Circle

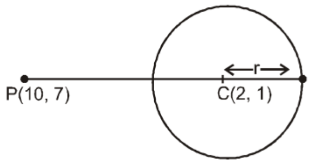
Exercise-3

* **Marked Questions may have more than one correct option.**

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. **Sol.** $S_1 \equiv 10^2 + 7^2 - 4 \times 10 - 2 \times 7 - 20 = 75 > 0$

Point (10, 7) lies out side the circle $x^2 + y^2 - 4x - 2y - 20 = 0$



greatest distance = CP + r

= 10 + 5

= 15 Unit.

2. **Sol.** Let equation of tangent is y = mx + c

Since it makes equal intercepts on positive coordinate axes.

Hence its slope is –1

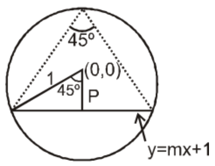
$\therefore$ $y = -x + c \Rightarrow x + y - c = 0$

centre (–2, 2), radius = $\sqrt{4+4-4}$ = 2

since it is tangent $\left|\frac{-2+2-c}{\sqrt{2}}\right| = 2 \Rightarrow c = \pm 2\sqrt{2}$

But c is positive $x + y = 2\sqrt{2}$.

3. **Sol.** $p = \cos 45^\circ = \frac{1}{\sqrt{2}}$



$\left|\frac{0-0+1}{\sqrt{1+m^2}}\right| = \frac{1}{\sqrt{2}}$

$\Rightarrow 1 + m^2 = 2$

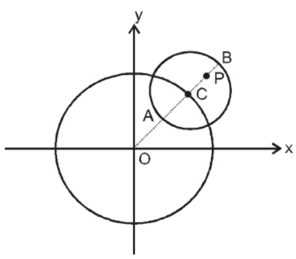
$\Rightarrow m = \pm 1$.

4. **Sol.** For any point P(x, y) in the circle

OA ≤ OP ≤ OB

$5 - 3 \le \sqrt{x^2+y^2} \le 5+3$

$4 \le x^2 + y^2 \le 64$



5. **Sol.** Let equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

it passes through (0, 0) and (1, 0)

$\Rightarrow c = 0,\ 1 + 0 + 2g + 0 + c = 0 \Rightarrow g = -\frac{1}{2}$

it touches $x^2 + y^2 = 9$ $\Rightarrow c_1c_2 = |r_1 \pm r_2|$

$\Rightarrow \sqrt{g^2 + f^2} = \left|\sqrt{g^2 + f^2 - c} \pm 3\right|$

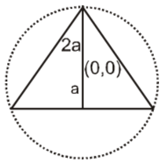
squarring both sides

$g^2 + f^2 = g^2 + f^2 + 9 \pm 6\sqrt{g^2 + f^2}$

$\Rightarrow \pm 6\sqrt{\frac{1}{4} + f^2} = -9 \Rightarrow \frac{1}{4} + f^2 = \frac{9}{4} \Rightarrow f = \pm\sqrt{2}$

Hence centre is $\left(\frac{1}{2}, \pm\sqrt{2}\right)$.

6. **Sol.** Since its equilateral triangle



Hence circumcentre is also centroid

which divides median in 2 : 1, therefore its radius is 2a.

$x^2 + y^2 = 4a^2$.

7. **Sol.** $S_1 : (x - 1)^2 + (y - 3)^2 = r^2$ C_1 (1, 3), $r_1 = r$

$S_2 : x^2 + y^2 - 8x + 2y + 8 = 0$ C_2 (4, –1), $r_2 = 3$

circles intersect $|r_1 - r_2| < C_1 C_2 < r_1 + r_2$

$|r - 3| < 5 < r + 3$

$\Rightarrow |r - 3| < 5 \Rightarrow -5 < r - 3 < 5 \Rightarrow -2 < r < 8$

$5 < r + 3 \Rightarrow r > 2$

After intersection $2 < r < 8$.

8. **Sol.** Point of intersection of $2x - 3y = 5$

$3x - 4y = 7$ is (1, – 1)

Hence centre (1, –1), Area = 154 = $\frac{22}{7}r^2$ $\Rightarrow r = 7$

equation of circle $(x - 1)^2 + (y + 1)^2 = 7^2$

$\Rightarrow x^2 + y^2 - 2x + 2y = 47$.

9. **Sol.** Let centre of circle is (h, k) and it passes through (a, b)

equation of circle is $(x - h)^2 + (y - k)^2 = (h - a)^2 + (k - b)^2$

This circle cuts $x^2 + y^2 - 4 = 0$ orthogonally

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$\Rightarrow 2g_1(0) + 2f_1(0) = -(h - a)^2 - (k - b)^2 + h^2 + k^2 - 4$

$\Rightarrow 2ah + 2kb - \left(a^2 + b^2 + 4\right)$

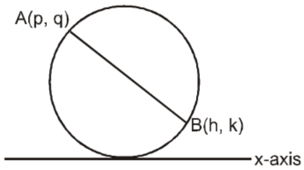
Hence locus of (h, k) is $2ax + 2by - \left(a^2 + b^2 + 4\right) = 0$.

10. **Sol.** equation of circle

$(x - p)(x - h) + (y - q)(y - k) = 0$

$\Rightarrow x^2 + y^2 - x(h + p) - y(q + k) + (ph + qk) = 0$

This circle touches x-axis $g^2 = c$



$$\Rightarrow \left(\frac{h+p}{2}\right)^2 = ph + qk$$

Locus of (h, k) is $(x - p)^2 = 4qy$.

11. Sol. Point of intersection of $2x + 3y + 1 = 0$

$3x - y - 4 = 0$ is $(1, -1)$

and circumference of circle $= 2\pi r = 10\pi \Rightarrow r = 5$

Hence equation of circle $(x - 1)^2 + (y + 1)^2 = 25$

$\Rightarrow x^2 + y^2 - 2x + 2y - 23 = 0.$

12. Sol. By family of circle $x^2 + y^2 - 2x + \lambda(x - y) = 0$

centre of this circle $\left(\frac{2-\lambda}{2}, \frac{\lambda}{2}\right)$

lies on y = x $\Rightarrow \frac{2-\lambda}{2} = \frac{\lambda}{2} \Rightarrow \lambda = 1$

Hence $x^2 + y^2 - x - y = 0$.

13. Sol. Let $S_1 : x^2 + y^2 + 2ax + cy + a = 0$

$S_1 : x^2 + y^2 - 3ax + dy - 1 = 0$

common chord $S_1 - S_2 = 0 \Rightarrow 5ax + y(c - d) + (a + 1) = 0$

given line is $5x + by - a = 0$

compare both $\frac{5a}{5} = \frac{c-d}{b} = \frac{a+1}{-a}$

$a = \frac{c-d}{b} = -1 - \frac{1}{a}$

(i) (ii) (iii)

From (i) & (iii) $a^2 + a + 1 = 0 \Rightarrow a = \omega, \omega^2$ no real a.

14. Sol. Let the centre be (α, β)

$\because$ it cut the circle $x^2 + y^2 = p^2$ orthogonally

$2(-\alpha) \times 0 + 2(-\beta) \times 0 = c_1 - p^2$

$c_1 = p^2$

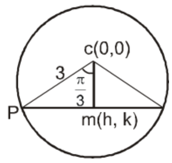
Let equation of circle is $x^2 + y^2 - 2\alpha x - 2\beta y + p^2 = 0$

It pass through (a, b) $\Rightarrow a^2 + b^2 - 2\alpha a - 2\beta b + p^2 = 0$

Locus

$\therefore 2ax + 2by - (a^2 + b^2 + p^2) = 0$

sol. $\cos\frac{\pi}{3} = \frac{cm}{cp} = \frac{\sqrt{h^2+k^2}}{3}$



Locus of (h, k) is $x^2 + y^2 = \frac{9}{4}$.

16. **Sol.** Point of intersection of lines

$3x - 4y - 7 = 0$

$2x - 3y - 5 = 0$ is $(1, -1)$

Area of circle $= \pi r^2 = 49\pi \Rightarrow r = 7$

Hence equation of circle $(x - 1)^2 + (y + 1)^2 = 7^2 \Rightarrow x^2 + y^2 - 2x + 2y = 47$

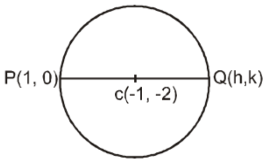
17. **Sol.** Let equation of circle is $(x - h)^2 + (y - k)^2 = (h + 1)^2 + (k - 1)^2$

it touches x-axis $g^2 = c$

$h^2 = 2k - 2h - 2 \Rightarrow k = \frac{h^2+2h+2}{2}$

$k \in \left[\frac{-D}{4a}, \infty\right) \Rightarrow k \in \left[\frac{1}{2}, \infty\right)$.

18. **Sol.**



$\frac{h+1}{2} = -1 \Rightarrow h = -3$

$\frac{k+0}{2} = -2 \Rightarrow k = -4$ Hence (vr%) Q(–3, –4).

19. **Sol.** $S_1 + \lambda S_2 = 0$ should satisfy (1, 1)

$(2 + 3 + 7 + 2p - 5) + \lambda(1 + 1 + 2 + 2 - p^2) = 0$

$\lambda = -\frac{7+2p}{6-p^2}$

$p^2 \neq 6 \Rightarrow p \neq \pm\sqrt{6}$

but at $p = \pm\sqrt{6}$ the 2nd circle is

$x^2 + y^2 + 2x + 2y - 6 = 0$

satisfies (1, 1) and obviously P and Q

so $p = \pm\sqrt{6}$ is also acceptable

$\lambda \neq -1 \Rightarrow \frac{7+2p}{6-p^2} \neq 1 \Rightarrow 7 + 2p \neq 6 - p^2$

$p^2 + 2p + 1 \neq 0$

$p \neq -1$

20. Sol. Equation of circumcircle is $x^2 + y^2 - (5/2)x + 1 = 0$

21. Ans. (1)

Sol. $r = \sqrt{4+16+5} = 5$

$\left|\frac{6-16-m}{5}\right| < 5$

$\Rightarrow -25 < m + 10 < 25$

$\Rightarrow -35 < m < 15$

Hence correct option is (1)

22. Sol. (2)

$x^2 + y^2 = ax$ ....(1)

$\Rightarrow$ centre $c_1\left(\frac{a}{2}, 0\right)$ and radius $r_1 = \left|\frac{a}{2}\right|$

$x^2 + y^2 = c^2$ ....(2)

$\Rightarrow$ centre c_2 (0, 0) and radius $r_2 = c$

both touch each other iff

$|c_1c_2| = |r_1 - r_2|$

$\frac{a^2}{4} = c - \left|\frac{a}{2}\right|$

$\Rightarrow \frac{a^2}{4} = \frac{a^2}{4} - |a|\, c + c^2$

$\Rightarrow |a| = c$

23. Sol. (2)

Circle whose diametric end points are (1, 0) and (0, 1) will be of smallest radius.

$(x - 1)(x - 0) + (y - 0)(y - 1) = 0$

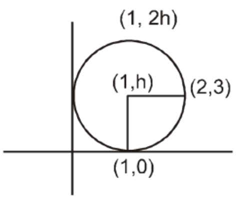
$x^2 + y^2 - x - y = 0$

24. Sol. Now

$h^2 = (1 - 2)^2 + (h - 3)^2$

$0 = 1 - 6h + 9$

$6h = 10$



$h = \frac{5}{3}$

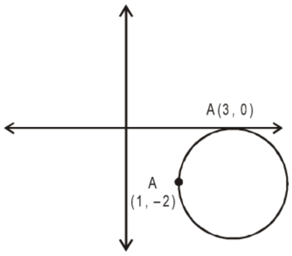
Now diameter is $2h = \frac{10}{3}$

25. **Sol.** (3)

Let the equation of circle be

$(x - 3)^2 + (y - 0)^2 + \lambda y = 0$

As it passes through (1, – 2)



$\therefore$ $(1 - 3)^2 + (-2)^2 + \lambda(-2) = 0$

$\Rightarrow$ $\lambda = 4$

$\therefore$ equation of circle is

$(x - 3)^2 + y^2 - 8 = 0$

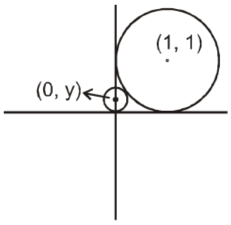
so (5, – 2) satisfies equation of circle

26. **Sol.** **Ans.** **(2)**

c_1 (1, 1) $r_1 = 1$

c_2 (0, y) $r_2 = |y|$

$c_1c_2 = r_1 + r_2$



$\sqrt{(1-0)^2 + (1-y)^2} = 1 + |y|$

$2 - 2y + y^2 = y^2 + 2|y| + 1$

$4|y| = 1$

$|y| = \frac{1}{4}$

$y = \frac{1}{4}$

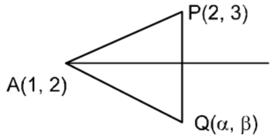
27. **Ans.** **(3)**

Sol. Line passing through (1, 2)

AP = AQ

$(\alpha - 1)^2 + (\beta - 2)^2 = (2 - 1)^2 + (3 - 2)^2$

$\alpha^2 + \beta^2 - 2\alpha - 4\beta + 3 = 0$



$x^2 + y^2 - 2x - 4y + 3 = 0$

$r = \sqrt{1+4-3} = \sqrt{2}$

28. **Ans.** **(3)**

Sol. $C_1(2, 3)$ $r_1 = 5$

$C_2(-3, -9)$ $r_2 = 8$

$C_1C_2 = \sqrt{25+144} = 13$

$C_1C_2 = r_1 + r_2 \Rightarrow$ externally touch

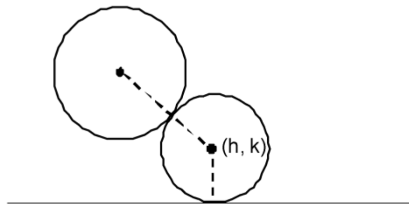
$\Rightarrow$ 3 common tangents

29. **Ans.** **(3)**

Sol. Parabola ¼ijoy;½

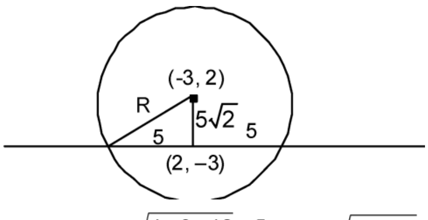
Property : distance from a fixed point & fixed line is equal

xq.k/keZ % fLFkj fcUnq rFkk fLFkj js[kk ls nwjh leku gS



30. **Ans.** **(1)**

Sol.



$r_1 = \sqrt{4+9+12} = 5 \Rightarrow R = \sqrt{25+50} = 5\sqrt{3}$

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. **Sol.** The line $5x - 2y + 6 = 0$ meets the y-axis at the point (0,3) and therefore the tangent has to pass through the point (0,3) and required length is

Sol. The line $5x - 2y + 6 = 0$ meets the y-axis at the point (0,3) and therefore the tangent has to pass through the point (0,3) and required length is therefore,

$= \sqrt{x_1^2 + y_1^2 + 6x_1 + 6y_1 - 2}$

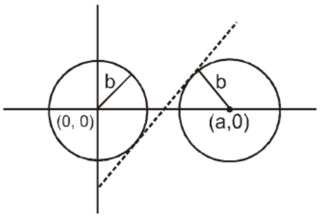
$= \sqrt{0 + 3^2 + 6(0) + 6(3) - 2}$

$= \sqrt{25} = 5$

2. **Solution**

$x^2 + y^2 = b^2$: centre (0,0) radius = b

$(x - a)^2 + y^2 = b^2$ centre (a,0) radius = b



y = $mx - b\sqrt{1+m^2}$ is tangent to both circle

Since, when x = 0 y = $-b\sqrt{1+m^2} < 0$

when y = 0 x = $\frac{b\sqrt{1+m^2}}{m} > 0$.

Now, perpendicular distance from centre (a,0) will be equal to radius b.

$$\left|\frac{ma - 0 - b\sqrt{1+m^2}}{\sqrt{m^2+1}}\right| = b \Rightarrow \left|\frac{ma - b\sqrt{1+m^2}}{\sqrt{m^2+1}}\right| = b$$

–ve sign $\frac{ma - b\sqrt{1+m^2}}{\sqrt{m^2+1}} = -b \Rightarrow$ $ma = 0 \Rightarrow m = 0$ or a = 0 not possible.

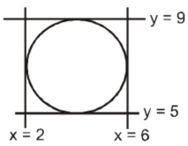
+ve sign $\frac{ma - b\sqrt{1+m^2}}{\sqrt{m^2+1}} = +b \Rightarrow$ $m = \frac{2b}{\sqrt{a^2 - 4b^2}}$.

3. **Sol.** The lines given by $x^2 - 8x + 12 = 0$ are x = 2 and x = 6.

The lines given by $y^2 - 14y + 45 = 0$ are y = 5 and y = 9

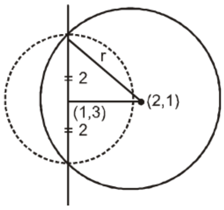
Centre of the required circle is the centre of the square.

∴ Required centre is



$\left(\frac{2+6}{2}, \frac{5+9}{2}\right) = (4, 7)$.

4. **Sol.** Clearly from the figure the radius of bigger circle



$r^2 = 2^2 + \{(2-1)^2 + (1-3)^2\}$

$r^2 = 9$ or $r = 3$

5. **Sol.** Statement-1 is true because point (17, 7) lies on the director circle and Statement-2 is equation of director circle of given circle.

6. **Sol.** $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$

$\Rightarrow$ $x = 3y$ or $x = 2y$ or $ax^2 + by^2 + c = 0$

If a = b and c is of opposite sign, then it will represent a circle

Hence (B) is correct option.

7. **Sol.** The distance between L_1 and L_2 is $\frac{6}{\sqrt{13}} < 2$

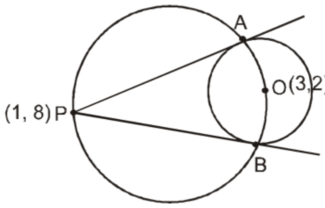
Statement '1' is True because distance between lines is less than radius but L_2 need not be a diameter.

Statement '2' is False because if

L_1 is diameter then L_2 has to be a chord of circle

Thus 'C' is correct

8. **Sol.** For required circle, P(1, 8) and O(3, 2) will be the end points of its diameter.



$\therefore$ $(x-1)(x-3) + (y-8)(y-2) = 0$ $\Rightarrow$ $\mathbf{x^2 + y^2 - 4x - 10y + 19 = 0}$

9. **Sol.** Let equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

as it passes through (-1,0) & (0,2)

$1 - 2g + c = 0$

and $4 + 4f + c = 0$

also $f^2 = c$

$\Rightarrow$ $f = -2$, $c = 4$; $g = \frac{5}{2}$

equation of circle is

$x^2 + y^2 + 5x - 4y + 4 = 0$

which passes through (–4, 0)

10. **Sol.** Circle $x^2 + y^2 = 9$

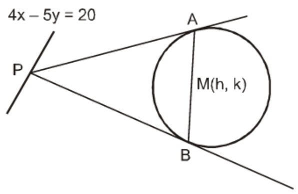
line $4x - 5y = 20$

$P\left(t, \frac{4t-20}{5}\right)$

equation of chord AB whose mid point is M (h, k)

$T = S_1$

$\therefore$ $hx + ky = h^2 + k^2$ ....(1)



equation of chord of contact AB with respect to P.

T = 0

$tx + \left(\frac{4t-20}{5}\right)y = 9$ ....(2)

comparing equation (1) and (2)

$$\frac{h}{t} = \frac{5k}{4t-20} = \frac{h^2+k^2}{9}$$

on solving

$45k = 36h - 20h^2 - 20k^2$

$\Rightarrow$ Locus is $20(x^2 + y^2) - 36x + 45y = 0$

Ans. (A)

Comprehension (Q. No. 11 to 12)

12 Equation of L is

$x - y\sqrt{3} + c = 0$

length of perpendicular dropped from centre = radius of circle

$\therefore \left|\frac{3+C}{2}\right| = 1 \Rightarrow C = -1, -5$

$\therefore x - \sqrt{3}y = 1$ or $x - \sqrt{3}y = 5$

13*. **Sol.** **(AC)**

Let $x^2 + y^2 + 2gx + 2fy + c = 0$

$g^2 - c = 0$

$g^2 = c$ ...(i)

$2\sqrt{f^2 - c} = 2\sqrt{7}$

$f^2 - c = 7$ ...(ii)

$9 + 0 + 6g + 0 + c = 0$

$9 + 6g + g^2 = 0$

$(g + 3)^2 = 0$

$g = -3 \quad \therefore \quad c = 9$

$f^2 = 16 \quad f = \pm 4$

$\therefore x^2 + y^2 - 6x \pm 8y + 9 = 0$

14*. Ans. (BC)

Sol. Let the cirlce be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(1)

given circles

$x^2 + y^2 - 2x - 15 = 0$ ...(2)

$x^2 + y^2 - 1 = 0$ ....(3)

(1) & (2) are orthogonal

$\Rightarrow \quad -g + 0 = \frac{c-15}{2}$

$0 + 0 = \frac{c-1}{2}$

$\Rightarrow \quad c = 1$ & $g = 7$

so the cirle is

$x^2 + y^2 + 14x + 2fy + 1 = 0$ it passes thrgouh

(0, 1) $\Rightarrow \quad 0 + 1 + 0 + 2f + 1 = 0$

$f = -1$

$\Rightarrow \quad x^2 + y^2 + 14x - 2y + 1 = 0$

Centre (–7, 1)

radius = 7

15. Ans. (A,B,C)

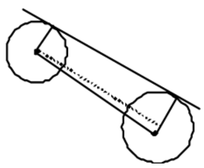
Sol. $y^2 + 2y - 3 = 0$

$y = 1$, $y = -3$

$p\left(\sqrt{2}, -1\right)$

tangent is $x\sqrt{2} + y = 3$

$C_2(0,\alpha) \perp$ distance $= 2\sqrt{3}$



$\frac{|\alpha - 3|}{3} = 2\sqrt{3}$

$\alpha - 3 = \pm 6$

$\alpha = 3, \pm 6$

$\alpha = 9, -3$

(0,9) (0,–3)

$L_{DCT} = \sqrt{(C_2C_1)^2 - (R+r)^2} = \sqrt{144 - 16 \times 3} = 4\sqrt{6}$

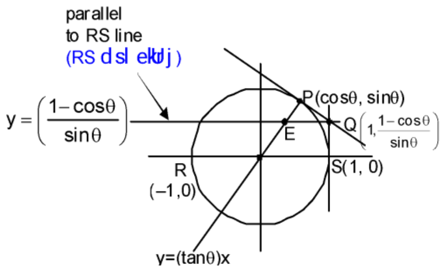
(C) $A = \frac{1}{2} R_3R_2 \times \perp$ from (0,0) $= 2\sqrt{6} \times \frac{3}{\sqrt{3}} = 6\sqrt{2}$

(D) Area $= \frac{1}{2}\begin{vmatrix} 0 & -3 & 1 \\ 0 & 9 & 1 \\ \sqrt{2} & 1 & 1 \end{vmatrix} = 6\sqrt{2}$

Area of ΔPQ_2Q_3 {ks=kQy = $\frac{1}{2}\begin{vmatrix}\sqrt{2} & 1 & 1\\ 0 & 9 & 1\\ 0 & -3 & 1\end{vmatrix} = \left|\frac{1}{2}\sqrt{2}(9+3)\right| = 6\sqrt{2}$

16. Ans. (A,C)

Sol.



$E\left(\left(\frac{1-\cos\theta}{\sin\theta\tan\theta}\right), \left(\frac{1-\cos\theta}{\sin\theta}\right)\right) \Rightarrow E\left(\frac{\tan\frac{\theta}{2}}{\tan\theta}, \tan\frac{\theta}{2}\right)$

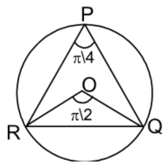
Let $h = \frac{\tan\frac{\theta}{2}}{\tan\theta}$ and $k = \tan\frac{\theta}{2}$ ∴ $h = \frac{k}{\tan\theta}$ ∴ $\tan\frac{\theta}{2} = \frac{k}{h}$

$\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \frac{k}{h} \Rightarrow \left(\frac{2k}{1-k^2}\right) = \frac{k}{h}$ ∴ $2xy = y(1 - y^2)$

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

1. Sol. $m_{OQ} = \frac{4}{3}$



$m_{OR} = \frac{-3}{4}$

∴ $m_{OQ} . m_{OQ} = -1$

⇒ $OP \perp OQ$ ∴ $\angle RPQ = \frac{\pi}{4}$

2. Sol. ∴ radius is minimum as possible as

∴ Equation of circle $(x-1)(x-0) + (y-1)(y-0) = 0$

⇒ $x_2 + y_2 - x - y = 0$

∴ g = 1, f = 1, c = 0 ∴ g + f + c = 2

3. Sol. Let centre is (a, 0)

$\therefore$ radius = $\left|\frac{a-0}{\sqrt{2}}\right| = \frac{a}{\sqrt{2}}$

circle cut off a chord of length 2

$\therefore CP^2 = CM^2+PM^2$

$\Rightarrow \frac{a^2}{2} = \left(\frac{a-\sqrt{3}.0}{\sqrt{1+3}}\right)^2 + 1$

$\Rightarrow a = 2 \therefore r = \sqrt{2}$ circle is $(x-2)^2 + (y-0)^2 = 2$

4. **Sol.** $(\alpha-1,\alpha+1)$ lies inside the circle

$\therefore(\alpha-1)^2 + (\alpha+1)^2 - (\alpha-1) - (\alpha+1) - 6 < 0$

$\Rightarrow (\alpha-2)(\alpha+1) < 0 \Rightarrow -1 < \alpha < 2$ ....(1)

Also $C\left(\frac{1}{2},\frac{1}{2}\right)$ & $(\alpha-1,\alpha+1)$ lies same side of line $x + y - 2 = 0$

$C\left(\frac{1}{2},\frac{1}{2}\right)$ & $(\alpha-1,\alpha+1)$ $x + y - 2 = 0$

$\therefore L_cL_p > 0 \Rightarrow \alpha < 1$ ....(2)

by (1) & (2)

5. **Sol.** Clearly $g^2 - c < 0$ & $f^2 - c < 0$

$\Rightarrow 9 < \lambda$ & $25 < \lambda$

Also point (1, 4) lies inside the circle

$\therefore 1+16 - 6 - 40 + \lambda < 0 \Rightarrow \lambda < 29$

$\therefore \lambda \in (25, 29)$

$\therefore$ maximum integral value λ is 28

6. **Sol.** Given circle $x^2 + y^2 + \lambda x + (1-\lambda) y + 5 = 0$

r = $\sqrt{g^2+f^2-c} = \sqrt{\frac{\lambda^2}{4}+\left(\frac{1-\lambda}{2}\right)^2-5} \le 5 \Rightarrow \lambda^2+1+\lambda^2-2\lambda-20 \le 100$

$\Rightarrow 2\lambda^2-2\lambda-119 \le 0 \Rightarrow (\lambda-1+\sqrt{239})(\lambda-1-\sqrt{239}) \le 0 \Rightarrow 1-\sqrt{239}) \le \lambda \le 1+\sqrt{239}$

$\Rightarrow -14.46 \le \lambda \le 16.46$

$\therefore$ Number of integral values of λ = 31

7. **Sol.** Circle passing through (1,0), (0,0) & (0,1)

$x^2 + y^2 - x - y = 0$

$(2k, 3k)$ lies on it if $4k^2 + 9k^2 - 2k - 3k = 0$

$\Rightarrow 13k^2 - 5k = 0$

$\Rightarrow k = 0$ or $\frac{5}{13}$

8. **Sol.** $A(1,2)$, $B(\alpha,\beta)$ are end of diameter

$\therefore 2\alpha + \beta = 5 \Rightarrow \beta = 5 - 2\alpha$ ....(1)

Let C (h,k) is centre then h = $\frac{\alpha+1}{2}$ $\Rightarrow \alpha = 2h - 1$

& k = $\frac{2+\beta}{2}$ $\Rightarrow$ k = $\frac{2+5-2\alpha}{2}$

$\Rightarrow 2k = 7 - 2\alpha$

$\Rightarrow 2k = 7 - 2(2h - 1)$ by (1)

$\Rightarrow 2y = 7 - 4x + 2 \Rightarrow 4x + 2y - 9 = 0$

9. **Sol.** We know that PA.PB = PT^2

PT = $\sqrt{100+49-40-14-20} = \sqrt{149-74} = \sqrt{75}$

10. **Sol.** Length of tangent = $\sqrt{S_1} = \sqrt{25+9+10+3k+17} = 7$

$\sqrt{61+3k} = 7 \Rightarrow 61 + 3k = 49$

$\Rightarrow 3k = -12 \Rightarrow k = -4$

11. **Sol.** Let p (h,k) is on $x^2+y^2 = a^2$

$\therefore h^2 + k^2 = a^2$ .....(1)

chord of contact w.r.t. $x^2+y^2 = b^2$ is $hx + ky = b^2$

$\Rightarrow y = -\frac{h}{k}x + \frac{b^2}{k}$ touches the circle, $x^2+y^2 = c^2$

$\therefore \frac{b^2}{k} = c\sqrt{\frac{h^2}{k^2}+1} \Rightarrow \frac{b^4}{k^2} = c^2\frac{(h^2+k^2)}{k^2}$

$\Rightarrow b^4 = a^2c^2 \Rightarrow b^2 = ac$

12. **Sol.** Given circles are $x^2 + y^2 + 2a'x + 2b'y + c' = 0$

& $x^2+y^2 + ax + by + \frac{c}{2} = 0$ they intersect orthogonally if $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$\Rightarrow 2\left(a'.\frac{a}{2} + b'.\frac{b}{2}\right) = c^1 + \frac{c}{2}$

$\Rightarrow 2(aa' + bb') = 2c' + c$, $\lambda + \mu + \delta = 5$

13. **Sol.** Here radical centre is (1,2)

$\therefore$ r = $\sqrt{S_1} = \sqrt{1+4-3-12+14} = 2$

$\therefore$ equation of circle is $(x-1)^2 + (y-2)^2 = 2^2$

$\Rightarrow x^2+y^2 - 2x - 4y + 1 = 0$

$\therefore g = -2, \lambda = 1 \therefore g + 5\lambda = -2 + 5 = 3$

14. **Sol.** Here common chord is $S_1 - S_2 = 0$

$\Rightarrow 3x + 4y + 3 = 0$

$\therefore$ Length = $2\sqrt{10-\frac{144}{25}} = \frac{2\sqrt{106}}{5}$

$\therefore \alpha = 2, \beta = 5 \therefore \alpha + \beta = 7$

15. **Sol.** Any circle is $x^2+y^2 - 9 + \lambda (x+y-1) = 0$

$\Rightarrow x^2+y^2 + \lambda x + \lambda y - 9 - \lambda = 0$

for smallest circle chord x +y = 1 circle will be diameter

$\therefore \frac{-\lambda}{2} - \frac{\lambda}{2} = 1 \Rightarrow \lambda = -1$

$\therefore$ equation of smallest circle is $x^2+y^2-x-y-8 = 0$

16. **Sol.** Let y = mx be a chord

$\therefore x^2(1+m^2) - x(3+4m) - 4 = 0$

$x_1+x_2 = \frac{3+4m}{1+m^2}$ and $x_1x_2 = \frac{-4}{1+m^2}$

(0,0) divides chord in the ratio 1 : 4 $\therefore x_2 = -4x_1$

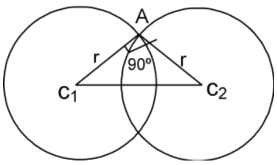
$\therefore -3x_1 = \frac{3+4m}{1+m^2}$ and $4x_1^2 = \frac{4}{1+m^2}$ $\Rightarrow$ m = 0 or $\frac{-24}{7}$

$\therefore$ lines the y = 0 or y = $\frac{-24}{7}$ x $\Rightarrow$ 24x + 7y= 0

17. **Sol.** $C_1C_2^2 = AC_1^2 + AC_2^2$ $\Rightarrow 9 + 9 = r^2 + r^2$

$2r^2 = 18$

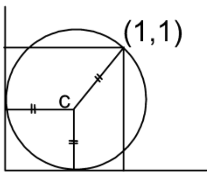
r = 3



18. **Sol.** $(r-1)^2 + (r-1)^2 = r^2$

$\Rightarrow r^2 - 4r + 2 = 0$

$\Rightarrow r = \frac{4+\sqrt{16-8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$



19. **Sol.** $\therefore$ Chord subtends an angle 45° at major segment it will subtends 90º at the centre (0,0)

$\therefore x^2+y^2 = 1.(y - mx)^2 \Rightarrow x^2+y^2 = y^2+m^2x^2 - 2mxy$

$\Rightarrow (m^2-1)x^2 - 2mxy = 0$

$a+b = 0 \Rightarrow m^2 - 1 = 0 \Rightarrow m = \pm 1$

20. **Sol.** Centre (1,0) , r = $\sqrt{1+0-0}$ = 1

$\therefore \left|\frac{\cos\theta + 0 - 2}{\sqrt{\cos^2\theta + \sin^2\theta}}\right| = 1$

$\Rightarrow \cos\theta - 2 = \pm 1$

$\Rightarrow$ $\cos\theta = 2 \pm 1$

$\Rightarrow$ $\cos\theta = 1$ or 3

$\theta = 2n\pi$, $n \in I$ $\therefore \cos\theta \neq 3$

21. **Sol.** $S+\lambda S' = 0$

$\Rightarrow x^2(\sin^2\theta + \lambda\cos^2\theta) + y^2(\cos^2\theta + \lambda\sin^2\theta) + 2xy\,(h + \lambda h') + x\,(32+16\lambda) + y\,(16+32\lambda) + 19\,(1+\lambda) = 0$

it will represent a circle it

$\sin^2\theta + \lambda\cos^2\theta = \cos^2\theta + \lambda\sin^2\theta$ & $h + \lambda h' = 0$

$\lambda = 1$ $\therefore h + h' = 0$

22. **Sol.** $ax+by = 2$ is normal for $x^2+y^2-4x-4y=0$

$\therefore 2a+2b = 2 \Rightarrow a+b = 1$

is tangent for $x^2+y^2=1$ $ax+by = 2$

$\therefore \dfrac{2}{\sqrt{a^2+b^2}} = 1 \Rightarrow a^2+b^2 = 4$

$\Rightarrow (a+b)^2 - 2ab = 4$

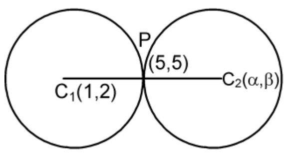
$\Rightarrow 2ab = -3$

$\therefore a = \dfrac{1+\sqrt{7}}{2}, b = \dfrac{1-\sqrt{7}}{2}$

23. **Sol.** C (1,2) $r = \sqrt{1+4+20} = 5$

P is mid point of C_1C_2

$\therefore \dfrac{\alpha+1}{2} = 5, \dfrac{\beta+2}{2} = 5$



$\Rightarrow \alpha = 9,\ \beta = 8$

$\therefore$ circle is $(x-9)^2 + (y-8)^2 = 25$

24. **Sol.** Let circle is $x^2+y^2+2gx+2gy+c= 0$

it passes through (a,b)

$\therefore a^2+b^2+2ga+2fb+c= 0$ ...(1)

by orthogonality we get

$2g.0+2f.0 = c - k^2 \Rightarrow c = k^2$

$\therefore a^2+b^2+2ga+2fb+k^2= 0$

$\therefore$ locus of $C(-g_1,-f)$ is $a^2+b^2-2ax - 2by + k^2 = 0$

$\therefore C(-g_1,-f)$ $a^2+b^2-2ax - 2by + k^2 = 0$

$\Rightarrow 2ax + 2by = a^2 + b^2 + k^2$

25. **Sol.** Equation of circle C is $(x-2)^2+(y-1)^2 = r^2$

C $(x-2)^2+(y-1)^2 = r^2$

$\Rightarrow x_2+y_2- 4x-2y + 5- r_2 = 0$

Equation of common chord

$(x_2+y_2-4x-2y+5-r_2) - (x_2+y_2-2x-6y + 6 = 0$

$\Rightarrow 2x - 4y + r_2 + 1 = 0$

it is diameter of 2nd circle

$\therefore 2.1 - 4.3 + r_2 + 1 = 0 \Rightarrow r_2 = 9 \;\; \Rightarrow r = 3$

26. **Sol.** Equation of common chord is

$(x_2+y_2-6x-4y+9)- (x_2+y_2-(\lambda+4) x - (\lambda+2)y +5\lambda+3 = 0$

$\Rightarrow (\lambda+4-6) x + (\lambda+2-4)y +9-5\lambda- 3 = 0$

$\Rightarrow (\lambda-2) x + (\lambda- 2) y + 6-5\lambda= 0$

centre $C_2 \left(\frac{\lambda+4}{2}, + \frac{(\lambda+2)}{2}\right)$ will satisfy it

$\therefore(\lambda- 2) \left(\frac{\lambda+4+\lambda+2}{2}\right) +6 -5\lambda = 0$

$\Rightarrow \lambda = 0$ or 4

27. **Sol.** Here Radical centre is point of intersection of

$S_1 -S_2 = 0$ & $S_2 -S_3 = 0$ i.e. $x+y = 2$ & $2x+3y = 5$

$\Rightarrow x = 1, y = 1$

$\therefore y = mx \Rightarrow m = 1$

28. **Sol.** Here $r_2 = 7$ or 8

Tangent at (α,β) is $\alpha x +\beta y = r_2$

$\therefore A\left(\frac{r^2}{\alpha}, 0\right), B\left(0, \frac{r^2}{\beta}\right)$

□ $\text{Area} = \frac{1}{2}.\frac{r^2}{\alpha}.\frac{r^2}{\beta} = \frac{r^4}{2\alpha\beta}$ □ $\text{Area} = \frac{1}{2}.\frac{r^2}{\alpha}.\frac{r^2}{\beta} = \frac{r^4}{2\alpha\beta}$

$\therefore$ area can be = $\frac{49}{2\alpha\beta}$ or $\frac{64}{2\alpha\beta}$

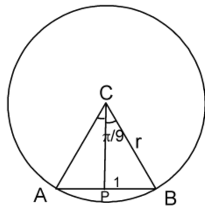
29. **Sol.** $\ell = 2\sqrt{g^2 - c} = 2\sqrt{16-16} = 0$

$m = 2\sqrt{f^2 - c} = 2\sqrt{25-16} = 6$

$y = -x$ intersects circle at $(8,-8)$ & $(1,-1)$

$\ell_2 + 10m_2 + 26n_2 = 0 + 360 + 2548 = 2908$

30. **Sol.** $\sin\frac{\pi}{9} = \frac{1}{r}$



$\Rightarrow$ r = cosec $\frac{\pi}{9}$

Circle

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

*** Marked Questions may have more than one correct option.**

1. **Sol.** From equation of circle it is clear that circle passes through origin. Let AB is chord of the circle.

A ≡ (p,q).C is mid-point and

Co-ordinate of C is (h,0)

Then coordinates of B are (–p + 2h, –q).

and B lies on the circle

$x^2 + y^2 = px + qy$, we have

$(-p + 2h)^2 + (-q)^2 = p(-p + 2h) + q(-q)$

⇒ $p^2 + 4h^2 - 4ph + q^2 = -p^2 + 2ph - q^2$

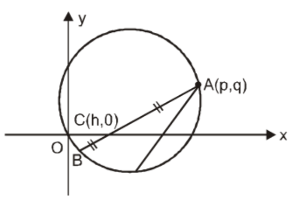
⇒ $2p^2 + 2q^2 - 6ph + 4h^2 = 0$

⇒ $2h^2 - 3ph + p^2 + q^2 = 0$ ....(1)

There are given two distinct chords which are bisected at x-axis then, there will be two distinct values of h satisfying (1).

So discriminant of this quadratic equation must be > 0

⇒ D > 0



⇒ $(-3p)^2 - 4.2\,(p^2 + q^2) > 0$

⇒ $9p^2 - 8p^2 - 8q^2 > 0$

⇒ $p^2 - 8q^2 > 0$

⇒ $p^2 > 8q^2$ Therefore, (4) is the answer.

⇒ D > 0

⇒ $(-3p)^2 - 4.2\,(p^2 + q^2) > 0$

⇒ $9p^2 - 8p^2 - 8q^2 > 0$

⇒ $p_2 - 8q_2 > 0$

⇒ $p_2 > 8q_2$ (4)

2*. **Solution**

Lines $L_1 \equiv y - mx = 0$

$L_2 \equiv y + x - 1 = 0$

If two lines make equal intercept on same circle then perpendicular distance from centre $\left(\frac{1}{2}, -\frac{3}{2}\right)$ to both the lines are same.

⇒ $\left|\frac{-3/2 + 1/2 - 1}{\sqrt{1^2 + 1^2}}\right| = \left|\frac{-3/2 - m/2}{\sqrt{1 + m^2}}\right|$

⇒ $\frac{2}{\sqrt{2}} = \frac{|m+3|}{2\sqrt{1+m^2}}$

⇒ $2\sqrt{2}\sqrt{1+m^2} = |m+3|$

Upon squaring : $8m_2 + 8 = m_2 + 6m + 9$

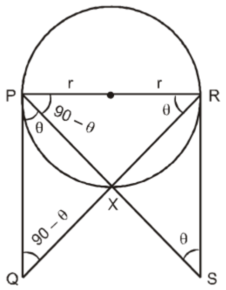
$7m_2 - 6m - 1 = 0$ ⇒ $m = -\frac{1}{7}, 1$

so lines are $y + \frac{1}{7}x = 0$ ⇒ $7y + x = 0$

and $y - 1.x = 0$ ⇒ $y - x = 0$

3. **Sol.** From figure it is clear that ΔPRQ and ΔRSP are similar.

$\frac{PR}{RS} = \frac{PQ}{RP}$



⇒ $PR_2 = PQ . RS$

⇒ $PR = \sqrt{PQ.RS}$

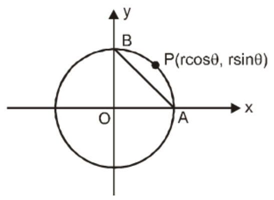
⇒ $2r = \sqrt{PQ.RS}$

Therefore, (1) is the answer.

4. **Solution**

Let A(r,0) and B(0,r) be end points of chord AB and moving point P(r cosθ, rsinθ) on the circle.

Let centroid is (x,y)



$\Rightarrow \quad x = \dfrac{r + r\cos\theta + 0}{3}$

$\Rightarrow \quad (3x - r)^2 = r^2\cos^2\theta$ ....(1)

$\Rightarrow \quad y = \dfrac{0 + r\sin\theta + r}{3}$

$\Rightarrow \quad (3y - r)^2 = r^2\sin^2\theta$ ....(2)

Equation (1) + (2)

$\Rightarrow \quad (3x - r)^2 + (3y - r)^2 = r^2$ which is a circle

5. **Sol.** The line 5x – 2y + 6 = 0 meets the y-axis at the point (0,3) and therefore the tangent has to pass through the point (0,3) and required length is therefore,

$= \sqrt{x_1^2 + y_1^2 + 6x_1 + 6y_1 - 2}$

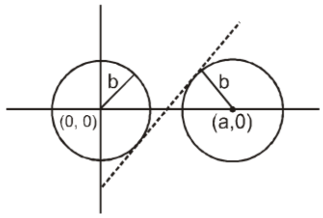
$= \sqrt{0 + 3^2 + 6(0) + 6(3) - 2}$

$= \sqrt{25} = 5$

6. **Solution**

$x^2 + y^2 = b^2$: centre (0,0) radius = b

$(x - a)^2 + y^2 = b^2$ centre (a,0) radius = b



y = $mx - b\sqrt{1+m^2}$ is tangent to both circle

Since, when x = 0 y = $-b\sqrt{1+m^2} < 0$

when y = 0 x = $\dfrac{b\sqrt{1+m^2}}{m} > 0$

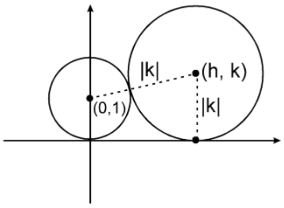
Now, perpendicular distance from centre (a,0) will be equal to radius b.

$$\left|\frac{ma - 0 - b\sqrt{1+m^2}}{\sqrt{m^2+1}}\right| = b \quad \Rightarrow \quad \left|\frac{ma - b\sqrt{1+m^2}}{\sqrt{m^2+1}}\right| = b$$

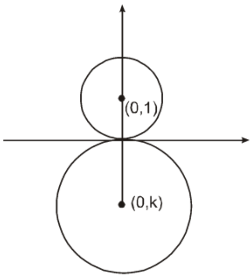
–ve sign $\dfrac{ma - b\sqrt{1+m^2}}{\sqrt{m^2+1}} = -b \Rightarrow ma = 0 \Rightarrow m = 0$ or a = 0 not possible.

+ve sign $\dfrac{ma - b\sqrt{1+m^2}}{\sqrt{m^2+1}} = +b$ ⇒ $m = \dfrac{2b}{\sqrt{a^2 - 4b^2}}$

7. **Sol.** $\sqrt{(h-0)^2 + (k-1)^2} = 1 + |k|$



or

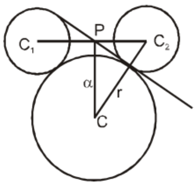


⇒ $h^2 + k^2 - 2k + 1 = 1 + 2|k| + k^2$

⇒ $h^2 = 2|k| + 2k$

⇒ $x^2 = 4y$ if $y \geq 0$ & $x = 0$ if $y \leq 0$

8. **Sol.**



$(r + 1)^2 = \alpha^2 + 9$

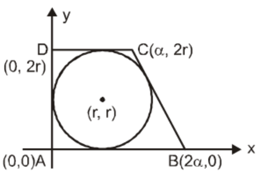
$r^2 + 8 = \alpha^2$

⇒ $r^2 + 2r + 1 = r^2 + 8 + 9$

$2r = 16$

$r = 8$

9. **Sol.** $18 = \frac{1}{2}(3\alpha)(2r)$ $\alpha r = 6$



Line, $y = -\dfrac{2r}{\alpha}(x - 2\alpha)$ is tangent to circle

$(x - r)^2 + (y - r)^2 = r^2$

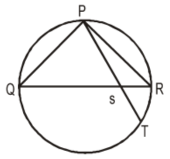
$2\alpha = 3r$ and $\alpha r = 6$

$r = 2$

10*. **Sol.** PS . ST = QS . SR

Now HM < GM

⇒ $\dfrac{2}{\frac{1}{PS}+\frac{1}{ST}} < \sqrt{PS\,.\,ST}$



⇒ $\dfrac{1}{PS} + \dfrac{1}{ST} > \dfrac{2}{\sqrt{QS.SR}}$

⇒ (2) is correct and (1) is wrong.

Now QR = QS + SR

Applying AM > GM

$\dfrac{QS+SR}{2} > \sqrt{QS.SR}$

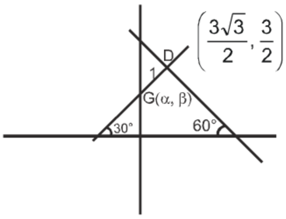
QR > 2 $\sqrt{QS.SR}$ $\dfrac{4}{QR} < \dfrac{2}{\sqrt{PS\,.\,ST}}$

⇒ $\dfrac{1}{PS}+\dfrac{1}{ST} > \dfrac{2}{\sqrt{PS\,.\,ST}} > \dfrac{4}{QR}$

∴ (4) is correct and (3) is wrong

⇒ (2) and (4) are correct.

11. Sol.



Let G (α, β) be the centre of C

α = $\dfrac{3\sqrt{3}}{2}$ – 1. cos 30 = $\sqrt{3}$

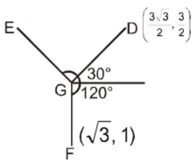
β = $\dfrac{3}{2}$ – 1 . sin 30 = 1

∴ equation of C is

$(x - \sqrt{3})^2 + (y - 1)^2 = 1$

∴ D is correct.

12. Sol.



$\angle FGD = \angle DGE = 120°$ $\Rightarrow$ $F = (\sqrt{3}, 0)$ and

$GF = GE = GD = 1$ $E = \left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$

13. **Sol.** Slope QR = $\sqrt{3}$

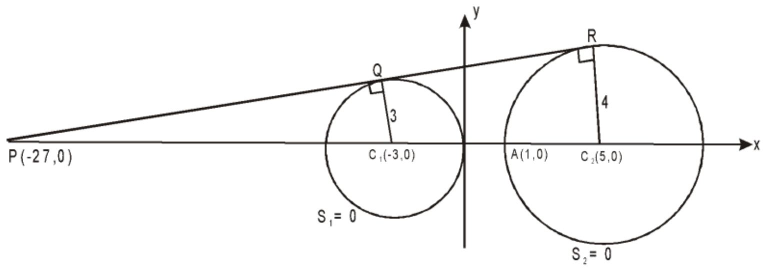
$\therefore$ equation of QR is $y - \frac{3}{2} = \sqrt{3}\left(x - \frac{\sqrt{3}}{2}\right)$

$y = \sqrt{3}\,x$

and slope of RP = 0

$\therefore$ equation RP is y = 0

14. ΔPQC_1 and ΔPRC_2 are similar



$\therefore \frac{\text{Area of } \Delta PQC_1}{\text{Area of } \Delta PRC_2} = \frac{\frac{1}{2}\times PQ\times r_1}{\frac{1}{2}\times PR\times r_2} = \frac{PQ}{PR}\cdot\frac{r_1}{r_2} = \frac{r_1}{r_2}\cdot\frac{r_1}{r_2} = \frac{r_1^2}{r_2^2} = \frac{9}{16}$

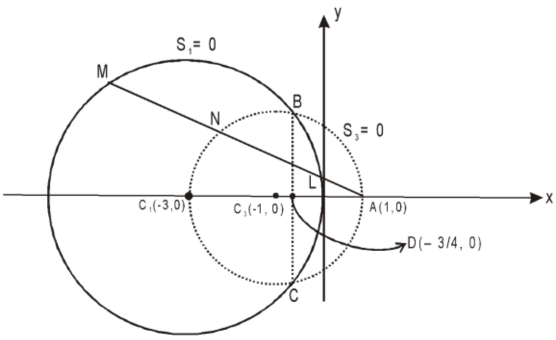
15. Let mid point be N(h, k). Now equation of chord LM is $T = S_1$

$\Rightarrow$ $hx + ky + 3(x + h) = h^2 + k^2 + 6h$

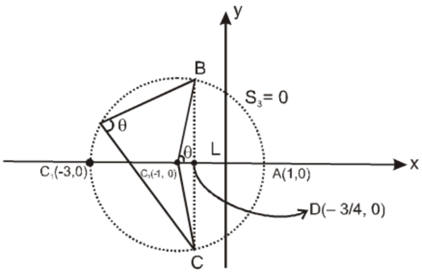
As it passes through (1, 0) $\Rightarrow$ $h + 3(1 + h) = h^2 + k^2 + 6h$

So locus is $x^2 + y^2 + 2x - 3 = 0$ which is a circle with centre (– 1, 0) and radius 2.

But it is clear from geometry that it will be major arc BC_1C as shown in figure.



16. Common chord of $S_1 = 0$ and $S_3 = 0$ is $4x + 3 = 0$ $\Rightarrow$ $x = -3/4$



At x = –3/4, $\left(-\frac{3}{4}+3\right)^2 + y^2 = 9$

$\Rightarrow \quad y^2 = 9 - \frac{81}{16} \quad \Rightarrow \quad y^2 = \frac{63}{16} \quad \Rightarrow \quad y = \pm\frac{3\sqrt{7}}{4}$

$\Rightarrow \quad B \equiv \left(\frac{-3}{4}, \frac{3\sqrt{7}}{4}\right)$ and $C \equiv \left(\frac{-3}{4}, \frac{-3\sqrt{7}}{4}\right)$

Hence $\tan\theta = \frac{BL}{C_3L} = \frac{\frac{3\sqrt{7}}{4}}{(3-3/4)} = \frac{\sqrt{7}}{3} \Rightarrow \cos\theta = \frac{3}{4}$