Exercise-1

Marked Questions may have for Revision Questions.

OBJECTIVE QUESTIONS

Section (A) : Fundamental principle of counting

A-1. Ans. $10 \times 9 = 90$

A-2. Sol. Number of four digit no. in which atleast one digit is repeated (i.e. all digit are not different) is

9	10	10	10	–	9	9	8	7	= 9000 – 4536 = 4464

A-3. Sol. Number of words which have at least one letter repeated = total words – number of words which have no letter repeated = $10^5 - 10 \times 9 \times 8 \times 7 \times 6 = 69760$

A-4. Sol. Total even numbers

		2	(numbers whose unit digit is 2)
5 ×	5		= 25
		4	(numbers whose unit digit is 4)
5 ×	5		= 25

on adding = 50

A-5. Sol. Total number of ways = $3 \times 5 \times 4 = 60$

A-6. Sol. The number of ways = ${}^{10}C_3 \times 3! = 720$

Section (B) :Permutation and combination of distinct objects gap and string method, Rank of a word

B-1. Sol. ${}^5P_3 = 5 \times 4 \times 3 = 60$

B-2. Sol. Even place

	E		E		E		E	

There are four even places and four odd digit number so total number of filling is $\frac{4!}{2!.2!}$ rest are also occupy in $\frac{5!}{3!.2!}$ ways

Hence total number of ways = $\frac{4!}{2!.2!} \times \frac{5!}{3!.2!} = 60$

B-3. Sol. Total no. of arrangement if all the girls do not seat side by side is

= [all arrangement – number of ways in which girls sit side by side]

$= 8! - (6! \times 3!) = 6!\,(56 - 6) = 6! \times 50$

$= 720 \times 50 = 36000$

B-4. **Sol.** Total number of ways = n! – 2! × (n– 1)!
= (n – 1)! (n – 2)

B-5. **Sol.** Total number of ways = 2! × 5! × 5!

B-6. **Sol.** Required sum = 3!(3 + 4 + 5 + 6)
= 108

B-7. **Sol.** Total number of possible arrangements is
$^4p_2 \times {}^6p_3$.

B-8. **Sol.** Total number of signals can be made from 3 flags each of different colour by hoisting 1 or 2 or 3 above.
i.e. $^3p_1 + {}^3p_2 + {}^3p_3 = 3 + 6 + 6 = 15$

B-9. **Sol.** Number of teams = $^{15-1}C_{11-1} = {}^{14}C_{10} = 1001$

B-10. **Sol.** There are 4 balls marked even digits {i.e. 2, 4, 6, 8}
and 5 balls marked odd digits {1, 3, 5, 7, 9}
sum is odd ⇒ one ball with even digit and other one is with odd digit
⇒ no. of way = $^4C_1 \cdot {}^5C_1 = 20$

B-11. **Sol.** Let there are n teams in championship
No of matches played = $^nC_2 = 153$
$\Rightarrow \dfrac{n(n-1)}{2} = 153$
⇒ n(n – 1) = 306
⇒ (n – 18) (n + 17) = 0
⇒ n = 18

B-12. **Sol.** First we select 4 consonant out of 7 consonant and 2 vowel out of 4 vowel then arranging
Hence total number of ways = $^7C_4 \times {}^4C_2 \times 6!$
= 210 × 720 = 151200

B-13. **Sol.** Number of bowlers = 4
Number of wicketkeeper = 2
Total number of required selection
$= {}^4C_3 \cdot {}^2C_1 \cdot {}^{10}C_7 + {}^4C_4 \cdot {}^2C_1 \cdot {}^{10}C_6 + {}^4C_3 \cdot {}^2C_2 \cdot {}^{10}C_6 + {}^4C_4 \cdot {}^2C_2 \cdot {}^{10}C_5$
= 960 + 420 + 840 + 252
= 2472

B-14. **Sol.** There are 9 married couple so first we select 2
man out of 9 and then we select 2 women out of
rest 7 then we arranged them, so required no. is $^9C_2 \times {}^7C_2 \times 2! = 36 \times 21 \times 2 = 1512$

B-15. **Sol.** First we select one married couple out of 6 married couple i.e.
$^6C_1$ ways
total number of required case = $^6c_1[{}^5c_1 \times {}^4c_1 + {}^5c_1 + {}^5c_2] = 6(20 + 10 + 10) = 240$

B-16. **Sol.** First we find 3 ball from 9 ball

$_9C_3 = 84$
Now number of ways if no black ball is selected = $_6C_3 = 20$
Hence required no is 84 - 20 = 64

B-17. Sol. upperdeck - 13 seats → 8 in upper deck.
lowerdeck - 7 seats → 5 in lower deck
Remaining passengers = 7
Now Remains 5 seats in upper deck and 2 seats
in lower deck
for upper deck number of ways = $_7C_5$
for lower deck number of ways = $_2C_2$
So total number of ways = $_7C_5 \times {_2C_2} = \frac{7.6}{2} = 21$

B-18. Sol. $\frac{5!\,{}^6C_2\,.2!}{6!2!} = \frac{5}{2}$

B-19. Sol. If 1 be unit digit then total no. of number is 3! = 6
Similarly so on if 3, 5, or 7 be unit digit number
then total no. of no. is 3! = 6
Hence sum of all unit digit no. is = 6× (1+3+5+7)
= 6× 16 = 96
Hence total sum is
$= 96 \times 10^3 + 96 \times 10^2 + 96 \times 10^1 + 96 \times 10^0$
= 96000 + 9600 + 960 + 96 = 106656
= 16 × 1111 × 3!

Section (C) : Permutation and combination of alike objects

C-1. Sol. NINETEEN
⇒ N → 3 : I, T
E → 3
First we arrange the word of N, N, N, I and T
then the number of ways = $\frac{5!}{3!}$.
Now total 6 number of place which are arrange E is $_6C_3$
Hence total number of ways = $\frac{5!}{3!}.\,{_6C_3}$

C-2. Sol.
mW + nR ——|—— mW + nR
Arrangements will be one side
∴ $^{m+n}C_m$

C-3. Sol. SERIES
S - 2, E - 2 , R, I
case-I when all letter distinct is
$_4C_3 \times 3\,! = 4 \times 6 = 24$
case-II when 2 letters are same

${}_2C_1 \,.\, {}_3C_1 \times \frac{3!}{2!} = 2 \,.\, 3 \,.\, 3 = 18$

total number is 24 + 18 = 42

C-4. **Sol.** Word QUEUE

E → 2, Q , U - 2

E				

$= \frac{4!}{2!} = 12$

Q	E			

$= \frac{3!}{2!} = 3$

Q	U	E	E	U

= 1

Q	U	E	U	E

= 1 17th rank

C-5. **Sol.**

					2

$= \frac{5!}{2!3!} = 10$

					3

$= \frac{5!}{3!} = 20$

					4

$= \frac{5!}{2!2!} = 30$

Hence sum of unit places is

$2 \times 10 + 3 \times 20 + 4 \times 30 = 200$

Hence required sum is

$= 200 \times (10_5 + 10_4 + 10_3 + 10_2 + 10_1 + 10_0)$

$= 200 \times (111111) = 22222200$

C-6. **Sol.** Total number of ways $= 3! \times \frac{4!}{3!} = 4!$

C-7. **Sol.**

T										T

Total arrangement is $\frac{9!}{2!.2!}$

$= 90720$

Section (D) : Division into groups, Selection of one or more objects, divisors, factorisation of numbers

D-1. **Sol.** Required number of ways $= (2 + 1)(3 + 1)(4 + 1) - 1 = 59$.

D-2. **Sol.** Here first three children receive 2 each and younger receives 3 toys

then total number of distribution is

${}_9C_2 \,.\, {}_7C_2 \,.\, {}_5C_2 \,.\, {}_3C_3$

$= \frac{9!}{2!.7!} . \frac{7!}{2!.5!} . \frac{5!}{2!.3!} . \frac{3!}{3!.0!} = \frac{9!}{3!.(2!)^3}$

D-3. **Sol.** Required number of ways

$_{36}C_9 \,.\, _{27}C_9 \,.\, _{18}C_9 \,.\, _{9}C_9 \,.\, 4\,! \;=\; \frac{36!}{(9!)^4} \times 4!$

D-4.

Sol.

2	27720
2	13860
5	6930
2	1386
3	693
3	231
7	77
	11

$2_3\ 3_2 \,.\, 5.\ 11 \,.\, 7$

Hence number of co-prime factor

$2_{5-1} = 2_4 = 16$

D-5. **Sol.** $94864 = 2_4 \,.\, 7_2 \,.\, 11_2$ number of ways $= \frac{5.3.3+1}{2} = 23$

D-6. **Sol.** Factorizing the given number, we have

$38808 = 2_3 \,.\, 3_2 \,.\, 7_2 \,.\, 11$

The total number of divisors of this number is same as the number of ways of selecting some or all of three 2's, two 3's, two 7's and one 11. Therefore, the total number of divisors

$= (3 + 1)\ (2 + 1)\ (2 + 1)\ (1 + 1) = 72$

Hence, the required number of proper divisors = 72 – 2 = 70

D-7. **Sol.** Total number of requried divisors is

$(p + 1)\ (q + 1)\ (r + 1)\ (s + 1) - 2$

D-8. **Sol.** Here $21600 = 2_5.\ 3_3.\ 5_2$

$(2 \times 5) \times 2_4 \times 3_3 \times 5_1$

Now numbers which are divisible by 10

$= (4 + 1)(3 + 1)(1 + 1) = 40$

$(2 \times 3 \times 5) \times (2_4 \times 3_2 \times 5_1)$ now numbers which are

divisible by both 10 and 15

$= (4 + 1)(2 + 1)(1 + 1) = 30$

So the numbers which are divisible by only 10 but not by 15 = 40 – 30 = 10

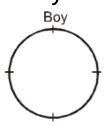
D-9. **Sol.** sum of the divisors of $2_5.\ 3_7 \,.\, 5_3 \,.\, 7_2$

$(2_0 + 2_1 + 2_2 + 2_3 + 2_4 + 2_5)\ (3_0 + 3_1 + ...+ 3_7)\ (5_0 + 5_1 + 5_2 + 5_3)\ (7_0 + 7_1 + 7_2 + 7_3)$

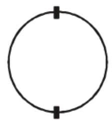
Section (E) : Circular permutation

E-1. **Sol.** Total number of ways is $\frac{6! \times 3!}{2!} = 720 \times 3 = 2160$

E-2. **Sol.** First we arrange all the boy so no. of ways of all the boy can stand is 3! now we arrange all the girl in 4 ! ways so total no. of ways is = 3! 4!



E-3. **Sol.** First we seat first two specified person in $2 \times 10 = 20$ ways and remaining 10 person can be arranged in 10! ways.



So total no. of ways is = $2 \times 10 \times 10! = 20.\ 10!$

E-4. **Sol.** As 4 particular flowers are together then the total number of ways is

$$\frac{4! \times 4!}{2} = 288$$

E-5. **Sol.** Indians - 2
Americans - 3
Italians - 3
Frenchmen - 4
number of ways of arranged in a row. while persons of same nationality are together is
$= 3! \times 2! \times 3! \times 3! \times 4!$
$= 2 . (3!)^3 . 4!$

Section (F) : Multinomial theorem, Distribution of objects (Method of fictitious partition)

F-1. **Sol.** Total number of positive integral solution of
$x_1 . x_2 . x_3 = 30 = 2 \times 3 \times 5$ is
$3 \times 3 \times 3 = 27$

F-2. **Sol.** $xyz = 21600$
$= 2^5 . 3^3 . 5^2$
Here if $x + y + z = 5 \Rightarrow {}^7C_2 = 21$
$x + y + z = 3 \Rightarrow {}^5C_2 = 10$
$x + y + z = 2 \Rightarrow {}^4C_2 = 6$
Hence required no. = $21 \times 10 \times 6 = 1260$

F-3. **Sol.** Using multinomial theorem
Total no. of ways of choosing 6 chocolates out of 8 different brand is = ${}^{8+6-1}C_6 = {}^{13}C_6$

F-4. **Sol.** Required number of ways
= Coefficient of x^{10} in $(1 + x + x^2 + ...)^4$
= Coefficient of x^{10} in $(1 - x)^{-4} = {}^{10+4-1}C_{4-1} = {}^{13}C_3 = 286$

F-5. **Sol.** Using multinomial theorem
Total no. of ways of choosing 6 chocolates out of 8 different brand is = ${}^{8+6-1}C_6 = {}^{13}C_6$

F-6. **Sol.** Required number of ways
$= \text{Coefficient of } x^{10} \text{ in } (1 + x + x^2 + ...)^4$
$= \text{Coefficient of } x^{10} \text{ in } (1 - x)^{-4} = {}^{10+4-1}C_{4-1} = {}^{13}C_3 = 286$

F-7. **Sol.** Using multinomial theorem
total number of required selection is
${}^{8+3}C_8 = {}^{11}C_8 = {}^{11}C_3$

F-8. **Sol.** Using multinomial theorem
co-efficient of x^{11} in the expansion of $(x + x^2 + x^3 + ...+ x^6)^3$
= coeff. of x'' in $x^3 (1 - x^6)^3.(1-x)^{-3}$ is
$= {}^{10}C_8 - 3 \,.\, {}^4C_2 = 45 - 18 = 27$

Section (G) : Geometrical Problems

G-1. **Sol.** The number of ways of selecting 3 points out of 12 points is ${}^{12}C_3$.
Three points out of 7 collinear points can be selected in ${}^7C_3$ ways.
Hence, the number of triangles formed is ${}^{12}C_3 - {}^7C_3 = 185$.

G-2. **Sol.** Total number of ways $= {}^8C_3 - {}^5C_3 - {}^3C_3 = 45$

G-3. **Sol.** Total number of diagonal $= {}^8C_2 - 8 = 20$

G-4. **Sol.** Let n be the sides of polygon then

$$\frac{n(n-3)}{2} = 44$$

$\Rightarrow$ $n^2 - 3n - 88 = 0$
$\Rightarrow$ $(n - 11)(n + 8) = 0$
$\Rightarrow$ $n = 11, \; n \neq -8$

G-5. **Sol.** Total number of triangle $= {}^4C_3 = 4$

G-6. **Sol.** Required straight lines $= {}^{20}C_2 - {}^4C_2 + 1$
$= 190 - 6 + 1 = 185$

G-7. **Sol.** Total number of straight lines $= {}^nC_2 - {}^pC_2 + 1$

G-8. **Sol.** Total number of parallelogram $= {}^4C_2 \times {}^3C_2 = 18$

G-9. **Sol.** Total number of points of
intersection $= {}^8C_2 + 2 \times {}^4C_2 + 2 \times {}^8C_1 \times {}^4C_1$
$= 104$

Section (H) : Exponent of prime number p in n, Derangement

H-1. $\Rightarrow$ $\left[\frac{20}{3}\right] + \left[\frac{20}{3^2}\right] + \left[\frac{20}{3^3}\right] + ...$
$= 6 + 2 + 0 = 8$

H-2. **Sol.** Exponent of 2 in 45! is

$$\left[\frac{45}{2}\right]+\left[\frac{45}{2^2}\right]+\left[\frac{45}{2^3}\right]+\left[\frac{45}{2^4}\right]+\left[\frac{45}{2^5}\right]+\left[\frac{45}{2^6}\right]$$

$= 22 + 11 + 5 + 2 + 1 + 0 \quad = 41$

Exponent of 5 in 45! is

$$\left[\frac{45}{5}\right]+\left[\frac{45}{5^2}\right]+\left[\frac{45}{5^3}\right] = 9 + 1 + 0 = 10$$

So no. of zeros at the end of 45! is 10

H-3. **Sol.** Required number of ways $= 5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right) = 44$

$$= 10 \times 3!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right) + 5 \times 4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)$$

$$+ 1 \times 5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right) = 109$$

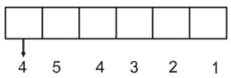
H-4. **Sol.** Required number of ways $= {}^5C_1\, D_4 + {}^5C_0\, D_5$

$$= 5 \times 4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right) + 1 \times 5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right) = 89$$

Exercise-2

1. **Sol.** possible outcomes are $= {}^{10}P_4$

2. **Sol.**



Hence total number of ways.

$= 4 \times 5! = 4 \times 120 = 480$

3. **Sol.** Number of one digit numbers = 9

Number of 2 digits numbers = 9 × 9 = 81

Number of 3 digits numbers = 9 × 9 × 8 = 648

total numbers = 9 + 81 + 648 = 738

4. **Sol.** Total number of ways is if there is no condition is

8 ! = 40320

Again if all vowels are together i.e.

A E U DGHTR.

so total number of ways = 6 ! × 3 ! = 4320

Hence total number of ways if all vowels do not together is

40320 – 4320 = 36000 ways

5. **Sol.** When 0 is not being used = 5!

When 3 is not being used = 5! – 4 !

Total number of ways = 5 ! + 5 ! – 4 ! = 216

6. **Sol.** Here $T_r = r.\ P_r = (r + 1 – 1)\ r\,!$

$= (r + 1)\ r! – r! = (r + 1)! – r!$

$\Rightarrow\ S_n = (n+1)! – 1$

here 49^{th} word

7. **Sol.** Total number of 6 digit number that ends with 2, 1

i.e.

				2	1
7	7	6	5		

Hence total number of ways is

$7 \times 7 \times 6 \times 5 = 7 \times {}_7P_3$

8. **Sol.**

A	B	A	B	A	B

A	B	A	B	A	B

total number of required ways is ${}_{12}C_6 \times 6\,! \times 6\,! \times 2\,!$

$= \frac{12!}{6! \times 6!} \times 6\,! \times 6\,! \times 2\,! = 2 \times 12\,!$

9. **Sol.**

A				

$4! = 24$ ways

G				

$4!/2! = 12$ ways

I				

$\frac{4!}{2!} = 12$ ways

N	A	A	G	I

and

N	A	A	I	G

50^{th} word

10. **Sol.** We have arrange all the letter except 'CCC' is

$\frac{12!}{5!.3!.2!}$ now there are 13 place where 'C' can be placed $= {}_{13}C_3$

Hence required number of ways is $= \frac{12!}{5!\,3!\,2!}\ {}_{13}C_3 = 11. \frac{13!}{6!}$

11. **Sol.** Total number of possible arrangements are

$\frac{8!}{3!4!} = 280$

12. **Sol.** Total no. of M are = 1

Total no. of I are = 4

Total no. of P are = 2

Total no. of S are = 4

First we arrange all the words other than I's are

Number of ways $= \frac{7!}{2!\,4!} = \frac{7 \times 6 \times 5}{1 \times 2} = 105$

Now, there are 8 places in between arranged letter where I can be placed to keep separated from each other. For doing this no. of ways $= {}_8C_4$

Total required no. $= 105 \times {}_8C_4 = \frac{105 \times 8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4}$

$= 105 \times 70 = 7350$

13. Sol. Case - I when all 5 match win by the india then total
no. of ways = ${}_5C_5$
Case - II when 6_{th} match win by the india then total
no. of ways = ${}_5C_4$
Case - III when 7_{th} match win by the india then total
no. of ways = ${}_6C_4$
Case - IV when 8_{th} match win by the india then total
no. of ways = ${}_7C_4$
Case - V when 9_{th} match win by the india then total
no. of ways = ${}_8C_4$
Hence required no. of ways
$= 1 + 5 + {}_6C_4 + {}_7C_4 + {}_8C_4$
$= 1 + 5 + 15 + 35 + 70 = 126$

14. Sol. Total No. of bowlers = 6
Now, (i) If 4 bowlers are including the no. of ways
selecting 11 players out of 15 players
$= {}_6C_4 \times {}_9C_7 = 15 \times 36 = 540$
(ii) If 5 bowlers are selected
$= {}_6C_5 \times {}_9C_6 = 6 \times 84 = 504$
(iii) If all 6 bowlers are selected
$= {}_6C_6 \times {}_9C_5 = 1 \times 126 = 126$
Hence total no. of ways = 540 + 504 + 126 = 1170

15. Sol. Total number is ${}_8C_5 = 56$
not required is ${}_6C_5 + {}_6C_5 = 12$
Hence required no of arrangement = 56 – 12 = 44

16. Sol. First we select 3 places out of 10 for speakers S_1, S_2 and S_3 put them in order S_1, S_3, S_2 or S_3, S_1, S_2 then arrange rest seven speakers at seven place without any restriction i.e. total number of ways

$= {}_{10}C_3 \,.\, 7!.2\,! = \dfrac{10!}{3}$

17. Sol. First we choose any three place out of 11 place
i.e. ${}_{11}C_3$ ways and rest 8 places are arranged by 8!
ways.
Hence required no. is ${}_{11}C_3 .\, 8! = 11!/3!$

18. Sol. Possible size of group of coins is
1, 2, 4
so the number of ways is
${}_7C_1 \,.\, {}_6C_2 \,.\, {}_4C_4 \,.\, 3! = 7 \times 15 \times 6 = 630$

19. Sol. $\dfrac{{}^{200}C_2 \,.\, {}^{198}C_2 \,.\, {}^{196}C_2 \ldots\ldots {}^{2}C_2}{100!}$

$= \dfrac{200!}{2^{100}.100!} = \dfrac{101.102.103\ldots\ldots200}{2^{100}}$

$= \left(\frac{101}{2}\right).\left(\frac{102}{2}\right).\left(\frac{103}{2}\right).....\left(\frac{200}{2}\right)$

And $\frac{1.2.3.4.5.6.7.8.....200}{2^{100}.100!}$

$= \frac{(1.3.5.7.....199)(2.4.6.8......200)}{2^{100}.100!}$

$= \frac{(1.3.5......199).2^{100}.100!}{2^{100}.100!}$ = 1.3.5 .199

20. **Sol.** $\frac{(8+8)!}{8!8!}$ = ${}_{16}C_8$

21. **Sol.** 14400 = $2^6 \times 3^2 \times 5^2$

Number of ways = $\frac{1}{2}\left[(6+1)(2+1)(2+1)+1\right]$ = 32

22. **Sol.** 10080 = $2^5 \times 3^2 \times 5^1 \times 7^1$

coprime factors M = 2^{4-1} = 2^3 = 8

23. **Sol.** First we select 5 beads from 8 different beads.

No. of ways = ${}_8C_5$

Now total number of arrangement is ${}_8C_5 \times \frac{4!}{2!}$ = 672

24. **Sol.** First find if all the person are sitting in a round table is 4! = 24 ways

if two of the person are sitting together i.e.

3! × 2! = 12 ways

Hence required number of ways = 24 – 12 = 12 ways

25. **Sol.** If women are sit together then total number of person is 5 hence required ways = 4! × 3!

= 24 × 6 = 144

26. **Sol.** Here B is always between A and C so i.e. either ABC or CBA

so total required number of ways is 4! × 2! = 24 × 2 = 48 ways

27. **Sol.** Desired no. of ways coefficient of x^{14} in the expansion of $(x^0 + x^2 + x^4)^5$

= coefficient of x^{14} in $(1 + x^2 + x^4)^5$

= coefficient of x^{14} in $\left(\frac{1-x^6}{1-x^2}\right)^5$

= coefficient of x^{14} in $(1 - x^6)^5 (1 - x^2)^{-5}$

= coefficient of x^{14} in $(1 - 5x^6 + 10x^{12}............)$ $\left(1+{}^5C_1x^2+{}^6C_2x^4+{}^7C_3x^6+........\right)$

= ${}_{11}C_7 - 5\ .\ {}_8C_4 + 10.5$

$= \frac{11\times10\times9\times8}{1\times2\times3\times4} - 5.\frac{8\times7\times6\times5}{1\times2\times3\times4}$ + 50

$= 330 - 350 + 50 = 30$

28. Sol. Here $-10 \le x, y, z \le -1$

Using multinomial theorem

Find the coefficient of x^{12} in this expansion of

$(x + x^2 + ...+ x^{10})^3$

$=$ coeff of x^{12} in $x^3 (1 + x + x^2 + ...+ x^9)^3$

$=$ coeff of x^{12} in $x^3 (1 - x^{10})^3 . (1 - x)^{-3} = {}^{11}C_9 = \frac{11 \times 10}{2} = 55$

29. Sol. $x_1 + x_2 + x_3 = 20 - t$

$t = 0, 1, 2, 3, 4$

Required value $= \sum_{t=0}^{4} {}^{19-t}C_2$

$= {}^{20}C_3 - {}^{15}C_3 = 1140 - 455 = 685$

30. Sol. Using multinomial theorem

Total no. of ways $= {}^{15+3-1}C_{15} \times {}^{10+3-1}C_{10}$

$= {}^{17}C_{15} \times {}^{12}C_{10} = \frac{17 \times 16}{2} \times \frac{12 \times 11}{2} = 8976$

31. Sol. $x_1 + x_2 + x_3 + x_4 \le n$ $\quad x_1 + x_2 + x_3 + x_4 + y = n$

(where y is known as pseudo variable) Total no. of required solution is

$= {}^{n+5-1}C_n = {}^{n+4}C_n$ or ${}^{n+4}C_4$

32. Sol. Let number be $x_1 x_2 x_3 x_4 x_5 x_6$

But Here $x_1 + x_2 + ... x_6 = 12$

so required no. of integers = coefficient of x^{12} in expansion $(1 + x + x^2 + ... + x^9)^6$

$= (1 - x^{10})^6 . (1 - x)^{-6}$

$\Rightarrow$ ${}^{17}C_{12} - {}^{6}C_1 . {}^{7}C_2$

$= 6188 - 126 = 6062$

33. Sol. Total number of required quadrilateral

${}^{7}C_4 + {}^{7}C_3 \times {}^{5}C_1 + {}^{7}C_2 \times {}^{5}C_2$

$= \frac{7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 4} + \frac{7 \times 6 \times 5}{1 \times 2 \times 3}.5 + \frac{7 \times 6}{1 \times 2} \times \frac{5 \times 4}{1 \times 2} = 35 + 175 + 210 = 420 = 2 . {}^{7}P_3$

34. Sol. $D_4 \times D_3$

$= 4! (1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}) \times 3! (1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}) = (12 - 4 + 1) \times (3 - 1)$

$= 9 \times 2 = 18$

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

A-1. Ans. (1)

Sol. **Statement-I** : $\frac{(n+1)!}{(n-1)!}$ = (n + 1) n is divisible by 6 for some n ∈ N like 2, 3 etc.

Statement-II : Product of three consecutive integers is always divisible by 3!.

both are true but statement-2 is NOT a correct explanation for statement-1.

A-2. Ans. (2)

Sol. a + b+ c = 8 – t

$0 \le t \le 5$

$\sum_{t=0}^{5} {}^{7-t}C_2$ = 56

A-3. Ans. (2)

Sol. $x_1.x_2.x_3.x_4 = 2 \times 5 \times 7 \times 11$

$N = 4^4 = 2^8$

N is divisible by only one prime number.

N dsoy ,d vHkkT; la[;k ls foHkkftr gSA

A-4. Ans. (1)

Sol. Statement -1**:** Two circles intersect in 2 points.

∴ Maximum number of points of intersection

= 2 × number of selections of two circles from 8 circles.

= 2 × ${}^8C_2$ = 2 × 28 = 56

Statement -2**:** 4 lines intersect each other in ${}^4C_2$ = 6 points.

4 circles intersect each other in 2 × ${}^4C_2$ = 12 points.

Further, one lines and one circle intersect in two points. So, 4 lines will intersect four circles in 32 points.

∴ Maximum number of intersecting points = 6 + 12 + 32 = 50.

A-5. Ans. (1)

Sol. L_1 L_3 L_5 L_2 L_4 L_6

E_1 E_3 E_5 E_2 E_4 E_6

Number of ways = $3!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right) . 3!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right)$ = 4

A-6. Ans. (3)

Sol. If power of P is l in n! then $l = \left[\frac{n}{P}\right] + \left[\frac{n}{P^2}\right] + ...$

Power of 5 in 100! is

$= \left[\frac{100}{5}\right] + \left[\frac{100}{5^2}\right] + \left[\frac{100}{5^3}\right] = 20 + 4 + 0 = 24$

Power of 2 in 100 ! is

$= \left[\frac{100}{2}\right] + \left[\frac{100}{2^2}\right] + \left[\frac{100}{2^3}\right] + \left[\frac{100}{2^4}\right] + \left[\frac{100}{2^5}\right] + \left[\frac{100}{2^6}\right] = 50 + 25 + 12 + 6 + 3 + 1 = 97$

So power of 50 in 100 ! is 12.

Maximum value of k is 12.

Section (B) : MATCH THE COLUMN

B-1. **Ans.** A → r, B → q, C → r, D → p

Sol. N = 249480 = $2^3 \times 3^4 \times 5^1 \times 7^1 \times 11^1$

(A) Number of ways N is divisible by 3 but not by 5 = 4 × 4 ×2 × 2 = 64

(B) Number of ways N is divisible by 5 but not by 7 = 4 × 5 × 1 × 2 = 40

(C) Number of ways N is divisible by 3 but not by 21 = 4 × 4×2×2 = 64

(D) Number of ways N is divisible by 35 but not by 77 = 4×5 × 1× 1= 20

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. **Sol.** $10 \geq x - 1$ and $10 \geq x$

$x \leq 11$ $\quad$ $x \leq 10$

$2 \cdot {}^{10}C_x < {}^{10}C_{x-1}$

$$\frac{{}^{10}C_x}{{}^{10}C_{x-1}} < \frac{1}{2}$$

$$\frac{10-x+1}{x} < \frac{1}{2}$$

$22 - 2x < x$

$3x > 22$

$x > \frac{22}{3}$

$\therefore x \in \left(\frac{22}{3}, 10\right]$

Integral values of x are 8, 9, 10.

C-2. **Sol.** Total number of required possibilities

${}^5C_3 \cdot {}^8C_7 + {}^5C_4 \cdot {}^8C_6 + {}^5C_5 \cdot {}^8C_5 \cdot {}^5C_5$

$= {}^5C_3 \cdot {}^8C_7 + {}^5C_4 \cdot {}^8C_6 + {}^8C_6$

$= {}^{13}C_{10} - {}^5C_3 = 276$

C-3. **Sol.** Required number of possibilities

$8! - 2 . 7! = 7! (8 - 2) = 6 . 7!$

$2 . 6! . {}^7C_2$

C-4. **Sol.** Total required number of teams is

$= \frac{10!}{(4!3!3!)2!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5}{6 \times 6 \times 2} = 2100$

C-5. **Sol.** $p = E_2(10!) = \left[\frac{10}{2}\right] + \left[\frac{10}{4}\right] + \left[\frac{10}{8}\right] = 5 + 2 + 1 = 8.$

$q = E_3(10!) = \left[\frac{10}{3}\right] + \left[\frac{10}{9}\right] = 3 + 1 = 4$

$r = E_5(10!) = \left[\frac{10}{5}\right] + \left[\frac{10}{25}\right] = 2 + 0 = 2$

$s = E_7(10!) = \left[\frac{10}{7}\right] = 1.$

Exercise-3

*** Marked Questions may have more than one correct option.**

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Sol. The number of triangles can be formed using n non-collinear points is ${}^nC_3$.

Since, $T_n = {}^nC_3$

Given, $T_{n+1} - T_n = 21$

$\Rightarrow$ ${}^{n+1}C_3 - {}^nC_3 = 21$

$\Rightarrow$ ${}^nC_2 + {}^nC_3 - {}^nC_3 = 21$

$(\because {}^nC_2 + {}^nC_3 = {}^{n+1}C_3\}$

$\Rightarrow$ ${}^nC_2 = 21$

$\Rightarrow$ $\frac{n(n-1)}{2} = 21$ $\Rightarrow$ $n^2 - n - 42 = 0$

$\Rightarrow$ $(n-7)(n+6) = 0$

$\Rightarrow$ $n = 7$ $(\because n \neq -6)$

2. Sol. ${}^5C_4 \cdot {}^8C_6 + {}^5C_5 \cdot {}^8C_5$

$= 140 + 56 = 196$

3. Sol. $6!\ 5!$

4. Sol. ${}^6C_2 \times 4! = 360$

5. Sol. 

$\frac{(5+2)!}{5!2!} = 21$

6. Sol. ACHINS

A ......5 !

C ......5 !

H ......5 !

I ......5 !

N ......5 !

SACHIN

5 . 5 ! + 1 = 601

7. Sol. ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 = 10 + 45 + 120 + 210 = 385$

8 Sol. $\frac{12!}{(4!)^3 3!} \times 3! = \frac{12!}{(4!)^3}$

9. Sol. $x_1 + x_2 + x_3 + x_4 + x_5 = 6$

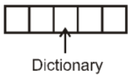
$\frac{(6+4)!}{6!4!} = {}^{10}C_4$

Statement - 1 is false while statement-2 is true
Ans. (4)

10. **Sol.** M I I I I P P

${}^{8}C_4 \cdot \frac{7!}{4!.2!} = 7. {}^{6}C_4 . {}^{8}C_4$

11. **Sol.** 6 different novels and 3 different dictionaries



$\therefore$number of ways = ${}^{3}C_1 \times {}^{6}C_4 \times 4! = 1080$

12. **Sol.** ${}^{3}C_2 \times {}^{9}C_2 = 3 \times \frac{9 \times 8}{2 \times 1} = 12 \times 9 = 108$

13. **Sol.** **Statement - 1 :**
$B_1 + B_2 + B_3 + B_4 = 10$
= coefficient of x^{10} in $(x^1 + x^2 + ...+ x^7)^4$
= coefficient of x^6 in $(1 - x^7)^4 (1 - x)^{-4}$
= ${}^{4+6-1}C_6 = {}^{9}C_3$
Statement - 2 :
Obviously ${}^{9}C_3$

14. **Sol.** ${}^{10}C_3 - {}^{6}C_3$

$= \frac{10 \times 9 \times 8}{6} - \frac{6 \times 5 \times 4}{6} = 120 - 20 = 100$

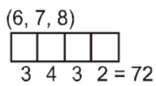
15. **Sol.** (10 + 1) (9 + 1) (7 + 1) – 1= 11.10.8 – 1 = 879

16. **Sol.** Every element has 3 options. Either set Y or set Z or none
so number of ordered pairs = 3^5

17. **Sol.** **(2)**
$T_n = {}^{n}C_3$
$T_{n+1} = {}^{n+1}C_3$
$T_{n+1} - T_n = {}^{n+1}C_3 - {}^{n}C_3$
$\Rightarrow$ ${}^{n}C_2 = 10$
$\Rightarrow$ n = 5.

18. **Ans.** **(2)**
Sol. 

Number of integer greater than 6000 may be 4 digit or 5 digit
C-1 when number is of 4 digit
C-2 when number is of 5 digit = 5! = 120



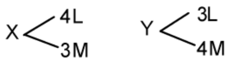
total = 120 + 72 = 192 digit

19. **Ans.** **(3)**
Sol. SMALL

A _ _ _ _ _ # $\frac{4!}{2!} = 12$
L _ _ _ _ _ # $4! = 24$
M _ _ _ _ _ # $\frac{4!}{2!} = 12$
SA _ _ _ _ # $\frac{3!}{2!} = 3$
SL _ _ _ _ # $3! = 6$

<u>S</u> <u>M</u> <u>A</u> <u>L</u> <u>L</u> # 1
58_{th} position

20. Ans. (1)
Sol.



X	Y		X	Y		X	Y		X	Y
0L	3L	+	1L	2L	+	2L	1L	+	3L	0L
3M	0M		2M	1M		1M	2M		0M	3M

${}_3C_3 \times {}_3C_3 + {}_4C_1 \times {}_3C_2 \times {}_3C_2 \times {}_4C_1 + {}_4C_2 \times {}_3C_1 \times {}_3C_1 \times {}_4C_2 + {}_4C_3 \times {}_4C_3$ = 1 + 144 + 324 + 16 = 485

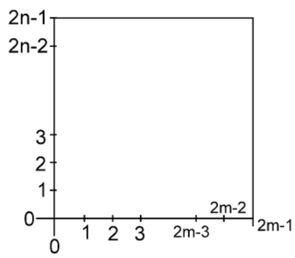
21. Sol. (3)
Number of ways : $x = {}^6C_4 \times {}^3C_1 \times 4! = 15 \times 3 \times 24 = 1080$

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. Sol. Total 6 letters word that can be formed = $\frac{6!}{2!3!} = 60$

total 6 letters word in which both N comes together = $\frac{5!}{3!} = 20$
number of arrangents in which both N donot appear together = 60 – 20 = 40

2. Sol.



No. of ways of choosing horizontal side of rectangle of one unit length = 2m – 1
No. of ways of choosing horizontal side of rectangle of 3 unit length = 2m – 3
∴Total no. of ways of choosing horizontal side of rectangle of odd length
= (2m – 1) + (2m – 3) + ... + 1 = m^2
similarly no. of ways of choosing the vertical side of rectangle of odd length = n^2.
∴Total no. of ways of choosing the rectangle = $n^2 m^2$

3. **Sol.** LCM of p and q is $r^2 s^2 t^4$. Therefore at least one of p and q is divisible by r^2. If p is divisible by r^2 and q is not divisible by r, then let us denoted by (2, 0). Therefore possible way are (2, 0), (2, 1), (2, 2), (1, 2), (0, 2)
which are 5 in number. Similarly 5 ways for s and 9 ways for t
$\therefore$ total number of ways = 5 × 5 × 9 = 225.

4. **Sol.** COCHIN ⇒ C - 2 N - 1
H - 1 O - 1
I - 1
Number of words = 24

C	C				

4! = 24

C	H				

4! = 24

C	I				

4! = 24

96

C	O	C	H	I	N

Before word of cochin the number of words is = 96

5. **Ans.** (A) → (p), (B) → (s), (C) → (q), (D) → (q)
Sol.
(A) Consider 'ENDEA' as one letter so there are five things which are ENDEA , N, O, E, L
$\therefore$number of permutations = 5!
Hint : Number of arrangements of n objects in a line in which r objects always remain together in a particular order are (n – r + 1)!

(B) E N D E A N O L E (first and last letters are f'ixed) Number of permutations = $\frac{7!}{2!}$ = 21 × 5!

Hint : Number of permutations of n objects in which r are alike = $\frac{n!}{r!}$

(C) N, N, D, L will come at first four places and E, E, E, O, A will come at last five places
$\therefore$ number of permutations = $\frac{4!}{2!} \times \frac{5!}{3!}$ = 2 × 5!

Hint : Number of permutations of n objects in which r are alike = $\frac{n!}{r!}$

(D) A, E, E, E, O, occur at odd positions and N, N, D, L at even positions = $\frac{5!}{3!} \times \frac{4!}{2!}$ = 2 × 5!

Hint : Number of permutations of n objects in which r are alike = $\frac{n!}{r!}$

6. **Sol.** There are two possible cases
Case 1 : Five 1's, one 2's, one 3's
Number of numbers = $\frac{7!}{5!}$ = 42
Case 2 : Four 1's, three 2's
Number of numbers = $\frac{7!}{4!3!}$ = 35

Total number of numbers = 42 + 35 = **77**

7. **Sol.**

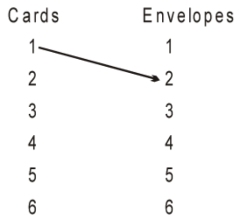
	B_1	B_2	B_3
Case-1:	1	1	3
Case-2:	2	2	1

Ways of distribution $= \frac{5!}{1!\,1!\,3!\,2!} . 3! + \frac{5!}{2!\,2!\,1!\,2!} .3!$

= 150

8. **Ans.** **(C)**

Sol.



If '2' goes in '1' then it is dearrangement of 4 things which can be done in $4!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)$ = 9 ways.

If '2' doen't go in 1, it is dearrangement of 5 things which can be done in 44 ways. Hence total 53 ways.

9. **Ans.** **(A)**

Sol. 1 Boy + 0 Boy

$\left({}^4C_1.{}^6C_3 + {}^6C_4\right) \times 4$ = (4 × 20 + 15) × 4 = 95 × 4 = 380

10. **Ans.** **(C)**

Sol. $N_1 = {}^5C_1.{}^4C_4 = 5$

$N_2 = {}^5C_2.{}^4C_3 = 40$

$N_3 = {}^5C_3.{}^4C_2 = 60$

$N_4 = {}^5C_4.{}^4C_1 = 20$

$N_5 = {}^5C_5.{}^4C_0 = 1$

$\therefore$ Total dqy = 126