CURRENT ELECTRICITY

1. ELECTRIC CURRENT

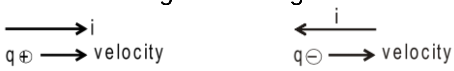
(a) Time rate of flow of copharge through a cross sectional area is called **Current**.
if Δq charge flows in time interval Δt then average current is given by

$I_{av} = \frac{\Delta q}{\Delta t}$ and

Instantaneous current

$i = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$.

(b) Direction of current is along the direction of flow of positive charge or opposite to the direction of flow of negative charge. But the current is a scalar quantity.



SI unit of current is ampere and

1 Ampere = 1 coloumb/sec

1 coloumb/sec = 1A

It is a scalar quantity because it does not obey the law of vectors.

2. CONDUCTOR

In some materials, the outer electrons of each atom or molecule are only weakly bound to it. These electrons are almost free to move throughout the body of the material and are called free electrons. They are also known as conduction electrons. When such a material is placed in an electric field, the free electrons drift in a direction opposite to the field. Such materials are called conductors.

3. INSULATOR

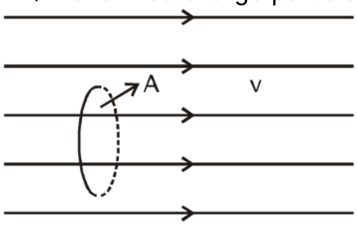
Another class of materials is called insulators in which all the electrons are tightly bound to their respective atoms or molecules. Effectively, there are no free electrons. When such a material is placed in an electric field, the electrons may slightly shift opposite to the field but they can't leave their parent atoms or molecules and hence can't move through long distances. Such materials are also called dielectrics.

4. SEMICONDUCTOR

In semiconductors, the behaviour is like an insulator at low levels of temperature. But at higher temperatures, a small number of electrons are able to free themselves and they respond to the applied electric field. As the number of free electrons in a semiconductor is much smaller than that in a conductor, its behaviour is in between a conductor and an insulator and hence, the name semiconductor. A free electron in a semiconductor leaves a vacancy in its normal bound position. These vacancies also help in conduction.

Current, velocity and current density

n → no. of free charge particles per unit volume



q → charge of each free particle

i → charge flow per unit time

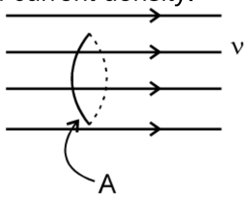
i = nqvA

Current density, a vector, at a point have magnitude equal to current per unit normal area at that point and direction is along the direction of the current at that point.

$$\vec{J} = \frac{di}{ds}\vec{n}$$

so $\qquad di = \vec{J}.\vec{ds}$

Current is flux of current density.



Due to principle of conservation of charge:

Charge entering at one end of a conductor = charge leaving at the other end, so current does not change with change in cross section and conductor remains uncharged when current flows through it.

Solved Examples

Example 1. Find free electrons per unit volume in a metallic wire of density 10^4 kg/m^3, atomic mass number 100 and number of free electron per atom is one.

Solution : Number of free charge particle per unit volume

$$(n) = \frac{\text{total free charge particle}}{\text{total volume}}$$

∴ Number of free electron per atom means
total free electrons = total number of atoms.

$$= \frac{N_A}{M_W} \times M$$

So $\quad n = \dfrac{\frac{N_A}{M_W} \times M}{V} = \dfrac{N_A}{M_W} \times d = \dfrac{6.023 \times 10^{23} \times 10^4}{100 \times 10^{-3}}$ $\qquad$ **$n = 6.023 \times 10^{28}\ m^{-3}$**

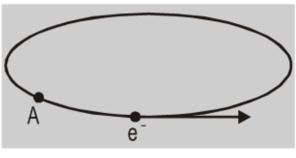
Example 2. What will be the number of electron passing through a heater wire in one minute, if it carries a current of 8 A.

Solution : $I = \dfrac{ne}{t} \quad n = \dfrac{It}{e} = \dfrac{8 \times 60}{1.6 \times 10^{-19}} = 3 \times 10^{21}$ electrons

Example 3. An electron moves in a circle of radius 10 cm. with a constant speed of 4×10^6 m/sec. Find the electric current at a point on the circle.

Solution : Consider a point A on the circle. The electron crosses this point once in every revolution. The number of revolutions made by electron in one second is

$$n = \frac{v}{2\pi r} = \frac{4 \times 10^6}{2\pi \times 10 \times 10^{-2}} = \frac{2}{\pi} \times 10^7 \text{ rot./s}$$



Current $\quad I = \dfrac{ne}{t} = \dfrac{2}{\pi} \times 10^7 \times 1.6 \times 10^{-19}$

(∵ t = 1 s)

$$= \frac{3.2}{\pi} \times 10^{-12} \cong 1 \times 10^{-12} \text{ A}$$

Example 4. The current through a wire depends on time as i =(2 + 3t)A. Calculate the charge crossed through a cross section of the wire in 10 s.

Solution : $i = \frac{dq}{dt}$ r dq = (2 + 3t)dt

$\int_0^{10} (2+3t)dt$ r $q = \left(2t + \frac{3t^2}{2}\right)_0^{10}$?

$q = 2\times10 + \frac{3}{2}\times100$ = 20 + 150 = 170 C

Example 5. Current through a wire decreases uniformly from 4 A to zero in 10 s. Calculate charge flown through the wire during this interval of time.

Solution : charge flown = average current × time

= $\left[\frac{4+0}{2}\right]\times10 = 20C$

Self Practice Problems

1. The expression for Ohm's law in terms of electric field E and current density J is:-

(1) $E = (\sigma J)^{1/2}$ (2) J = σ/E (3) J = σE (4) $\sigma = (J/E)^{1/2}$

2. A potential difference V is applied across a copper wire of diameter d and length ℓ. When d is doubled, the drift velocity:-

(1) Increases two times (2) decreases $\frac{1}{2}$ times (3) Does not change (4) Decreases $\frac{1}{4}$ times

3. Through a tube of radius R, 10, 000 α-particles pass per minute. The value of electric current through the tube is:-

(1) 0.5×10^{-12}A (2) 2×10^{-12}A (3) 0.5×10^{-16}A (4) 2×10^{-16}A

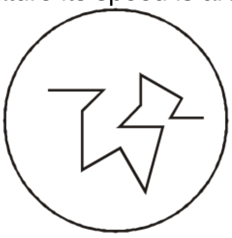
Answer : 1. (3) **2.** (3) **3.** (3)

5. MOVEMENT OF ELECTRONS INSIDE CONDUCTOR

All the free electrons are in random motion due to the thermal energy and relationship in given by

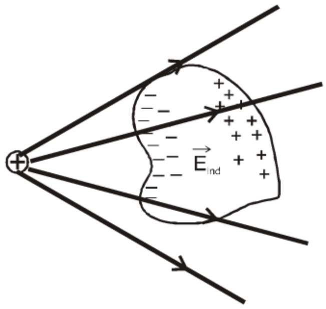
$\frac{3}{2}$ KT = $\frac{1}{2}$ mv^2

At room temperature its speed is around 10^6 m/sec or 10^3 km/sec



but the average velocity is zero so current in any direction is zero.

When a conductor is placed in an electric field. Then for a small duration electrons, do have an average velocity but its average velocity becomes zero within short interval of time.



THERMAL SPEED

Conductor contain a large number of free electrons, which are in continuous random motion.

Due to random motion, the free electrons collide with positive metal ions with high frequency and undergo change in direction at each collision. So, the thermal velocities are randomly distributed in all possible directions.

$\vec{u}_1, \vec{u}_2, ...\vec{u}_N$ are the individual thermal velocities of the free electrons at any given time.

the total number of free electrons in the conductor = N

average velocity $\vec{u}_{ave} = \left[\dfrac{\vec{u}_1 + \vec{u}_2 + ...\vec{u}_N}{N}\right] = 0$

The average velocity is zero but average speed is non zero.

DRIFT VELOCITY $(\vec{V}_d)$

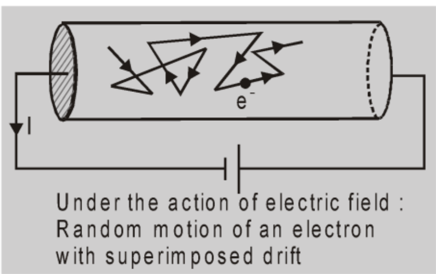
Drift velocity is defined as the velocity with which the free electrons get drifted towards the positive terminal under the effect of the applied electric field.

When the ends of a conductor are connected to a source of emf, an electric field E is established in the conductor, such that $E = \dfrac{V}{\ell}$,

where V = the potential difference across the conductor and ℓ = the length of the conductor.

The electric field $\vec{E}$ exerts an electrostatic force $-e\vec{E}$ on each electron in the conductor.

The acceleration of each electron $\vec{a} = \dfrac{-e\vec{E}}{m}$



m = mass of electron e = charge of electron

In addition to its thermal velocity, due to this acceleration,

the electron acquires, a velocity component in a direction opposite to the direction of the electric field.

The gain in velocity due to the applied field is very small

and is lost in the next collision.

At any given time, an electron has a velocity $\vec{v}_1 = \vec{u}_1 + \vec{a}\,\tau_1$

Where $\vec{u}_1$ = the thermal velocity

$\vec{a}\tau_1$ = the velocity acquired by the electron under the influence of the aplied electric field.
τ_1 = the time that has elapsed since the last collision. Similarly, the velocities of the other electrons are

$\vec{v}_2 = \vec{u}_2 + \vec{a}\tau_2,\ \vec{v}_3 = \vec{u}_3 + \vec{a}\tau_3, \ldots,\ \vec{v}_N = \vec{u}_N + \vec{a}\tau_N$

The average velocity of all the free electrons in the conductor is equal to the drift velocity $\vec{v}_d$ of the free electrons.

$$\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \ldots \vec{v}_N}{N} = \frac{(\vec{u}_1 + \vec{a}\tau_1) + (\vec{u}_2 + \vec{a}\tau_2) + \ldots + (\vec{u}_N + \vec{a}\tau_N)}{N}$$

or $\vec{v}_d = \dfrac{(\vec{u}_1 + \vec{u}_2 + \ldots + \vec{u}_N)}{N} + \vec{a}\dfrac{(\tau_1 + \tau_2 + \ldots + \tau_N)}{N}$ order of drift velocity is 10^{-4} m/s

$\because \dfrac{\vec{u}_1 + \vec{u}_2 + \ldots + \vec{u}_N}{N} = 0$ $\quad \therefore \vec{v}_d = \vec{a}\dfrac{\tau_1 + \tau_2 + \ldots + \tau_N}{N}$ r $\vec{v}_d = \vec{a}\tau$ $\quad \Rightarrow \vec{v}_d = -\dfrac{e\vec{E}}{m}\tau$

RELAXATION TIME (τ) :

Average time elapsed between two successive collisions.

It is of the order of 10^{-14} s

It is a temperature dependent characteristic of the material of the conductor.

It decreases with increases in temperature.

MEAN FREE PATH (λ)

The distance travelled by a conduction electron during relaxation time is known as mean free path λ.

Mean free path of conduction electron = Thermal velocity × Relaxation time

Solved Examples

Example 6. Find the approximate total distance travelled by an electron in the time-interval in which its displacement is one meter along the wire.

Solution : time = $\dfrac{\text{displacement}}{\text{drift velocity}} = \dfrac{S}{V_d}$

∴ V_d = 1 mm/s = 10^{-3} m/s (normally the value of drift velocity is 1 mm/s)

S = 1 m

time $= \dfrac{1}{10^{-3}} = 10^3$ s

distance travelled = speed × time

∴ speed = 10^6 m/s

So required distance = $10^6 \times 10^3$ m = **10^9 m**

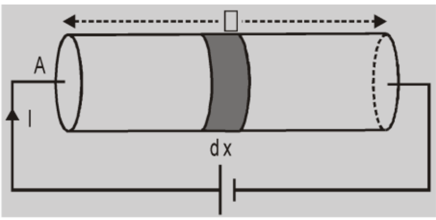
6. RELATION BETWEEN I & V IN A CONDUCTOR

Let the number of free electrons per unit volume in a conductor = n

Total number of electrons in dx distance = n (Adx)

Total charge dQ = n (Adx)e

Cross sectional area = A



Current $I = \frac{dQ}{dt} = nAe\frac{dx}{dt}$ ⇒ $I = neAv_d$

Current density $J = \frac{I}{A} = nev_d$ ⇒ $J = ne\left(\frac{eE}{m}\right)\tau$ ∵ $v_d = \left(\frac{eE}{m}\right)\tau$

$J = \left(\frac{ne^2\tau}{m}\right)E$ ⇒ $J = \sigma E$ conductivity $\sigma = \frac{ne^2\tau}{m}$

In vector form $\vec{J} = \sigma\vec{E}$

σ depends only on the material of the conductor and its temperature.

As temperature (T) ↑ , τ ↓

Example 7. A current of 1.34 A exists in a copper wire of cross - section 1.0 mm^2. Assuming each copper atom contributes one free electron. Calculate the drift speed of the free electrons in the wire. The density of copper is 8990 kg/m^3 and atomic mass = 63.50.

Solution : Mass of $1m^3$ volume of the copper is = 8990 kg = 8990 × 10^3 g

Number of moles in $1m^3$ $= \frac{8990\times10^3}{63.5} = 1.4\times10^5$

Since each mole contains 6 × 10^{23} atoms therefore number of atoms in $1m^3$

n = (1.4 × 10^5) × (6 × 10^{23})

= 8.4 × 10^{28} = electron density

i = $neAv_d$

$v_d = \frac{i}{neA} = \frac{1.34}{8.4\times10^{28}\times1.6\times10^{-19}\times10^{-6}}$

(∵ 1 mm^2 = 10^{-6} m^2) = 10^{-4} m/s

7. ELECTRICAL RESISTANCE

The property of a substance by virtue of which it opposes the flow of electric current through it is termed as electrical resistance. Electrical resistance depends on the size, geometry, temperature and internal structure of the conductor.

We have $i = \frac{nAe^2\tau}{2m\ell}V$

Here $i \propto V$

it is known as Ohm's law

$i = \frac{V}{R}$

$R = \frac{2m\ell}{nAe^2\tau}$ ⇒ V = IR

hence $R = \frac{2m}{ne^2\tau}.\frac{\ell}{A}$

So, Here $R = \frac{\rho\ell}{A}$ ⇒ $V = I\times\frac{\rho\ell}{A}$

⇒ $\frac{V}{\ell} = \frac{I}{A}\rho$ ⇒ E = J ρ ⇒ $J = \frac{I}{A}$ = current density

ρ is called resistively (it is also called specific resistance), and $\rho = \frac{2m}{ne^2\tau} = \frac{1}{\sigma}$, σ is called conductivity. Therefore current in conductors is proportional to potential difference applied across its ends. This is **Ohm's Law**. Units: $R \to ohm(\Omega)$, $\rho \to ohm - meter(\Omega - m)$ also called siemens, $\sigma \to \Omega^{-1}m^{-1}$.

IMPORTANT POINTS

◎ 1 ampere of current means the flow of 6.25×10^{18} electrons per second through any cross section of conductor.

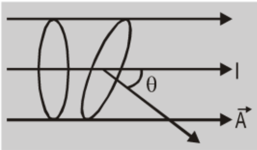
◎ Electric field outside a current carrying conductor is zero but inside a conductor is $\frac{V}{\ell}$.

◎ Current is a scalar quantity but current density is a vector quantity.

◎ If A is not normal to I but makes an angle θ with the normal to current then.

$$I = JA\cos\theta$$

$$J = \frac{I}{A\cos\theta}$$



◎ Order of free e^- density in conductors = 10^{28} electrons/m^3, while in semi conductors = 10^{16} e^-/m^3

◎

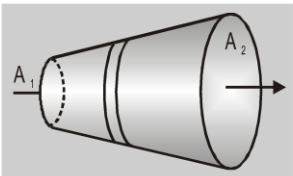
Terms	Thermal speed v_T	Mean free path λ	Relaxation time τ	Drift speed v_d
	10^5 m/s	10 Å	10^{-14} s	10^{-4} m/s

◎ If a steady current flows in a metallic conductor of non uniform cross section.

(i) Along the wire I is same.

(ii) Current density, drift velocity depends on area inversly so $J_1 > J_2$, $E_1 > E_2$, $v_{d_1} > v_{d_2}$

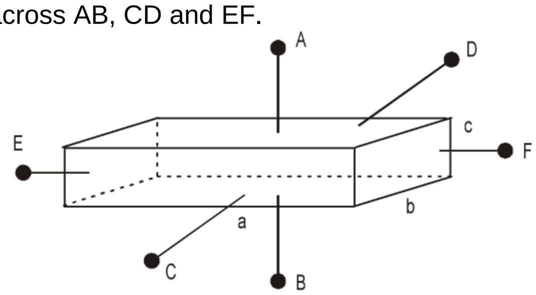
$I_1 = I_2$, $A_1 < A_2$



◎ If the temperature of the conductor increases, the amplitude of the vibrations of the positive ions in the conductor also increase. Due to this, the free electrons collide more frequently with the vibrating ions and as a result, the average relaxation time decreases.

Solved Examples

Example 8. The dimensions of a conductor of specific resistance ρ are shown below. Find the resistance of the conductor across AB, CD and EF.



Answer : $R_{AB} = \frac{\rho c}{ab}$, $R_{CD} = \frac{\rho b}{ac}$, $R_{EF} = \frac{\rho a}{bc}$

Solution : For a condition

$R = \frac{\rho \ell}{A} = \frac{\text{Resistivity} \times \text{length}}{\text{Area of cross section}}$ $\Rightarrow R_{AB} = \frac{\rho c}{ab}$, $R_{CD} = \frac{\rho b}{ac}$, $R_{EF} = \frac{\rho a}{bc}$

7.1 Dependence of Resistance on various factors

$$R = \rho \frac{\ell}{A} = \frac{2m}{ne^2 \tau} \cdot \frac{\ell}{A}$$

Therefore R depends as

(1) $\propto \ell$ (2) $\propto \frac{1}{A}$ (3) $\propto \frac{1}{n}$ $\propto \frac{1}{\tau}$

(4) and in metals τ decreases as T increases $\Rightarrow$ R also increases.

Results

(a) On stretching a wire (volume constant)

If length of wire is taken into account then $\frac{R_1}{R_2} = \frac{\ell_1^2}{\ell_2^2}$

If radius of cross section is taken into account then $\frac{R_1}{R_2} = \frac{r_2^4}{r_1^4}$, where R_1 and R_2 are initial and final resistances and ℓ_1, ℓ_2, are initial and final lengths and r_1 and r_2 initial and final radii respectively. (if elasticity of the material is taken into consideration, the variation of area of cross-section is calculated with the help of Young's modulus and Poison's ratio)

(b) Effect of percentage change in length of wire

$$\frac{R_2}{R_1} = \frac{\ell^2 \left[1 + \frac{x}{100}\right]^2}{\ell^2}$$ where ℓ - original length and x- % increment

if x is quite small (say < 5%) then % change in R is

$$\frac{R_2 - R_1}{R_1} \times 100 = \left(\frac{\left(1 + \frac{x}{100}\right)^2 - 1}{1} \right) \times 100 \cong 2x\%$$

Solved Examples

Example 9. If a wire is stretched to double its length, find the new resistance if original resistance of the wire was R.

Solution : As we know that $R = \frac{\rho \ell}{A}$ $\Rightarrow$ in case $R' = \frac{\rho \ell'}{A'}$

$\ell' = 2\ell$

$A'\ell' = A\ell$ (volume of the wire remains constant)

$A' = \frac{A}{2}$ $\Rightarrow$ $R' = \frac{\rho \times 2\ell}{A/2} = 4 \frac{\rho \ell}{A} = 4R$

Example 10. The wire is stretched to increase the length by 1% find the percentage change in the Resistance.

Solution : As we known that

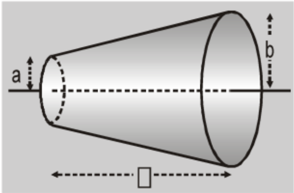
$\therefore$ $R = \frac{\rho \ell}{A}$ $\Rightarrow$ $\frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} + \frac{\Delta \ell}{\ell} - \frac{\Delta A}{A}$ and $\frac{\Delta \ell}{\ell} = -\frac{\Delta A}{A}$

$\frac{\Delta R}{R} = O + 1 + 1 = 2$

Hence percentage increase in the Resistance = 2%

Note : Above method is applicable when % change is very small.

Example 11. Figure shows a conductor of length ℓ carrying current i and having a circular cross - section. The radius of cross section varies linearly from a to b. Assuming that (b – a) << ℓ calculate current density at distance x from left end.



Solution : Since radius at left end is a and that of right end is b, therefore increase in radius over length ℓ is (b – a).

Hence rate of increase of radius per unit length = $\left(\frac{b-a}{\ell}\right)$

Increase in radius over length x = $\left(\frac{b-a}{\ell}\right)x$

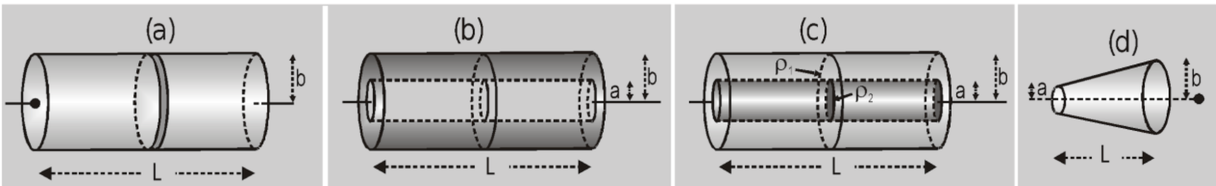
Since radius at left end is a, radius at distance x = r = a + $\left(\frac{b-a}{\ell}\right)x$

Area at this particular section A = πr^2 = $\pi\left[a+\left(\frac{b-a}{\ell}\right)x\right]^2$

Hence current density J = $\frac{i}{A}$ = $\frac{i}{\pi r^2}$ = $\frac{i}{\pi\left[a+\frac{x(b-a)}{\ell}\right]^2}$

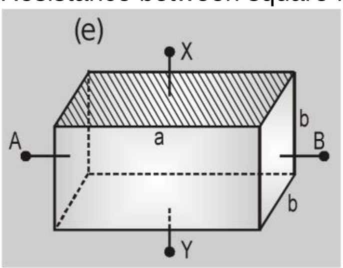
Note :

- Resistance of different shaped conductors.



R = $\frac{\rho L}{\pi b^2}$ R = $\frac{\rho L}{\pi (b^2 - a^2)}$ $R_1 = \frac{\rho_1 L}{\pi(b^2 - a^2)}$ R = $\frac{\rho L}{\pi ab}$

(i) Resistance between square faces



R_{AB} = $\rho\frac{\text{distance between faces}}{\text{area of square}} = \rho\frac{a}{b^2}$

(ii) Resistance between rectangular faces

$R_{xy} = \rho\frac{b}{a.b} = \frac{\rho}{a}$ (does not depends on b)

Temperature Dependence of Resistivity and Resistance :

The resistivity of a metallic conductor nearly increases with increasing temperature. This is because, with the increase in temperature the ions of the conductor vibrate with greater amplitude, and the collision between electrons and ions become more frequent. Over a small temperature range (upto 100ºC), the resistivity of a metal can be represented approximately by the equation,

$$\rho(T) = \rho_0 [1 + \alpha (T - T_0)] \qquad ...(i)$$

where, ρ_0 is the resistivity at a reference temperature T_0 (often taken as 0ºC or 20ºC) and $\rho(T)$ is the resistivity at temperature T, which may be higher or lower than T_0. The factor α is called the temperature coefficient of resistivity.

The resistance of a given conductor depends on its length and area of cross-section besides the resistivity. As temperature changes, the length and area also change. But these changes are quite small and the factor ℓ/A may be treated as constant.

Then, $R \propto \rho$

and hence, $R(T) = R_0 [1 + \alpha(T - T_0)]$ ...(ii)

In this equation R(T) is the resistance at temperature T and R_0 is the resistance at temperature T_0, often taken to be 0ºC or 20ºC. The temperature coefficient of resistance α is the same constant that appears.

Note :

- The ρ-T equation written above can be derived from the relation,
  α = fractional change in resistivity per unit change in temperature
  $= \frac{d\rho}{\rho dT} = \alpha$ or, $\frac{d\rho}{dT} = \alpha\rho$

  $\therefore \frac{d\rho}{\rho} = \alpha dT$ (α can be assumed constant for small temperature variation)

  $\therefore \int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = \alpha \int_{T_0}^{T} dT$ .....(iii)

$\therefore$ $\ln\left(\frac{\rho}{\rho_0}\right) = \alpha (T - T_0)$

$\therefore$ $\rho = \rho_0 e^{\alpha(T-T_0)}$

if $\alpha (T - T_0) << 1$ then

$e^{\alpha(T-T_0)}$ can approximately be written as $1 + \alpha(T - T_0)$. Hence,

In the above discussion we have assumed α to be constant. If it is a function of temperature it will come inside the integration in Eq. (iii).

IMPORTANT POINTS

◎ If a wire is stretched to n times of it's original length, its new resistance will be n_2 times.

◎ If a wire is stretched such that it's radius is reduced to $\frac{1}{n}th$ of it's original values, then resistance will increases n_4 times similarly resistance will decrease n_4 time if radius is increased n times by contraction.

◎ The equivalent resistance of parallel combination is lower than the value of lowest resistance in the combination.

◎ In general :

(i) Resistivity of alloys is greater than their metals.
(ii) Temperature coefficient of alloys is lower than pure metals.
(iii) Resistance of most of non metals decreases with increase in temperature. (e.g.carbon)
(iv) The resistivity of an insulator (e.g. amber) is greater then the metal by a factor of 10^{22}

◎ Temperature coefficient (α) of semi conductor including carbon (graphite), insulator and electroytes is negative.

Solved Examples

Example 12. The resistance of a thin silver wire is 1.0 Ω at 20ºC. The wire is placed in liquid bath and its resistance rises to 1.2 Ω. What is the temperature of the bath ? (Here $\alpha = 10^{-2}$ /ºC)

Solution : Here change in resistance is small so we can apply

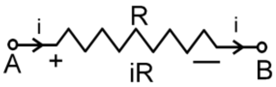
$R = R_0(1 + \alpha\Delta\theta)$

$\Rightarrow \quad 1.2 = 1 \times (1 + 10^{-2}\,\Delta\theta) \quad \Rightarrow \quad \Delta\theta = 20^\circ C$

$\Rightarrow \quad \theta - 20 = 20 \quad \Rightarrow \quad \theta = 40^\circ\ C$ **Ans.**

Electric current in resistance

In a resistor current flows from high potential to low potential

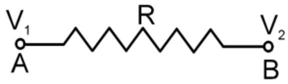


High potential is represented by positive (+) sign and low potential is represented by negative (–) sign.

$V_A - V_B = iR$

If $V_1 > V_2$

then current will flow from A to B

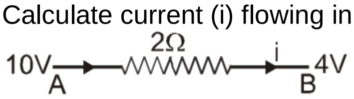


and $i = \dfrac{V_1 - V_2}{R}$

If $V_1 < V_2$

then current will go from B to A and $i = \dfrac{V_2 - V_1}{R}$

Example 13. Calculate current (i) flowing in part of the circuit shown in figure?



Solution : $V_A - V_B = i \times R \quad \Rightarrow \quad i = \dfrac{6}{2} = 3A$ **Ans.**

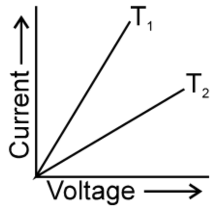
Self Practice Problems

4. Two rods A and B made up of same metal have same length. The ratio of their resistance is 1:2 if these wires are immersed in water then loss in weight will be:-
(1) More in A (2) More in B (3) Same in A and B (4) In the ratio 1 : 2

5. For two wires A and B of same material and of same mass, the radius of A is doubles that of B. If the resistance of wire A is 34 ohm than that of B will be:-
(1) 544 ohm (2) 272 ohm (3) 68 ohm (4) 17 ohm

6. A square rod of aluminimum of length 1 m and length of the side of cross sectional surface 5×10^{-3} m will have a resistance (Resistivity of aluminimum 2.8×10^{-8} ohm-meter):-
(1) $1.24 \times 10^{-4}\Omega$ (2) $2.42 \times 10^{-3}\Omega$ (3) $1.12 \times 10^{-3}\Omega$ (4) $11.2 \times 10^{-3}\Omega$

7. The following graph shows the relation between the voltage and the current for the temperature T_1 and T_2 in a metal wire. Then the relation between T_1 and T_2 is:-



(1) $T_1 = T_2$ (2) $T_1 > T_2$ (3) $T_1 < T_2$ (4) data insufficient

8. If the temperatures of iron and silicon wires are increased from $30^{0}C$ to $50^{0}C$, the correct statement is:-
(1) Resistance of both wires increase
(2) Resistance of both wires decrease
(3) Resistance of iron wire increases and the resistance of silicon wire decreases
(4) Resistance of iron wire decreases and the resistance of silicon wire increases

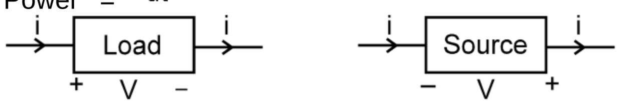
Answer : 4. (1) **5.** (1) **6.** (3) **7.** (3) **8.** (3)

8. ELECTRICAL POWER :

Energy liberated per second in a device is called its power. The electrical power P delivered or consumed by an electrical device is given by P = VI, where V = Potential difference across the device and I = Current.

If the current enters the higher potential point of the device then electric power is consumed by it (i.e. acts as load). If the current enters the lower potential point then the device supplies power (i.e. acts as source).

Power $= \frac{V.dq}{dt}$



$= V I$

P = V I

If power is constant then energy = P t

If power is variable then Energy = $\int p dt$

Power consumed by a resistor

$P = I^2R = VI = \frac{V^2}{R}$.

When a current is passed through a resistor energy is wasted in overcoming the resistance of the wire. This energy is converted into heat.

$W = VIt = I^2Rt = \frac{V^2}{R}t$

The heat generated (in joules) when a current of I ampere flows through a resistance of R ohm for t second is given by:

$H = I^2Rt$ Joule $= \frac{I^2Rt}{4.2}$ Calorie

1 unit of electrical energy = 1 Kilowatt hour = 1 KWh = 3.6×10^6 Joule.

Solved Examples

Example 14. If bulb rating is 100 watt and 220 V then determine
(a) Resistance of filament
(b) Current through filament
(c) If bulb operate at 110 volt power supply then find power consumed by bulb.

Solution : Bulb rating is 100 W and 220 V bulb means when 220 V potential difference is applied between the two ends then the power consumed is 100 W

Here V = 220 Volt
P = 100 W

$\frac{V^2}{R} = 100$ So $R = 484\ \Omega$

Since Resistance depends only on material hence it is constant for bulb

$I = \frac{V}{R} = \frac{220}{22 \times 22} = \frac{5}{11}$ Amp.

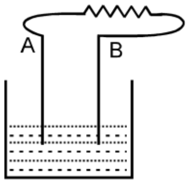
power consumed at 110 V

$\therefore$ power consumed = $\frac{110 \times 110}{484}$ = 25 W

9. BATTERY (CELL)

A battery is a device which maintains a potential difference across its two terminals A and B. Dry cells, secondary cells, generator and thermocouple are the devices used for producing potential difference in an electric circuit. Arrangement of cell or battery is shown in figure.

Electrolyte provides continuity for current.



It is often prepared by putting two rods or plates of different metals in a chemical solution. Some internal mechanism exerts force $(\vec{F}_n)$ on the ions (positive and negative) of the solution. This force drives positive ions towards positive terminal and negative ions towards negative terminal. As positive charge accumulates on anode and negative charge on cathode a potential difference and hence an electric field $\vec{E}$ is developed from anode to cathode. This electric field exerts an electrostatic force $\vec{F} = q\vec{E}$ on the ions. This force is opposite to that of $\vec{F}_n$. In equilibrium (steady state)

$F_n = F_e$ and no further accumulation of charge takes place.

When the terminals of the battery are connected by a conducting wire, an electric field is developed in the wire. The free electrons in the wire move in the opposite direction and enter the battery at positive terminal. Some electrons are withdrawn from the negative terminal. Thus, potential difference and hence, F_e decreases in magnitude while F_n remains the same. Thus, there is a net force on the positive charge towards the positive terminal. With this the positive charge rush towards positive terminal and negative charge rush towards negative terminal. Thus, the potential difference between positive and negative terminal is maintained.

Internal resistance (r) :

The potential difference across a real source in a circuit is not equal to the emf of the cell. The reason is that charge moving through the electrolyte of the cell encounters resistance. We call this the internal resistance of the source.

The internal resistance of a cell depends on the distance between electrodes ($r \propto d$), area of electrodes ($r \propto \frac{1}{s}$) and nature, concentration ($r \propto c$) and temperature of electrolyte ($r \propto \frac{1}{\text{Temp.}}$).

Solved Example

Example 15. What is the meaning of 10 Amp. hr ?

Solution : It means if the 10 A current is withdrawn then the battery will work for 1 hour.

10 Amp $\longrightarrow$ 1 hr

1 Amp $\longrightarrow$ 10 hr

½ Amp $\longrightarrow$ 20 hr

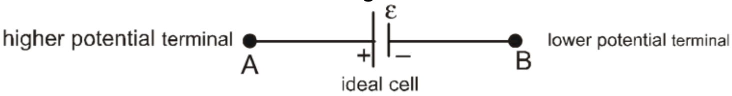
10. ELECTROMOTIVE FORCE : (E.M.F.)

Definition I : Electromotive force is the capability of the system to make the charge flow.
Definition II : It is the work done by the battery for the flow of 1 coloumb charge from lower potential terminal to higher potential terminal inside the battery.

10.1 Representation for battery :

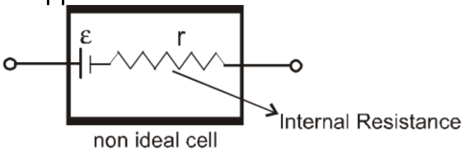
Ideal cell :

Cell in which there is no heating effect.



Non ideal cell :

Cell in which there is heating effect inside due to opposition to the current flow internally



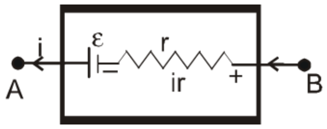
Case I : **Battery acting as a source (or battery is discharging)**

$V_A - V_B = \varepsilon - ir$

$V_A - V_B$

⇒ it is also called terminal voltage.

The rate at which the chemical energy of the cell is consumed = εi



The rate at which heat is generated inside the battery or cell = i^2r

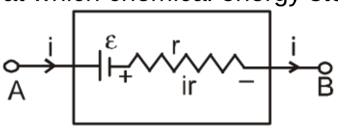
electric power output = $\varepsilon i - i^2r$

= $(\varepsilon - ir)\, i$

Case II : Battery acting as a load (or battery charging) :

$V_A - V_B = \varepsilon + ir$

the rate at which chemical energy stored in the cell = εi



thermal power inside the cell = i^2r

electric power input = $\varepsilon i + i^2r$ = $(\varepsilon + ir)\, i$ = $(V_A - V_B)\, i$

Definition III :

Electromotive force of a cell is equal to potential difference between its terminals when no current is passing through the circuit.

Case III :

When cell is in open circuit

i = 0 as resistance of open circuit is infinite (∞).

So V = ε, so open circuit terminal voltage difference is equal to emf of the cell.

Case IV :

Short circuiting : Two points in an electric circuit directly connected by a conducting wire are called short circuited, under such condition both points are at same potential.

When cell is short circuited

$i = \frac{\varepsilon}{r}$ and V = 0, short circuit current of a cell is maximum.

Note :

- The potential at all points of a wire of zero resistance will be same.

Earthing : If some point of circuit is earthed then its potential is assumed to be zero.

IMPORTANT POINTS

◎ At the time of charging a cell. When current is supplied to the cell, the terminal voltage is greater than the e.m.f. E

$$V = E + Ir$$

◎ Series combination is useful when internal resistance is less than external resistance of the cell.

◎ Parallel combination is useful when internal resistance is greater than external resistance of the cell.

◎ Power in R (given resistance) is maximum, if its value is equal to net resistance of remaining circuit.

◎ Internal resistance of ideal cell = 0

◎ if external resistance is zero than current given by circuit is maximum.

Value of External Resistance	Current from Cell potenial difference	Terminal	Power consumed in external resistance. $P_R = I^2 R = V^2/R$
R	$I = \frac{E}{R+r}$	V = E – Ir	$P = I^2R$
R = 0 Short circuit	$I = \frac{E}{r}$ Maximum	$V = E - \frac{E}{r}r$ V = 0	P = 0
R = r	$I = \frac{E}{2r}$	$V = E - \frac{E}{2r}r$ $V = \frac{E}{2}$	$P = \frac{E^2}{4r}$ Maximum
Open circuit R = ∞	I = 0	V = E – 0 V = E (TPD = EMF)	P = 0

11 RELATIVE POTENTIAL

While solving an electric circuit it is convenient to chose a reference point and assigning its voltage as zero, then all other potentials are measured with respect to this point. This point is also called the common point.

Solved Examples

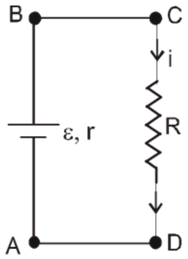
Example 16. In the given electric circuit find

(a) current

(b) power output

(c) relation between r and R so that the electric power output (that means power given to R) is maximum.

(d) value of maximum power output.



(e) plot graph between power and resistance of load

(f) From graph we see that for a given power output there exists two values of external resistance, prove that the product of these resistances equals r^2.

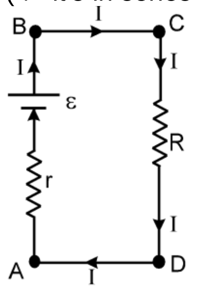
(g) what is the efficiency of the cell when it is used to supply maximum power.

Solution : (a) In the circuit shown if we assume that potential at A is zero then potential at B is $\varepsilon - ir$. Now since the connecting wires are of zero resistance

$\therefore \quad V_D = V_A = 0 \quad \Rightarrow \quad V_C = V_B = \varepsilon - ir$

Now current through CD is also i

($\because$ it's in series with the cell).



$\therefore \quad i = \dfrac{V_C - V_D}{R} = \dfrac{(\varepsilon - ir) - 0}{R} \qquad$ Current $i = \dfrac{\varepsilon}{r+R}$

Note : After learning the concept of series combination we will be able to calculate the current directly

(b) Power output $\quad P = i^2 R = \dfrac{\varepsilon^2}{(r+R)^2} . R$

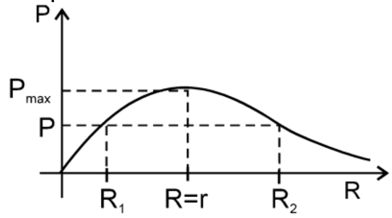
(c) $\dfrac{dP}{dR} = \dfrac{\varepsilon^2}{(r+R)^2} - \dfrac{2\varepsilon^2 R}{(r+R)^3} = \dfrac{\varepsilon^2}{(R+r)^3}[R + r - 2R]$ for maximum power supply

$\dfrac{dP}{dR} = 0 \Rightarrow \quad r + R - 2R = 0 \quad \Rightarrow \quad$ **r = R**

Here for maximum power output outer resistance should be equal to internal resistance

(d) $P_{max} = \dfrac{\varepsilon^2}{4r}$

(e) Graph between 'P' and R



maximum power output at R = r

$P_{max} = \dfrac{\varepsilon^2}{4r} \quad \Rightarrow \quad i = \dfrac{\varepsilon}{r+R}$

(f) Power output

$P = \dfrac{\varepsilon^2 R}{(r+R)^2}$

$P(r^2 + 2rR + R^2) = \varepsilon^2 R$

$R^2 + \left(2r - \dfrac{\varepsilon^2}{P}\right) R + r^2 = 0$

above quadratic equation in R has two roots R_1 and R_2 for given values of ε, P and r such that

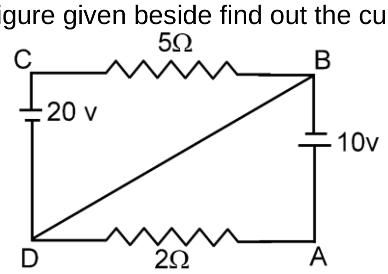
$\therefore \quad R_1 R_2 = r^2 \qquad$ (product of roots)

$\mathbf{r^2 = R_1 R_2}$

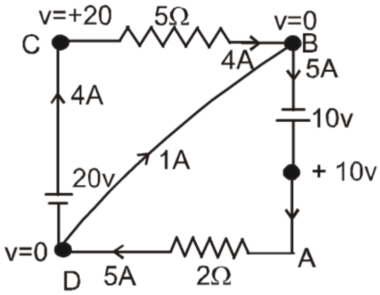
(g) Power of battery spent $= \dfrac{\varepsilon^2}{(r+r)^2}.2r = \dfrac{\varepsilon^2}{2r}$ power (output) $= \left(\dfrac{\varepsilon}{r+r}\right)^2 \times r = \dfrac{\varepsilon^2}{4r}$

Efficiency $= \dfrac{\text{power output}}{\text{total power spent by cell}} = \dfrac{\frac{\varepsilon^2}{4r}\times 100}{\frac{\varepsilon^2}{2r}} = \dfrac{1}{2} \times 100 = 50\%$

Example 17. In the figure given beside find out the current in the wire BD

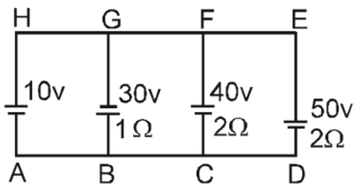


Solution : Let at point D potential = 0 and write the potential of other points then current in wire AD $= \dfrac{10}{2}$ = 5 A from A to D current in wire CB $= \dfrac{20}{5}$ = 4A from C to F



∴ current in wire BD = 1 A from D to B

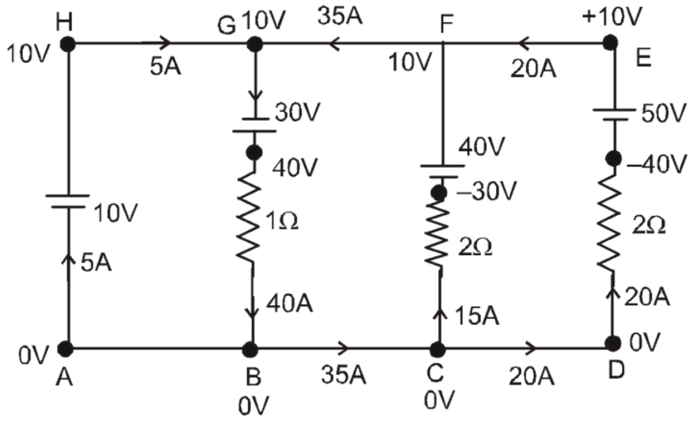
Example 18. Find the current in each wire



Solution : Let potential at point A is 0 volt then potential of other points is shown in figure.

current in BG $= \dfrac{40-0}{1} =$ 40 A from G to B

current in FC $= \dfrac{0-(-30)}{2} =$ 15 A from C to F



current in DE = $\frac{0-(-40)}{2} =$ 20 A from D to E

current in wire AH = 40 – 35 = 5 A from A to H

12. KIRCHHOFF'S LAWS

121- Kirchhoff's Current Law (Junction law)

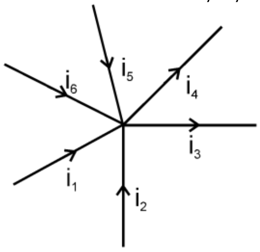
This law is based on law of conservation of charge. It states that " The algebraic sum of the currents meeting at a point of the circuit is zero " or total currents entering a junction equals total current leaving the junction.

$$\Sigma I_{in} = \Sigma I_{out}.$$

It is also known as KCL (Kirchhoff's current law).

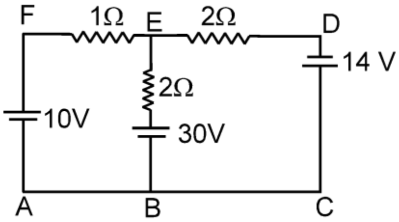
Solved Examples

Example 19. Find relation in between current i_1, i_2, i_3, i_4, i_5 and i_6 .

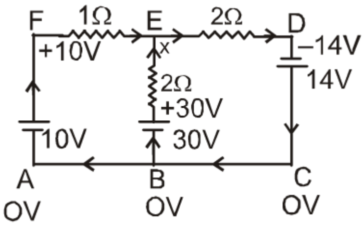


Solution : $i_1 + i_2 - i_3 - i_4 + i_5 + i_6 = 0$

Example 20. Find the current in each wire



Solution :



Let potential at point B = 0. Then potential at other points are mentioned.

∴ Potential at E is not known numerically.

Let potential at E = x

Now applying kirchhoff's current law at junction E. (This can be applied at any other junction also).

$$\frac{x-10}{1} + \frac{x-30}{2} + \frac{x+14}{2} = 0$$

$4x = 36 \Rightarrow \quad x = 9$

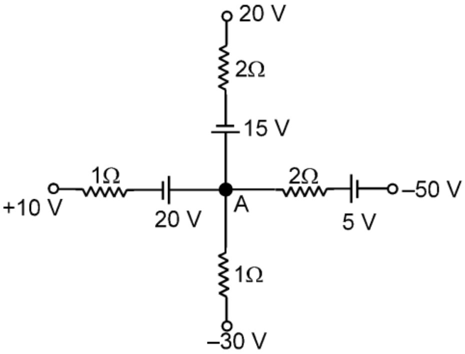
Current in EF = $\frac{10-9}{1}$ = 1 A from F to E

Current in BE = $\frac{30-9}{2} = 10.5\,A$ from B to E

Current in DE = $\frac{9-(-14)}{2} = 11.5\,A$ from E to D

Solved Example

Example 21. Find the potential at point A



Solution : Let potential at A = x, applying kirchhoff current law at juction A

$$\frac{x-20-10}{1}+\frac{x-15-20}{2}+\frac{x+45}{2}+\frac{x+30}{1}=0$$

$\Rightarrow$ $$\frac{2x-60+x-35+x+45+2x+60}{2}=0$$

$\Rightarrow$ 6x + 10 = 0 $\Rightarrow$ x = –5/3

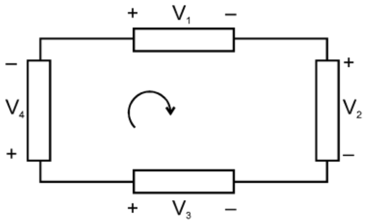
Potential at A = $\frac{-5}{3}$ V

12.2 Kirchhoff's Voltage Law (Loop law) :

"The algebraic sum of all the potential differences along a closed loop is zero.

So IR + Σ EMF = 0".

The closed loop can be traversed in any direction. hile traversing a loop if potential increases, put a positive sign in expression and if potential decreases put a negative sign. (Assume sign convention)



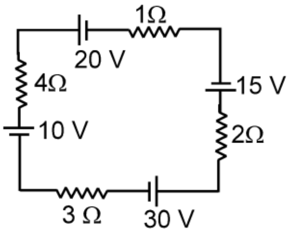
$$-V_1 - V_2 + V_3 - V_4 = 0.$$

Boxes may contain resistor or battery or any other element (linear or nonlinear).

It is also known as **KVL**

Solved Examples

Example 22. Find current in the circuit

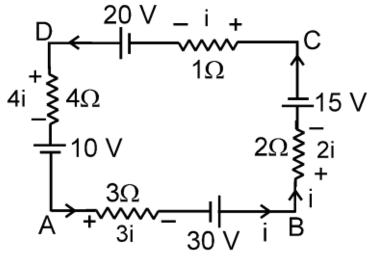


Solution : ∵ all the elements are connected in series

current is all of them will be same

let current = i

Applying kirchhoff voltage law in ABCDA loop

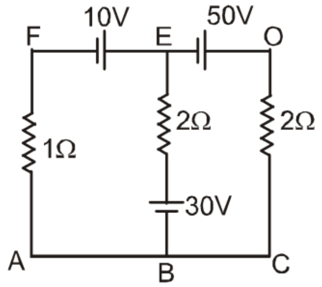


10 + 4i – 20 + i + 15 + 2i – 30 + 3i = 0

10 i = 25

i = 2.5 A

Example 23. Find the current in each wire applying only kirchhoff voltage law



Solution : Applying kirchhoff voltage law in loop ABEFA

$i_1 + 30 + 2(i_1 + i_2) - 10 = 0$

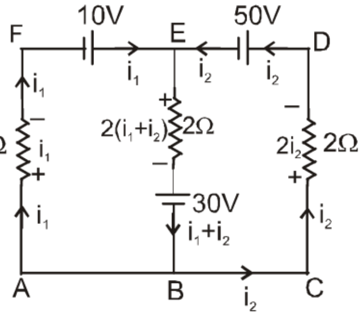
$3I_1 + 2I_2 + 20 = 0$ -------------- (i)

Applying kirchoff voltage law in BEDCB

$+ 30 + 2(i_1 + i_2) + 50 + 2i_2 = 0$

$4i_2 + 2i_1 + 80 = 0$

$2i_2 + i_1 + 40 = 0$ -------------- (ii)



Solving (i) and (ii)

$3\left[-40-2i_2\right] + 2i_2 + 20 = 0$

$-120 - 4i_2 + 20 = 0$

$i_2 = -25$ A

and $i_1 = 10$ A

∴ $i_1 + i_2 = -15$ A

current in wire AF = 10 A from A to F

current in wire EB = 15 A from B to E

current in wire DE = 25 A from E to D.

Self Practice Problems

9. For a cell terminal P.D. is 2.2V when circuit is open and reduces to 1.8V when cell is connected to a resistance of R = 5Ω. Determine internal resistance of cell (r) :-

(1) $\frac{10}{9}\Omega$ (2) $\frac{9}{10}\Omega$ (3) $\frac{11}{9}\Omega$ (4) $\frac{5}{9}\Omega$

10. Krichoff;s Ist law based on:-

(1) Energy conservation (2) Charge conservation (3) Current conservation (4) None

11. A car battery of emf 12 V and internal resistance 5×10^{-2} Ω, receives a current of 60 amp. from external source, then terminal p.d of battery is :-

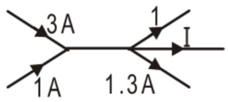
(1) 12 V (2) 9 V (3) 15 V (4) 20 V

12. Two bulbs of (40 W, 200 V), and (100 W, 200 V). Then correct relation for their resistances:-

(1) $R_{40} < R_{100}$ (2) $R_{40} > R_{100}$

(3) $R_{40} = R_{100}$ (4) No relation can be predicted

13. For the circuit shown the value of current I is:-



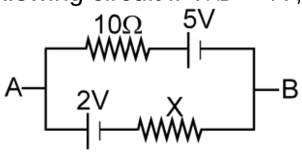
(1) 1.3 A (2) 1.7 A (3) 3.7 A (4) 1 A

14. The terminal voltage across a cell is more than its emf if an another cell of:-

(1) Higher emf is connected in parallel to it (2) Less emf is connected in parallel to it

(3) Less emf is connected in series to it (4) Higher emf is connected in series to it

15. In the following circuit if V_{AB} = 4V, then the value of resistance X in ohm's will be:-



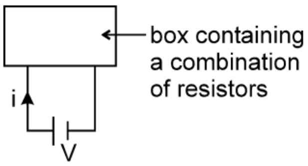
(1) 5 (2) 10 (3) 15 (4) 20

Answer : 9. (1) **10.** (2) **11.** (3) **12.** (2) **13.** (2) **14.** (1) **15.** (4)

13. COMBINATION OF RESISTANCES :

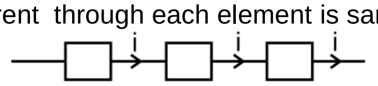
A number of resistances can be connected and all the complicated combinations can be reduced to two different types, namely series and parallel.

The equivalent resistance of a combination is defined as $R_{eq} = \frac{V}{i}$



13.1 Resistances in Series:

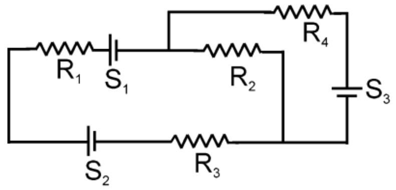
When the resistances (or any type of elements) are connected end to end then they are said to be in series. The current through each element is same.



Resistances in series carry equal current but reverse may not be true.

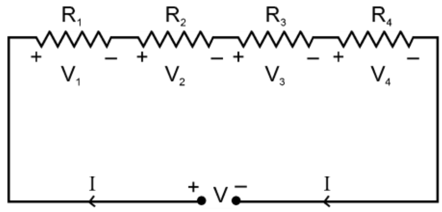
Solved Example

Example 24. Which electrical elements are connected in series.



Solution : Here S_1, S_2, R_1, R_3 connected in one series and R_4, S_3 connected in different series

Equivalent of Resistors :



The effective resistance appearing across the battery (or between the terminals A and B) is

$R = R_1 + R_2 + R_3 + \ldots\ldots\ldots + R_n$ (this means R_{eq} is greater then any resistor)

and $V = V_1 + V_2 + V_3 + \ldots\ldots\ldots + V_n$.

The potential difference across a resistor is proportional to the resistance. Power in each resistor is also proportional to the resistance

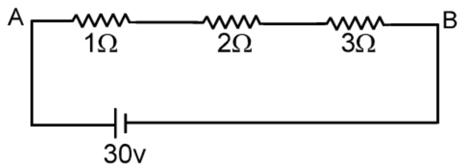
$\because \quad V = IR$ and $P = I^2R$

where I is same through any of the resistor.

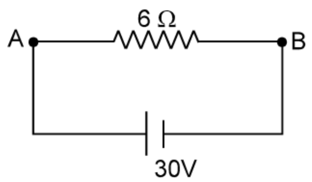
$V_1 = \dfrac{R_1}{R_1 + R_2 + \ldots\ldots\ldots + R_n} V$; $V_2 = \dfrac{R_2}{R_1 + R_2 + \ldots\ldots\ldots + R_n} V$; etc

Solved Examples

Example 25. Find the current in the circuit

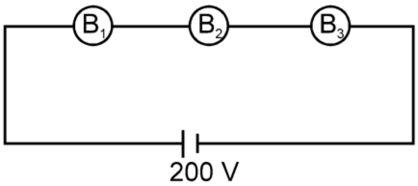


Solution : $R_{eq} = 1 + 2 + 3 = 6\ \Omega$ the given circuit is equivalent to



current $i = \dfrac{v}{R_{eq}} = \dfrac{30}{6}$ = **5 A** **Ans.**

Example 26. In the figure shown B_1, B_2 and B_3 are three bulbs rated as (200V, 50 W), (200V, 100W) and (200 V, 25W) respectively. Find the current through each bulb and which bulb will give more light?



Solution : $R_1 = \frac{(200)^2}{50}$; $R_2 = \frac{(200)^2}{100}$; $R_3 = \frac{(200)^2}{25}$

the current following through each bulb is

$$= \frac{200}{R_1 + R_2 + R_3} = \frac{200}{(200)^2\left[\frac{2+1+4}{100}\right]}$$

$$= \frac{100}{200 \times 7} = \frac{1}{14} \text{ A}$$

Since $R_3 > R_1 > R_2$

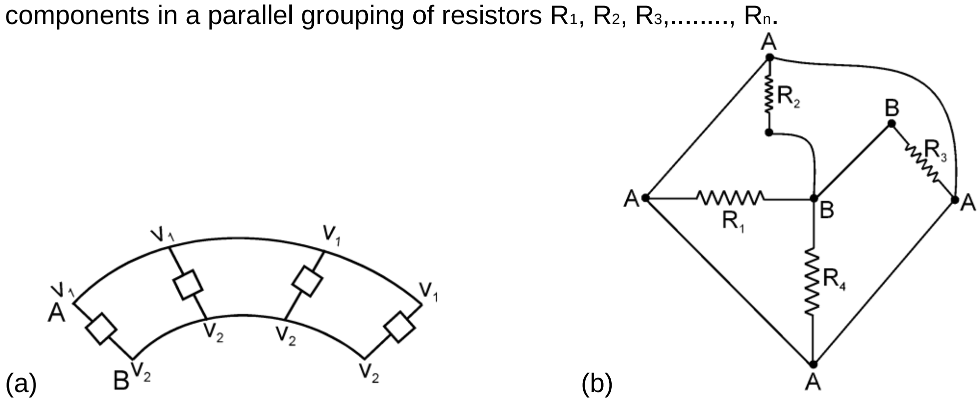
∴ Power consumed by bulb = i^2R

∴ if the resistance is of higher value then it will give more light.

∴ Here Bulb B_3 will give more light.

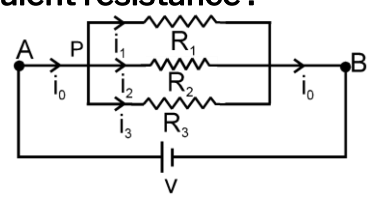
13.2 Resistances in Parallel :

A parallel circuit of resistors is one in which the same voltage is applied across all the components in a parallel grouping of resistors R_1, R_2, R_3,........, R_n.



In the figure (a) and (b) all the resistors are connected between points A and B so they are in parallel.

Equivalent resistance :

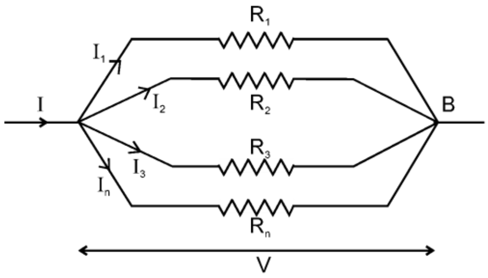


Applying kirchhoff's junction law at point P

$i_0 = i_1 + i_2 + i_3$

Therefore, $\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$ $\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$

in general,



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$$

Conclusions: (about parallel combination)

(a) Potential difference across each resistor is same.

(b) $I = I_1 + I_2 + I_3 + ... I_n$.

(c) Effective resistance (R) then. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$

(R is less than each resistor).

(d) Current in different resistors is inversely proportional to the resistance.

$I_1 : I_2 : .... I_n = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} : .... : \frac{1}{R_n}$.

$I_1 = \frac{G_1}{G_1 + G_2 + ... + G_n} I$, $I_2 = \frac{G_2}{G_1 + G_2 + ... + G_n} I$, etc.

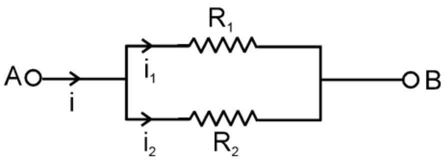
where $G = \frac{1}{R}$ = Conductance of a resistor. [Its unit is Ω^{-1} or ℧ (mho)]

Solved Example

Example 27. When two resistors are in parallel combination then determine i_1 and i_2, if the combination carries a current i ?

Solution : ∴ $i_1R_1 = i_2R_2$

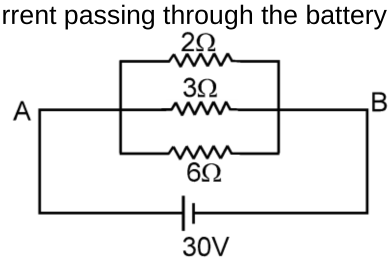
or $\frac{i_1}{i_2} = \frac{R_2}{R_1}$



$i_1 = \frac{R_2 i}{R_1 + R_2}$ ⇒ $i_2 = \frac{R_1 i}{R_1 + R_2}$,

Note : Remember this law of $i \propto \frac{1}{R}$ in the resistors connected in parallel. It can be used in problems.

Example 28. Find current passing through the battery and each resistor.



Solution : **Method (I) :**

It is easy to see that potential difference across each resistor is 30 V.

$\therefore$ current is each resistors are $\frac{30}{2}$ = 15 A, $\frac{30}{3} = 10\,A$ and $\frac{30}{6} = 5\,A$

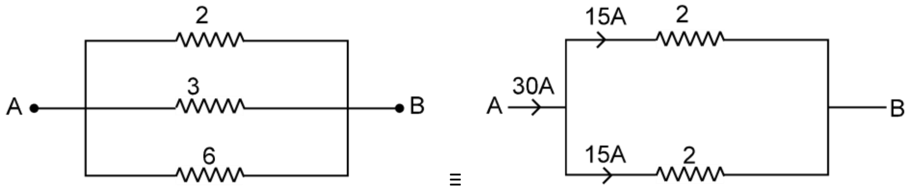
$\therefore$ Current through battery is = 15 + 10 + 5 =30 A.

Method (II) :

By ohm's law $i = \frac{V}{R_{eq}}$ $\Rightarrow$ $\frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1\Omega$

$R_{eq} = 1\ \Omega$ $\Rightarrow$ $i = \frac{30}{1} = 30$ A

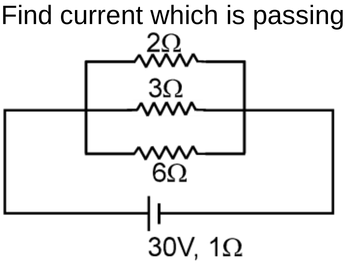
Now distribute this current in the resistors in their inverse ratio.



Current total in 3 Ω and 6 Ω is 15 A it will be divided as 10 A and 5 A.

Note : The method (I) is better. But you will not find such an easy case every where.

Exercise 29. Find current which is passing through battery.



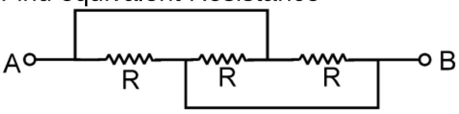
Solution : Here potential difference across each resistor is not 30 V

$\because$ battery has internal resistance. Here the concept of combination of resistors is useful.

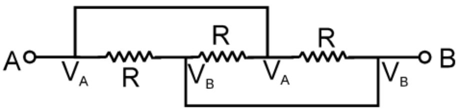
$R_{eq} = 1 + 1 = 2\ \Omega$

$i = \frac{30}{2} = 15$ A.

Example 30. Find equivalent Resistance



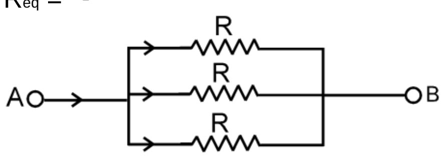
Solution :



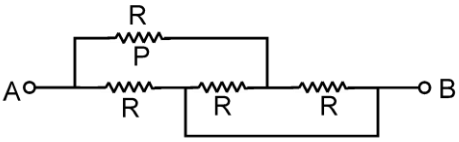
Here all the Resistance are connected between the terminals A and B

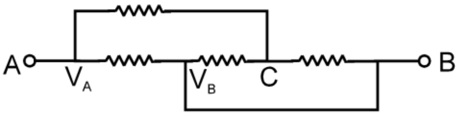
Modified circuit is

So $R_{eq} = \frac{R}{3}$

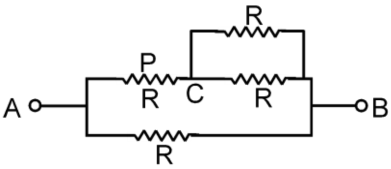


Example 31. Find the current in Resistance P if voltage supply between A and B is V volts

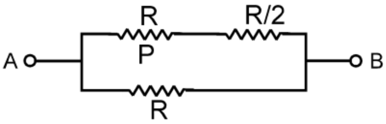




Solution : $R_{eq} = \frac{3R}{5}$



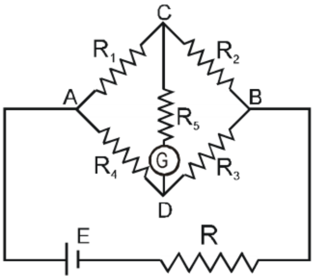
$I = \frac{5V}{3R}$ Modified circuit



Current in P = $\frac{R \times \frac{5V}{3R}}{1.5R + R}$

$= \frac{2V}{3R}$

14. WHEATSTONE NETWORK : (4 TERMINALNETWORK)



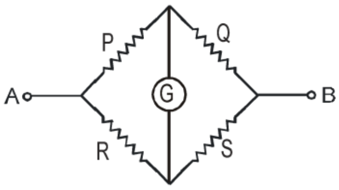
The arrangement as shown in figure, is known as Wheat stone bridge

Here there are four terminals in which except two all are connected to each other through resistive elements.

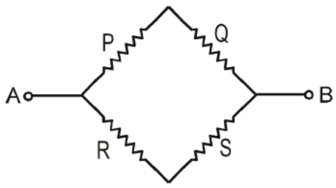
In this circuit if $R_1 R_3 = R_2 R_4$ then $V_C = V_D$ and current in $R_5 = 0$ this is called balance point or null point

When current through the galvanometer is zero (null point or balance point) $\frac{P}{Q} = \frac{R}{S}$, then PS = QR ⇒ Here in this case products of opposite arms are equal. Potential difference between C and D at null point is zero. The null point is not affected by resistance R_5, E and R. **It is not affected even if the positions of Galvanometer and battery (E) are interchanged.**

hence, here the circuit can be assumed to be following,

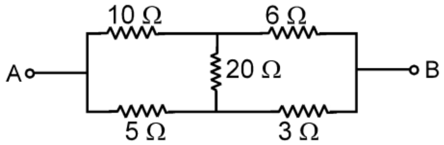


or



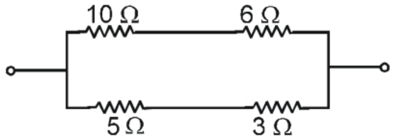
Solved Examples

Example 32. Find equivalent resistance of the circuit between the terminals A and B.



Solution : Since the given circuit is wheat stone bridge and it is in balance condition.

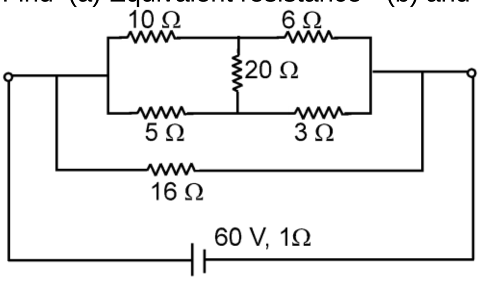
$\because$ $10\times3 = 30 = 6\times5$



hence this is equivalent to

$R_{eq} = \frac{16\times8}{16+8} = \frac{16}{3}\Omega$

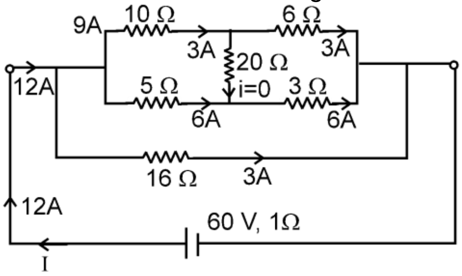
Example 33. Find (a) Equivalent resistance (b) and current in each resistance



Solution : (a) $R_{eq} = \left(\frac{1}{16}+\frac{1}{8}+\frac{1}{16}\right)^{-1} + 1 = 5\Omega$

(b) $i = \frac{60}{4+1}$ = 12 A

Hence 12 A will flow through the cell.



By using current distribution law.

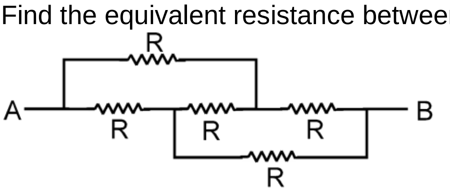
Current in resistance 10Ω and 6Ω = 3A

Current in resistance 5Ω and 3Ω = 6A

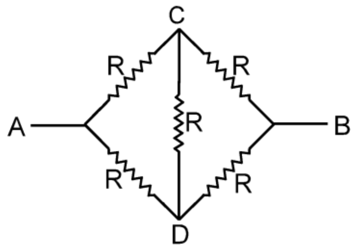
Current in resistance 20Ω = 0

Current in resistance 16Ω = 3A

Example 34. Find the equivalent resistance between A and B

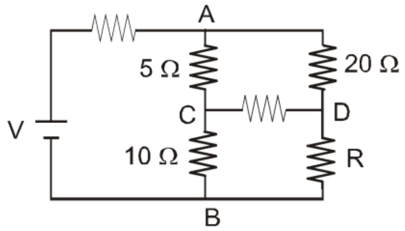


Solution : This arrangement can be modified as shown in figure since it is balanced wheat stone bridge



$$R_{eq} = \frac{2R \times 2R}{2R + 2R} = R$$

Example 35. Determine the value of R in the circuit shown in figure, when the current is zero in the branch CD.



Solution : The current in the branch CD is zero, if the potential difference across CD is zero.
That means, voltage at point C = voltage at point D.
Since no current is flowing, the branch CD is open circuited. So the same voltage is applied across ACB and ADB

$V_{10} = V \times \frac{10}{15}$ $\Rightarrow$ $V_R = V \times \frac{R}{20+R}$

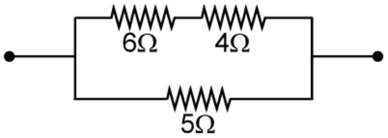
$\therefore$ $V_{10} = V_R$ and $V \times \frac{10}{15} = V \times \frac{R}{20+R}$

$\therefore$ $R = 40\ \Omega$ **Ans.**

Self Practice Problems

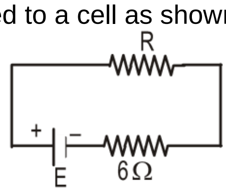
16. The resistance of each arm of the wheat stone bridge is 10 ohm. A resistance of 10 ohm is connected in series with galvenometer then the equivalent resistance across the battery will be :-
(1) 10 ohm (2) 15 ohm (3) 20 ohm (4) 40 ohm

17. In the circuit shown the 5 Ω resistor develops 20 W due to current flowing through it. Then power dissiptad in 4W resistor is:-



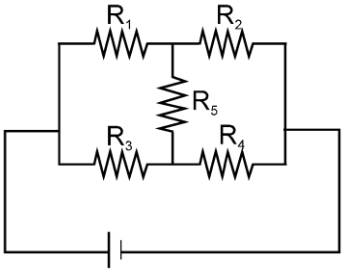
(1) 4 W (2) 6W (3) 10 w (4) 20 W

18. A resistor R is connected to a cell as shown in the figure. The value of R for which in its is maximum is:-



(1) 3Ω (2) 6Ω (3) 9Ω (4) 12Ω

19. The bridge show in figure will be balanced when:-

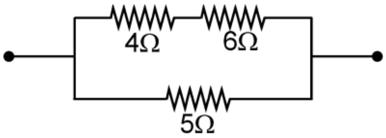


(1) $R_1/R_2 = R_3/R_4$ (2) $R_1/R_2 = R_4/R_3$ (3) $R_1R_3 = R_2R_4$ (4) $R_5 = 0$

20. In a Wheatstone bridge P = Q = 10 ohm and R = S = 15 Ω and G = 20 Ω and a cell of e.m.f. 1.5 V is connected in circuit. The current drawn from the cell is:-

(1) 0.125 A (2) 0.060 A (3) 0.025 A (4) 0.021 A

21. For the circuit shown rate of heat produced in the 5 ohm resistance is 10 cal/sec. then the rate of heat produced in 4 ohm resistance will be:



(1) 10 cal/sec. (2) 2 cal/sec. (3) 3 cal/sec. (4) 4 cal/sec.

Answer : 16. (1) **17.** (1) **18.** (2) **19.** (1) **20.** (1) **21.** (2)

15. GROUPING OF CELLS

15.1 Cells in Series:

Equivalent EMF

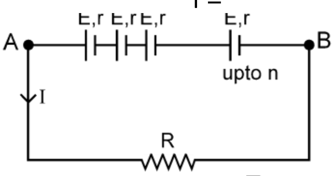
$E_{eq} = E_1 + E_2 + ... + E_n$ [write EMF's with polarity]

Equivalent internal resistance

$r_{eq} = r_1 + r_2 + r_3 + r_4 + ... + r_n$

If n cells each of emf E, arranged in series and if r is internal resistance of each cell, then total emf = n E so current in the circuit

$I = \frac{nE}{R+nr}$



If nr << R then $I = \frac{nE}{R}$ → Series combination is advantageous.

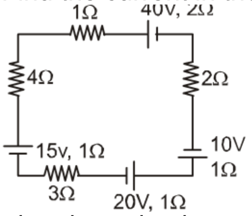
If nr >> R then $I = \frac{E}{r}$ → Series combination is not advantageous.

Note: If polarity of m cells is reversed, then equivalent emf = (n-2m)E while the equivalent resistance is still nr+R, so current in R will be

$i = \frac{(n-2m)E}{nr+R}$.

Solved Examples

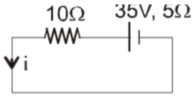
Example 36. Find the current in the loop.



Solution : The given circuit can be simplified as

$i = \frac{35}{10+5} = \frac{35}{15}$

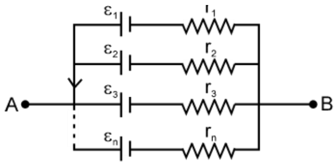
$= \frac{7}{3}$ A $\Rightarrow$ $I = \frac{7}{3}$ A



15.2 Cells in Parallel :

$$E_{eq} = \frac{\varepsilon_1/r_1 + \varepsilon_2/r_2 + ... + \varepsilon_n/r_n}{1/r_1 + 1/r_2 + ... + 1/r_n}$$

[Use emf's with polarity]

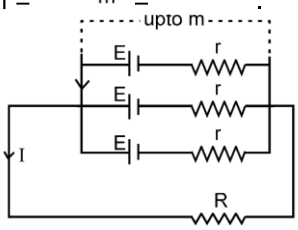


$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + ... + \frac{1}{r_n}$$

If m cells each of emf E and internal resistance r be connected in parallel and if this combination is connected to an external resistance then equivalent emf of the circuit = E.

Internal resistance of the circuit = $\frac{r}{m}$.

and $I = \frac{E}{R + \frac{r}{m}} = \frac{mE}{mR + r}$.



If mR << r ; $I = \frac{mE}{r}$ –→ Parallel combination is advantageous.

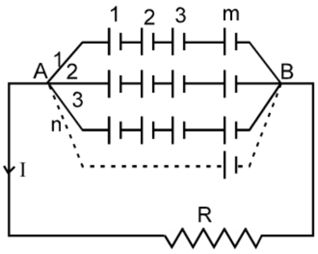
If mR >> r ; $I = \frac{E}{R}$ –→ Parallel combination is not advantageous.

15.3 Cells in Multiple Arc :

mn = number of identical cells.
n = number of rows
m = number of cells in each row.
The combination of cells is equivalent to single cell of

emf = mE

and internal resistance = $\frac{mr}{n}$



Current $I = \dfrac{mE}{R + \frac{mr}{n}}$

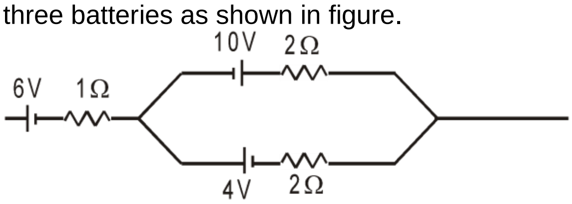
For maximum current nR = mr

or $R = \dfrac{mr}{n}$ = internal resistance of the equivalent battery.

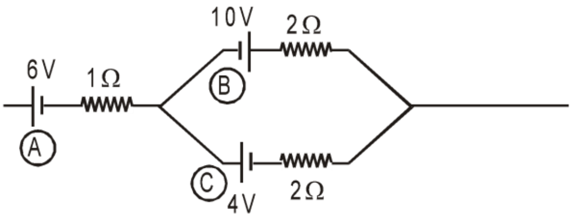
$I_{max} = \dfrac{nE}{2r} = \dfrac{mE}{2R}$.

Solved Examples

Example 37. Find the emf and internal resistance of a single battery which is equivalent to a combination of three batteries as shown in figure.



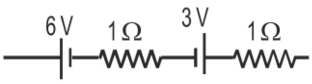
Solution :



Battery (B) and (C) are in parallel combination with opposite polarity. So, their equivalent

$\varepsilon_{BC} = \dfrac{\frac{10}{2} + \frac{-4}{2}}{\frac{1}{2} + \frac{1}{2}} = \dfrac{5-2}{1} = 3V \quad \Rightarrow \quad r_{BC} = 1\Omega.$

Now,



$\varepsilon_{ABC} = 6 - 3 = 3V \qquad r_{ABC} = 2\Omega.$ **Ans.**

Self Practice Problems

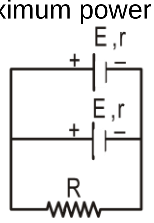
22. Two e.m.f. source of e.m.f. E_1 and E_2 and internal resistance r_1 and r_2 are connected in parallel. The e.m.f. of this combination is:-

(1) $\dfrac{E_1 + E_2}{2}$ (2) $\dfrac{E_1r_1 + E_2r_2}{r_1 + r_2}$ (3) $\dfrac{E_2r_1 + E_1r_2}{r_1 + r_2}$ (4) $\dfrac{E_1 + E_2}{E_1 + E_2}$

23. In a torch there are two cells each of 1.45 volt and of internal resistance 0.15Ω. Each cell gives a current to the filament of a lamp length of resistance 1.5Ω, then the value of current in ampere is:-

(1) 16.11 (2) 1.611 (3) 0.1611 (4) 2.6

24. Two cells of same emf E and internal resistance r are connected in in parallel with a resistance of R. To get maximum power in the external circuit, the value of R is:-

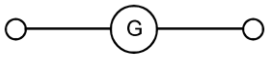


(1) R = $\frac{r}{2}$ (2) R = r (3) R = 2r (4) R = 4r

Answer : 22. (3) **23.** (2) **24.** (1)

16. GALVANOMETER

Galvanometer is represented as follow :



It consists of a pivoted coil placed in the magnetic field of a permanent magnet. Attached to the coil is a spring. In the equilibrium position, with no current in the coil, the pointer is at zero and spring is relaxed. When there is a current in the coil, the magnetic field exerts a torque on the coil that is proportional to current. As the coil turns, the spring exerts a restoring torque that is proportional to the angular displacement. Thus, the angular deflection of the coil and pointer is directly proportional to the coil current and the device can be calibrated to measure current.
When coil rotates the spring is twisted and it exerts an opposing torque on the coil.
There is a resistive torque also against motion to damp the motion. Finally in equilibrium

$\tau_{magnetic} = \tau_{spring} \Rightarrow BINA \sin\theta = C\phi$

But by making the magnetic field radial θ = 90º.

$\therefore$ BINA = C ϕ

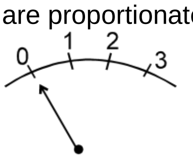
I ∝ ϕ

here B = magnetic field, A = Area of the coil
I = Current, C = torsional constant
N = Number of turns, ϕ = angle rotate by coil.

- **Current sensitivity**
  The ratio of deflection to the current i.e. deflection per unit current is called current sensitivity (C.S.) of the galvanometer CS = $\frac{\phi}{I} = \frac{BNA}{C}$

Note :

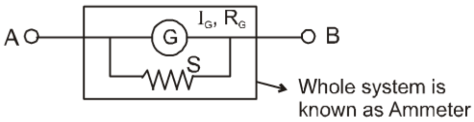
Shunting a galvanometer decreases its current sensitivity.
A linear scale is obtained. The marking on the galvanometer are proportionate.

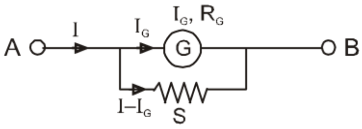


The galvanometer coil has some resistance represented by R_g . It is of the order of few ohms. It also has a maximum capacity to carry a current known as I_g. I_g is also the current required for full scale deflection. This galvanometer is called moving coil galvanometer.

17. AMMETER

A shunt (small resistance) is connected in parallel with galvanometer to convert it into ammeter; An ideal ammeter has zero resistance.
Ammeter is represented as follow -





If maximum value of current to be measured by ammeter is I then

$I_G \cdot R_G = (I - I_G)S$

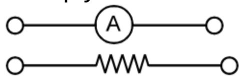
$$S = \frac{I_G . R_G}{I - I_G}$$

$$S = \frac{I_G \times R_G}{I} \quad \text{when} \quad I >> I_G.$$

where I = Maximum current that can be measured using the given ammeter.

For measuring the current the ammeter is connected is series.

In calculation it is simply a resistance



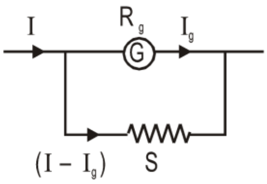
Resistance of ammeter

$$R_A = \frac{R_G . S}{R_G + S} \quad \text{for} \quad S << R_G \quad \Rightarrow \quad R_A = S$$

Solved Examples

Example 38. What is the value of shunt which passes 10% of the main current through a galvanometer of 99 ohm ?

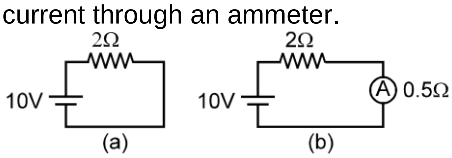
Solution :



As in figure $R_g I_g = (I - I_g)S$

$$\Rightarrow \quad 99 \times \frac{I}{10} = \left(I - \frac{I}{10}\right) \times S \quad \Rightarrow \quad S = 11\ \Omega.$$

Example 39. Find the current in the circuit (a) & (b) and also determine percentage error in measuring the current through an ammeter.



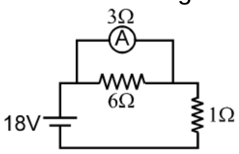
Solution : In A $I = \frac{10}{2} = 5A$

In B $I = \frac{10}{2.5} = 4A$

Percentage error is = $\frac{i - i'}{i} \times 100 = 20\%$ **Ans.**

Here we see that due to ammeter the current has reduced. A good ammeter has very low resistance as compared with other resistors, so that due to its presence in the circuit the current is not affected.

Example 40. Find the reading of ammeter ? Is this the current through 6 Ω ?



Solution : $R_{eq} = \frac{3\times 6}{3+6} + 1 = 3\ \Omega$

Current through battery

$I = \frac{18}{3} = 6\ A$

So, current through ammeter

$= 6 \times \frac{6}{9} = 4\ A$

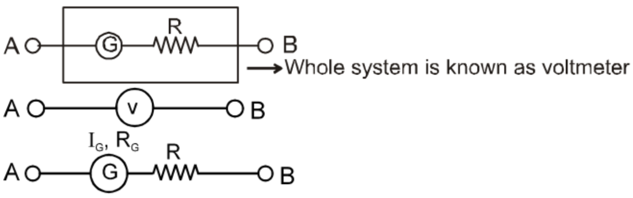
No, it is not the current through the 6 Ω resistor.

Note :

- Ideal ammeter is equivalent to zero resistance wire for calculation potential difference across it is zero.

18. VOLTMETER

A high resistance is put in series with galvanometer. It is used to measure potential difference across a resistor in a circuit.



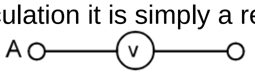
For maximum potential difference

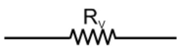
$V = I_G \cdot R + I_G R_G$ $\quad\quad R = \frac{V}{I_G} - R_G$

If $R_G << R$ $\quad \Rightarrow \quad R_S \approx \frac{V}{I_G}$

For measuring the potential difference a voltmeter is connected across that element. (parallel to that element it measures the potential difference that appears between terminals 'A' and 'B'.)

For calculation it is simply a resistance





Resistance of voltmeter $R_V = R_G + R \approx R$

$I_g = \frac{V_o}{R_g + R}$. $\quad R \to \infty \Rightarrow$ Ideal voltmeter.

A good voltmeter has high value of resistance.

Ideal voltmeter → which has high value of resistance.

Note :

- For calculation purposes the current through the ideal voltmeter is zero.
- Percentage error in measuring the potential difference by a voltmeter is $= \frac{V - V'}{V} \times 100$

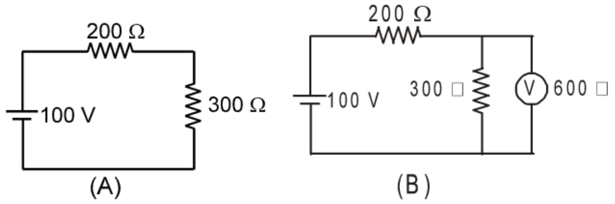
Solved Example

Example 41. A galvanometer has a resistance of G ohm and range of V volt. Calculate the resistance to be used in series with it to extend its range to nV volt.

Solution : Full scale current $i_g = \frac{V}{G}$
to change its range

$$V_1 = (G + R_s)i_g \Rightarrow nV = (G + R_s)\frac{V}{G} \Rightarrow R_s = G(n-1)$$ **Ans.**

Example 42. Find potential difference across the resistance 300 Ω in A and B.



Solution : In (A) : Potential difference = $\frac{100}{200+300} \times 300$ = 60 volt

In (B) : Potential difference = $\frac{100}{200 + \frac{300 \times 600}{300+600}} \times \frac{300 \times 600}{300+600}$ = 50 volt

We see that by connecting voltmeter the voltage which was to be measured has changed. Such voltmeters are not good. If its resistance had been very large than 300 Ω then it would not have affected the voltage by much amount.

Current sensitivity

The ratio of deflection to the current i.e. deflection per unit current is called current sensitivity (C.S.) of the galvanometer CS = $\frac{\theta}{I}$

Note :

- Shunting a galvanometer decreases its current sensitivity.

Solved Examples

Example 43. A galvanometer with a scale divided into 100 equal divisions, has a current sensitivity of 10 division per mA and voltage sensitivity of 2 division per mV. What adoptions are required to use it (a) to read 5A full scale and (b) 1 division per volt ?

Solution : Full scale deflection current $i_g = \frac{\theta}{cs} = \frac{100}{10}$ mA = 10 mA

Full scale deflection voltage $V_g = \frac{\theta}{vs} = \frac{100}{2}$ mv = 50 mv

So galvanometer resistance $G = \frac{V_g}{i_g} = \frac{50mV}{10mA} = 5\ \Omega$

(a) to convert the galvanometer into an ammeter of range 5A, a resistance of value SΩ is connected in parallel with it such that

$(I - i_g)\,S = i_g\,G$

$(5 - 0.01)\,S = 0.01 \times 5$

$S = \frac{5}{499} \cong 0.01\ \Omega$ **Ans.**

(b) To convert the galvanometer into a voltmeter which reads 1 division per volt, i.e. of range 100 V,

$V = i_g (R + G)$

$100 = 10 \times 10^{-3} (R + 5)$

$R = 10000 - 5$

$R = 9995\ \Omega \cong 9.995\ k\Omega$ **Ans.**

19. POTENTIOMETER

Necessity of potentiometer

Practically voltameter has a finite resistance. (ideally it should be) in other words it draws some current from the circuit. To overcome this problem potentiometer is used because at the instant of measurement, it draws no current from the circuit.

Working principle of potentiometer

Any unknown potential difference is balanced on a known potential difference which is uniformly distributed over entire length of potentiometer wire.

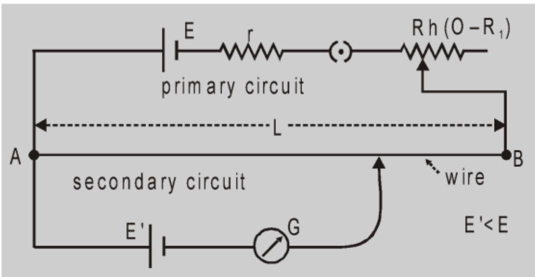
This process is named as zero deflection or null deflection method.

Note :

(i) Potentiometer wire : Made up of alloys of magnin, constantan. Eureka.

(ii) Specific properties of these alloys are high specific resistance, negligible temprature co-efficient of resistance (α). Invariability of resistance of potentiometer wire over a long period.

Circuits of potentiometer



Primary circuit contains constant source of voltage rheostat or Resistance Box

Secondary, Unknown or galvanometer circuit

Let ρ = Resistance per unit length of potentiometer wire

Potential gradient (x)

◎ Potential gradient corresponding to unit length of potentiometer is also called potential gradient.

◎ Rate of growth of potential per unit length of potentiometer wire is equal to potential gradiant.

Let r = 0 and $R_1 = 0$ then $V_{AB} = E$ (max. ideal) then $x = \frac{E}{L}$ (V/m ; $MLT^{-3}A^{-1}$)

◎ Always $V_{AB} < E$;

$x = \frac{V_{AB}}{L}$ Now $V_{AB} = I R_P$ (R_P = resistance of potentiometer wire)

So $x = \frac{I R_P}{L} = I\rho$ $\rho = \frac{R_P}{L}$

current in primary circuit $I = \frac{E}{R_1 + r + R_P}$; $x = \frac{E}{R_1 + R_P + r}\left(\frac{R_P}{L}\right)$

◎ If radius is uniform = x is uniform over entire length of potentiometer wire.

◎ If I constant $x \propto \frac{1}{(radius)^2}$

◎ 'x' directly depends on → ρ, r , σ etc.

Factor affecting 'x'

◎ If V_{AB} = const. and L = const. then for any change → x remains unchanged.

◎ If there is no information about V_{AB} then Always take V_{AB} as constant so ($x \propto \frac{1}{L}$)

◎ If V_{AB} and L are constant :

For any change like radius of wire, substance of wire (σ) there is no change in x.

◎ Any change in secondary circuit causes no change in x because x is an element of primary circuit.

Note : $x = \frac{E}{R_P + r + R_1}\left(\frac{R_P}{L}\right)$

x_{max} or x_{min} on the basis of range of rheostat or resistance box (R.B.)

If $R_1 = 0 \Rightarrow x_{max} = \frac{E}{R_P} \times \frac{R_P}{L}$ (r ˜– 0)

If $R_1 = R \Rightarrow x_{min} = \frac{E}{R_P + R}\left(\frac{R_P}{L}\right)$

then $\frac{x_{max}}{x_{min}} = \frac{R_P + R}{R_P}$

Standarization and sensitivity of potentiometer

Standardization process of evaluating x experimentally

If balanced length for standard cell (emf E) is = ℓ_0 then potential gradient $x = \frac{E}{\ell_0}$

Sensitivity :

(i) x also indicates about sensitivity of potentiometer.

(ii) If x ↓ ⇒ sensitivity ↑

(iii) To increase sensitivity → R_h ↑ (current in primary ckt should be reduced), L ↑

(iv) Any change in secondary ckt, no effect on sensitivity.

(v) Balanced length for unknown potential difference ↑ ⇒ sensitivity ↑

Applications of potentiometer

(1) To measure potential difference across a resistance.

(2) To find out emf of a cell.

(3) Comparision of two emfs $\frac{E_1}{E_2}$

(4) To find out internal resistance of a primary cell.

(5) Comparision of two resistances.

(6) To find out an unknown resistance which is connected in series with the given resistance.

(7) To find out current in a given circuit.

(8) Calibration of an ammeter or to have a check on reading of (A)

(9) Calibration of a voltmeter or to have a check on reading of (V)

(10) To find out thermocouple emf (e_t) (mV or μV)

Note :

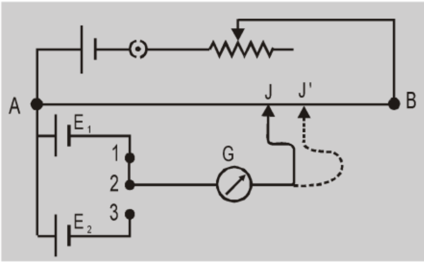
(i) For application 3-6 no need of standard cell and no need of value of x.

(ii) For 7, 8, 9, 10 - Always require a standard cell ($E_0 = x\ell_0$)

(iii) For 1 – 9 order of voltage drop (0.1 to 1v)

(a) Comparision of emf of two cells

plug only in (1– 2) plug only in (2 – 3)

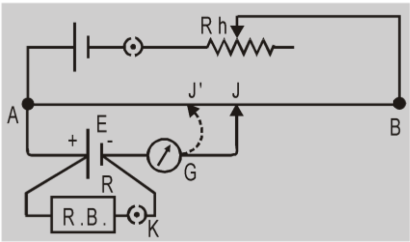


Jocky is at position J — balance length AJ = ℓ_1 — $E_1 = x\ell_1$

Jocky is at position J' — balance length AJ' = ℓ_2 — $E_2 = x\ell_2$ $\Rightarrow \dfrac{E_1}{E_2} = \dfrac{\ell_1}{\ell_2}$

(b) Internal resistance of a given primary cell

E = V + I r $\Rightarrow$ $r = \dfrac{E-V}{I}$ or $r = \left(\dfrac{E-V}{V}\right)R$



Key K open — $E = x\ell_1$ (AJ = ℓ_1)

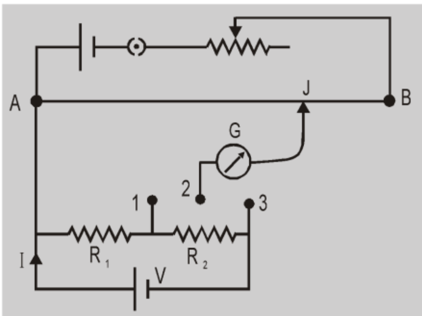
Key K closed T.P.D. — $V = x\ell_2$ (AJ' = ℓ_2)

$$\boxed{r = \left(\frac{\ell_1 - \ell_2}{\ell_2}\right)R}$$

(c) Comparision of two resistances

Plug only in (1–2)

potential difference across R_1 is balanced



$I R_1 = x\ell_1$

Plug only in (2-3)

potential difference across (R_1+R_2) is balanced

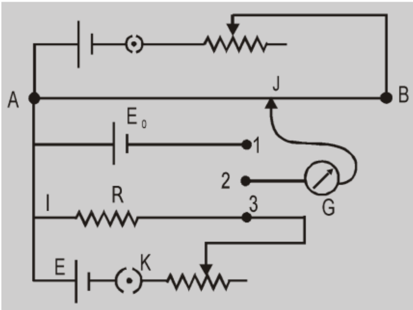
$I (R_1+R_2) = x\ell_2$ $\dfrac{R_1+R_2}{R_1} = \dfrac{\ell_2}{\ell_1}$ $\Rightarrow$ $\dfrac{R_2}{R_1} = \dfrac{\ell_2-\ell_1}{\ell_1}$

(d) Measurement of current

Plug only in (1–2)

$E_0 = x\ell_0$

Plug only in (2–3)



$V = I\ R = x\ell_1$ $\qquad I = \frac{\ell_1}{R} \times \frac{E_0}{\ell_0}$

Solved Examples

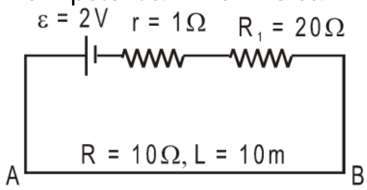
Example 44. Primary circuit of potentiometer is shown in figure determine :

(a) current in primary circuit

(b) potential drop across potentiometer wire AB

(c) potential gradient (means potential drop per unit length of potentiometer wire)

(d) maximum potential which we can measure above potentiometer



Solution : (a) $i = \frac{\varepsilon}{r + R_1 + R} = \frac{2}{1+20+10}$ $\Rightarrow$ $i = \frac{2}{31}$ A **Ans.**

(b) $V_{AB} = iR = \frac{2}{31} \times 10$ $\Rightarrow$ $V_{AB} = \frac{20}{31}$ volt **Ans.**

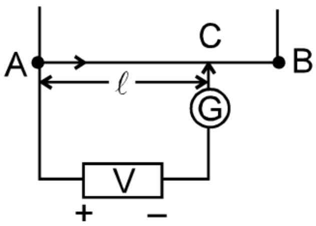
(c) $x = \frac{V_{AB}}{L} = \frac{2}{31}$ volt/m **Ans.**

(d) Maximum potential which we can measure by it = potential drop across wire AB

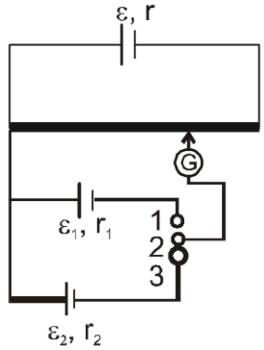
$= \frac{20}{31}$ volt **Ans.**

Example 45. How to measure an unknown voltage using potentiometer.

Solution : The unknown voltage V is connected across the potentiometer wire as shown in figure. The positive terminal of the unknown voltage is kept on the same side as of the source of the top most battery. When reading of galvanometer is zero then we say that the meter is balanced. In that condition $V = x\ \ell$.



Example 46. In an experiment to determine the em$f$ of an unknown cell, its emf is compared with a standard cell of known emf ε_1 = 1.12 V. The balance point is obtained at 56cm with standard cell and 80 cm with the unknown cell. Determine the emf of the unknown cell.



Solution Here, ε_1 = 1.12 V; ℓ_1 = 56 cm; ℓ_2 = 80 cm

Using equation

$\varepsilon_1 = x\ell_1$ ....(1)

$\varepsilon_2 = x\ell_2$ ....(2)

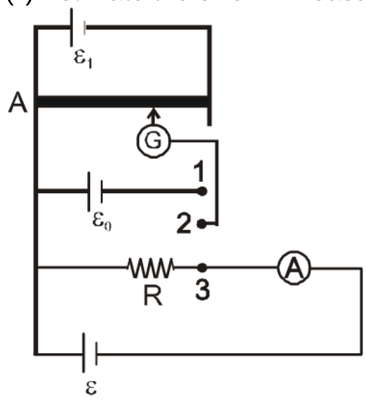
we get $\frac{\varepsilon_1}{\varepsilon_2} = \frac{\ell_1}{\ell_2}$ ⇒ $\varepsilon_2 = \varepsilon_1\left(\frac{\ell_2}{\ell_1}\right)$

or $\varepsilon_2 = 1.12\left(\frac{80}{56}\right)$ = 1.6 V Ans

Example 47. A standard cell of emf ε_0 = 1.11 V is balanced against 72 cm length of a potentiometer. The same potentiometer is used to measure the potential difference across the standard resistance R = 120 Ω. When the ammeter shows a current of 7.8 mA, a balanced length of 60 cm is obtained on the potentiometer.

(i) Determine the current flowing through the resistor.

(ii) Estimate the error in measurement of the ammeter.



Solution : Here, ℓ_0 = 72 cm ; ℓ = 60 cm; R = 120 Ω and ε_0 = 1.11 V

(i) By using equation $\varepsilon_0 = x\,\ell_0$ ....(i)

$V = IR = x\,\ell$ ....(ii)

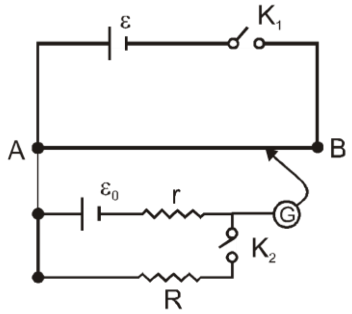
From equation (i) and (ii)

$I = \frac{\varepsilon_0}{R}\left(\frac{\ell}{\ell_0}\right)$ ∴ $I = \frac{1.11}{120}\left(\frac{60}{72}\right)$ = 7.7 mA

(ii)Since the measured reading 7.8 mA (> 7.7 mA) therefore, the instrument has a positive error.

ΔI = 7.8 – 7.7 = 0.1 mA, $\frac{\Delta I}{I} = \frac{0.1}{7.7} \times 100 = 1.3\ \%$

Example 48. The internal resistance of a cell is determined by using a potentiometer. In an experiment, an external resistance of 60Ω is used across the given cell. When the key is closed, the balance length on the potentiometer decreases from 72 cm to 60 cm. calculate the internal resistance of the cell.



Solution : According to equation $\varepsilon_0 = x\ell_0$ ....(i)

$V = IR = x\ell$ ....(ii)

$I = \dfrac{\varepsilon_0}{R+r}$ .....(iii)

From equation (i), (ii) and (iii) we get

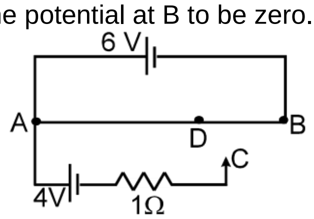
$r = R\left(\dfrac{\ell_0}{\ell} - 1\right)$

here $\ell_0 = 72$ cm; $\ell = 60$ cm; $R = 60\ \Omega$

$\therefore$ $r = (60)\left(\dfrac{72}{60} - 1\right)$ or $r = 12\ \Omega$.

Example 49.
Comprehension

A 6 volt battery of negligible internal resistance is connected across a uniform wire AB of length 100 cm. The positive terminal of another battery of emf 4V and internal resistance 1Ω is joined to the point A as shown in figure. Take the potential at B to be zero.



1. The potentials at the points A and C

(1) $V_A = 6$ V, $V_C = 2$V (2) $V_A = 3$V, $V_C = 2$V (3) $V_A = 2$V, $V_C = 3$V (4) None of these

2. Which point D of the wire AB, the potential is equal to the potential at C.

(1) AD = 200 (2) AD = $\dfrac{200}{3}$ (3) AD = $\dfrac{100}{3}$ (4) None of these

3. If the 4V battery is replaced by 7.5 V battery, what would be the potentials at the points A and C

(1) $V_A = 6$ V, $V_C = 2$V (2) $V_A = 6$ V, $V_C = 1.5$V
(3) $V_A = -6$ V, $V_C = 1.5$V (4) $V_A = 6$ V, $V_C = -1.5$V

Solution (Q. 1 to 3) :

(1) $V_A = 6$ V
$V_C = 2$V **Ans.**

(2) $E = x\,\ell$ $\Rightarrow$ $4 = \ell \times \dfrac{6}{100}$ $\Rightarrow$ $\ell = \dfrac{200}{3}$ **Ans.**

(3) 6 V, 6 – 7.5 = – 1.5 V, no such point D exists **Ans.**
6 V, 6 – 7.5 = – 1.5 V

20. METRE BRIDGE (USE TO MEASURE UNKNOWN RESISTANCE)

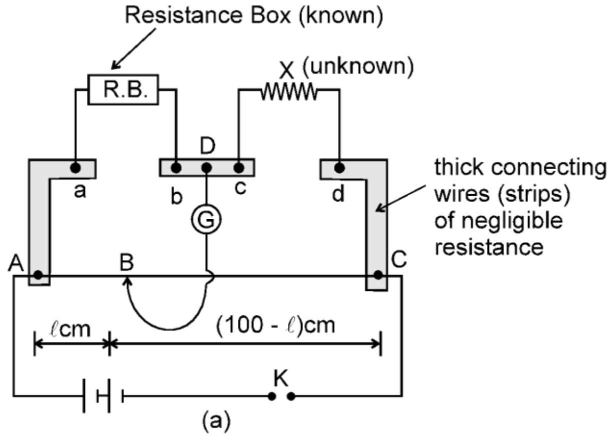
If AB = ℓ cm, then BC = $(100 - \ell)$ cm.

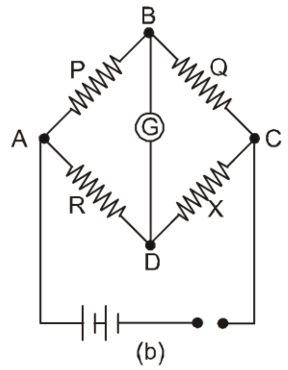
Resistance of the wire between A and B $R \propto \ell$

[∵ Specific resistance ρ and cross-sectional area A are same for whole of the wire]

or $R = \sigma\ell$ ...(1)

where σ is resistance per cm of wire.





Similarly, if Q is resistance of the wire between B and C, then

$$Q \propto 100 - \ell$$

∴ $Q = \sigma(100 - \ell)$ ....(2)

Dividing (1) by (2), $\dfrac{P}{Q} = \dfrac{\ell}{100-\ell}$

Applying the condition for balanced Wheatstone bridge, we get

$R\ Q = P\ X$ ∴ $x = R\dfrac{Q}{P}$ or $X = \dfrac{100-\ell}{\ell}R$

Since R and ℓ are known, therefore, the value of X can be calculated.

Note : For better accuracy, R is so adjusted that ℓ lies between 40 cm and 60 cm.

Solved Example

Example 50. In a meter bridge experiment, the value of unknown resistance is 2Ω. To get the balancing point at 40cm distance from the same end, the resistance in the resistance box will be :

(1) 0.5 Ω (2) 3 Ω (3) 20 Ω (4) 80 Ω

Solution : Apply condition for balance wheat stone bridge,

$\dfrac{P}{Q} = \dfrac{\ell}{100-\ell} = \dfrac{P}{2} = \dfrac{100-40}{40}$

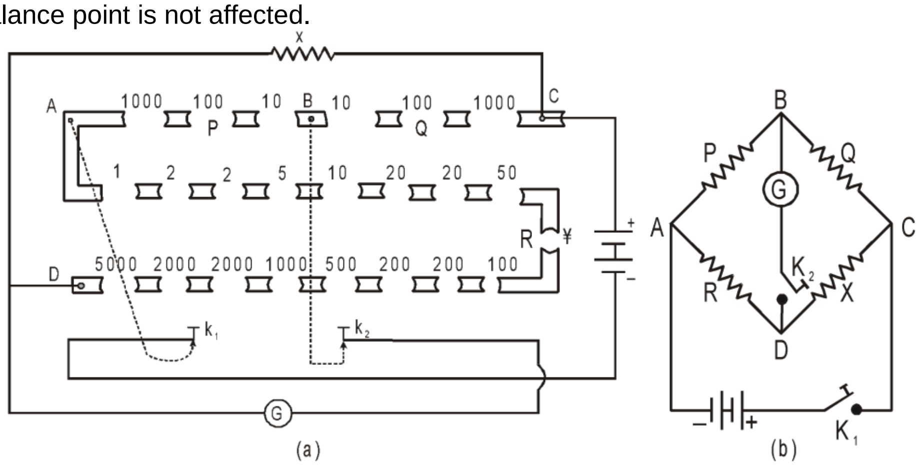
Ans. : P = 3Ω.

21. POST-OFFICE BOX (EXPERIMENT IN CBSE)

Introduction. It is so named because its shape is like a box and it was originally designed to determine the resistances of electric cables and telegraph wires. It was used in post offices to determine the resistance of transmission lines.

Construction. A post office box is a compact form of Wheatstone bridge with the help of which we can measure the value of the unknown resistance correctly up to 2nd decimal place, i.e., up to 1/100th of an ohm correctly. Two types of post office box are available - plug type and dial type. In the plug-type instrument shown in figure (a), each of the arms AB and BC contains three resistances of 10, 100 and

1000 ohm. These arms are called the ratio arms. While the resistance P can be introduced in the arm AB, the resistance Q can be introduced in the arm BC. The third arm AD, called the resistance arm, is a complete resistance box containing resistances from 1 Ω to 5,000 Ω. In this arm, the resistance R is introduced by taking out plugs of suitable values. The unknown resistance X constitutes the fourth arm CD. Thus, the four arms AB, BC, CD and AD are infect the four arms of the Wheatstone bridge (figure (b)). Two tap keys K_1 and K_2 are also provided. While K_1 is connected internally to the terminal A, K_2 is connected internally to B. These internal connections are shown by dotted lines in figure (a).
A battery is connected between C and key K_1 (battery key). A galvanometer is connected between D and key K_2 (galvanometer key). Thus, the circuit is exactly the same as that shown in figure (b). It is always the battery key which is pressed first and then the galvanometer key. This is because a self-induced current is always set up in the circuit whenever the battery key is pressed or released. If we first press the galvanometer key, the balance point will be disturbed on account of induced current. If the battery key is pressed first, then the induced current becomes zero by the time the galvanometer key is pressed. So, the balance point is not affected.



(a) (b)

Working : The working of the post office box involves broadly the following four steps :

I. Keeping R zero, each of the resistances P and Q are made equal to 10 ohm by taking out suitable plugs from the arms AB and BC respectively. After pressing the battery key first and then the galvanometer key, the direction of deflection of the galvanometer coil is noted. Now, making R infinity, the direction of deflection is again noted. If the direction is opposite to that in the first case, then the connections are correct.

II. Keeping both P and Q equal to 10Ω, the value of R is adjusted, beginning from 1Ω, till 1 Ω increase reverses the direction of deflection. The 'unknown' resistance clearly lies somewhere between the two final values of R.

$$\left[X = R\frac{Q}{P} = R\frac{10}{10} = R\right]$$

As an illustration, suppose with 3Ω resistance in the arm AD, the deflection is towards left and with 4Ω, it is towards right. The unknown resistance lies between 3Ω and 4Ω.

III. Making P 100 Ω and keeping Q 10 Ω, we again find those values of R between which direction of deflection is reversed. Clearly, the resistance in the arm AD will be 10 times the resistance X of the wire.

$$\left[X = R\frac{Q}{P} = R\frac{10}{100} = \frac{R}{10}\right]$$

In the illustration considered in step II, the resistance in the arm AD will now lie between 30 Ω, and 40 Ω. So, in this step, we have to start adjusting R from 30 Ω onwards. If 32 Ω and 33 Ω are the two values of R which give opposite deflections, then the unknown resistance lies between 3.2 Ω and 3.3 Ω.

IV. Now, P is made 1000 Ω and Q is kept at 10 Ω. The resistance in the arm AD will now be 100 times the 'unknown' resistance.

$$\left[X = R\frac{10}{1000} = \frac{R}{100}\right]$$

In the illustration under consideration, the resistance in the arm AD will lie between 320 Ω and 330Ω. Suppose the deflection is to the right for 326 ohm, towards left for 324 ohm and zero deflection for 325Ω Then, the unknown resistance is 3.25 Ω.

The post office box method is a less accurate method for the determination of unknown resistance as compared to a metre bridge. This is due to the fact that it is not always possible to arrange resistance in the four arms to be of the same order. When the arms ratio is large, large resistance are required to be introduced in the arm R.

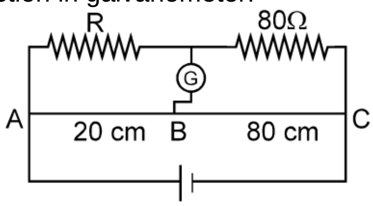
Self Practice Problems

26. The post office box works on the principle of :
(1) Potentiometer (2) Wheatstone bridge
(3) Matter waves (4) Ampere's law

27. While using a post office box the keys should be switched on in the following order :
(1) first cell key the and then galvanometer key. (2) first the galvanometer key and then cell key.
(3) both the keys simultaneously. (4) any key first and then the other key.

28. In a post office box if the position of the cell and the galvanometer are interchanged, then the :
(1) null point will not change (2) null point will change
(3) post office box will not work (4) Nothing can be said.

29. What is value of R for zero defliction in galvanometer:-



(1) 20 Ω (2) 80 Ω (3) 10 Ω (4) 40 Ω

30. A cell, an ammeter and a variable resistance R, are connected in series and a voltmeter is connected across R. For a certain value of R ammeter and voltmeter readings are 0.3 amp and 0.9 V respectively, and for some other value of R these readings are 0.25 amp. and 1.0 V. The internal resistance of the cell is:-
(1) 3.4 Ω (2) 4.3 Ω (3) 2.0 Ω (4) 4.6 Ω

31. In the measurement of a resistance by the Wheatstone bridge the known and the unknown resistance are interchanged to eliminate:-
(1) Minor error (2) Observational error
(3) Error due to thermo electric effect (4) Connection error

32. Which of the statement is wrong:-
(1) When all the resistance are equal, then the sensitivity of Wheatstone bridge is maximum.
(2) When the galvanometer and the cell are interchanged, then the balancing of Wheatstone bridge will be effected.
(3) Kirchhoff's first law for the current meeting at the junctions in an electric circuit shows the conservation of charge.
(4) Rheostat can be used as a potential divider

Answer : 26. (2) **27.** (1) **28.** (1) **29.** (1) **30.** (3) **31.** (1) **32.** (3)

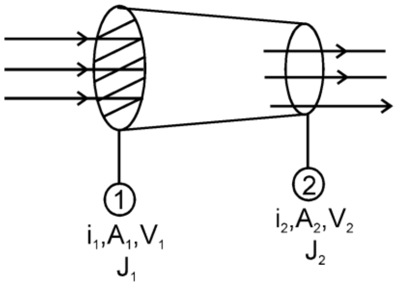
Solved Miscellaneous Problems

Problem 1. Current is flowing from a conductor of non-uniform cross section area if $A_1 > A_2$ then find relation between

(a) i_1 and i_2

(b) j_1 and j_2

(c) v_1 and v_2 (drift velocity)
where i is current, j is current density and V is drift velocity.



Answer : $i_1 = i_2$, $V_1 < V_2$, $J_1 < J_2$

Solution : (a) i = charge flowing through a cross-section per unit time.

$\therefore i_1 = i_2$

(b) $j = \frac{i}{A}$

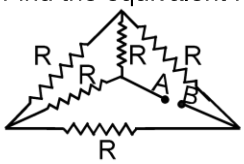
as $A_1 > A_2$ then $j_1 < j_2$

(c) $j = nev_d$

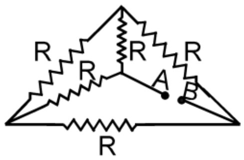
$v_d = \frac{j}{ne}$

as $j_1 < j_2$ then, $v_1 < v_2$

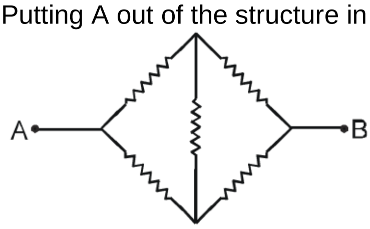
Problem 2. Find the equivalent Resistance between A and B

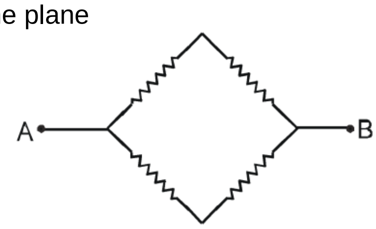


Solution :



Putting A out of the structure in the same plane





$R_{eq} = \frac{2R \times 2R}{2R + 2R} = R$

Ans. : $R_{eq} = R$

Problem 3. What shunt resistance is required to convert the 1.0 mA, 20Ω galvanometer into an ammeter with a range of 0 to 50mA ?

Answer : $S = \frac{20}{49} = 0.408\ \Omega$

Solution : $i_g R_g = (i - i_g)S$

$i_g = 1.0 \times 10^{-3}$ A , G = 20Ω

$i = 50 \times 10^{-3}$ A

$S = \frac{i_g R_g}{i - v_g} = \frac{1 \times 10^{-3} \times 20}{49 \times 10^{-3}} = 0.408\ \Omega$

Problem 4. How can we convert a galvanometer with R_g = 20 Ω and i_g = 1.0 mA into a voltmeter with a maximum range of 10 V ?

Answer : A resistance of 9980 Ω is to be connected in series with the galvanometer.

Solution : $v = i_g R_S + i_g R_g$

$10 = 1 \times 10^{-3} \times R_s + 1 \times 10^{-3} \times 20$

$R_S = \frac{10 - 0.02}{1 \times 10^{-3}} = \frac{9.98}{10^{-3}}$ = **9980 Ω**

Problem 5. A Potentiometer wire of 10 m length and having 10 ohm resistance, emf 2 volts and a rheostat. If the potential gradient is 1 micro volt/mm, the value of resistance in rheostat in ohms will be :

(1) 1.99 (2) 19.9 (3) 199 (4) 1990

Solution : d = 10 m , R = 10Ω ,

E = 2volts , $\frac{dv}{d\ell}$ = 1μ v/mm

$\frac{dv}{d\ell} = \frac{1 \times 10^{-6}}{1 \times 10^{-3}}$ v/m = 1×10^{-3} v/m

Across wire potential drop ,

$\frac{dv}{d\ell} \times \ell = 1 \times 10^{-3} \times 10$ = 0.01 volts

$i = \frac{0.01}{10} = 0.001 = \frac{E}{R + R'}$ (R′ = resistance of rheostat)

$R' = \frac{E}{0.001} - R = \frac{2}{0.001} - 10$ = 2000 – 10 = 1990 Ω

SUMMARY
CURRENT ELECTRICITY

- $I_{av} = \frac{\Delta q}{\Delta t}$ and $i_{inst.} = \frac{dq}{dt}$ ⇒ $q = \int i\,dt$ = area between current – time graph on time axis.
- Current $i = neAV_d$ n = no. of free electron per unit volume, A = cross–section area of conductor, V_d = drift velocity, e = charge on electron = 1.6×10^{-19} C
- Ohm's law V = IR
- $R = \frac{\rho\ell}{A}$ ρ = resistivity = $\frac{1}{\sigma}$, σ = conductivity
- Power P = VI ⇒ $P = I^2R = \frac{V^2}{R}$
- Energy = power × time (if power is constant.) otherwise energy, $E = \int P.dt$ where P is power.
- The rate at which the chemical energy of the cell in consumed = Ei
- The rate at which heat is generated inside the battery = i^2r
- Electric power output = (ε – ir) i
- Maximum power output when net internal resistance = net external resistance, R = r

Maximum power output = $\frac{\varepsilon^2}{4r}$

☞ In series combination $R = R_1 + R_2 + R_3 + \dots\dots\dots$

☞ In parallel combination $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots\dots$

☞ Cell in series combination

☞ $E_{eq} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \dots\dots + \varepsilon_n$ (write Emf's with polarity)

$r_{eq} = r_1 + r_2 + r_3 + \dots\dots\dots$

☞ Cells in parallel combination

$$E_{eq} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \dots\dots + \frac{\varepsilon_n}{r_n}}{\frac{1}{r_1} + \frac{1}{r_2} + \dots\dots\dots + \frac{1}{r_n}}$$ (Use proper sign before the EMFs for polarity)

and $\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \dots\dots + \frac{1}{r_n}$

☞ In ammeter shunt (S) = $\frac{I_G \times R_G}{I - I_G}$

☞ In voltmeter $V = I_G R_S + I_G R_G$

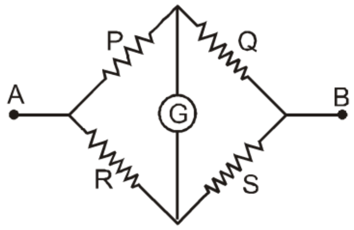
☞ Potential gradient in potentiometer : $x = \frac{\varepsilon}{R + r} \times \frac{R}{L}$

☞ $\frac{\theta_i + \theta_c}{2} = \theta_n$ where, θ_i = inversion temperature

θ_c = Temperature of cold junction

θ_n = Neutral temperature

☞ In balanced wheat stone bridge



$\frac{P}{R} = \frac{Q}{S}$