Exercise-1

Marked Questions may have for Revision Questions.

OBJECTIVE QUESTIONS

Section (A) : Classical definition of probability

A-1.
Sol. Max sum = 12

$$\left.\begin{array}{l}6+6=12\\6+5=11\\6+4=10\\5+5=10\end{array}\right\}$$ 6 cases

P = $\frac{6}{36} = \frac{1}{6} = \frac{1}{6}$

A-2. **Sol.** Total number of multiple of 5=24. Total
number of multiple of 15 = 8 i.e., n(A) = 24, n(B) = 8 and n(A∩B) = 8
∴ n(A∪B) = n(A) + n(B) – n(A∩B)
= 24+8–8 = 24
n(S) = 120,
∴ Reueired probability = $\frac{24}{120} = \frac{1}{5}$

A-3. Sol. P(A) = $\frac{1}{3}$ P(B) = $\frac{1}{4}$ and P(C) = $\frac{1}{5}$
Required probability
= $P(A\bar{B}\bar{C}$ or $\bar{A}B\bar{C}$ or $\bar{A}\bar{B}C)$
= P(A). P($\bar{B}$). P($\bar{C}$) + P($\bar{A}$) P(B) P($\bar{C}$)+ P($\bar{A}$) P($\bar{B}$) P(C)
= $\frac{1}{3}\times\frac{3}{4}\times\frac{4}{5}+\frac{2}{3}\times\frac{1}{4}\times\frac{4}{5}+\frac{2}{3}\times\frac{3}{4}\times\frac{1}{5} = \frac{12+8+6}{60} = \frac{13}{30}$

A-4. **Sol.** Total ways = 14–1! = 13!
favorable ways = 7! . 6!

A-5. **Sol.** 1(S) + 1(D) or 1(D) + 1(S)
$\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}=\frac{1}{2}$

A-6. **Sol.** ${}^{4}C_{1}$ × ${}^{13}C_{9}$ × ${}^{39}C_{4}$ = Formula card
Suit any 9 cards any 4 cards from 39 cards
${}^{52}C_{13}$ = total case

A-7. Sol. 1st coupon can be selected in 9 ways

2nd coupon can be selected in 9 ways

3rd coupon can be selected in 9 ways

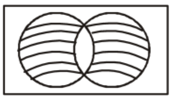
9^7 ways – when 9 is not take

$= 9^7 - 8^7$

Total = 15^7.

A-8. Sol. $\dfrac{\dfrac{2n-2!}{n-1!\,n-1!2!}\times 2!}{\dfrac{2n!}{n!\,n!2!}} = P$

A-9. Sol.



P = P(M∪N) – P(M∩N)

= P(M) + P(N) – 2P(M∩N)

(c) $P(\overline{M}\cup\overline{N}) - P(\overline{M}\cap\overline{N})$

$\overline{M}\cup\overline{N}$ $\overline{M}\cap\overline{N}$

A-10. Sol. Favourable case = (12–1)! × 2!

Total (13 – 1)!

$p = \dfrac{11!\times 2!}{12!} = \dfrac{2}{12} = \dfrac{1}{6}$

A-11. Sol. Favourable no. of ways = 12

total no. of ways = 220

$P = \dfrac{12}{220} = \dfrac{3}{55}$

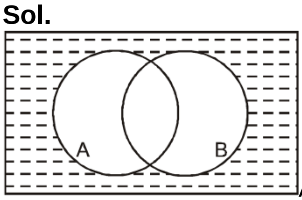
A-12. Sol. Let A : card is spade

B : card is an ace.

$P(A) = \dfrac{13}{52}$ $P(B) = \dfrac{4}{52}$

$P(A\cap B) = \dfrac{1}{52}$

A-13. Sol.



$A^c - B$

A-14. **Sol.** p (A) = $\frac{3}{6}$, p (B) = $\frac{2}{6}$
A ≡ {1, 3, 5 } B≡ { 3, 6 }

B $\not\subset$ A .
B – A = {6} as follows

A-15. **Sol.** $p_1 + p_2 + p_3 + p_4 = 1$ in in D obvious solution follows

A-16. **Sol.** ${}_2C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^0 = \frac{1}{4}$

A-17. **Sol.** P(A∪B) = $\frac{3}{4}$, P (A∩B) = $\frac{1}{4}$, P(A) = $\frac{1}{3}$
P(A∩B) = P(A)+ P(B) – P(A∪B)
$\frac{1}{4} = \frac{1}{3} + P(B) - \frac{3}{4} \Rightarrow P(B) = \frac{2}{3}$
$P(\bar{A} \cap B) = P(B) - P(A \cap B) = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$

Section (B) : Addition theorem

B-1. **Sol.** P(atleast one W) = P(1W, 1M) + P(2W, 0M)
= $\frac{5 \times 8}{{}^{13}C_2} + \frac{{}^5C_2}{{}^{13}C_2}$

B-2. **Sol.** P(A) = $\frac{3}{11}$
P(B) = $\frac{2}{7}$
P(C) = P
Now P(A) + P(B) + P(C) = 1
$\frac{3}{11} + \frac{2}{7} + P = 1$ ⇒ $P = 1 - \frac{43}{77} = \frac{34}{77}$
odds against C = 43 : 34

B-3. **Sol.** Since, 0 ≤ P(A) ≤ 1, 0 ≤ P(B) ≤ 1, 0 ≤ P(C) ≤ 1 and 0 ≤ P(A) + P(B) + P(C) ≤ 1
∴ $0 \le \frac{3p+1}{3} \le 1$
⇒ $-\frac{1}{3} \le p \le \frac{2}{3}$ ...(i)
$0 \le \frac{1-p}{4} \le 1$
⇒ $-3 \le p \le 1$ ...(ii)
$0 \le \frac{1-2p}{2} \le 1$

$\Rightarrow \quad -\frac{1}{2} \le p \le \frac{1}{2}$ ...(iii)

and $0 \le \frac{3p+1}{3} + \frac{1-p}{4} + \frac{1-2p}{2} \le 1$

$\Rightarrow 0 \le 13 - 3p \le 12$

$\Rightarrow \frac{1}{3} \le p \le \frac{13}{3}$ ...(iv)

From Eqs. (i), (ii), (iii) and (iv), we get

$\frac{1}{3} \le p \le \frac{1}{2}$.

B-4. **Sol.** if events are not exclusive (and are exhaustive) then p (A) + P (B) + p (C) ≥ 1

Section (C) : Conditional probability, dependent and independent events

C-1. **Sol.** Let event A : 6 comes on Ist die

B: sum is 7

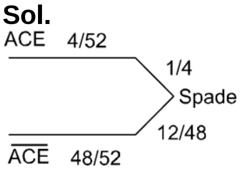
$p(A) = \frac{1}{6}$, $p(B) = \frac{6}{36} = \frac{1}{6}$ $\left.\begin{matrix} 6+1 \\ 5+2 \\ 4+3 \end{matrix}\right\}$

$A \cap B = \frac{1}{36}$; $p(A \cap B) = p(A) \times p(B)$

C-2. **Sol.** odd —— 1, 3, 5.

$p(\text{prime/ odd}) = \frac{2}{3}$

C-3. **Sol.**



Fav., case = 1

Total case = 13

Probability = $\frac{1}{13}$

C-4. **Sol.** 2 + 6 = 8 $p = \frac{1}{5}$

3 + 5 = 8

4 + 4 = 8

5 + 3 = 8

6 + 2 = 8

C-5. **Sol.** 2W & 4B

$P = {}^5C_4 \times \left(\frac{2}{6}\right)^4 \left(\frac{4}{6}\right)^1 + {}^5C_5 \left(\frac{2}{6}\right)^5$

C-6. **Sol.** 1 – P(BB)

1 – 1/2 × 1/2 = 1 – 1/4 = 3/4

C-7. **Sol.** $P(S_1) . P(\bar{S}_2) . P(\bar{S}_3) + P(\bar{S}_1) P(S_2) P(\bar{S}_3) + P(\bar{S}_1) P(\bar{S}_2) P(S_3)$

$= \frac{1}{3} \times \frac{5}{7} \times \frac{5}{8} + \frac{2}{3} . \frac{2}{7} . \frac{5}{8} + \frac{2}{3} . \frac{5}{7} . \frac{3}{8} = \frac{25}{56}$

C-8. **Sol.** $p(A / B) = \frac{p(A \cap B)}{p(B)} = \frac{0.1 + 0.1}{0.3} = \frac{2}{3}$.

similarly evaluate others

C-9. **Sol.** $P(A \cap B) = \frac{1}{6} \Rightarrow P(A).P(B) = \frac{1}{6}$

$P(\bar{A}) = \frac{2}{3} \Rightarrow P(A) = \frac{1}{3} \Rightarrow P(B) = \frac{1}{2}$

$\therefore$ 6P(B/A) = 6P(B) = 3

C-11._ **Sol.** Use venn diagram

C-12. **Sol.** (ii) $p\left(A/B\right) = \frac{P(A \cap B)}{P(B)}$

C-13. **Sol.** A & B are independent

$P(A \cup B)^c = 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A \cap B)$

$= P(\bar{A}) - P(B) + P(A) P(B) = P(\bar{A}) - P(\bar{A})P(B)$

$= P(\bar{A})P(\bar{B})$

C-14. **Sol.** $P(\bar{M} \cap \bar{N}) = 1 - P(M \cup N) = 1 - P(M) - P(N) + P(M) P(N)$

$= (1 - P(M)) (1 - P(N)) = \overline{P(M)}\ \overline{P(N)}$

$P(M \cap \ ) = P(M) - P(M \cap N) = P(M) - P(M) P(N) = P(M) P()$

and $P\left(\frac{M}{N}\right) + P\left(\frac{\bar{M}}{N}\right) = \frac{P(M \cap N)}{P(N)} + \frac{P(N) - P(M \cap N)}{P(N)} = 1$

Section (D) : Total probability theorem, Baye's theorem

D-1. **Sol.** p (Ist class) = $\frac{2}{3} \times 0.28 + \frac{1}{3} \times 0.25 = 0.27$

D-2. **Sol.** $P(E) = P(A) P(E/A) + P(B) P(E/B) = \frac{1}{2}.\frac{2}{5} + \frac{1}{2}.\frac{4}{6} = \frac{8}{15}$

D-3. **Sol.** Required probability = $\frac{^5C_1}{^{12}C_1} \times \frac{^4C_1}{^{12}C_1} + \frac{^7C_1}{^{12}C_1} \times \frac{^8C_1}{^{12}C_1} = \frac{76}{144}$

D-4. **Sol.** $P(R) = P(P).\,P\left(\frac{V.C}{P}\right) + P(Q).P\left(\frac{V.C}{Q}\right) = \frac{1}{3} \times \frac{{}^2C_1.{}^3C_1}{{}^5C_2} + \frac{2}{3}.\frac{{}^1C_1.{}^4C_1}{{}^5C_2}$

$= \frac{1}{3}\times\frac{6}{10}+\frac{2}{3}\times\frac{4}{10} = \frac{6+8}{30}=\frac{7}{15}$

D-5. **Sol.** U1 – 1W + 1B U2 → 2W + 3B

U3 → 3W + 5B U4 → 4W + 7B

$P(W) = \sum_{i=1}^{4}(u_1)\,P(w/u_i) = \sum_{i=1}^{4}\frac{i^2+1}{34}$ $P(w/v_i)$

$= \frac{1^2+1}{34}\times\frac{1}{2}+\frac{2^2+1}{34}\times\frac{2}{5}+\frac{3^2+1}{34}\times\frac{3}{8}+\frac{4^2+1}{34}\times\frac{4}{11} = \frac{569}{1496}$

D-6. **Sol.** Number of kings left are 3.

cards are 51 $p = \frac{3}{51} = \frac{1}{17}$

D-7. **Sol.** $\frac{7}{12} = \frac{1}{n+1}\times 1 + \frac{n}{n+1}\times\frac{1}{2}$ solve for n of get

D-8. **Sol.** A : 1 ball is W & 3 black balls

B_1 : Urn 1 is chosen

B_2 : Urn 2 is chosen

$P(B_1/A) = \frac{P(A/B_1)P(B_1)}{P(A/B_1)P(B_1)+P(A/B_2)P(B_2)}$

$P(B_1/A) = \frac{\frac{1}{2}\times\frac{4}{9}\times\left(\frac{5}{9}\right)^3\times{}^4C_3}{\frac{1}{2}\times\frac{4}{9}\times\left(\frac{5}{9}\right)^3\times{}^4C_3+\frac{1}{2}\times\frac{3}{9}\times\left(\frac{6}{9}\right)^3\times{}^4C_3} = \frac{125}{287}$

D-9. **Sol.** A = 2 nd ball in white

B_1 = 1st ball in white

B_2 = 1st is black

$P(B_1/A) = \frac{p(A/B_1)p(B_1)}{p(A/B_1)p(B_1)+p(A/B_2)p(B_2)} = \frac{\frac{4}{7}\times\frac{3}{6}}{\frac{4}{1}\times\frac{3}{6}+\frac{3}{7}\times\frac{4}{6}}$

Section (E) : Probability distribution and binomial probability distribution

E-1. **Sol.** $\frac{np}{npq} = \frac{3}{2}$ ⇒ $q = \frac{2}{3}$ ⇒ $p = \frac{1}{3}$

$r \le \frac{11}{1+2}$ ⇒ $r \le \frac{1}{3}$ ⇒ $r \le 3.66$

thus 3 succes is most probable.

E-2. **Sol.** $n = 3,\ p = \frac{2}{6} = \frac{1}{3}$

mean = np = $3 \times \frac{1}{3} = 1$

variance σ_2 = npq = $\sqrt{3\times\frac{1}{3}\times\frac{2}{3}}$ = $3\times\frac{1}{3}\times\frac{2}{3}$

$\sigma_2 = \frac{2}{3}$

E-3. **Sol.** ${}_3C_2\,P_2$ (1–P) = 12 ${}_3C_3$ P_3

1 – P = 4 P ⇒ $\frac{1}{5}$ = p

E-4. **Sol.** P(atleast 4) = P(4) + P(5)

$= {}_5C_4 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^4 + {}^5C_5\left(\frac{9}{10}\right)^5$

E-5. **Sol.** $(P + q)_{99}$ r ≤ $\frac{99+1}{1+\left|\frac{1/2}{1/2}\right|}$ ⇒ r ≤ $\frac{100}{2}$ ⇒ r ≤ 50

Terms 50 or 51 are highest

E-6. **Sol.**

$$\left.\begin{array}{l} p(x=4) = {}^n c_4\left(\frac{1}{2}\right)^n \\ p(x=5) = {}^n c_5\left(\frac{1}{2}\right)^n \\ p(x=6) = {}^n c_6\left(\frac{1}{2}\right)^n \end{array}\right]$$

2 ${}_nC_5$ = ${}_nC_4$ + ${}_nC_6$

4 ${}_nC_5$ = ${}_{n+1}C_5$ + ${}_{n+1}C_6$

4 ${}_nC_5$ = ${}_{n+2}C_6$

4 . $\frac{n!}{5!\,(n-5)!}$ = $\frac{(n+2)!}{6!(n-4)!}$

4 = $\frac{(n+2)(n+1)}{6(n-4)}$ ⇒ 24 (n – 4) = (n + 2) (n + 1)

n = 7, 14

E-7. **Sol.** We need to calculate here

Probability that out of n bombs at least two strike > 0.99

i.e. 1 – prob (none strikes) – prob (exactly one strikes) > 0.99

i.e $1 - {}_nC_0\left(\frac{1}{2}\right)^n - {}_nC_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{n-1} > 0.99$

i.e. $0.01 \geq \frac{1+n}{2^n}$

i.e. $2_n \geq 100 + 100n$

Least values of n is 11

E-8. **Sol.** $^5C_2 . \left(\frac{1}{6}\right)^2 . \left(\frac{5}{6}\right)^3 = \frac{625}{3888}$

Exercise-2

Marked Questions may have for Revision Questions.

PART - I : OBJECTIVE QUESTIONS

1. **Sol.** Since sum of 1+2+3 +......9 = $\frac{9\times10}{2}$ = 45 is disisible by 9, hance all no. will be divisible by 9.

2. **Sol.** p = $\frac{4}{12} . \frac{3}{11} . 2$

3. **Sol.** $\frac{6.7.5}{^{18}C_3} = \frac{35}{136}$

4. **Sol.** Favourable case : (3,3,3,3) or (3,3,3,5) = 1+ $\frac{4!}{3!}$ = 5
total number of way → 2^4
p = $\frac{5}{2^4}$

5. **Sol.** Coefficient of x^8 $(x^0 + x^1 + ...x^6)^4 = \left(\frac{1-x^7}{1-x}\right)^4$
= $(1 - x^7)^4 (1 - x)^{-4}$
= $(1 - x^7)^4 (1 - x)^{-4} = (1 - 4x^7) (1 - x)^{-4}$
Total ways a = $^{4+8-1}C_8 - 4\ ^{4+1-1}C_1 = {}^{11}C_8 - 4 \times 4$
= 165 – 16 = 149.
P = $\frac{149}{7^4}$

6. **Sol.**

Unit digit in number	Unit digit in number	Unit digit in product
Odd	Odd	Odd
Odd	Even	Even
Even	Odd	Even
Even	Even	Even

p = $\frac{3}{4}$ ⇒ q = $\frac{1}{4}$ ⇒ $\frac{p}{q}$ = 3

7. **Sol.** Required probability = 1 – both number are not divisible by 5 = 1 – $\frac{8}{10}\times\frac{8}{10} = \frac{9}{25}$

8. **Sol.** $\frac{6}{10}\times\frac{5}{9}+\frac{4}{10}\times\frac{6}{9}=\frac{54}{90}=\frac{3}{5}$

9. **Sol.** Even integers ends in 0, 2, 4, 6, 8. Square of an even integer ends in 4 only when the integer ends either in 2 or 8.

∴ probability = $\frac{2}{5}$

10. **Sol.** KRISHNAGIRI or DHARMAPURI
A = RI is visible
B_1 = its from KRISHNAGIRI
B_2 = its from DHARMAPURI

$$P(B_1/A) = \frac{P(A/B_1)P(B_1)}{P(A/B_1)P(B_1)+P(A/B_2)P(B_2)} = \frac{\frac{1}{2}\times\frac{2}{10}}{\frac{1}{2}\times\frac{2}{10}+\frac{1}{2}\times\frac{1}{9}} = \frac{9}{14}$$

11. **Sol.** $\left(\frac{13}{52}\right)^2\times\left(\frac{13}{52}\right)^2\times\left(\frac{13}{52}\right)^2\times\frac{6!}{2!2!2!}=P$

12. **Sol.** E_1 = {(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5) }
E_2 = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) }
E_3 = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}
clearly (1), (2) and (3) are correct.

13. **Sol.** P(T 1) = p
P(T 2) = q
P(T 3) = 1/2
$\frac{1}{2}$ = P(T 1, T 2) + P(T 1, T3) + P(T 1 T 2 T 3)
$\frac{1}{2}$ = pq 1/2 + p(1 – q) $\frac{1}{2}$ + pq $\frac{1}{2}$
$\frac{1}{2}=\frac{pq}{2}+\frac{p}{2}$ ⇒ 1 = pq + p.
Now, check options

14. **Sol.** A = {1,3,5}
B = {2,4,6}
C = {4,5,6}
D = {1,2}

15. **Sol.** 10 coins 9 5 paisa 10 coins 5 paisa
1 1 Rs.

p = p (1 Rs. transfered + Back transfered) + p (1 Rs. not transfered) $\frac{{}^9c_8}{{}^{10}c_9}\times\frac{{}^{18}c_8}{{}^{19}c_9}+\frac{{}^9c_9}{{}^{10}c_9}=\frac{10}{19}$

method 2 when 1 Rs coin is in second purse and did not came back in first purse this prob. = $\frac{{}^{9}C_8 \times {}^{1}C_1}{{}^{10}C_9}$ $\times \frac{{}^{18}c_9}{{}^{19}c_9} = \frac{9}{19}$ ⇒ Required probability = $1 - \frac{9}{19} = \frac{10}{19}$

16. Sol. $p(A) = \frac{13}{52} + \left(\frac{39}{52}\right)^3\left(\frac{13}{52}\right) + \left(\frac{39}{52}\right)^6\left(\frac{13}{52}\right) + ...$

$p(B) = \left(\frac{39}{52}\right)\frac{13}{52} + \left(\frac{39}{52}\right)^4\frac{13}{52} + \left(\frac{39}{52}\right)^7\left(\frac{13}{52}\right) + ...$

$p(C) = \left(\frac{39}{52}\right)^2\left(\frac{13}{52}\right) + \left(\frac{39}{52}\right)^5\left(\frac{13}{52}\right) + ...$

17. Sol. Required probability = $p = {}_2C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{2}$

∴ $6p = 3$

18. Sol. Since line are more ${}_NC_M$ are those lines where telegrams will go

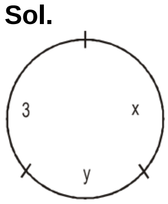
${}_NC_M \times M!$ = farvourable case

Total = N_M [As first telegram can go in any ore of n lies]

[As 2nd telegram can go in any ore of n lies]

$P = \frac{{}^{N}C_M\, M!}{N^M}$

19. Sol.



$x + y + z = 8$ $\qquad$ $x, y, z > 0$

$x' + y' + z' = 5$

$\frac{{}^{5+3-1}C_3 - 1\times{}^{11}c_1}{3}$

Total = ${}_{11}C_3$

20. Sol. $\frac{2}{5} = (1 - P)P + (1 - P)_3 P + (1-P)_5 p + ...$

$\frac{2}{5} = P(1 - P)\{1 + (1 - P)_2 + (1 - P)_4 + ...\}$

$\frac{2}{5} = P(1 - P)\left[\frac{1}{1-(1-P)^2}\right]$

$\frac{2}{5} = P\,(1 - P)\left[\frac{1}{P(2-P)}\right]$

$3P = 1 \Rightarrow P = 1/3$

21. **Sol.** $625p^2 - 175p + 12 < 0$ gives $p \in \left(\frac{3}{25}, \frac{4}{25}\right)$

$\left(\frac{4}{5}\right)^{n-1} \cdot \frac{1}{5} = p$

$\therefore \frac{3}{25} < \left(\frac{4}{5}\right)^{n-1} \cdot \frac{1}{5} < \frac{4}{25}$

i.e. $\frac{3}{5} < \left(\frac{4}{5}\right)^{n-1} < \frac{4}{5}$

value of n is 3

22. **Sol.** $1 - \left(\frac{3}{5}\right)^0 \cdot \left(\frac{2}{5}\right)^3 = \frac{117}{125}$

PART - II : MISCELLANEOUS QUESTIONS

A-1. **Ans. (3)**
Sol. **Statement - 2 :** True (By definition)
Statement -1 : False because the sample points are not equally likely.

A-2. **Ans. (1)**
Sol. Statement-2 $P(A/B) = P(A)$ $\Leftrightarrow$ $P(A) = \frac{P(A \cap B)}{P(B)}$ $\Leftrightarrow$ $P(A)\,P(B) = P(A \cap B)$
A and B are mutually exclusive $\Rightarrow$ $P(A \cap B) = 0$
$\Rightarrow$ $P(B)\,P(A/B) = 0$ $\Rightarrow$ $P(A/B) = 0$ $\Rightarrow$ $P(A/B) \neq P(A)$
$\therefore$ statement-2 is true
Statement-1 Suppose A and B are mutually exclusive, then by statement-2 $P(A/B) \neq P(A)$ which is a contradiction.
$\therefore$ statement-1 is true.

A-3. **Ans. (1)**
Sol. **(2)**
(2) We must have

$0 \le \frac{1+4P}{4} \le 1,\ 0 \le \frac{1-P}{4} \le 1$ and $0 \le \frac{1-2P}{4} \le 1$

$\Rightarrow$ $-\frac{1}{4} \le P \le \frac{3}{4},\ -3 \le P \le 1,\ -\frac{3}{2} \le P \le \frac{1}{2}$

Again the events are pair-wise mutually exclusive so

$$0 \le \frac{1+4P}{4} + \frac{1-P}{4} + \frac{1-2P}{4} \le 1$$

$\Rightarrow$ $-3 \le P \le 1$
Taking intersection of all four intervals of 'P'
We get $-\frac{1}{4} \le P \le \frac{1}{2}$

A-4. **Ans. (1)**
Sol. $ax + by = 0$ $a, b, c, d \in \{0, 1\}$
$cx + dy = 0$
system has unique solution if and only if $ad - bc \neq 0$
For which $a = d = 1$ and $bc = 0$ $\Rightarrow$ 3 combination
Similarly if $bc = 1$, $ad = 0$ $\Rightarrow$ 3 combination
Total choice for a, b, c, d is 2^4

Hence probability of unique solution is $\frac{6}{16} = \frac{3}{8}$
Statement-2 is also true since (0, 0) is a solution
Aliter : $ad - bc \neq 0$
If (i) $ad = 1, bc = 0$
(ii) $ad = 0, bc = 1$

$P(ad = 1) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$P(bc = 0) = 1 - P(bc = 1) = 1 - \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}$

$\therefore$ $P(ad = 1 \text{ and } bc = 0) = \frac{3}{16}$

P(ad = 0 and bc = 1) = $\frac{3}{16}$

$\therefore$ required probability = $\frac{3}{8}$

Section (B) : MATCH THE COLUMN

B-1. **Sol.** Total cases = 5! = 120

(A) Favourable cases = ${}_5C_2 \times 2$

required probability = $\frac{20}{120} = \frac{1}{6}$

(B) Favourable cases = ${}_5C_3 \times 1 = 10$

required probability = $\frac{10}{120} = \frac{1}{1/12}$

(C) Favourable cases 5! – (44+${}_5C_1 \times 9$)

= 31

required probability = $\frac{31}{120}$

(D) Favourable cases = 3! $\left(\frac{1}{2!} - \frac{1}{3!}\right) + 4!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)$ = 2 +9 = 11

required probability =

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. **Sol.** $P(A \cup B \cup C) = P(A) + P(B) + P(C) – P(A \cap B) – P(B \cap C) – P(C \cap A) + P(A \cap B \cap C)$

$= 0.4 + 0.6 + 0.5 + 0.75 – (0.4 + 0.6) – P(B \cap C) – 0.35 + 0.2$

$= 1.1 – P(B \cap C)$ [Since $P(A \cup B) = P(A) + P(B) – P(A \cap B)$]

But, $0.75 \le P(A \cup B \cup C) \le 1$

$\Rightarrow 0.75 \le 1.1 – P(B \cap C) \le 1$

$\Rightarrow 0.1 \le P(B \cap C) \le 0.35$, but from when diagram $P(B \cap C) \ne .1$

C-2. **Sol.** $P(E_1) = \frac{2}{4} = \frac{1}{2}$

$P(E_2) = \frac{2}{4} = \frac{1}{2}$

$P(E_3) = \frac{2}{4} = \frac{1}{2}$

$P(E_1 \cap E_2) = \frac{1}{4} = P(E_1)\,P(E_2)$

$\therefore$ E_1 and E_2 are independent E_1 E_2

$P(E_2 \cap E_3) = \frac{1}{4} = P(E_2)\,P(E_3)$

$\therefore$ E_2 and E_3 are independent E_2 E_3

$P(E_3 \cap E_1) = \frac{1}{4} = P(E_3)\,P(E_1)$

$\therefore$ E_3 and E_1 are indendpent E_3 E_1

$P(E_1 \cap E_2 \cap E_3) = \frac{1}{4} \ne P(E_1)\,P(E_2)\,P(E_3)$

$\therefore$ E_1, E_2, E_3 are not independent. E_1, E_2, E_3

C-3. Sol. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{5}{8} = \frac{3}{8} + \frac{4}{8} - P(A \cap B) \Rightarrow P(A \cap B) = \frac{2}{8} = \frac{1}{4}$

Now $P(A^c/B) = \frac{P(A^C \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$

$= 1 - \frac{2}{8} \cdot \frac{8}{4} = \frac{1}{2}$

$2P(A/B^c) = \frac{2P(A \cap B^c)}{P(B^c)} = \frac{2(P(A) - P(A \cap B))}{1 - P(B)} = 4\left(\frac{3}{8} - \frac{2}{8}\right) = \frac{1}{2} \Rightarrow$ (1) is correct

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{4} \cdot \frac{2}{1} = \frac{1}{2} = P(B) \Rightarrow$ (2) is correct

again $P(A^c/B^c) = \frac{P(A^c \cap B^c)}{P(B^c)} = \frac{1 - P(A \cup B)}{1 - P(B)} = 2\left(1 - \frac{5}{8}\right) = \frac{3}{4} \neq P(A \cup B)$. So (3) is incorrect.

again $2P(A/B^c) = \frac{1}{2}$ from (1) $\Rightarrow$ $P(A/B^c) = \frac{1}{4} = P(A \cap B)$

hence (4) is correct

C-4. Sol. P(A wins the game)

= P (H or TH or TTTH or TTTTH or TTTTTTH or ....

$= \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^7} + \ldots\ldots\ldots\ldots\ldots$

$= \left(\frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \ldots\ldots\ldots\right) + \left(\frac{1}{2^2} + \frac{1}{2^5} + \frac{1}{2^8} + \ldots\ldots\ldots\right) = \frac{\frac{1}{2}}{1 - \frac{1}{8}} + \frac{\frac{1}{4}}{1 - \frac{1}{8}} = \frac{4}{7} + \frac{2}{7} = \frac{6}{7}.$

$\beta = 1 - x = 1 - \frac{6}{7} = \frac{1}{7}$

C-5. Sol. $P(E_1) = 1 - P$(unit's place in both is 1, 2, 3, 4, 6, 7, 8, 9)

$P(E_1 = 0 \text{ or } 5) = 1 - \frac{8}{10} \cdot \frac{8}{10} = \frac{9}{25}$

$P(E_2 : 5) = P(1\ 3\ 5\ 7\ 9) - P(1\ 3\ 7\ 9)$ for both numbers

$= \frac{5}{10} \cdot \frac{5}{10} - \frac{4}{10} \cdot \frac{4}{10} = \frac{25 - 16}{100} = \frac{9}{100}$

$\frac{P(E_2)}{P(E_1)} = \frac{9}{100} \cdot \frac{25}{9} = \frac{1}{4}$

$P(E_1) = 4\,P(E_2) \Rightarrow$ (1) is not correct

$P(E_2/E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{P(E_2)}{P(E_1)} = \frac{9}{100} \cdot \frac{25}{9} = \frac{1}{4}$

$P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{P(E_2)}{P(E_2)} = 1$

Exercise-3

*** Marked Questions may have more than one correct option.**

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. **Sol.** Since, the probabilities of solving the problem by A, B and C are $\frac{1}{2}, \frac{1}{3}$, and $\frac{1}{4}$ respectively.

$\therefore$ Probability that the problem is not solved

$= P(\bar{A})P(\bar{B})P(\bar{C})$

$= \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)$

$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$

Hence, the probability that the problem is solved

$= 1 - \frac{1}{4} = \frac{3}{4}$

2. **Sol.** The total number of ways in which numbers can be choosed = 25 × 25 = 625

The number of ways in which either players can choose same numbers = 25

$\therefore$ Probability that they with a prize = $\frac{25}{625} = \frac{1}{25}$

Thus, the probability that they will not win a prize in a single trial = $1 - \frac{1}{25} = \frac{24}{25}$

3. **Sol.** Since, A and B are two mutually exclusive events.

$\therefore$ $A \cap B = \varphi$

$\Rightarrow$ either $A \subseteq \bar{B}$ or $B \subseteq \bar{A}$

$\Rightarrow P(A) \leq P(\bar{B})$

or $P(B) \leq P(\bar{A})$.

4. **Sol.** Let A_1, A_2 and A_3 be the events of match winning in first, second and third match respectively. And whose probabilities are

$P(A_1) = P(A_2) = P(A_3) = \frac{1}{2}$

$\therefore$ Required probability

$= P(A_1 A'_2, A_3) + P(A'_1, A_2 A_3)$

$= P(A_1)\,P(A'_2)\,P(A_3) + P(A'_1)\,P(A_2)\,P(A_3)$

$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$.

5. **Sol.** Let the probability of getting a head is p and not getting a head is q.

Since, head appears first time in an even throw 2 or 4 or 6....

$\therefore \frac{2}{5} = qp + q^3p + q^5p + ...$

$\Rightarrow \frac{2}{5} = \frac{qp}{1-q^2}$

$\Rightarrow \frac{2}{5} = \frac{(1-p)p}{1-(1-p)^2}$ $\quad (\because\ q = 1 - p)$

$\Rightarrow \frac{2}{5} = \frac{1-p}{2-p}$

$\Rightarrow 4 - 2p = 5 - 5p$

$\Rightarrow p = \frac{1}{3}$

6. **Sol.** Probability of getting success, $p = \frac{1}{2}$ and probability of failure, $q = \frac{1}{2}$

$\therefore$ Required probability $= {}^7C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^5 \times \frac{1}{6} = \frac{{}^7C_2 \times 5^5}{6^8}$

8. **Sol.** Since, $0 \le P(A) \le 1$, $0 \le P(B) \le 1$, $0 \le P(C) \le 1$ and $0 \le P(A) + P(B) + P(C) \le 1$

$\therefore 0 \le \frac{3x+1}{3} \le 1$

$\Rightarrow -\frac{1}{3} \le x \le \frac{2}{3}$ ...(i)

$0 \le \frac{1-x}{4} \le 1$

$\Rightarrow -3 \le x \le 1$ ...(ii)

$0 \le \frac{1-2x}{2} \le 1$

$\Rightarrow -\frac{1}{2} \le x \le \frac{1}{2}$ ...(iii)

and $0 \le \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$

$\Rightarrow 0 \le 13 - 3x \le 12$

$\Rightarrow \frac{1}{3} \le x \le \frac{13}{3}$ ………...(iv)

From Eqs. (i), (ii), (iii) and (iv), we get

$\frac{1}{3} \le x \le \frac{1}{2}$

9. Sol. Given that, for binomial distribution mean, np = 4 and variance, npq = 2

$\therefore$ q = $\frac{1}{2}$, but p + q = 1 $\Rightarrow$ p = $\frac{1}{2}$

and n × $\frac{1}{2}$ = 4 $\Rightarrow$ n = 8

We know, P(X = r) = $^nC_r p^r q^{n-r}$

$\therefore$ P(X = 1) = $^8C_1 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^1$ = 8 × $\frac{1}{2^8}$ = $\frac{1}{2^5}$ = $\frac{1}{32}$

10. Sol. Given probabilities of speaking truth are

P(A) = $\frac{4}{5}$ and P(B) = $\frac{3}{4}$

And their corresponding probabilities of not speaking truth are

P($\bar{A}$) = $\frac{1}{5}$ and P($\bar{B}$) = $\frac{1}{4}$

The probability that they contradict each other

= P(A) × P($\bar{B}$) + P($\bar{A}$) × P(B)

= $\frac{4}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{3}{4}$

= $\frac{1}{5}+\frac{3}{20}$ = $\frac{7}{20}$

11. Sol. Given, E = {X is a prime number}

= {2, 3, 5, 7}

P(E) = P(X = 2) + P(X = 3) + P(X = 5) + P(X = 7)

$\Rightarrow$ P(E) = 0.23 + 0.12 + 0.20 + 0.07 = 0.62

and F = {X < 4} = {1, 2, 3}

$\Rightarrow$ P(F) = P(X = 1) + P(X = 2) + P(X = 3)

$\Rightarrow$ P(F) = 0.15 + 0.23 + 0.12 = 0.5

and E $\cap$ F = {X is prime number as well as < 4}

= {2, 3}

P(E$\cap$ F) = P(X = 2) + P(X = 3)

= 0.23 + 0.12 = 0.35

$\therefore$ Required probability

P(E $\cup$ F) = P(E) + P(F) – P(E $\cap$ F)

$\Rightarrow$ P(E $\cup$ F) = 0.62 + 0.5 – 0.35

$\Rightarrow$ P(E $\cup$ F) = 0.77

12. Sol. Given that, mean = 4

$\Rightarrow$ np = 4

and variance = 2

$\Rightarrow$ npq = 2 $\Rightarrow$ 4q = 2

$\Rightarrow$ q = $\frac{1}{2}$

$\therefore$ p = 1 – q = 1 – $\frac{1}{2}$ = $\frac{1}{2}$

Also, n = 8

Probability of 2 successes = P(X = 2)

$= {}^8C_2 p^2 q^6$

$= \frac{8!}{2! \times 6!} \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^6$

13. **Sol.** Given that,

$P(A \cap B) = \frac{1}{4}$, $P(\bar{A}) = \frac{1}{4}$ and $\overline{P(A \cup B)} = \frac{1}{6}$

$\Rightarrow 1 - P(A \cup B) = \frac{1}{6}$ $\Rightarrow 1 - P(A) - P(B) + P(A \cup B) = \frac{1}{6}$ $\Rightarrow P(\bar{A}) - P(B) + \frac{1}{4} = \frac{1}{6}$

$\Rightarrow P(B) = \frac{1}{4} + \frac{1}{4} - \frac{1}{6}$ $\Rightarrow P(B) = \frac{1}{3}$ and $P(A) = \frac{3}{4}$

Now, $P(A \cap B) = \frac{1}{4} = \frac{3}{4} \times \frac{1}{3} = P(A)\,P(B)$.

Hence, the events A and B are independent events but not equally likely.

14. **Sol.** All the three persons has three options to apply a house.

∴ Total number of cases = 3_3

Now, favourable cases = 3 (An either all has applied for house 1 or 2 or 3)

∴ Required probability = $\frac{3}{3^3} = \frac{1}{9}$.

15. **Sol.** Probability of getting score 9 in a single throw

$= \frac{4}{36} = \frac{1}{9}$

Probability of getting score 9 exactly in double throw

$= {}^3C_2 \times \left(\frac{1}{9}\right)^2 \times \frac{8}{9} = \frac{8}{243}$.

16. **Sol.** Let the events,

A = Ist aeroplane hit the target

B = IInd aeroplane hit the target

And their corresponding probabilites are

$P(A) = 0.3$ and $P(B) = 0.2$

$\Rightarrow$ $P(\bar{A}) = 0.7$ and $P(\bar{B}) = 0.8$

∴ Required probability

$= P(\bar{A})\,P(B)$

$= (0.7)(0.2) = 0.14$

17. **Sol.** Given that, $P(A) = \frac{1}{4}$, $P\left(\frac{A}{B}\right) = \frac{1}{2}$ and $P\left(\frac{B}{A}\right) = \frac{2}{3}$

we know, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ ...(i)

and $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$ ...(ii)

$\therefore\ P(B) = \dfrac{P\left(\frac{B}{A}\right).P(A)}{P\left(\frac{A}{B}\right)} = \dfrac{\left(\frac{2}{3}\right)\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)} = \dfrac{1}{3}$

18. **Sol.** $\because$ A = {4, 5, 6} and B = {1, 2, 3, 4}

$\therefore\ A \cap B = \{4\}$

$\because\ P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \dfrac{3}{6} + \dfrac{4}{6} - \dfrac{1}{6} = 1.$

19. **Sol.** By binomial distribution, $\left(\dfrac{1}{4} + \dfrac{3}{4}\right)^n$

$\therefore$ probability of at least one success = 1 – no. of success = $1 - {}_nC_n \left(\dfrac{3}{4}\right)^n = 1 - \left(\dfrac{3}{4}\right)^n \geq \dfrac{9}{10}$

$\Rightarrow \left(\dfrac{3}{4}\right)^n \leq \dfrac{1}{10}$

Taking $\log_{10}$ on both sides

$n(\log_{10}3 - \log_{10}4) \leq -1$ or $n \geq \dfrac{-1}{\log_{10}3 - \log_{10}4}$ $\Rightarrow$ $n \geq \dfrac{1}{\log_{10}4 - \log_{10}3}$

20. **Sol.** Case in which sum of digits in 8 are 08, 17, 26, 35, 44

Total cases : 00, 01, 02, ..., 09, 10, 20, 30, 40

Required probability $= \dfrac{1}{14}$

21. **Ans.** **(2)**

Sol. Statement-1 Total ways = ${}_{20}C_4$

number of AP's of common difference 1 is = 17

number of AP's of common difference 2 is = 14

number of AP's of common difference 3 is = 11

number of AP's of common difference 4 is = 8

number of AP's of common difference 5 is = 5

number of AP's of common difference 6 is = 2

total = 57

probability $= \dfrac{57}{{}^{20}C_4} = \dfrac{1}{85}$

Statement-2 common difference can be ± 6 , so statement -2 is false

Hence correct option is (2)

vr% lgh fodYi (2) gSA

22. **Ans.** **(1)**

Sol. $= \dfrac{{}^3C_1\ {}^4C_1\ {}^2C_1}{{}^9C_3} = \dfrac{\frac{3\,.\,4\,.\,2}{9\,.\,8\,.\,7}}{3\,.\,2\,.\,1} = \dfrac{2}{7}$

Hence correct option is (1)

23. **Sol.** **(3)**

$1 - P_5 \geq \frac{31}{32}$

$P_5 \leq \frac{1}{32}$

$P \leq \frac{1}{2}$

$P \in \left[0, \frac{1}{2}\right]$

24. **Sol.** **(2)**

$P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)} \Rightarrow \frac{1}{P(D)} \geq 1$

$\frac{P(C)}{P(D)} \geq P(C) \Rightarrow P(C) \leq P\left(\frac{C}{D}\right)$

25. **Sol.** **(4)**

$P(A_c \cap B_c / C) = \frac{P((A^c \cap B^c) \cap C)}{P(C)} = \frac{P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}{P(C)}$

$= \frac{P(C) - P(A).P(C) - P(B)P(C) + 0}{P(C)} = 1 - P(A) - P(B) = P(A_c) - P(B)$

26. **Sol.** Let Event (Given : {1, 2, 3,.........8})

A : Maximum of three numbers is 6.

B : Minimum of three numbers is 3

$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{{}^2C_1}{{}^5C_2} = \frac{2}{10} = \frac{1}{5}$

27. **Sol.** (3)

$p = \frac{1}{3}$, $q = \frac{2}{3}$

${}_5C_4 \left(\frac{1}{3}\right)^4 . \frac{2}{3} + {}_5C_5 \left(\frac{1}{3}\right)^5 = 5. \frac{2}{3^5} + \frac{1}{3^5} = \frac{11}{3^5}$

28. **Sol.** **Ans.** **(1)**

Given $P\left(\overline{AUB}\right) = \frac{1}{6}$, P = , P

$1 - P(A \cup B) = \frac{1}{6}$

$\Rightarrow 1 - P(A) - P(B) + P(A \cap B) = \frac{1}{6}$

$\Rightarrow 1 - \frac{3}{4} - P(B) + \frac{1}{4} = \frac{1}{6}$ $(P(A) = 1 - P(\overline{A}))$

$\Rightarrow P(B) = \frac{1}{2} - \frac{1}{6} = \frac{3-1}{6} = \frac{1}{3}$

A and B are not equally likely.

Further $P(A).\ P(B) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4} = P(A \cap B)$

A and B are independent events

29. Ans. (1)

Sol. There seems to be ambiguity in the question. It should be mentained that boxs are different and one particular box has 3 balls :

then

number of ways = $\frac{{}^{12}C_3 \times 2^9}{3^{12}} = \frac{55}{3}\left(\frac{2}{3}\right)^{11}$

Alter

${}_3C_1\ {}_{12}C_3\ ({}_9C_0 + {}_9C_1 + {}_9C_2 + {}_9C_4 + {}_9C_5 + {}_9C_5 + {}_9C_7 + {}_9C_8 + {}_9C_9)$

$+ \frac{\lfloor 12 \times \lfloor 3}{\lfloor 3 \lfloor 3 \lfloor 6 \lfloor 3}$

$= {}_3C_1\ {}_{12}C_3\ (2_9 - 2_9C_3) + \frac{\lfloor 12}{\lfloor 3 \lfloor 2 \lfloor 6}$

correct answer should have been $\frac{{}^3C_1{}^{12}C_3(2^9 - 2.{}^9C_3) + \frac{\lfloor 12}{\lfloor 3 \lfloor 2 \lfloor 6}}{3^{12}}$

30. Ans. (3)

Sol. E_1 : {(4, 1) ,................ (4,6,)} 6 cases

E_2 : {(1,2), ... (6,2)} 6 cases

E_3 : 18 cases (sum of both are odd)}

$P(E_1) = \frac{6}{36} = \frac{1}{6} = P(E_2)$

$P(E_3) = \frac{18}{36} = \frac{1}{2}$

$P(E \cap E_2) = \frac{1}{36}$

$P(E \cap {}_2E_3) = \frac{1}{12}$

$P(E \cap {}_3E_1) = \frac{1}{12}$

$P(E \cap {}_1E \cap {}_2E_3) = 0$

$\therefore E_1, E_2, E_3$ are not independent

31. Ans. (2)

Sol. $P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$

$P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$

$P(A) + P(C) - 2P(A \cap C) = \frac{1}{4}$

$P(A \cap B \cap C) = \frac{1}{16}$

$\therefore P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{1}{16} = \frac{6+1}{16} = \frac{7}{16}$

32. **Ans. (1)**

Sol. $P = \frac{6}{^{11}C_2} = \frac{6}{55}$

$x_1 - x_2 = \pm 4\lambda$

$x_1 + x_2 = 4\alpha$

$2x_1 = 4(\lambda \pm \alpha)$

$x_1 = 2(\lambda \pm \alpha)$

x_1	x_2
0	4, 8
2	6, 10
4	0, 8
6	2, 10
8	0, 4
10	2, 6

33. **Ans. (1)**

Sol. 15 green + 10 yellow = 25 balls

P(green) = $\frac{3}{5} = p_1$

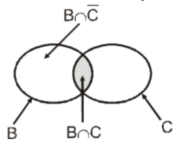
P(yellow) = $\frac{2}{5} = q$

n = 10

$\therefore$ Variance = npq $= 10 \frac{3}{5} \cdot \frac{2}{5} = \frac{60}{25} = \frac{12}{5}$

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. **Sol.** We have $P(B \cap \bar{C}) = P[(A \cup \bar{A}) \cap (B \cap \bar{C})] = P(A \cap B \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$



$P(B \cap C) = P(B) - P(B \cap \bar{C}) = \frac{3}{4} - \frac{2}{3} = \frac{1}{12}$

2. **Sol.** Let E denote the event that minimum of two numbers is less than 4, then E' denote the event that minimum number ≥ 4.

$P(E') = \frac{{}^3C_2}{{}^6C_2} = \frac{3}{15} = \frac{1}{5}$

⇒ P(E) = 4/5.

3. **Sol.** Such numbers are 6, 12, 18, ... 96

i.e. 16 such numbers. Hence required probability = $\frac{{}^{16}C_3}{{}^{100}C_3} = \frac{16 \times 15 \times 14}{100 \times 99 \times 98} = \frac{4}{1155}$

Hence (D)

4. **Sol.** $P\left(\frac{E^c \cap F^c}{G}\right) = \frac{P(E^c \cap F^c \cap G)}{P(G)} = \frac{P(G) - P(E \cap G) - P(G \cap F)}{P(G)}$

$= \frac{P(G)[1 - P(E) - P(F)}{P(G)}$ [∴P(G)≠ 0] $= 1 - P(E) - P(F) = P(E^c) - P(F)$.

5. **Sol.** If A : Indian Men sit with their wife

B : American men sit with their wife

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{n(A \cap B)}{n(B)} = \frac{4!(2!)^5}{5!(2!)^4} = \frac{2}{5}$

6. **Sol.** n(A) = 4

Let n(B) = x > 0

∵ A & B are independent events

⇒ P(A) . P(B) = P(A ∩ B)

$\frac{4}{10} \cdot \frac{x}{10} = \frac{y}{10}$ where y = n (A ∩ B) ≤ min (4, x)

$x = \frac{5y}{2}$

y	x
0	0
1	not an integer
2	5
4	10

So x = 5 or 10

7. Sol. $P(X = 3) = \frac{5}{6} . \frac{5}{6} . \frac{1}{6} = \mathbf{\frac{25}{216}}$

8. Sol. $P(X \geq 3) = \frac{5}{6}\ \frac{5}{6}\ 1 = \mathbf{\frac{25}{36}}$

9. Sol. $P((X \geq 6) / (X > 3)) = \frac{P((X > 3)/(X \geq 6)) . P(X \geq 6)}{P(X > 3)} = \frac{1.\left[\left(\frac{5}{6}\right)^5 . \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^6 . \left(\frac{1}{6}\right) + ...\infty\right]}{\left[\left(\frac{5}{6}\right)^3 . \frac{1}{6} + \left(\frac{5}{6}\right)^4 . \frac{1}{6} + ...\infty\right]} = \mathbf{\frac{25}{36}}$

10. Sol. Probability $(P) = \frac{P(GGG) + P(GRG)}{P(GGG) + P(GRG) + P(RGG) + P(RRG)}$

$\Rightarrow$ $P = \frac{\frac{4}{5} \times \frac{3}{4} \times \frac{3}{4} + \frac{4}{5} \times \frac{1}{4} \times \frac{1}{4}}{\frac{4}{5} \times \frac{3}{4} \times \frac{3}{4} + \frac{4}{5} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{4} \times \frac{3}{4} + \frac{1}{5} \times \frac{3}{4} \times \frac{1}{4}}$ $\Rightarrow$ $P = \frac{36 + 4}{36 + 4 + 3 + 3} = \frac{40}{46} = \frac{20}{23}$

11. Sol. $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$; r_1, r_2, r_3 are to be selected from {1, 2, 3, 4, 5, 6}

As we know that $1 + \omega + \omega_2 = 0$

$\therefore$ from r_1, r_2, r_3 , one has remainder 1, other has remainder 2 and third has remainder 0 when divided by 3.

$\therefore$we have to select r_1, r_2, r_3 from (1, 4) or (2, 5) or (3, 6) which can be done in ${}_2C_1 \times {}_2C_1 \times {}_2C_1$ ways value of r_1, r_2, r_3 can be interchanged in 3! ways.

$\therefore$ required probability = $\frac{({}^2C_1 \times {}^2C_1 \times {}^2C_1) \times 3!}{6 \times 6 \times 6} = \frac{2}{9}$

12*. Sol. $P(E \cap F) = P(E) . P(F)$ ....(1)

$P(E \cap \bar{F}) + P(\bar{E} \cap F) = \frac{11}{25}$ ....(2)

$P(\bar{E} \cap \bar{F}) = \frac{2}{25}$ ....(3)

by (2)

$P(F) + P(E) - 2P(E \cap F) = \frac{11}{25}$ ....(4)

by (3)

$1 - [P(E) + P(F) - P(E \cap F)] = \frac{2}{25}$

$[P(E) + P(F) - P(E \cap F)] = \frac{23}{25}$ ....(5)

by (4) & (5) $P(E)\,P(F) = \frac{12}{25}$ ....(6)

and $P(E) + P(F) = \frac{7}{5}$ ....(7)

By (6) and (7) $P(E) = \frac{4}{5}$, $P(F) = \frac{3}{5}$ or $P(E) = \frac{3}{5}$, $P(F) = \frac{4}{5}$

13*. **Sol.** $P(X/Y) = \frac{1}{2}$

$\frac{P(X \cap Y)}{P(Y)} = \frac{1}{2} \Rightarrow P(Y) = \frac{1}{3}$

$P(Y/X) = \frac{1}{3}$

$\frac{P(X \cap Y)}{P(X)} = \frac{1}{3} \Rightarrow P(X) = \frac{1}{2}$

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = \frac{2}{3}$ A is correct

$P(X \cap Y) = P(X) \cdot P(X) \Rightarrow$ X and Y are independent B is correct

$= P(Y) - P(X \cap Y)$

$= \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$ D is not correct

14. **Sol.** Favourable : D_4 shows a number and
only 1 of $D_1D_2D_3$ shows same number
or only 2 of $D_1D_2D_3$ shows same number
or all 3 of $D_1D_2D_3$ shows same number

Required Probability = $\frac{{}^6C_1({}^3C_1 \times 5 \times 5 + {}^3C_2 \times 5 + {}^3C_3)}{216 \times 6}$

$= \frac{6 \times (75 + 15 + 1)}{216 \times 6}$

$= \frac{6 \times 91}{216 \times 6}$

$= \frac{6 \times 91}{216 \times 6}$

15. **Sol.** **(A)**
P (problem solved by at least one) = 1 – P(problem is not solved by by all)

$= 1 - P(\bar{A})\,P(\bar{B})\,P(\bar{C})\,P(\bar{D})$

$$= 1 - \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{7}{8}\right) = 1 - \frac{21}{256} = \frac{235}{256}$$

16. Sol. (A)

Bag 1	Bag 2	Bag 3
1 W	2 W	3 W
3 R	3 R	4 R
2 B	4 B	5 B

P(W W W) + P(R R R) + P(B B B)

$$\left(\frac{1}{6}\times\frac{2}{9}\times\frac{3}{12}\right) + \left(\frac{3}{6}\times\frac{3}{9}\times\frac{4}{12}\right) + \left(\frac{2}{6}\times\frac{4}{9}\times\frac{5}{12}\right) \Rightarrow \frac{6+36+40}{6\times9\times12} \Rightarrow \frac{82}{648}$$

17. Sol. (D)

P (Ball drawn from box 2 / one is W one is R) = $\frac{P(A\cap B)}{P(B)}$

$$= \frac{\frac{1}{3}\times\frac{2\times3}{{}^9C_2}}{\frac{1}{3}\left[\frac{1\times3}{{}^6C_2}+\frac{2\times3}{{}^9C_2}+\frac{3\times4}{{}^{12}C_2}\right]} = \frac{\frac{2\times3\times2}{9\times8}}{\frac{3\times2}{6\times5}+\frac{2\times6\times2}{9\times8}+\frac{3\times4\times2}{12\times11}}$$

$$= \frac{\frac{1}{6}}{\frac{1}{5}+\frac{1}{6}+\frac{2}{11}} = \frac{\frac{1}{6}}{\frac{66+55+60}{55\times60}} = \frac{55}{181}$$

18. Ans. (A)

Sol. 3 Boys & 2 Girls..................

(1) B (2) B (3) B (4)

Girl can't occupy 4_{th} position. Either girls can occupy 2 of 1, 2, 3 position or they can both be a position (1) or (2).

Hence total number of ways in which girls can be seated is ${}_3C_2 \times 2! \times 3! + {}_2C_1 \times 2! \times 3! = 36 + 24 = 60$.

Number of ways in which 3 B & 2 A can be seated = 5 !

Hence required prob. = $\frac{60}{5!} = \frac{1}{2}$.

19. Ans. (A,B)

Sol. $\text{Box} - \text{I} < \begin{matrix}\text{Red} \to n_1 \\ \text{Black} \to n_2\end{matrix}$ $\qquad$ $\text{Box} - \text{II} < \begin{matrix}\text{Red} \to n_3 \\ \text{Black} \to n_4\end{matrix}$

P(R) = $\frac{1}{2}.\frac{n_1}{n_1+n_2} + \frac{1}{2}.\frac{n_3}{n_3+n_4}$

R(II/R) = $\dfrac{\frac{1}{2}.\frac{n_3}{n_3+n_4}}{\frac{1}{2}.\frac{n_1}{n_1+n_2}+\frac{1}{2}.\frac{n_3}{n_3+n_4}} = \dfrac{\frac{n_3}{n_3+n_4}}{\frac{n_1}{n_1+n_2}+\frac{n_3}{n_3+n_4}}$

by option $\quad n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$

$$P(II/R) = \frac{\frac{5}{20}}{\frac{3}{6}+\frac{5}{20}} = \frac{n_4}{\frac{1}{2}+\frac{1}{4}} = \frac{1}{4}\times\frac{4}{2+1} = \frac{1}{3}$$

20. **Ans.** **(C,D)**

Sol. Given $\frac{n_1}{n_1+n_2}.\frac{n_1-1}{n_1+n_2-1}+\frac{n_2}{n_1+n_2}.\frac{n_1}{n_1+n_2-1}=\frac{1}{3}$

$3(n_1^2 - n_1 + n_1n_2) = (n_1 + n_2)(n_1 + n_2 - 1)$

$3n_1(n_1 + n_2 - 1) = n_1 + n_2(n_1 + n_2 - 1)$

$2n_1 = n_2$

21. **Ans.** **(C)**

Sol. Let x = P(computer turns out to be defective given that it is produced in Plant T_2),

$\frac{7}{100}=\frac{1}{5}\times(10x)+\frac{4}{5}x \Rightarrow \quad 7 = 200x + 80x \quad \Rightarrow \quad x = \frac{7}{280}$

P(produced in T_2/ not defective) = $\frac{P(A\cap B)}{P(B)}$

$$\frac{4/5(1-x)}{\frac{1}{5}(1-10x)+\frac{4}{5}(1-x)} = \frac{\frac{4}{5}\left(\frac{273}{280}\right)}{\frac{1}{5}\left(\frac{280-70}{280}\right)+\frac{4}{5}\left(\frac{273}{280}\right)} = \frac{4\times273}{210+4\times273} = \frac{2\times273}{105+2\times273} = \frac{546}{651} = \frac{78}{93}$$

22. **Ans.** **(B)**

Sol. $P(X > Y) = T_1T_1 + DT_1 + T_1D$ (Where T_1 represents wins and D represents draw)

$\frac{1}{2}.\frac{1}{2}+\frac{1}{6}.\frac{1}{2}+\frac{1}{2}.\frac{1}{6} = \frac{5}{12} = \Rightarrow$ (B) is correct

23. **Ans.** **(C)**

Sol. $P(X = Y) = DD + T_1T_2 + T_2T_1 = \frac{1}{6}.\frac{1}{6}+\frac{1}{2}.\frac{1}{3}+\frac{1}{3}.\frac{1}{2} = \frac{1}{36}+\frac{1}{3} = \frac{39}{36\times3} = \frac{13}{36} \Rightarrow$ (C) is correct

24. **Ans.** **(A,B)**

Sol. $\frac{P(X\cap Y)}{P(Y)} = \frac{1}{2}$

$\frac{P(Y\cap X)}{P(X)} = \frac{2}{5}$

$P(X \cap Y) = \frac{P(Y)}{2} = \frac{2}{5}$ $P(X) = \frac{2}{5}.\frac{1}{3} = \frac{2}{15} \Rightarrow P(Y) = \frac{4}{15}$

$$\frac{P(\bar{X}\cap Y)}{P(Y)} = \frac{P(Y)-P(X\cap Y)}{P(Y)} = \frac{\frac{4}{15}-\frac{2}{15}}{\frac{4}{15}} = \frac{2}{4}=\frac{1}{2}$$

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = \frac{1}{3}+\frac{4}{15}-\frac{2}{15}=\frac{7}{15}$

25. **Ans.** **(C)**

Sol. x + y + z = 10

Total number of non-negative solutions = ${}_{10+3-1}C_{3-1} = {}_{12}C_2 = 66$

Now Let z = 2n.

x + y + 2n = 10 ; n ≥ 0

Total number of non-negative solutions = 11 + 9 + 7 + 5 + 3 + 1 = 36

Required probability = $\frac{36}{66} = \frac{6}{11}$