. **Sol.** Series is $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... \infty$

Here $T_n = \frac{1}{n(n+1)} = \left[\frac{1}{n} - \frac{1}{n+1}\right]$

$\therefore$ $T_1 = \frac{1}{1} - \frac{1}{2}$ $\Rightarrow$ $T_2 = \frac{1}{2} - \frac{1}{3}$

$T_n = \frac{1}{n} - \frac{1}{n+1}$

∴ $S_n = T_1 + T_2 ...... + T_n = 1 - \frac{1}{n+1}$

when (tc) $n \to \infty$ $\frac{1}{n+1} \to 0$

∴ $S_\infty = 1 - 0 = 1$

E-12. Sol. $S = \frac{1}{1.3.5} + \frac{1}{3.5.7} + \frac{1}{5.7.9} + ...n$

$T_n = \frac{1}{(2n-1)(2n+1)(2n+3)}$

$\therefore T_n = \frac{1}{4}\left[\frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n+1)(2n+3)}\right]$

$T_1 = \frac{1}{4}\left[\frac{1}{1.3} - \frac{1}{3.5}\right]$, $T_2 = \frac{1}{4}\left[\frac{1}{3.5} - \frac{1}{5.7}\right]$,

$T_3 = \left[\frac{1}{5.7} - \frac{1}{7.9}\right]$

$T_n = \left[\frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n+1)(2n+3)}\right]$

sum of all terms gives S_n

$S_n = \frac{1}{4}\left[\frac{1}{3} - \frac{1}{(2n+1)(2n+3)}\right]$

E-13. Sol. $t_r = \frac{1}{1+2+3+......+r} = \frac{2}{r(r+1)} = 2\left[\frac{1}{r} - \frac{1}{r+1}\right]$

Hence $\sum_{r=1}^{n} t_r = 2\sum_{r=1}^{n}\left(\frac{1}{r} - \frac{1}{r+1}\right) = 2\left[\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + ... + \left(\frac{1}{n} - \frac{1}{n+1}\right)\right] = \frac{2n}{n+1}$ Ans.

Exercise-2

Marked Questions may have for Revision Questions.

PART - I : OBJECTIVE QUESTIONS

Note : Tough Questions

1. Sol. (4) $a_1 + 4a_2 + 6a_3 - 4a_4 + a_5 = 0$
$a - 4(a + d) + 6(a + 2d) - 4(a + 3d) + (a + 4d)$
$= 0 - 0 = 0$
Like wise we can check other options

2. Sol. $2\log_5(2^x - 5) = \log_5 2 + \log_5 (2^x - 7/2)$

$\Rightarrow$ $(2^x - 5)^2 = 2\left(2^x - \frac{7}{2}\right)$

$\Rightarrow$ $t^2 - 10t + 25 = 2t - 7$ {put $2^x = t$ } {$2^x = t$ j[kus ij}

$\Rightarrow$ $t^2 - 12t + 32 = 0$

$\Rightarrow$ $t = 8 , 4$

$\therefore$ $2^x = 4$ or $2^x = 8$

$\therefore$ $x = 2, 3$ ($\because$ $2^x - 5 > 0 \Rightarrow 2^x > 5$)

so only solution $x = 3$

$\therefore 2x = 6$

3. Sol. $S = \log a + \log \frac{a^2}{b} + \log \frac{a^3}{b^2} + \ldots.$ n terms $= \log a + (2 \log a - \log b) + (3 \log a - 2 \log b) + \ldots$ n terms

Which is an A.P. with $d = \log a - \log b = \log \frac{a}{b}$ and $A = \log a$

$\therefore$ $S_n = \frac{n}{2}\left[2 \log a + (n-1) \log \frac{a}{b}\right]$

$= \frac{n^2}{2} \log \frac{a}{b} + \frac{n}{2}[2 \log a - \log a + \log b]$

$= \frac{n^2}{2} \log \frac{a}{b} + \frac{n}{2} \log ab$

4. Sol. Let the first term and common difference of first A.P. is a_1 and d_1 and of second A.P. is a_2 and d_2

$\therefore$ $\frac{S_1}{S_2} = \frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{3n+8}{7n+15}$

$\frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{3n+8}{7n+15}$

Ratio of 12th terms $= \frac{a_1 + 11d_1}{a_2 + 11d_2}$

So $\frac{n-1}{2} = 11$, n = 23

So ratio of 12th terms = $\frac{3\times 23+8}{7\times 23+15} = \frac{77}{176} = \frac{7}{16}$

5. **Sol.** a and be first and last term

$\therefore$ S = $\frac{n}{2}$ (a + ℓ) or $\frac{2S}{a+\ell}$ = n ....(1)

= a + (n – 1)d so d = $\frac{\ell-a}{n-1}$ ....(2)

putting the value of n from (1) in (2)

d = $\frac{\ell-a}{\frac{2S}{a+\ell}-1}$, d = $\frac{\ell^2-a^2}{2S-\ell-a}$

6. **Sol.** b_1, b_2, b_3 are in G.P. $\therefore$ $b_3 > 4b_2 - 3b_1$ $\Rightarrow$ $r_2 > 4r - 3$

$\Rightarrow$ $r_2 - 4r + 3 > 0$

$\Rightarrow$ (r – 1) (r – 3) > 0 So 0 < r < 1 and r > 3

7. **Sol.** Let $T_{k+1} = ar_k$ and $T'_{k+1} = br_k$. **[S & S, M]**

Since $T''_{k+1} = ar_k + br_k = (a + b)\, r_k$, $\therefore$ T''_{k+1} is general term of a G.P.

8. **Sol.** Let b + c − a, c + a − b, a + b − c are in A.P.

$\Rightarrow$ 2(c + a– b) = (b + c – a) + (a + b – c)

$\Rightarrow$ 2c + 2a = 4b

$\Rightarrow$ 2b = a + c $\Rightarrow$ a, b, c are in A.P.

9. **Sol.** x, y, z are in G.P.

$\therefore$ $y_2 = xz$ ....(i)

$\Rightarrow$ x(x + z), y (x + z), z (x + z) are in G.P.

$\Rightarrow$ $x_2 + xz$, $xy + yz$, $z_2 + xz$ are in G.P.

$\Rightarrow$ $x_2 + y_2$, $xy + yz$, $y_2 + z_2$ are in G.P.

[putting $y_2 = xz$ from (i)]

10. **Sol.** Let b = ar, c = ar_2 and d = ar_3

So $a_2\,(1-r_2)$, $a_2(r_2)(1-r_2)$, $a_2r_4(1-r_2)$ these are in G.P.

So $(a_2 - b_2)$, $(b_2 - c_2)$, $(c_2 - d_2)$ are in G.P.

11. **Sol.** Let three positive numbers which are in G.P. be a, b, c

$\therefore$ $b_2 = ac$ .....(i)

By first condition a, b + 8, c are in A.P.

$\therefore$ b = $\frac{a+c}{2}$ – 8 ....(ii)

By second condition, a, b + 8, c + 64 are in G.P.

$\therefore$ $(b + 8)_2 = a\,(c + 64)$

$\therefore$ $b_2 + 64 + 16b = ac + 64a$

$\therefore$ $b_2 = ac$

$\therefore$ 64 + 16b = 64a

$\Rightarrow$ b = 4(a – 1) ....(iii)

putting the value of b in (ii)

4(a – 1) = $\frac{a+c}{2}$ – 8

⇒ $c = 7a + 8$ ....(iv)

Now putting the value of both b and c in (i)

$16(a-1)^2 = a(7a+8)$

⇒ $9a^2 - 4a + 16 = 0$

$(9a-4)(a-4) = 0$

⇒ $a = \frac{4}{9}$, $a = 4$

$a = \frac{4}{9}$ gives negative value of b, but it is given that b is positive, so a = 4 is acceptable value

$b = 4(4-3) = 12$

∴ common ratio = $\frac{12}{4} = 3$

12. Sol. If a be the side of a square then d = $a\sqrt{2}$

by given condition → $a_n = \sqrt{2}\ a_{n+1}$ or $a_{n+1} = \frac{a_n}{\sqrt{2}} = \frac{a_{n-1}}{(\sqrt{2})^2} = \frac{a_{n-2}}{(\sqrt{2})^3} = ... = \frac{a_1}{(\sqrt{2})^n}$

Replacing n by n – 1, we get

$a_n = \frac{a_1}{(\sqrt{2})^{n-1}} = \frac{10}{2^{\frac{(n-1)}{2}}}$

Area of $S_n < 1$ ⇒ $a_n^2 < 1$

⇒ $\frac{100}{2^{n-1}} < 1$ or $200 < 2^n$ or $2^n > 200$

Now $2^7 = 128 < 200$, $2^8 = 256 > 200$

∴ n = 8, 9, 10

13. Sol. ∵ a,b,c are in H.P.

∴ $b = \frac{2ac}{a+c}$

Now $-\frac{b}{2}, \frac{b}{2}, c - \frac{b}{2}$

$a - \frac{ac}{a+c}, \frac{ac}{a+c}, c - \frac{ac}{a+c}$

$\frac{a^2}{a+c}, \frac{ac}{a+c}, \frac{c^2}{a+c}$

∴ $\left(\frac{ac}{a+c}\right)^2 = \left(\frac{a^2}{a+c}\right)\left(\frac{c^2}{a+c}\right)$

so given numbers are in G.P.

14. Sol. $\frac{a+be^y}{a-be^y} + 1 = \frac{b+ce^y}{b-ce^y} + 1 = \frac{c+de^y}{c-de^y} + 1$

$\frac{2a}{a-be^y} = \frac{2b}{a-ce^y} = \frac{2c}{c-de^y}$

$1 - \frac{be^y}{a} = 1 - \frac{c}{b}e^y = 1 - \frac{d}{c}e^y$ ⇒ $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

⇒ a, b, c, d are in G.P. a, b, c, d xq.kksÙkj Js<h esa gSA

15. Sol. $\frac{1}{a_1}, \frac{1}{a_2}, ...\frac{1}{a_n}$, are in A.P.

$\Rightarrow$ $\frac{1}{a_2} - \frac{1}{a_1} = ... = \frac{1}{a_n} - \frac{1}{a_{n-1}} = d$

$\Rightarrow$ $a_1a_2 = \frac{a_1 - a_2}{d}$, $a_2a_3 = \frac{a_2 - a_3}{d}$ ...., $\frac{a_{n-1} - a_n}{d} = a_{n-1}.a_n$

$\Rightarrow$ $a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$

$= \frac{a_1 - a_2}{d} +........+ \frac{a_{n-1} - a_n}{d} = \frac{a_1 - a_n}{d} = (n - 1)a_1a_n$

16. Sol. In an A.P. $t_1 = \log_{10}a$, $t_{n+1} = \log_{10}b$ and $t_{2n+1} = \log_{10}c$

t_1, t_{n+1}, t_{2n+1} form an A.P. of common difference nd as they are a, a + nd, a + 2nd

$2t_{n+1} = t_1 + t_{2n+1}$

or $2 \log b = \log a + \log c$

or $\log b_2 = \log ac$

$\therefore$ $b_2 = ac$

$\therefore$ a, b, c are in G.P.

17. Sol. $a_1, a_2, a_3 ...... a_{2n+1}$ are in A.P.

Let common difference is d

$a_{2n+1} = a_1 + 2nd \Rightarrow a_{2n+1} - a_1 = 2nd$

Similarly $a_{2n} - a_2 = 2(n - 1) d$

In denominator, by properties of A.P. $a_{2n+1} + a_1 = a_{2n} + a_2 = a_{n+2} + a_n$

So all terms in denominator is same

sum = $\frac{2d[n + (n-1) + (n-2) + ...1]}{a_{2n+1} + a_1}$

sum = $\frac{2dn(n+1)}{2(a_1 + a_{2n+1})} = \frac{n(n+1)(a_2 - a_1)}{a_1 + a_{2n+1}}$

But $a_1 + a_{2n+1} = a_1 + a_1 + 2nd = 2(a_1 + nd)$

$\because$ $a_1 + nd = a_{n+1}$

$\therefore$ $a_1 + a_{2n+1} = 2a_{n+1}$

$\therefore$ sum = $\frac{n(n+1)}{2a_{n+1}}(a_2 - a_1)$

18. Sol. $x + y + z = 15$ ....(i)

a, x, y, z, b are in AP

Suppose d is common difference

$d = \frac{b - a}{4}$

$\therefore$ $x = a + \frac{b-a}{4} = \frac{b+3a}{4}$, $y = \frac{2b+2a}{4}$ and $z = \frac{3b+a}{4}$

on substituting the values of x, y and z in (i), we get

$\Rightarrow$ $\frac{6a+6b}{4} = 15$

$\Rightarrow$ $a + b = 10$ ..... (ii)

$\because$ $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{5}{3}$ ....(iii)

and a, x, y, z, b are in H.P.

$\therefore$ $\frac{1}{a}, \frac{1}{x}+\frac{1}{y}+\frac{1}{z}, \frac{1}{b}$ are in A.P.

$\therefore$ $\frac{1}{x}=\frac{1}{a}+\frac{\left(\frac{1}{b}-\frac{1}{a}\right)}{4}$, $\frac{1}{y}=\frac{1}{a}+\frac{2}{4}\left(\frac{1}{b}-\frac{1}{a}\right)$ and $\frac{1}{z}=\frac{1}{a}+\frac{3}{4}\left(\frac{1}{b}-\frac{1}{a}\right)$

on substituting the value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ in (iii), we get

$\frac{3}{a}+\frac{6}{4}\left(\frac{1}{b}-\frac{1}{a}\right)=\frac{5}{3}$

$\Rightarrow$ $\frac{3}{2a}+\frac{3}{2b}=\frac{5}{3}$

$\frac{1}{a}+\frac{1}{b}=\frac{10}{9}$ ..... (iv)

By equations (ii) & (iv), we get

a = 9, b = 1 or a = 1, b = 9

19. **Sol.** A.P. T_1 $T_n = a$ T_{2n-1}

G.P. T_1 $T_n = b$ T_{2n-1}

H.P. T_1 $T_n = c$ T_{2n-1}

The n^{th} term is equidistant from the first and $(2n-1)^{th}$ term. In other words it is the middle term of a series of (2n – 1) terms. Also it is given that T_1 is same and T_{2n-1} is same for all the series. If they be p and q respectively, then p, a, q ; p, b, q ; p, c, q are in A.P., G.P. and H.P. respectively.

Therefore a, b, c are A.M., G.M. and H.M. respectively of the same quantities p and q.

We know that A.H. = G^2

$\therefore$ $ac = b^2$

$\therefore$ $ac - b^2 = 0$

20. **Sol.** $g(n) - g(n-1) = 1^2 + 2^2 + 3^2 + \ldots\ldots + (n-1)^2 + n^2 - (1^2 + 2^2 + 3^2 + \ldots\ldots + (n-1)^2) = n^2$

21. **Sol.** $x_1 + x_2 + x_3 + \ldots\ldots + x_{50} = 50$

AM ≥ HM

$$\frac{x_1+x_2+\ldots\ldots+x_{50}}{50} \geq \frac{1}{\frac{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\ldots.+\frac{1}{x_{50}}}{50}}$$

$$\Rightarrow \frac{x_1+x_2+\ldots\ldots+x_{50}}{50} \geq \frac{50}{\frac{1}{x_1}+\frac{1}{x_2}+\ldots\ldots+\frac{1}{x_{50}}}$$

$$\Rightarrow \frac{1}{x_1}+\frac{1}{x_2}+\ldots\ldots+\frac{1}{x_{50}} \geq 50$$

so minimum value of $\frac{1}{x_1}+\frac{1}{x_2}+\ldots\ldots+\frac{1}{x_{50}} = 50$

22. **Sol.** 0 < x , y and ab < 1 given series is $\sqrt{x}(\sqrt{a}+\sqrt{x})+\sqrt{x}(\sqrt{ab}+\sqrt{xy})+\sqrt{x}(b\sqrt{a}+y\sqrt{x})+\ldots\ldots\infty$.

$\therefore$ S = $(\sqrt{ax}+x+\sqrt{axb}+x\sqrt{y}+\sqrt{ax}b+xy \ldots\ldots \infty)$.

$\Rightarrow$ S = $(\sqrt{ax}+\sqrt{axb}+\sqrt{ax}b\ldots\ldots\infty) + (x+x\sqrt{y}+xy+\ldots\ldots\infty)$.

$\Rightarrow S = \frac{\sqrt{ax}}{1-\sqrt{b}} + \frac{x}{1-\sqrt{y}}$

$\Rightarrow S = \frac{\sqrt{ax}}{1-\sqrt{b}} + \frac{x}{1-\sqrt{y}}$

23. **Sol.** $S = \frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + ...$ n terms

$= \left(1-\frac{1}{2}\right) + \left(1-\frac{1}{2^2}\right) + \left(1-\frac{1}{2^3}\right) + ...+ \left(1-\frac{1}{2^n}\right) \quad = n - \left(\frac{1}{2}+\frac{1}{2^2}+......\frac{1}{2^n}\right)$

$= n - \frac{1}{2}\frac{1-\left(\frac{1}{2}\right)^n}{1-\left(\frac{1}{2}\right)} = n + 2^{-n} - 1$

24. **Sol.** $S = 1 + \frac{3}{4} + \frac{7}{16} + \frac{15}{64} + \frac{31}{256} + ...\infty$ ..... (i)

$\frac{1}{4}S = \frac{1}{4} + \frac{3}{16} + \frac{7}{64} + ...\infty$ ..... (ii)

(i) – (ii), we get

$\frac{3}{4}S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\infty$

$\frac{3}{4}S = \frac{1}{1/2} \Rightarrow S = \frac{8}{3}$

25. **Sol.** $T_n = \frac{n}{1+n^2+n^4} = \frac{1}{2}\left[\frac{(2n)}{(1+n+n^2)(1-n+n^2)}\right]$

$T_n = \frac{1}{2}\left[\frac{1}{1-n+n^2} - \frac{1}{1+n+n^2}\right]$

$\Rightarrow \quad T_1 = \frac{1}{2}\left[\frac{1}{1}-\frac{1}{3}\right]$,

$T_2 = \frac{1}{2}\left[\frac{1}{3}-\frac{1}{7}\right]$,

$T_3 = \frac{1}{2}\left[\frac{1}{7}-\frac{1}{13}\right]$,

:

$T_n = \frac{1}{2}\left[\frac{1}{1-n+n^2} - \frac{1}{1+n+n^2}\right]$

$S_n = \sum T_n = \frac{1}{2}\left[1-\frac{1}{1+n+n^2}\right] = \frac{n+n^2}{2(1+n+n^2)}$

$\therefore \quad S_\infty = \frac{1}{2}$

26. **Sol.** $S = 1^2 - 2^2 + 3^2 - 4^2 + ...- 2002^2 + 2003^2$

$= \sum_{n=1}^{2003} n^2 - 2\,[2^2 + 4^2 + \ldots\ldots\ldots\ldots + 2002^2]$

$= \sum_{n=1}^{2003} n^2 - 2 \times 4 \left[\sum_{n=1}^{1001} n^2\right]$

$= \frac{2003 \times 2004 \times 4007}{6} - \frac{8(1001)(1002)(2003)}{6}$

$= \frac{2003 \times 2004}{6}\,[4007 - 4004] = 2007006$ **Ans**

27. **Sol.** $S = 1 + (1 + x) + (1 + x + x^2) + \ldots.. + \frac{1-x^n}{1-x}$

$Sx = \quad x + (x + x^2) + \ldots. + \frac{x(1+x^{n-1})}{1-x} + \frac{x(1-x^n)}{1-x}$

$S(1 - x) = 1 + 1 + \ldots\ldots.$ to n terms

$\therefore S(1 - x) = n - \frac{x(1-x^n)}{1-x}$

$S = \frac{n}{1-x} - \frac{x(1-x^n)}{1-x}$ or $S = \frac{n(1-x)-x(1-x^n)}{(1-x)^2}$

28. **Sol.** $H_n = 1 \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{n}$

$1 + \frac{3}{2} + \frac{5}{3} + \ldots\ldots. + \frac{2n-1}{n}$

Here $T_n = \frac{2n-1}{n}$

$\therefore\ T_n = 2 - \frac{1}{n}$

$T_1 = 2 - \frac{1}{1}$

$T_2 = 2 - \frac{1}{2}$

$T_3 = 2 - \frac{1}{3}$

$T_n = 2 - \frac{1}{n}$

$\therefore \quad S_n = T_1 + T_2 + T_3 + \ldots\ldots. T_n$

$S_n = 2n - \left(1+\frac{1}{2}+\frac{1}{3}+\ldots..+\frac{1}{n}\right)$

$\therefore \quad S_n = 2n - H_n$

PART - II : MISCELLANEOUS QUESTIONS

A-1. **Ans.** **(1)**

Sol. Sum of n terms of A.P is always of the form An^2+Bn ; i.e. a quadratic expression in n, in such case the common difference is twice the co-efficient of n^2 i.e. 2A

A-2. Ans. (1)

Sol. Let a, b, c in G.P. then $b^2 = ac$

Now a + b , 2b, b + c in HP

$\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c}$ in AP

$\frac{2}{2b} = \frac{1}{a+b} + \frac{1}{b+c}$

(a + b) (b + c) = (a + c + 2b) b

$ab + b^2 + ac + bc = ab + bc + 2b^2$

∴ $b^2 = ac$

So statement (1) and (2) is true

A-3. Ans. (3)

Sol. $S = 1 + 2 + 4 + 7 + 11 + 16 \ldots\ldots T_n$ ...(i)

$S = 1 + 2 + 4 + 7 + 11 \ldots\ldots T_n$ ...(ii)

(i) – (ii) we get

O = 1+(1+2+3+4+5............(n–1) term) – T_n

$T_n = 1 + \frac{(n-1)n}{2} = \frac{n^2}{2} - \frac{n}{2} + 1$

∴ General term = $T_n = an^2 + bn + c$

here a = 1/2 , b= –1/2 , c = 1

$S_n = \sum T_n = \frac{n(n+1)(2n+1)}{12} - \frac{n(n+1)}{4} + n$

$S_{30} = \frac{30.31.61}{12} - \frac{30.31}{4} + 30$

= 4727.5 – 232.5 + 30

= 4525

A-4. Ans. (2)

Sol. $b_1 = a_1$, $b_2 = a_1(1 + r)$, $b_3 = a_1(1 + r + r^2)$, $b_4 = a_1(1 + r + r^2 + r^3)$

Statement 1 is correct as the numbers are neither in A.P. nor in G.P.

Now, $\frac{2b_1b_3}{b_1+b_3} = \frac{2a_1\, a_1(1+r+r^2)}{a_1(2+r+r^2)} \neq b_2$.

Hence statement '2' is false

Correct Answer is (C)

Section (B) : MATCH THE COLUMN

Sol. (P) $\frac{a-b}{b-c} = \frac{a}{a}$ ⇒ 2b = a + c

(Q) $\frac{a-b}{b-c} = \frac{a}{b}$ ⇒ $b^2 = ac$

(R) $\frac{a-b}{b-c} = \frac{a}{c}$ ⇒ $b = \frac{2ac}{a+c}$

(S) $t_p = aR^{p-1}$

$t_q = aR^{q-1}$

$t_r = aR^{r-1}$ ⇒ $t_q^2 = t_p t_r$

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. **Sol.** $x = \sum_{n=0}^{\infty} \cos^{2n}\phi = \frac{1}{1-\cos^2\phi} = \frac{1}{\sin^2\phi}$,

$y = \sum_{n=0}^{\infty} \sin^{2n}\phi = \frac{1}{1-\sin^2\phi} = \frac{1}{\cos^2\phi}$

$z = \sum_{n=0}^{\infty} \cos^{2n}\phi \sin^{2n}\varphi = \frac{1}{1-\cos^2\phi\sin^2\phi}$

$z = \frac{1}{\left(1-\frac{1}{x}\cdot\frac{1}{y}\right)}$ ⇒ $z = \frac{xy}{xy-1}$ ⇒ $xyz - z = xy$

⇒ $xyz = xy + z$

Since $xy = x + y$ ⇒ $xyz = x + y + z$

C-2. **Sol.** $a = \underbrace{111\ldots1}_{55\text{ times}} = 1 + 10 + 10^2 + \ldots\ldots. + 10^{54} = \frac{10^{55}-1}{9}$

$b = 1 + 10 + 10^2 + 10^3 + 10^4 = \frac{10^5-1}{9}$

$c = 1 + 10^5 + 10^{10} + \ldots\ldots\ldots + 10^{50}$

$= \frac{(10^5)^{11}-1}{10^5-1} = \frac{10^{55}-1}{10^5-1} = \frac{\frac{10^{55}-1}{9}}{\frac{10^5-1}{9}} = \frac{a}{b}$

$a = bc$ ∴ b, $\sqrt{a}$, c are in G.P.

C-3. **Sol.** a, b, c are in GP

$\frac{2ab}{a+b} = 20$, $\frac{b+c}{2} = 5$

$\frac{2a\,.\,ar}{a+ar} = 20$, $ar + ar^2 = 10$

$ar = 10(1 + r)$, $ar(1 + r) = 10$

i.e. $10(1 + r)^2 = 10$ ⇒ $(1 + r)^2 = 1$

∴ $1 + r = \pm 1$ ⇒ $r = 0$ or $r = -2$

∴ $r = -2$

if $r = -2$, then $a = 5$

∴ the GP is 5, – 10, 20, – 40, 80, ...

clearly no term of the series is square of an integer and AM of a, b, c is $= \frac{5-10+20}{3} = 5$

C-4. **Sol.** Let the roots be $\frac{A}{r}$, A , Ar

then A = 3

∴ $27 + 9a + 3b - 27 = 0$ i.e. $b = -3a$

∴ $a + b + 6 = 0$, $b = -3a$ ⇒ $a = 3$, $b = -9$

∴ $r = -1$

Exercise-3

Marked Questions may have for Revision Questions.

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. **Sol.** $\dfrac{\frac{p}{2}(2a_1+(p-1)d)}{\frac{q}{2}(2a_1+(q-1)d)} = \dfrac{p^2}{q^2}$

$\Rightarrow \dfrac{a_1+\left(\frac{p-1}{2}\right)d}{a_1+\left(\frac{q-1}{2}\right)d} = \dfrac{p}{q} \Rightarrow \dfrac{a+5d}{a+20d} = \dfrac{11}{41}$

2. **Sol.** $\dfrac{1}{a_1}, \dfrac{1}{a_2}, ..., \dfrac{1}{a_n}$, are in A.P.

$\Rightarrow \dfrac{1}{a_2} - \dfrac{1}{a_1} = ... = \dfrac{1}{a_n} - \dfrac{1}{a_{n-1}} = d$

$\Rightarrow a_1a_2 = \dfrac{a_1-a_2}{d}$, $a_2a_3 = \dfrac{a_2-a_3}{d}$ ...., $\dfrac{a_{n-1}-a_n}{d} = a_{n-1} - a_n$

$\Rightarrow a_1a_2 + a_2a_3 + ... + a_{n-1}\, a_n$

$= \dfrac{a_1-a_2}{d} +........+ \dfrac{a_{n-1}-a_n}{d} = \dfrac{a_1-a_n}{d} = (n-1)\, a_1a_n$

3. **Sol.** $a = ar + ar^2$

$\Rightarrow r^2 + r - 1 = 0 \quad \Rightarrow r = \dfrac{-1 \pm \sqrt{5}}{2}$

$\Rightarrow r = \dfrac{\sqrt{5}-1}{2}$ (–ve not permissible

4. **Sol.** Using AM ≥ GM

$\dfrac{p^2+q^2}{2} \ge pq$

$\Rightarrow pq \le$ $(\because p^2 + q^2 = 1)$

Now $(p + q)^2 = p^2 + q^2 + 2pq$

$\Rightarrow (p + q)^2 = 1 + 2pq$

$\Rightarrow (p + q)^2 \le 1 + 1$

$\Rightarrow p + q \le \sqrt{2}$

5. **Sol.** Let a, ar, ar^2

$a + ar = 12$

$ar^2 + ar^3 = 48$

dividing the two equations, we get

$r^2 = 4$ if $r \ne -1$

$\therefore \quad r = -2$

also $\quad a = -12$

6. **Sol.** Using formula for AGP sum = 3

7. **Sol.** $a_1 + a_2 + ... + a_n = 4500$ notes

$a_1 + a_2 + ... + a_{10} = 150 \times 10 = 1500$ notes

= 4500 – 1500 = 3000 notes

$a_{11} + a_{12} + \ldots + a_n = 3000$

148 + 146 ...... = 3000

$\frac{(n-10)}{2}$ [2 × 148 + (n – 10 – 1) (–2)] = 3000

n = 34, 135

a_{34} = 148 + (34 – 1) (–2) = 148 – 66 = 82

a_{135} = 148 + (135 – 1) (–2) = 148 – 268 = – 120 < 0

so answer is 34 minutes are taken

Hence correct option is (1)

8. **Sol.** a = Rs. 200

d = Rs. 40

savings in first two months = Rs. 400

remained savings = 200 + 240 + 280 + ... upto n terms

= $\frac{n}{2}\left[400+(n-1)40\right]$ = 11040 – 400

$200n + 20n^2 - 20n = 10640$

$20n^2 + 180n - 10640 = 0$

$n^2 + 9n - 532 = 0$

(n + 28) (n – 19) = 0

n = 19

∴ no. of months = 19 + 2 = **21** .

9. **Sol.** Let A.P. be a, a + d, a + 2d, ...

$a_2 + a_4 + \ldots + a_{200} = \alpha$

⇒ $\frac{100}{2}$ [2(a + d)+ (100 – 1)d] = α .... (i)

and $a_1 + a_3 + a_5 + \ldots + a_{199} = \beta$

⇒ $\frac{100}{2}$ [2a + (100 – 1)d] = β .....(ii)

on solving (i) and (ii)

d = $\frac{\alpha-\beta}{100}$

10. **Sol.** **Ans. (2)**

$T_n = (n-1)^2 + (n-1)n + n^2$

= $\frac{\left((n-1)^3-n^3\right)}{(n-1)-n}$ $= n^3 - (n-1)^3$

$T_1 = 1^3 - 0^3$

$T_2 = 2^3 - 1^3$

:

$T_{20} = 20^3 - 19^3$

$S_{20} = 20^3 - 0^3 = 8000$

11. **Sol.** 100 (a + 99d) = 50 (a + 49d)

2a + 198 d = a + 49d

a + 149d = 0

T_{150} = a + 149 d = 0

12. **Sol.** **(3)**

$\frac{7}{10}+\frac{77}{100}+\frac{777}{10^3}$ + ... + up to 20 terms

$= 7\left[\frac{1}{10}+\frac{11}{100}+\frac{111}{10^3}+\text{.....up to 20 terms}\right]$

$= \frac{7}{9}\left[\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+\text{.....up to 20 terms}\right]$

$= \frac{7}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{10^2}\right)+\left(1-\frac{1}{10^3}\right)+\text{..... up to 20 terms}\right]$

$= \frac{7}{9}\left[20-\frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^{20}\right)}{1-\frac{1}{10}}\right] = \frac{7}{9}\left[20-\frac{1}{9}\left(1-\left(\frac{1}{10}\right)^{20}\right)\right]$

$\frac{7}{9}\left[\frac{179}{9}+\frac{1}{9}\left(\frac{1}{10}\right)^{20}\right] = \frac{7}{81}\left[179+(10)^{-20}\right]$

13. Sol. Ans. (1)

Let $s = (10)_9 + 2(11)_1\,(10)_8 + 3\,(11)_2(10)_7 + \ldots\ldots\ldots\ldots\ldots\ldots + 10(11)_9$

$\therefore \frac{11}{10}\,s = \quad (11)_1\,(10)_8 + 2(11)_2\,(10)_7 + \ldots\ldots\ldots\ldots\ldots + 9(11)_9 + (11)_{10}$

substract

$\left(1-\frac{11}{10}\right)s = (10)_9 + (11)_1\,(10)_8 + (11)_2\,(10)_7 + \ldots\ldots\ldots\ldots\ldots + (11)_9 - (11)_{10}$

$\Rightarrow \frac{1}{10}\,s = \frac{10^9\left\{1-\left(\frac{11}{10}\right)^{10}\right\}}{1-\frac{11}{10}} - (11)_{10}$

$\Rightarrow \frac{1}{10}\,s = 10_9\,\frac{\left\{10^{10}-11^{10}\right\}}{10^{10}}\times\frac{10}{-1} - (11)_{10}$

$\Rightarrow \frac{1}{10}\,s = -10_{10} + 11_{10} - 11_{10}$

$s = 10_{11}$

given $10_{11} = k(10)_9$

$k = 100$

14. Sol. Ans. (2)

$a \qquad a_1 \qquad ar_2 \rightarrow$ G. P.

$a \qquad 2ar \qquad ar_2 \rightarrow$ A. P.

$L_1 ar = a + ar_2$

$4r = 1 + r_2$

$r_2 - 4r + 1 = 0$

$r = \frac{4 \pm 2\sqrt{3}}{2} = 2 + \sqrt{3}, \qquad 2-\sqrt{3}$

But $r > 1$

$r = 2+\sqrt{3}$

15. 4) 4 $l_2m_2n_2$

Ans. **(2)**

Sol. $M = \dfrac{\ell+n}{2}$

ℓ, G_1 , G_2 , G_3 , n are in G.P.

$r = \left(\dfrac{n}{\ell}\right)^{\frac{1}{4}}$

$G_1 = \ell\left(\dfrac{n}{\ell}\right)^{\frac{1}{4}}$ $\quad G_2 = \ell\left(\dfrac{n}{\ell}\right)^{\frac{1}{2}}$ $\quad G_3 = \ell\left(\dfrac{n}{\ell}\right)^{\frac{3}{4}}$

$G_{14} + 2G_{24} + G_{34}$

$= \ell_4 \times \dfrac{n}{\ell} + 2\ell_4 \dfrac{n^2}{\ell^2} + \ell_4 \times \dfrac{n^3}{\ell^3} = \ell_3 n + 2\ell_2 n_2 + \ell n_3 = n\ell\,(\ell_2 + 2n\ell + n_2)$

$= n\ell(\ell + n)_2 = 4m_2 n\ell$

16. **Ans.** **(2)**

Sol. $T_n = \dfrac{\dfrac{n^2(n+1)^2}{4}}{n^2}$

$T_n = \dfrac{1}{4}\,(n + 1)_2$

$T_n = \dfrac{1}{4}\,[n_2 + 2n + 1]$

$S_n = \sum_{n=1}^{n} T_n$

$S_n = \dfrac{1}{4}\left[\dfrac{n(n+1)\,(2n+1)}{6} + n(n+1) + n\right]$

n = 9

$S_9 = \dfrac{1}{4}\left[\dfrac{9\times10\times19}{6} + 9\times10 + 9\right]$

$\dfrac{1}{4} = \ [285 + 90 + 9] \ = \dfrac{384}{4} = 96.$

17. **Ans.** **(1)**

Sol. a + d, a + 4d, a + 8d $\rightarrow$ G.P

$\therefore$ $\quad (a + 4d)_2 = a_2 + 9ad + 8d_2$

$\Rightarrow$ $\quad 8d_2 = ad \qquad \Rightarrow a = 8d$

$\therefore$ $\quad$ 9d, 12d, 16d $\rightarrow$ G.P.

common ratio $r = \dfrac{12}{9} = \dfrac{4}{3}$

18. **Ans.** **(1)**

Sol. $\left(\frac{8}{5}\right)^2+\left(\frac{12}{5}\right)^2+\left(\frac{16}{5}\right)^2+\left(\frac{20}{5}\right)^2+\left(\frac{24}{5}\right)^2+.....=\frac{8^2}{5^2}+\frac{12^2}{5^2}+\frac{16^2}{5^2}+\frac{20^2}{5^2}+\frac{24^2}{5^2}+......$

$T_n = \frac{(4n+4)^2}{5^2}$

$S_n = \frac{1}{5^2}\sum_{n=1}^{10}16(n+1)^2 = \frac{16}{25}\sum_{n=1}^{10}(n^2+2n+1)$

$= \frac{16}{25}\left[\frac{10\times 11\times 21}{6}+\frac{2\times 10\times 11}{2}+10\right] = \frac{16}{25}\times 505 = \frac{16}{5}m \Rightarrow m = 101$

19. Ans. (2)

Sol. $225a^2 + 9b^2 + 25c^2 - 75ac - 45ab - 15bc = 0$

$(15a)^2 + (3b)^2 + (5c)^2 - (15a)(3b) - (3b)(5c) - (15a)(5c) = 0$

$\frac{1}{2}[(15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2] = 0$

$15a = 3b$, $3b = 5c$, $5c = 15a$

$5a = b$, $3b = 5c$, $c = 3a$

$\frac{a}{1}=\frac{b}{5}=\frac{c}{3}=\lambda$

$a = \lambda$, $b = 5\lambda$, $c = 3\lambda$

a, c, b are in AP

b, c, a are in AP

20. Sol. $f(x) = ax^2 + bx + c$

$f(x + y) = f(x) + f(y) + xy$

$a(x + y)^2 + b(x+ y) + c = ax^2 + bx + c + ay^2 + by + c + xy$

$2axy = c + xy \quad \forall x, y\in R$

$(2a - 1)xy - c = 0 \quad \forall\ x, y\in R$

$\Rightarrow c = 0, \quad a = \frac{1}{2}$

$a + b + c = 3$

$\frac{1}{2} + b + 0 = 3$

$b = \frac{5}{2}$

$\therefore f(x) = \frac{1}{2}x^2 + \frac{5}{2}x$

$\sum_{n=1}^{10} f(n) = \frac{1}{2}\sum_{n=1}^{10}n^2+\frac{5}{2}\sum_{n=1}^{10}n \Rightarrow \frac{1}{2}\times\frac{10\times 11\times 21}{6}+\frac{5}{2}\times\frac{10\times 11}{2} = 330$

21. Ans. (4)

Sol. The given quadratic equation is

$nx^2 + x(1 + 3 + 5 +.......+(2n - 1)) + (1.2 + 2.3 +.......+ (n - 1).n) - 10n = 0$

$\Rightarrow nx^2 + x(n^2) + \frac{n(n^2-1)}{3} - 10n = 0 \quad \Rightarrow x^2 + x(n) + \frac{(n^2-1)}{3} - 10 = 0 \begin{cases}\alpha\\ \beta\end{cases}$

$(\alpha - \beta)^2 = 1 \quad \Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 1 \quad \Rightarrow n^2 - 4\left(\frac{n^2-1}{3}-10\right) = 1 \Rightarrow n = 11$

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. Sol. $a_1 \cdot a_2 \cdot a_3 \cdot ....a_n = c$

$\frac{a_1 + a_2 + a_3 + ...2a_n}{n} \geq (a_1a_2a_3 \ldots 2a_n)^{1/n} \geq (2c)^{1/n}$

$\Rightarrow \quad a_1 + a_2 + a_3 + ...2a_n \geq n(2\,c)^{1/n}$

2. **Sol.** $2b = a + c$ and $b^2 = \pm ac$

case-I

if $b^2 = ac$ and $a + c + b = \frac{3}{2}$ $\Rightarrow$ $b = \frac{1}{2}$

$a + c = 1$ $\Rightarrow$ $ac = \frac{1}{4}$ $\Rightarrow$ $(1 - c)\,c = \frac{1}{4}$

$c^2 - c + \frac{1}{4} = 0$ $\Rightarrow$ $c = \frac{1}{2}$ $\Rightarrow$ $a = \frac{1}{2}$

$a = b = c$ so not valid $a = b = c$

case-II

$b^2 = -ac$ and $b = \frac{1}{2}$

$a + c = 1$ $\Rightarrow$ $ac = -\frac{1}{4}$

$(1 - c)\,c = -\frac{1}{4}$ $\Rightarrow$ $c^2 - c - \frac{1}{4} = 0$

$c = \frac{1 \pm \sqrt{1+1}}{2} = \frac{1+\sqrt{2}}{2}$

$c = \frac{1+\sqrt{2}}{2}$ $\Rightarrow$ $a = \frac{1-\sqrt{2}}{2}$

3. **Sol.** If $\alpha \in \left(0, \frac{\pi}{2}\right)$, $\tan\alpha$ is positive

$\because$ AM $\geq$ GM

$$\therefore \quad \frac{\sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}}}{2} \geq \sqrt{\sqrt{x^2+x} \cdot \frac{\tan^2\alpha}{\sqrt{x^2+x}}}$$

$$\Rightarrow \quad \sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}} \geq 2\tan\alpha.$$

4. **Sol.** $S_\infty = 5$, $a = x$ $\Rightarrow$ $S_\infty = \frac{x}{1-r} = 5$

$x = 5 - 5r$

$r = \frac{5-x}{5}$ $\because$ $-1 < r < 1$

$-5 < 5 - x < 5$ $\Rightarrow$ $-10 < -x < 0$

$0 < x < 10$

5. **Sol.** $\alpha + \beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$

$(\alpha + \beta),\ \alpha^2 + \beta^2,\ \alpha^3 + \beta^3$ are in G.P

then $(\alpha^2 + \beta^2)^2 = (\alpha + \beta)(\alpha^3 + \beta^3)$

$\Rightarrow \quad 2\alpha^2\beta^2 = \alpha\beta^3 + \beta\alpha^3$

$\Rightarrow \quad \alpha\beta[\alpha^2 + \beta^2 - 2\alpha\beta] = 0$

so $\quad \alpha\beta(\alpha - \beta)^2 = 0$

$\Rightarrow \quad \frac{c}{a} \cdot \frac{b^2 - 4ac}{a^2} = 0, \quad a \neq 0 \Rightarrow c \cdot \Delta = 0$

6. Sol. Corresponding A.P.

$\frac{1}{5}$,..................... $\frac{1}{25}$ (20^{th} term)

$\frac{1}{25} = \frac{1}{5} + 19d \quad \Rightarrow \quad d = \frac{1}{19}\left(\frac{-4}{25}\right) = -\frac{4}{19 \times 25}$

$a_n < 0$

$\frac{1}{5} - \frac{4}{19 \times 25} \times (n - 1) < 0$

$\frac{19 \times 5}{4} < n - 1$

$n > 24.75$

7. Sol. $4\alpha x^2 + \frac{1}{x} \geq 1 \quad \Rightarrow \quad y = 4\alpha x^2 + \frac{1}{x} \quad \Rightarrow \quad y' = \frac{dy}{dx} = 8\alpha x - \frac{1}{x^2} = 0 \Rightarrow x = \left(\frac{1}{8\alpha}\right)^{1/3}$

$\Rightarrow \quad f(x) = \frac{4\alpha x^3 + 1}{x} = \frac{1/2 + 1}{1/(8\alpha)^{1/3}} \Rightarrow \frac{3}{2} \times (8\alpha)^{1/3} \geq 1 \quad \Rightarrow \quad \alpha^{1/3} \geq 1/3 \Rightarrow \alpha \geq \frac{1}{27}$

8. Sol. $\log_e b_1, \log_e b_2, \log_e b_3, ... \log_e b_{101}$ are in A.P.

$b_1, b_2, b_3, ..., b_{101}$ are in G.P

Given : $\log_e(b_2) - \log_e(b_1) = \log_e(2) \Rightarrow \frac{b_2}{b_1} = 2 = r$ (common ratio of G.P.)

$a_1, a_2, a_3, ... a_{101}$ are in A.P.

$a_1 = b_1 = a$

$b_1 + b_2 + b_3 + ... b_{51} = t$,

$S = a_1 + a_2 + ... + a_{51}$

t = sum of 51 terms of G.P = $b_1 = \frac{(r^{51} - 1)}{r - 1} = \frac{a(2^{51} - 1)}{2 - 1} = a(2^{51} - 1)$

s = sum of 51 terms of A.P. $= \frac{51}{2}[2a_1 + (n-1)d] = \frac{51}{2}(2a + 50d)$

Given $a_{51} = b_{51}$

$a + 50d = a(2)^{50}$

$50d = a(2^{50} - 1)$

Hence $s = \frac{51}{2} a[2^{50} + 1] \Rightarrow s = a\left(51 . 2^{49} + \frac{51}{2}\right)$

$s = 2\left(4.2^{49} + 47.2^{49} + \frac{51}{2}\right) \quad \Rightarrow \quad s = a\left((2^{51} - 1) + 47.2^{49} + \frac{53}{2}\right)$

$s - t = a\left(47 . 2^{49} + \frac{53}{2}\right)$

Clearly $s > t$

$a_{101} = a_1 + 100d = a + 2a.2^{50} - 2a = a(2^{51} - 1)$

$b_{101} = b_1 r^{100} = a.2^{100} \quad$ Hence $b_{101} > a_{101}$