WAVE OPTICS

1. WAVEFRONTS

Consider a wave spreading out on the surface of water after a stone is thrown in. Every point on the surface oscillates. At any time, a photograph of the surface would show circular rings on which the disturbance is maximum. Clearly, all points on such a circle are oscillating in phase because they are at the same distance from the source. Such a locus of points which oscillate in phase is an example of a wavefront.

A wavefront is defined as a surface of constant phase. The speed with which the wavefront moves outwards from the source is called the phase speed. The energy of the wave travels in a direction perpendicular to the wavefront.

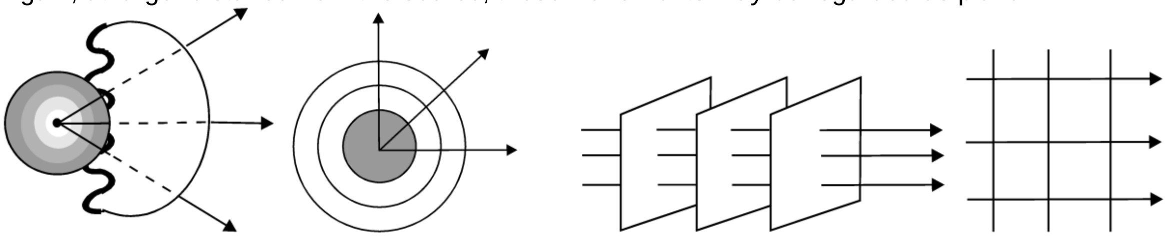
Figure (a) shows light waves from a point source forming a spherical wavefront in three dimensional space. The energy travels outwards along straight lines emerging from the source. i.e.. radii of the spherical wavefront. These lines are the rays. Notice that when we measure the spacing between a pair of wavefronts along any ray, the result is a constant. This example illustrates two important general principles which we will use later:

(i) Rays are perpendicular to wavefronts.

(ii) The time taken by light to travel from one wavefront to another is the same along any ray.

If we look at a small portion of a spherical wave, far away from the source, then the wavefronts are like parallel planes. The rays are parallel lines perpendicular to the wavefronts. This is called a plane wave and is also sketched in Figure (b)

A linear source such as a slit Illuminated by another source behind it will give rise to cylindrical wavefronts. Again, at larger distance from the source, these wave fronts may be regarded as planar.



(a) (b)

Figure : Wavefronts and the corresponding rays in. two cases: (a) diverging spherical wave. (b) plane wave. The figure on the left shows a wave (e.g.. light) in three dimensions. The figure on the right shows a wave in two dimensions (a water surface).

2. PRINCIPLE OF SUPERPOSITION :

When two or more waves simultaneously pass through a point, the disturbance of the point is given by the sum of the disturbances each wave would produce in absence of the other wave(s). In case of wave on string disturbance means displacement, in case of sound wave it means pressure change, in case of E.M.W. it is electric field or magnetic field. Superposition of two light traveling in almost same direction results in modification in the distribution of intensity of light in the region of superposition. This phenomenon is called *interference*.

2.1 SUPERPOSITION OF TWO SINUSOIDAL WAVES :

Consider superposition of two sinusoidal waves (having same frequency), at a particular point.

Let, $x_1(t) = a_1 \sin \omega t$

and, $x_2(t) = a_2 \sin (\omega t + \phi)$

represent the displacement produced by each of the disturbances. Here we are assuming the displacements to be in the same direction. Now according to superposition principle, the resultant displacement will be given by,

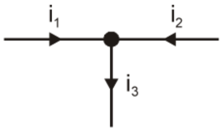
$x(t) = x_1(t) + x_2(t) = a_1 \sin \omega t + a_2 \sin (\omega t + \phi) = A \sin (\omega t + \phi_0)$

where $A^2 = a_1^2 + a_2^2 + 2a_1 . a_2 \cos \phi$ .... (1)

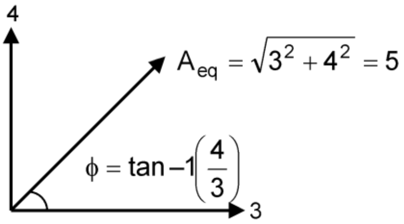
and $\tan \phi_0 = \dfrac{a_2 \sin\phi}{a_1 + a_2 \cos\phi}$ ....(2)

Solved Examples

Example 1. If $i_1 = 3\sin \omega t$ and $i_2 = 4 \cos \omega t$, find i_3.



Solution:



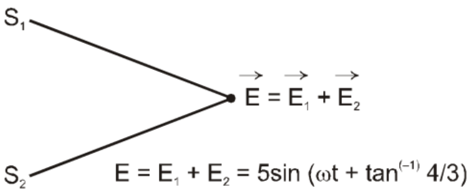
from kirchoff's current law,

$i_3 = i_1 + i_2$.

$= 3 \sin \omega t + 4 \sin (\omega t + \frac{\pi}{2}) = 5 \sin (\omega t + \tan^{-1}\left(\frac{4}{3}\right))$

Example 2. S_1 and S_2 are two source of light which produce individually disturbance at point P given by $E_1 = 3\sin \omega t$, $E_2 = 4 \cos \omega t$. Assuming $\vec{E_1}$ & $\vec{E_2}$ to be along the same line, find the result of their superposition.

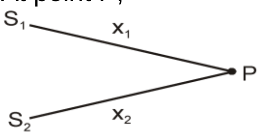
Solution :



3. SUPERPOSITION OF PROGRESSIVE WAVES; PATH DIFFERENCE :

Let S_1 and S_2 be two sources producing progressive waves (disturbance travelling in space given by y_1 and y_2)

At point P,



$y_1 = a_1 \sin (\omega t - kx_1 + \theta_1)$

$y_2 = a_2 \sin (\omega t - kx_2 + \theta_2)$

$y = y_1 + y_2 = A \sin(\omega t + \Delta\phi)$

Here, the phase difference,

$\Delta\phi = (\omega t - kx_1 + \theta_1) - (\omega t - kx_2 + \theta_2)$

$= k(x_2 - x_1) + (\theta_1 - \theta_2) = k\Delta p + \Delta\theta$

Here $\Delta p = \Delta x$ is the path difference

Clearly, phase difference due to path difference = k (path difference)

where $k = \frac{2\pi}{\lambda}$

$\Rightarrow \quad \Delta\phi = k\Delta p = \frac{2\pi}{\lambda}\Delta x \qquad$ ..... (1)

For Constructive Interference :

$\Delta\phi = 2n\pi, \qquad n = 0, 1, 2$ ....

or, $\Delta x = n\lambda$

$A_{max} = A_1 + A_2$

Intensity, $\sqrt{I_{max}} = \sqrt{I_1} + \sqrt{I_2}$ $\Rightarrow$ $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ ... (2)

For Destructive interference :

$\Delta\phi = (2n + 1)\pi$, n = 0, 1, 2 ....

or, $\Delta x = (2n + 1)\lambda/2$

$A_{min} = |A_1 - A_2|$

Intensity, $\sqrt{I_{min}} = \sqrt{I_1} - \sqrt{I_2}$ $\Rightarrow$ $I_{max} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ ... (3)

Solved Example

Example 3. Light from two sources, each of same frequency and travelling in same direction, but with intensity in the ratio 4 : 1 interfere. Find ratio of maximum to minimum intensity.

Solution :

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1}\right)^2 = \left(\frac{2+1}{2-1}\right)^2 = 9 : 1.$$

4. COHERENCE :

Two sources which vibrate with a fixed phase difference between them are said to be coherent. The phase differences between light coming form such sources does not depend on time.

In a conventional light source, however, light comes from a large number of individual atoms, each atom emitting a pulse lasting for about 1 ns. Even if atoms were emitting under similar conditions, waves from different atoms would differ in their initial phases. Consequently light coming from two such sources have a fixed phase relationship for about 1ns, hence interference pattern will keep changing every billionth of a second. The eye can notice intensity changes which lasts at least one tenth of a second. Hence we will observe uniform intensity on the screen which is the sum of the two individual intensities. Such sources are said to be incoherent. Light beam coming from two such independent sources do not have any fixed phase relationship and they do not produce any stationary interference pattern. For such sources, resultant intensity at any point is given by

$I = I_1 + I_2$ ...... (1)

5. YOUNG'S DOUBLE SLIT EXPERIMENT (Y.D.S.E.)

In 1802 Thomas Young devised a method to produce a stationary interference pattern. This was based upon division of a single wavefront into two; these two wavefronts acted as if they emanated from two sources having a fixed phase relationship. Hence when they were allowed to interfere, stationary interference pattern was observed.

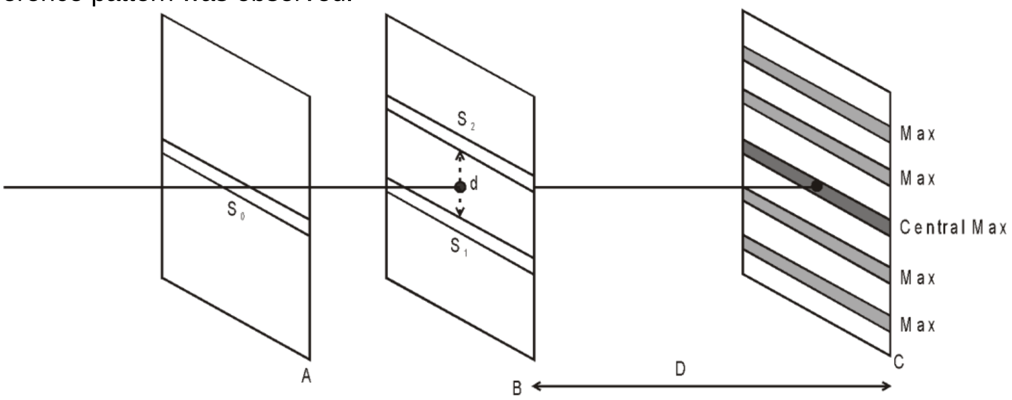


Figure : *Young's Arrangement to produce stationary interference pattern by division of wave front S_0 into S_1 and S_2*

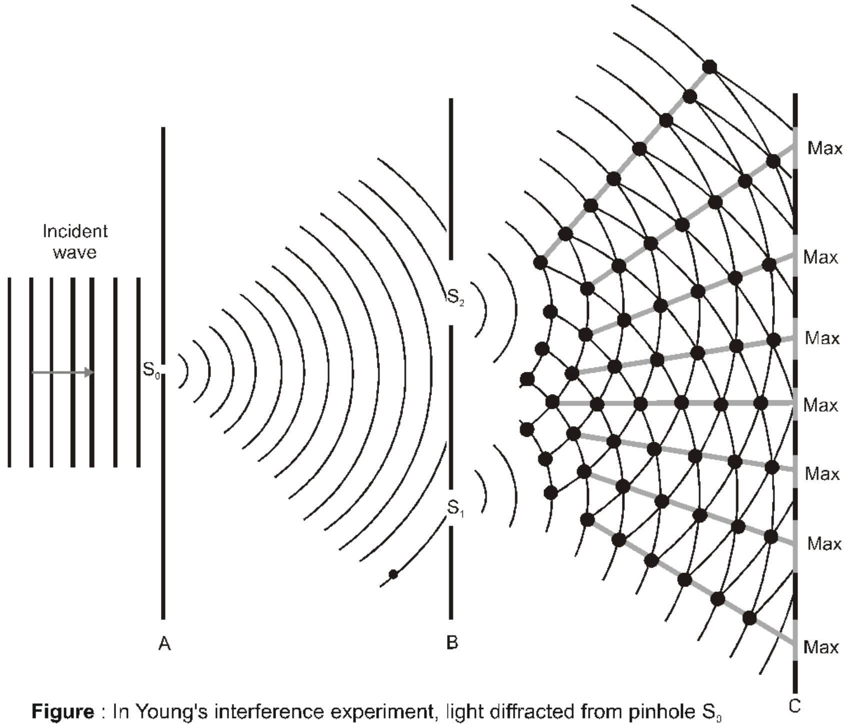
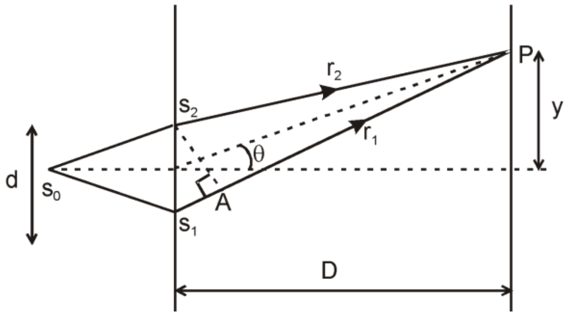


Figure : In Young's interference experiment, light diffracted from pinhole S_0 excounters pinholes S1 and S2 in screen B. Light diffracted from these two pinholes overlaps in the region between screen B and viewing screen C, producing an interference pattern on screen C.

5.1 ANALYSIS OF INTERFERENCE PATTERN

We have insured in the above arrangement that the light wave passing through S_1 is in phase with that passing through S_2. However the wave reaching P from S_2 may not be in phase with the wave reaching P from S_1, because the latter must travel a longer path to reach P than the former. We have already discussed the phase-difference arising due to path difference. If the path difference is equal to zero or is an integral multiple of wavelengths, the arriving waves are exactly in phase and undergo constructive interference.

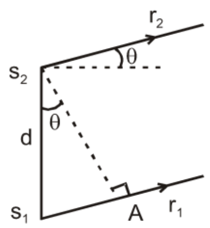


If the path difference is an odd multiple of half a wavelength, the arriving waves are out of phase and undergo fully destructive interference. Thus, it is the path difference Δx, which determines the intensity at a point P.

Path difference $\Delta p = S_1P - S_2P = \sqrt{\left(y+\frac{d}{2}\right)^2 + D^2} - \sqrt{\left(y-\frac{d}{2}\right)^2 + D^2}$ ...(1)

Approximation I :

For D >> d, we can approximate rays $\vec{r_1}$ and $\vec{r_2}$ as being approximately parallel, at angle θ to the principle axis.



Now, $S_1P - S_2P = S_1A = = S_1S_2 \sin\theta$

$\Rightarrow$ path difference = $d \sin\theta$ ...(2)

Approximation II :

further if θ is small, i.e.y << D, $\sin\theta = \tan\theta = \frac{y}{D}$

and hence, path difference = $\frac{dy}{D}$ ...(3)

for maxima (constructive interference),

$$\Delta p = \frac{d.y}{D} = n\lambda$$

$\Rightarrow$ $y = \frac{n\lambda D}{d}$, n = 0, ± 1, ± 2, ± 3 .....(4)

Here n = 0 corresponds to the central maxima

n = ±1 correspond to the 1st maxima

n = ±2 correspond to the 2nd maxima and so on.

for minima (destructive interference).

$$\Delta p = \pm \frac{\lambda}{2}, \pm \frac{3\lambda}{2} \pm \frac{5\lambda}{2} \Rightarrow \quad \Delta p = \begin{cases} (2n-1)\frac{\lambda}{2} & n = 1, 2, 3............ \\ (2n+1)\frac{\lambda}{2} & n = -1, -2, -3....... \end{cases}$$

consequently, $y = \begin{cases} (2n-1)\frac{\lambda D}{2d} & n = 1, 2, 3............ \\ (2n+1)\frac{\lambda D}{2d} & n = -1, -2, -3....... \end{cases}$ .... (5)

Here n = ± 1 corresponds to first minima,

n = ± 2 corresponds to second minima and so on.

5.2 FRINGE WIDTH :

It is the distance between two maxima of successive order on one side of the central maxima. This is also equal to distance between two successive minima.

fringe width $\beta = \frac{\lambda D}{d}$ ... (1)

· Notice that it is directly proportional to wavelength and inversely proportional to the distance between the two slits.

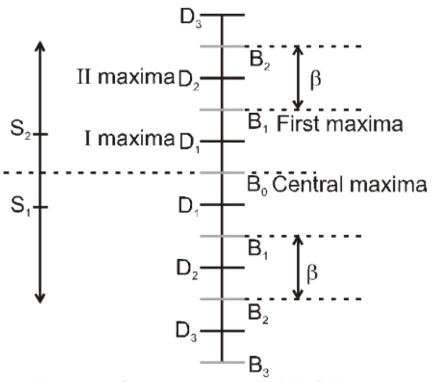


Figure : fringe pattern in YDSE

5.3 INTENSITY :

Suppose the electric field components of the light waves arriving at point P (in the Figure) from the two slits S_1 and S_2 vary with time as

$E_1 = E_0 \sin \omega t$

and $E_2 = E_0 \sin (\omega t + \phi)$

Here $\phi = k\Delta x = \frac{2\pi}{\lambda} \Delta x$

and we have assumed that intensity of the two slits S_1 and S_2 are same (say I_0); hence waves have same amplitude E_0.

then the resultant electric field at point P is given by,

$E = E_1 + E_2 = E_0 \sin \omega t + E_0 \sin (\omega t + \phi) = E_0' \sin (\omega t + \phi')$

where $E_0'^2 = E_0^2 + E_0^2 + 2E_0 . E_0 \cos \phi = 4 E_0^2 \cos^2 \phi/2$

Hence the resultant intensity at point P,

$$I = 4I_0 \cos^2 \frac{\phi}{2} \qquad ....(2)$$

$I_{max} = 4I_0$ when $\frac{\phi}{2} = n\pi$, $\quad n = 0, \pm1, \pm2, ...,$

$I_{min} = 0$ when $\frac{\phi}{2} = \left(n - \frac{1}{2}\right)\pi \quad n = 0, \pm 1, \pm 2$ ....

Here $\phi = k\Delta x = \frac{2\pi}{\lambda} \Delta x$

If $D >> d$, $\quad \phi = \frac{2\pi}{\lambda} d \sin \theta$

If $D >> d$ & $\quad y << D$, $\phi = \frac{2\pi}{\lambda} d \frac{y}{D}$

However if the two slits were of different intensities I_1 and I_2,

say $E_1 = E_{01}, \sin \omega t$

and $E_2 = E_{02}, \sin (\omega t + \phi)$

then resultant field at point P,

$E = E_1 + E_2 = E_0 \sin (\omega t + \phi)$

where $E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01} E_{02} \cos \phi$

Hence resultant intensity at point P,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi \qquad .... (3)$$

Solved Examples

Example 4. In a YDSE, D = 1m, d = 1mm and λ = 1/2 mm

(i) Find the distance between the first and central maxima on the screen.

(ii) Find the no of maxima and minima obtained on the screen.

Solution : (i) D >> d

Hence $\Delta P = d \sin \theta$

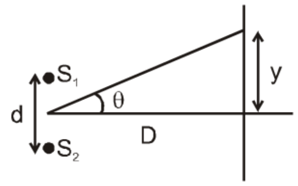
$\frac{d}{\lambda} = 2,$

clearly, $n << \frac{d}{\lambda} = 2$ is not possible for any value of n.

Hence $\Delta p = \frac{dy}{D}$ cannot be used for Ist maxima,

$\Delta p = d \sin \theta = \lambda$

$\Rightarrow \quad \sin \theta = \frac{\lambda}{d} = \frac{1}{2} \Rightarrow \quad \theta = 30^o$



Hence, $y = D \tan\theta = \frac{1}{\sqrt{3}}$ meter

(ii) Maximum path difference

$\Delta P_{max} = d = 1$ mm

⇒ Highest order maxima, $n_{max} = \left[\frac{d}{\lambda}\right] = 2$ and highest order minima $n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right] = 2$

Total no. of maxima = $2n_{max} + 1^* = 5$ *(central maxima).

Total no. of minima = $2n_{min} = 4$

Example 5. Monochromatic light of wavelength 5000 Aº is used in Y.D.S.E., with slit-width, d = 1mm, distance between screen and slits, D = 1m. If intensity at the two slits are, $I_1 = 4I_0$, $I_2 = I_0$, find

(i) fringe width β

(ii) distance of 5th minima from the central maxima on the screen

(iii) Intensity at $y = \frac{1}{3}$ mm

(iv) Distance of the 1000^{th} maxima

(v) Distance of the 5000^{th} maxima

Solution : (i) $\beta = \frac{\lambda D}{d} = \frac{5000 \times 10^{-10} \times 1}{1 \times 10^{-3}} = 0.5$ mm

(ii) $y = (2n - 1)\frac{\lambda D}{2d}$, n = 5 ⇒ y = 2.25 mm

(iii) At $y = \frac{1}{3}$ mm, y << D Hence $\Delta p = \frac{d.y}{D}$

$\Delta\phi = \frac{2\pi}{\lambda}\Delta p = 2\pi\frac{dy}{\lambda D} = \frac{4\pi}{3}$ Now resultant intensity

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi = 4I_0 + I_0 + 2\sqrt{4I_0^2}\cos\Delta\phi = 5I_0 + 4I_0\cos\frac{4\pi}{3} = 3I_0$

(iv) $\frac{d}{\lambda} = \frac{10^{-3}}{0.5 \times 10^{-6}} = 2000$ n = 1000 is not << 2000

Hence now $\Delta p = d \sin\theta$ must be used

Hence, $d\sin\theta = n\lambda = 1000\,\lambda$

⇒ $\sin\theta = 1000\frac{\lambda}{d} = \frac{1}{2}$ ⇒ θ = 30º

$y = D\tan\theta = \frac{1}{\sqrt{3}}$ meter

(v) Highest order maxima $n_{max} = \left[\frac{d}{\lambda}\right] = 2000$

Hence, n = 5000 is not possible.

5.4 SHAPE OF INTERFERENCE FRINGES IN YDSE :

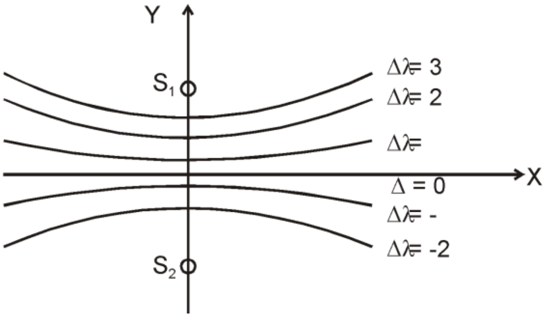
We discuss the shape of fringes when two pinholes are used instead of the two slits in YDSE.
Fringes are locus of points which move in such a way that its path difference from the two slits remains constant.

$S_2P - S_1P = \Delta = \text{constant}$ ....(1)

If $\Delta = \pm \frac{\lambda}{2}$, the fringe represents 1st minima.

If $\Delta = \pm \frac{3\lambda}{2}$ it represents 2nd minima

If $\Delta = 0$ it represents central maxima,



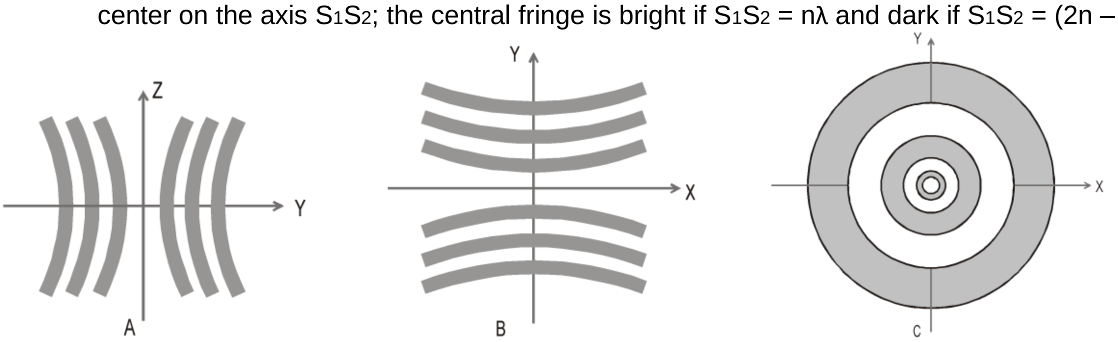
If $\Delta = \pm \lambda$, it represents 1st maxima etc.

Equation (1) represents a hyperbola with its two foci at S_1 and S_2
The interference pattern which we get on screen is the section of hyperboloid of revolution when we revolve the hyperbola about the axis S_1S_2.

A. If the screen is $\perp$er to the X axis, i.e. in the YZ plane, as is generally the case, fringes are hyperbolic with a straight central section.

B. If the screen is in the XY plane, again fringes are hyperbolic.

C. If screen is $\perp$ er to Y axis (along S_1S_2), ie in the XZ plane, fringes are concentric circles with center on the axis S_1S_2; the central fringe is bright if $S_1S_2 = n\lambda$ and dark if $S_1S_2 = (2n - 1)\frac{\lambda}{2}$.



5.5 YDSE WITH WHITE LIGHT :

The central maxima will be white because all wavelengths will constructively interference here. However slightly below (or above) the position of central maxima fringes will be coloured. or example if P is a point on the screen such that

$$S_2P - S_1P = \frac{\lambda_{violet}}{2} = 190 \text{ nm},$$

completely destructive interference will occur for violet light. Hence we will have a line devoid of violet colour that will appear reddish. And if

$$S_2P - S_1P = \frac{\lambda_{red}}{2} = 350 \text{ nm},$$

completely destructive interference for red light results and the line at this position will be violet. The coloured fringes disappear at points far away from the central white fringe; for these points there are so many wavelengths which interfere constructively, that we obtain a uniform white illumination. for example if

$$S_2P - S_1 P = 3000 \text{ nm},$$

then constructive interference will occur for wavelengths $\lambda = \frac{3000}{n}$ nm. In the visible region these wavelength are 750 nm (red), 600 nm (yellow), 500 nm (greenish–yellow), 430 nm (violet). Clearly such a light will appear white to the unaided eye.

Thus with white light we get a white central fringe at the point of zero path difference, followed by a few coloured fringes on its both sides, the color soon fading off to a uniform white.

In the usual interference pattern with a monochromatic source, a large number of identical interference fringes are obtained and it is usually not possible to determine the position of central maxima. Interference with white light is used to determine the position of central maxima in such cases.

Solved Examples

Example 6. A beam of light consisting of wavelengths 6000Å and 4500Å is used in a YDSE with D = 1m and d = 1 mm. Find the least distance from the central maxima, where bright fringes due to the two wavelengths coincide.

Solution: $\beta_1 = \frac{\lambda_1 D}{d} = \frac{6000 \times 10^{-10} \times 1}{10^{-3}} = 0.6$ mm

$\beta_2 = \frac{\lambda_2 D}{d} = 0.45$ mm

Let n_1th maxima of λ_1 and n_2th maxima of λ_2 coincide at a position y.

then, $y = n_1P_1 = n_2P_2$ = LCM of β_1 and β_2

$\Rightarrow$ y = LCM of 0.6 cm and 0.45 mm $\Rightarrow$ y = 1.8 mm **Ans.**

At this point 3rd maxima for 6000 Å & 4th maxima for 4500 Å coincide

Example 7. White light is used in a YDSE with D = 1m and d = 0.9 mm. Light reaching the screen at position y = 1 mm is passed through a prism and its spectrum is obtained. Find the missing lines in the visible region of this spectrum.

Solution : $\Delta p = \frac{yd}{D} = 9 \times 10^{-4} \times 1 \times 10^{-3}$ m $= 900$ nm

for minima $\Delta p = (2n - 1)\lambda/2$

$\Rightarrow \lambda = \frac{2\Delta P}{(2n-1)} = \frac{1800}{(2n-1)} = \frac{1800}{1}, \frac{1800}{3}, \frac{1800}{5}, \frac{1800}{7}$ ....

of these 600 nm and 360 nm lie in the visible range. Hence these will be missing lines in the visible spectrum.

6. GEOMETRICAL PATH & OPTICAL PATH :

Actual distance travelled by light in a medium is called geometrical path (Δx). Consider a light wave given by the equation

$$E = E_0 \sin (\omega t - kx + \phi)$$

If the light travels by Δx, its phase changes by $k\Delta x = \frac{\omega}{v}\Delta x$, where ω, the frequency of light does not depend on the medium, but v, the speed of light depends on the medium as $v = \frac{c}{\mu}$.

Consequently, change in phase

$$\Delta\phi = k\Delta x = \frac{\omega}{c}(\mu\Delta x)$$

It is clear that a wave travelling a distance Δx in a medium of refractive index μ suffers the same phase change as when it travels a distance $\mu\Delta x$ in vacuum. i.e. a path length of Δx in medium of refractive index μ is equivalent to a path length of $\mu\Delta x$ in vacuum.
The quantity $\mu\Delta x$ is called the optical path length of light, Δx_{opt} . And in terms of optical path length, phase difference would be given by,

$$\Delta\phi = \frac{\omega}{c}\,\Delta x_{opt} = \frac{2\pi}{\lambda_0}\,\Delta x_{opt} \qquad .... (1)$$

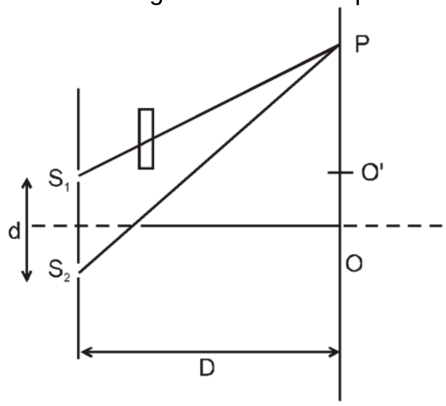
where λ_0 = wavelength of light in vacuum.
However in terms of the geometrical path length Δx,

$$\Delta\phi = \frac{\omega}{c}\,(\mu\Delta x) = \frac{2\pi}{\lambda}\,\Delta x \qquad .....(2)$$

where λ = wavelength of light in the medium ($\lambda = \frac{\lambda_0}{\mu}$).

6.1 DISPLACEMENT OF FRINGE :

On introduction of a glass slab in the path of the light coming out of the slits–



On introduction of the thin glass-slab of thickness t and refractive index μ, the optical path of the ray S_1P increases by $t(\mu - 1)$. Now the path difference between waves coming form S_1 and S_2 at any point P is

$\Delta p = S_2P - (S_1P + t(\mu - 1))$
$= (S_2P - S_1P) - t(\mu - 1)$

$\Rightarrow \quad \Delta p = d \sin\theta - t(\mu - 1) \quad$ if $\quad d << D$

and $\quad \Delta p = \frac{yd}{D} - t(\mu - 1) \quad$ If $\quad y << D$ as well.

for central bright fringe,

$\Delta p = 0 \quad \Rightarrow \quad \frac{yd}{D} = t(\mu - 1)$.

$\Rightarrow \quad y = OO' = (\mu - 1)t\,\frac{D}{d} = (\mu - 1)\,t\,.\,\frac{\beta}{\lambda}$

The whole fringe pattern gets shifted by the same distance

$\Delta = (\mu - 1)\,.\,\frac{D}{d} = (\mu - 1)t\,.\,\frac{\beta}{\lambda}$

***** Notice that this shift is in the direction of the slit before which the glass slab is placed. If the glass slab is placed before the upper slit, the fringe pattern gets shifted upwards and if the glass slab is placed before the lower slit the fringe pattern gets shifted downwards.

Solved Example

Example 8. In a YDSE with d = 1mm and D = 1m, slabs of ($t = 1\mu m$, $\mu = 3$) and ($t = 0.5\ \mu m$, $\mu = 2$) are introduced in front of upper and lower slit respectively. Find the shift in the fringe pattern.

Solution : Optical path for light coming from upper slit S_1 is

$S_1P + 1\mu m\ (2 - 1) = S_2P + 0.5\ \mu m$

Similarly optical path for light coming from S_2 is

$S_2P + 0.5\ \mu m\ (2 - 1) = S_2P + 0.5\ \mu m$

Path difference : $\Delta p = (S_2P + 0.5\ \mu m) - (S_1P + 2\mu m) \qquad = (S_2P - S_1P) - 1.5\ \mu m.$

$= \dfrac{yd}{D} - 1.5\ \mu m$

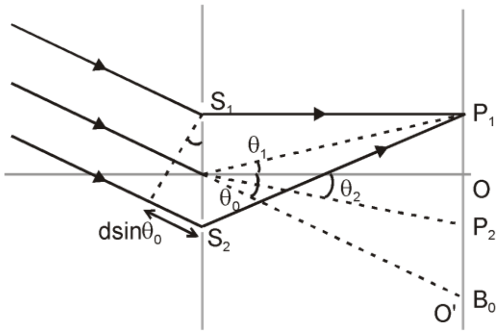
for central bright fringe $\Delta p = 0$

$\Rightarrow \qquad y = \dfrac{1.5\mu m}{1mm} \times 1m = 1.5\ mm.$

The whole pattern is shifted by 1.5 mm upwards. **Ans.**

7. YDSE WITH OBLIQUE INCIDENCE :

In YDSE, ray is incident on the slit at an inclination of θ_0 to the axis of symmetry of the experimental set-up



for points above the central point on the screen, (say for P_1)

$\Delta p = d \sin\theta_0 + (S_2P_1 - S_1P_1)$

$\Rightarrow \qquad \Delta p = d \sin\theta_0 + d\sin\theta_1 \qquad$ (If $d << D$)

and for points below O on the screen, (say for P_2)

$\Delta p = |(d\sin\theta_0 + S_2P_2) - S_1P_2|$

$= |d \sin\theta_0 - (S_1P_2 - S_2P_2)|$

$\Rightarrow \qquad \Delta p = |d \sin\theta_0 - d \sin\theta_2|$ (if $d << D$)

We obtain central maxima at a point where, $\Delta p = 0$.

$(d \sin\theta_0 - d \sin\theta_2) = 0$

or $\qquad \theta_2 = \theta_0$.

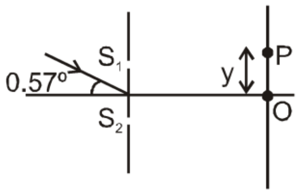
This corresponds to the point O' in the diagram.

Hence we have finally for path difference.

$$\Delta p = \begin{cases} d(\sin\theta_0 + \sin\theta) - \text{for points above O} \\ d(\sin\theta_0 - \sin\theta) - \text{for points between O\&O'} \\ d(\sin\theta - \sin\theta_0) - \text{for points below O'} \end{cases} \qquad ...(1)$$

Solved Example

Example 9. In YDSE with D = 1m, d = 1mm, light of wavelength 500 nm is incident at an angle of 0.57º w.r.t. the axis of symmetry of the experimental set up. If centre of symmetry of screen is O as shown.



(i) find the position of central maxima
(ii) Intensity at point O in terms of intensity of central maxima I_0.
(iii) Number of maxima lying between O and the central maxima.

Solution :. (i) $\theta = \theta_0 = 0.57^o$

$\Rightarrow \quad y = -D \tan\theta \simeq -D\theta = -1 \text{ meter} \times \left(\frac{0.57}{57}\text{rad}\right)$

$\Rightarrow \quad y = -1\text{cm}.$

(ii) for point 0, $\theta = 0$

Hence, $\Delta p = d \sin\theta_0$; $d\theta_0 = 1 \text{ mm} \times (10^{-2} \text{ rad})$

$= 10{,}000 \text{ nm} = 20 \times (500 \text{ nm})$

$\Rightarrow \quad \Delta p = 20\,\lambda$

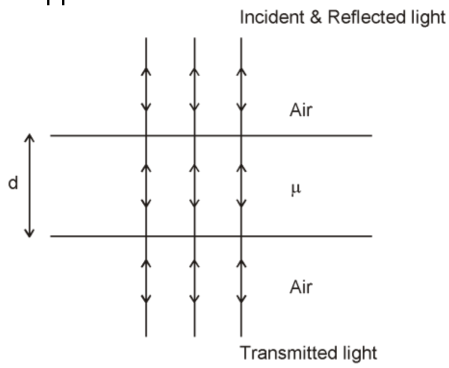
Hence point O corresponds to 20th maxima

$\Rightarrow$ intensity at O = I_0

(iii) 19 maxima lie between central maxima and O, excluding maxima at O and central maxima.

8. THIN-FILM INTERFERENCE :

In YDSE we obtained two coherent source from a single (incoherent) source by division of wave-front. Here we do the same by division of Amplitude (into reflected and refracted wave).
When a plane wave (parallel rays) is incident normally on a thin film of uniform thickness d then waves reflected from the upper surface interfere with waves reflected from the lower surface.



Clearly the wave reflected from the lower surface travel an extra optical path of $2\mu d$, where μ is refractive index of the film.

Further if the film is placed in air the wave reflected from the upper surface (from a denser medium) suffers a sudden phase change of π, while the wave reflected from the lower surface (from a rarer medium) suffers no such phase change.
Consequently condition for constructive and destructive interference in the reflected light is given by,

$2\mu d = n\lambda$ for destructive interference

and $2\mu d = (n + \frac{1}{2})\lambda$ for constructive interference ....(1)
where $n = 0, 1, 2$ ....,
and λ = wavelength in free space.

Interference will also occur in the transmitted light and here condition of constructive and destructive interference will be the reverse of (9.1)

i.e. $2\mu d = \begin{cases} n\lambda & \text{for constructive interference} \\ (n + \frac{1}{2}\lambda) & \text{for destructive interference} \end{cases}$ ....(2)

This can easily be explained by energy conservation (when intensity is maximum in reflected light it has to be minimum in transmitted light) However the amplitude of the directly transmitted wave and the wave transmitted after one reflection differ substantially and hence the fringe contrast in transmitted light is poor. It is for this reason that thin film interference is generally viewed only in the reflected light.

In deriving equation (1) we assumed that the medium surrounding the thin film on both sides is rarer compared to the medium of thin film.
If medium on both sides are denser, then there is no sudden phase change in the wave reflected from the upper surface, but there is a sudden phase change of π in waves reflected from the lower surface. The conditions for constructive and destructive interference in reflected light would still be given by equation (1).

However if medium on one side of the film in denser and that on the other side is rarer, then either there is no sudden phase in any reflection, or there is a sudden phase change of π in both reflection from upper and lower surface. Now the condition for constructive and destructive interference in the reflected light would be given by equation 2 and not equation (1).

Solved Examples

Example 10. White light, with a uniform intensity across the visible wavelength range 430–690 nm, is perpendicularly incident on a water film, of index of refraction μ = 1.33 and thickness d = 320 nm, that is suspended in air. At what wavelength λ is the light reflected by the film brightest to an observer?

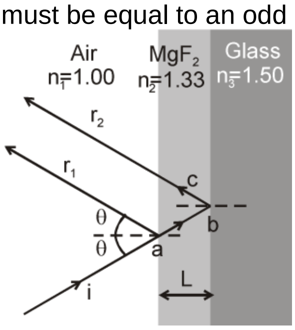
Solution : This situation is like that of Figure, for which equation gives the interference maxima. Solving for λ and inserting the given data, we obtain

$$\lambda = \frac{2\mu d}{m+1/2} = \frac{(2)(1.33)(320\,nm)}{m+1/2} = \frac{851\,nm}{m+1/2}$$

for m = 0, this give us λ = 1700 nm, which is in the infrared region. For m = 1, we find l = 567 nm, which is yellow-green light, near the middle of the visible spectrum. For m = 2, λ = 340 nm, which is in the ultraviolet region. So the wavelength at which the light seen by the observer is brightest is
λ = 567 nm. **Ans.**

Example 11. A glass lens is coated on one side with a thin film of magnesium fluoride (MgF_2) to reduce reflection from the lens surface (figure). The index of refraction of MgF_2 is 1.38; that of the glass is 1.50. What is the least coating thickness that eliminates (via interference) the reflections at the middle of the visible spectrum (λ = 550 nm)? Assume the light is approximately perpendicular to the lens surface.

Solution : The situation here differs from figure in that $n_3 > n_2 > n_1$. The reflection at point a still introduces a phase difference of π but now the reflection at point b also does the same (see figure 9.2). Unwanted reflections from glass can be, suppressed (at a chosen wavelength) by coating the glass with a thin transparent film of magnesium fluoride of a properly chosen thickness which introduces a phase change of half a wavelength. For this, the path length difference 2L within the film must be equal to an odd number of half wavelengths:



$$2L = (m + 1/2)\lambda_{n2},$$

or, with $\lambda_{n2} = \lambda/n_2$,

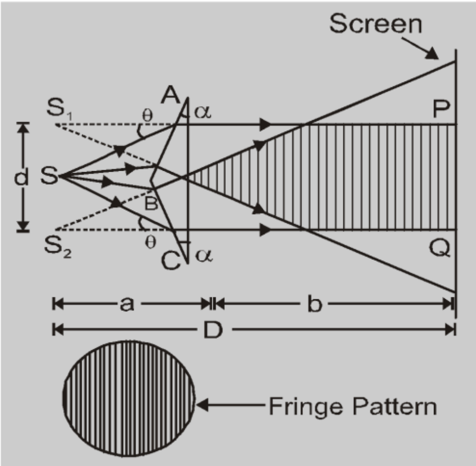
$$2n_2 L = (m + 1/2)\lambda.$$

We want the least thickness for the coating, that is, the smallest L. Thus we choose m = 0, the smallest value of m. Solving for L and inserting the given data, we obtain

$$L = \frac{\lambda}{4n_2} = \frac{550\,nm}{(4)(1.38)} = 96.6\ nm$$ **Ans.**

9. FRESNEL'S BIRPISM EXPERIMENT

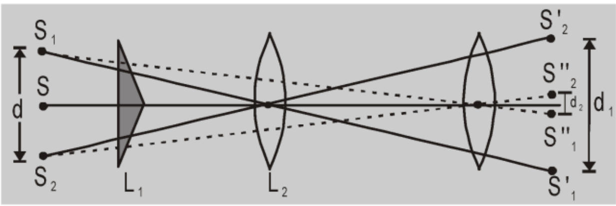
(1) It is optical device to obtain two coherent sources by refraction of lights.
(2) The angle of biprism is 179º & refracting angle is $\alpha = 1/2^o$.
(3) Distance between source & screen D = a + b.
Distance between two coherent source = $d = 2a(\mu - 1)\alpha$
Where a = distance between source & Biprism



b = distance between screen & Biprism
μ = refractive index of the material of prism.

$$\lambda = \frac{d\beta}{D} = \frac{2a(\mu - 1)\alpha\beta}{(a+b)} = \frac{\sqrt{d_1 d_2}\,.\beta}{(a+b)}$$

Note- α is in radian $\alpha^o = \alpha \times \frac{3.14}{180}$. Suppose refracting angle & refractive index is not known then d can be calculate by convex lens.



One convex lens whose focal length (f) and 4f < D.
First convex lens is kept near biprism & d_1 is calculated then it is kept near eyepiece & d_2 is calculated.

$$d = \sqrt{d_1 d_2}$$

Application :
With the help of this experiment the wavelength of monochromatic light, thickness of thin films and their refractive index & distance between apparent coherent sources can be determined.
When **Fresnel's arrangement** is immersed in water
(1) Effect on d
$d_{water} < d_{air}$. Thus when the Fresnel's biprism experiment is immersed in water, then the separation between the two virtual sources decreases but in young's double slit experiment it does not change.
(2) In young's double slit experiment β decrease and in fresnel's biprism experiment β increases.

Solved Example

Example 12 In Fresnel's biprism experiment the width of 10 fringes is 2cm which are formed at a distance of two 2 meter from the slit. If the wavelength of light is 5100 Å then the distance between two coherent sources.

Solution. $d = \frac{D\lambda}{\beta}$ ....(1)

According to question $\lambda = 5100 \times 10^{-10}$ m

$\beta = \frac{2}{10} \times 10^{-2}$m ....(2)

D = 2m d = ?

From eqs. (1) and (2) $d = \frac{2 \times 51 \times 10^{-8}}{2 \times 10^{-3}}$ = 5.1 × 10^{-4} m

10. DIFFRACTION

10.1. MEANING OF DIFFRACTION

It is the spreading of waves round the corners of an obstacle, of the order of wave length.

10.2. DEFINITION OF DIFFRACTION

The phenomenon of bending of light waves around the sharp edges of opaque obstacles or aperture and their encroachment in the geometrical shadow of obstacle or aperture is defined as diffraction of light.

10.3. NECESSARY CONDITIONS OF DIFFRACTION OF WAVES

The size of the obstacle (a) must be of the order of the wavelength of the waves (λ).

$$\frac{a}{\lambda} \approx 1$$

Note : Greater the wave length of wave higher will be its degree of diffraction. This is the reason that diffraction of sound & radio waves is easily observed but for diffraction of light, additional arrangement have to be arrange.

$\lambda_{sound} > \lambda_{light}$

Wave length of sound is nearly equal to size of obstacle. If size of obstacle is a & wavelength of light is λ then,

S.No.	**a** V/S **λ**	**Diffraction**
[1]	a << λ	Not possible
[2]	a >> λ	Not possible
[3]	$a \simeq \lambda$	Possible

10.4. INTERPRETATION OF DIFFRACTION

As a result of diffraction, maxima & minima of light intensities are found which has unequal intensities. Diffraction is the result of superposing of waves from infinite number of coherent sources on the same wavefront after the wavefront has been distorted by the obstacle.

10.5. EXAMPLE OF DIFFRACTION

- When an intense source of light is viewes with the partially opened eye, colours are observed in the light.
- Sound produced in one room can be heard in the nearby room.
- Appearance of a shining circle around the section of sun just before sun rise.
- Coloured spectrum is observed if a light source at far distance is seen through a thin cloth.

10.6. TWO TYPE OF DIFFRACTION

Fresnel Diffraction : Fresenel diffraction which involves non-plane (spherical) wavefronts, so that the sources and the point p (where diffraction effect is to be observed) are to be at a finite distance from the diffracting obstacle.

Fraunhoffer Diffraction : Fraunhofer diffraction deals with wavefronts that are plane on arrival and an effective viewing distance of infinity. If follows that fraunhofer diffraction is an important special case of fresnel diffraction. In youngs double slit experiment, we assume the screen to be relatively distance, that we have fraunhofer conditions.

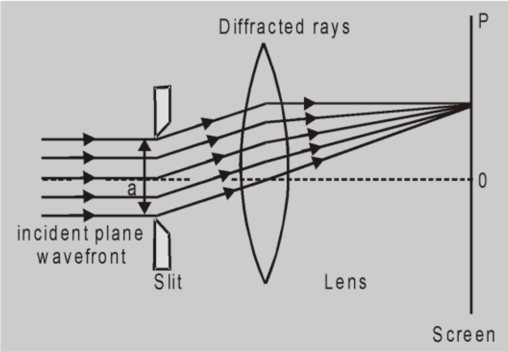
[1] Fresnel Diffraction : According to fresnel principal to determine the intensity of light at any point, a wavefront can be divided into a number of small parts which are known as fresnel's half period zones. Each point on the wavefront is a source of secondary wavelets, so that the wave from two consecutive zones reach the point of observation in opposite phase corresponding to a path difference of λ/2.

10.7. DIFFERENCE BETWEEN INTERFERENCE & DIFFRACTION OF LIGHT

	Interference	**Diffraction**
1.	Two coherent sources are necessary	One coherent source is necessary
2.	All fringes has same width	Fringes has unequal width
3.	Width of bright fringes is equal to other fringes	Width of first bright fringes is just doubled to second bright fringe

4.	All bright fringes has equal intensity	As order of bright fringes increases, intensity goes down
5.	For bright fringe : [a] Path difference D = nλ [b] Phase difference δ = 2nπ	For bright fringe : D = (2n – 1)λ/2 δ = (2n – 1) π
6.	For dark fringes : [a] Path difference D = (2n – 1)λ/2 [b] Phase difference δ = (2n – 1)π	For dark fringes : D = 2nλ/2 δ = 2nπ

10.8. FRAUNHOFER DIFFRACTION FOR SINGLE SLIT



In this diffraction pattern central maxima is bright on the both side of it, maxima & minima occurs symmetrically

For Diffraction Maxima :

$a \sin \theta = (2n + 1) \lambda/2$

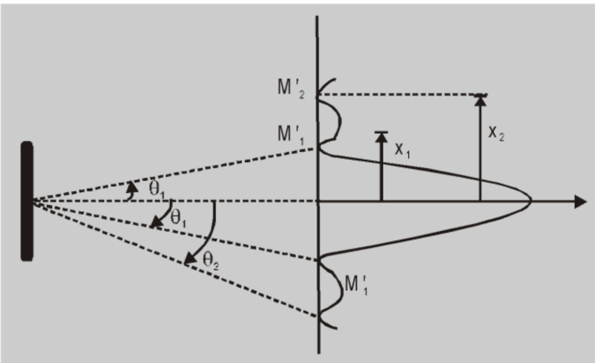
For Diffraction Minima :

$a \sin \theta = n\lambda$

The maxima or minima is observed due to the superposition of waves emerging from infinite secondary sources between A & B points of slit.

Fringe width :

The distance between two secondary minima formed on two sides of central maximum is known as the width of central maximum



$$W = \frac{2f\lambda}{a}$$

f = focal distance of convex lense

a = width of slit

Angular width = $W_\theta = \frac{2\lambda}{a}$

10.9. RESOLVING POWER (R.P.)

A large number of images are formed as a consequence of light diffraction from a source. If two sources are separated such that their central maxima do not overlap, their images can be distinguished and are said to be resolved R.P. of an optical instrument is its ability to distinguish two neighbouring points.

Linear R.P. = d/λD **Angular** R.P. = d/λ

D = Observed distance

d = Distance between two points

Telescope :

Limit of resolution = θ = $\sin^{-1}\dfrac{1.22\lambda}{a}$; For small angles $\theta = \dfrac{1.22\lambda}{a}$

Resolving power = $\dfrac{1}{\text{limit of resolution}}$

Microscope :

Limit of resolution (the smallest distance between two object) = $x_{min} = \dfrac{1.22\lambda}{2\mu\sin\theta}$

Prism : R.P. = t (dμ/dλ) = λ/dλ.

Diffraction Grating :

R.P. = λ/dλ = N × n (N is total number of lines & n is the order of spectrum)

Eye : The limit of resolution of human eye is 1' of arc (One minute of arc)

10.10. DIFFERENCE BETWEEN FRESNEL & FRAUNHOFFER DIFFRACTION

	Fresnel	Fraunhoffer
1.	The source is near and on one side of obstacle and screen also near to obstacle and on the other side.	Source & screen are both effectively at infinite distances from obstacle.
2.	Wavefront : spherical or cylindrical is used.	Plane wavefront is used.
3.	No sophisticated equipment is required.	Spectrometer is required.
4.	No lense is required.	Convex lense is required.
5.	Diffraction pattern may have both bright & dark central fringe.	Central fringe always bright.
6.	Example : Circular aperture, disc, ring etc.	Single slit, double slit, grating etc.

10.11. COMPARATIVE STUDY OF DIFFRACTION OF LIGHT & SOUND

- Sound travels in form of waves, that's why it is also diffracted. Generally diffraction of sound waves is easily observed rather than light because wavelength of sound waves is the order of obstacle, but wavelength of light is very small in comparison to obstacle.
- [a] Ordinary audible sound has wavelength of the order of 1m & size of ordinary obstacle has same order that's why diffraction is easily observed.
  [b] Ordinary light has wavelength of 10^{-7} m & ordinary obstacle has greater size in comparison to its wavelength that's why diffraction pattern is not observed.
- Generally diffraction of ultrasonic waves are not observed because its wavelength has order of 1 cm.

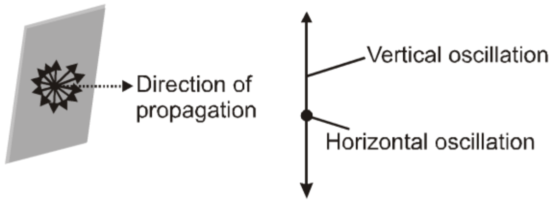
10.12 RECTLINEAR MOTION OF LIGHT

- Rectilinear motion of light can be explained by diffraction of light.
- If size of obstacle is the order of wavelength of light, then diffraction of light takes place & its rectilinear motion of light is not possible.
- If size of obstacle is much greater than wave length of light, then rectilinear motion of light is observed.

11. POLARISATION OF LIGHT

Light propagates as transverse EM waves. The magnitude of electric field is much larger as compared to magnitude of magnetic field. We generally prefer to described light as electric field oscillations.

(1) **Unpolarised light** : In ordinary light (light from sun, bulb etc.) the electric field vectors are distributed in all directions in a light is called unpolarised light. The oscillation of propagation of light wave resolved into horizontal and vertical component.



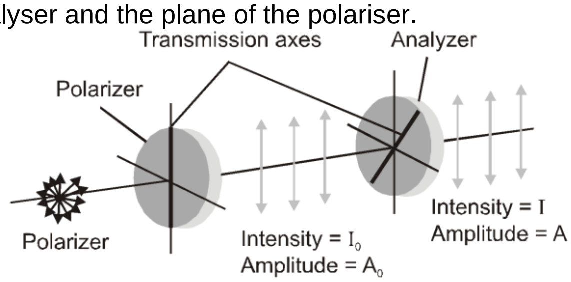
(2) **Polarised light :** The phenomenon of limiting the vibrating of electric field vector in one direction in a plane perpendicular to the direction of propagation of light wave is called polarization of light

(i) The plane in which oscillation occurs in the polarised light is called plane of oscillation.

(ii) The plane perpendicular to the plane of oscillation is called plane of polarisation.

(iii) Light can be polarised by transmitting through certain crystals such as tourmaline or polaroids.

(3) **Malus law :** This law states that the intensity of the polarised light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polariser.



(i) $I = I_0 \cos^2\theta$ and $A^2 = A_0^2 \cos^2\theta \Rightarrow A = A_0 \cos\theta$

If $\theta = 0^o$, $I = I_0$, $A = A_0$. If $\theta = 90^o$, $I = 0$, $A = 0$

(ii) If I_1 = intensity of unpolarised light.

So $I_0 = \frac{I_1}{2}$ i.e. if an unpolarised light is converted into plane polarised light (say by passing it though a Polariod or a Nicolprism, its intensity becomes half. and $I = \frac{I_1}{2}\cos^2\theta$.

Solved Miscellaneous Problems

Problem 1. Consider interference between waves from two sources of intensities I & 4I. Find intensities at points where the phase difference is π.

Solution. $I = R^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta = I + 4I + 4I \cos\pi$

$I = 5I - 4I = I$

Problem 2. The width of one of the two slits in a Young's double slits experiment is double of the other slit. Assuming that the amplitude of the light coming from a slit is proportion to slit-width. Find the ratio of the maximum to the minimum intensity in the interference pattern.

Solution. Suppose the amplitude of the light wave coming from the narrow slit is A and that coming from the wider slit is 2A. The maximum intensity occurs at a place where constructive interference takes place. Then the resultant amplitude is the sum of the individual amplitudes. Thus,

$A_{max} = 2A + A = 3A$

The minimum intensity occurs at a place where destructive interference takes place. The resultant amplitude is then difference of the individual amplitudes.

Thus, $A_{min} = 2A - A = A$. $\therefore \frac{I_{max}}{I_{min}} = \frac{(A_{max})^2}{(A_{min})^2} = \frac{(3A)^2}{(A)^2} = 9$

Problem 3. In a Young's experiment, the separation between the slits is 0.10mm, the wavelength of light used is 600nm and the interference pattern is observed on a screen 1.0 m away. Find the separation between the successive bright frignes.

Solution. The separation between the successive bright fringes is -

$$\beta = \frac{D\lambda}{d} = \frac{1\times 600\times 10^{-9}}{.1\times 10^{-3}}$$

$\beta = 6.0$ mm

Problem 4. Two waves originating from source S_1 and S_2 having zero phase difference and common wavelength λ will show completely destructive interference at a point P if (S_1 P – S_2 P) is

(1) 0 (2) 5λ (3) 11λ/4 (4*) 11λ/2

Solution. For destructive interference :

Path difference = $S_1P - S_2P = (2n - 1)\lambda/2$

For
$n = 1,\quad S_1P - S_2P = (2 \times 1 - 1)\lambda/2 = \lambda/2$
$n = 2,\quad S_1P - S_2P = (2 \times 2 - 1)\lambda/2 = 3\lambda/2$
$n = 3,\quad S_1P - S_2P = (2 \times 3 - 1)\lambda/2 = 5\lambda/2$
$n = 4,\quad S_1P - S_2P = (2 \times 4 - 1)\lambda/2 = 7\lambda/2$
$n = 5,\quad S_1P - S_2P = (2 \times 5 - 1)\lambda/2 = 9\lambda/2$
$n = 6,\quad S_1P - S_2P = (2 \times 6 - 1)\lambda/2 = 11\lambda/2$

So, destructive pattern is possible only for path difference = 11λ/2.

Problem 5. In Young's experiment the wavelength of red light is 7.5×10^{-5} cm. and that of blue light 5.0×10^{-5} cm. The value of n for which $(n+1)^{th}$ the blue bright band coincides with n^{th} red band.

Solution. $n_1 \lambda_1 = n_2 \lambda_2$ for bright fringe

$n(7.5 \times 10^{-5}) = (n + 1)(5 \times 10^{-5})$

$$n = \frac{5.0\times10^{-5}}{2.5\times10^{-5}} = 2$$

Problem 6. In Young's slit experiment, carried out with lights of wavelength λ = 5000 Aº, the distance between the slit is 0.2 mm and the screen is at 200 cm from the slits. The central maximum is at x = 0. The third maximum will be at x equal to.

Solution. $X_n = \dfrac{n\lambda D}{d}$ or $X_3 = \dfrac{3\lambda D}{d}$

$$x_3 = \frac{3x\,(5000\times10^{-8})\times200}{0.02} = 1.5 \text{ cm}$$

Problem 7. Two slits separated by a distance of 1mm are illuminated with red light of wavelength 6.5×10^{-7} m. The interference fringes are observed on a screen placed 1m from the slits. The distance between third dark fringe & the fifth bright fringe.

Solution. $\beta = \dfrac{\lambda D}{d} = \dfrac{6.5\times10^{-7}\times1}{10^{-3}}$

$\beta = .65 \times 10^{-3}$ m = .65 mm

The distance between the fifth bright fringe from third dark fringe

$= 5\beta - 2.5\beta \Rightarrow 2.5\beta = 2.5 \times .65 = 1.63$ mm

Problem 8. In an experiment the two slits are 0.5 mm apart and the fringes are observed to 100 cm from the plane of the slits. The distance of the 11th bright fringe from the Ist bright fringe is 9.72 mm. Calculate the wavelength.

Solution. Given d = .5 mm = 5×10^{-2} cm,

D = 100 cm

$X_n = X_{11} - X_1 = 9.72$ mm $\quad\therefore\quad X_n = \dfrac{n\lambda D}{d} \quad$ n = 11 – 1 = 10

$$\Rightarrow \lambda = \frac{X_n d}{nD} = \frac{.972\times5\times10^{-2}}{10\times100} \quad\Rightarrow\quad \lambda = 4.86 \times 10^{-5} \text{ cm}$$

Problem 9. In a Young's experiment, two coherent sources are placed 0.90 mm apart and the fringes are observed one meter away. If it produces the second dark fringe at a distance of 1mm from the central fringe, the wavelength of monochromatic light used.

Solution. D =1m, d = .90 mm = $.9 \times 10^{-3}$ m

The distance of the second dark ring from centre = 10^{-3} m

$$\therefore \quad X_n = (2n-1)\frac{\lambda}{2}\,\frac{D}{d}$$

for n = 2,

$$X_n = \frac{3\lambda}{2}\frac{D}{d} \Rightarrow \lambda = \frac{2X_n d}{3D} = \frac{2\times10^{-3}\times.9\times10^{-3}}{3}$$

$\lambda = 6 \times 10^{-7}$ m $\Rightarrow$ $\lambda = 6 \times 10^{-5}$ cm

Problem 10. A beam of light consisting of two wavelength 6500Aº & 5200Aº is used to obtain interference fringes in a young's double slit experiment. The distance between the slits is 2.0 mm and the distance between the plane of the slits and the screen is 120 cm. What is the least distance from the central maximum where the bright fringes due to both the wave length coincide ?

Solution. Suppose the m_{th} bright fringe of 6500Aº coincides with the n_{th} bright fringe of 5200Aº.

$$X_n = \frac{m\lambda_1 D}{d} = \frac{n\lambda_2 D}{d}$$

$$\Rightarrow \frac{m\times6500\times D}{d} = \frac{n\times5200\times D}{d} \Rightarrow \frac{m}{n} = \frac{5200}{6500} = \frac{4}{5}$$

$\therefore$ distance y is $y = \frac{m\lambda_1 D}{d}$ $\Rightarrow$ y = 0.156 cm .

Problem 11. Interference fringes were produced in young's double slit experiment using light of wave length 5000 Aº. When a film of material 2.5×10^{-3} cm thick was placed over one of the slits, the fringe pattern shifted by a distance equal to 20 fringe width. The refractive index of the material of the film is -

Solution.

$$n = \frac{(\mu-1)tD}{d} \quad \text{but } \beta = \frac{\lambda D}{d} \Rightarrow \frac{D}{d} = \frac{\beta}{\lambda}$$

$$n = (\mu-1)t\beta/\lambda$$

$20\beta = (\mu - 1)\, 2.5 \times 10^{-3} (\beta/5000 \times 10^{-8})$

$$\mu - 1 = \frac{20\times5000\times x10^{-8}}{2.5\times10^{-3}} \Rightarrow \mu = 1.4$$

Problem 12. The path difference between two interfering waves at a point on screen is 171.5 times the wavelength. If the path difference is 0.01029 cm. Find the wavelength.

Solution. Path difference = 171.5 λ

$= \frac{343}{2}\lambda$ = odd multiple of half wavelength .

It means dark finge is observed

According to question .

$$0.01029 = \frac{343}{2}\lambda \Rightarrow \lambda = \frac{0.01029\times2}{343} = 6\times10^{-5}\text{cm} \Rightarrow \lambda = 6000\text{A}^o.$$

Problem 13. Find the minimum thickness of a film which will strongly reflect the light of wavelength 589 nm. The refractive index of the material of the film is 1.25

Solution. For strong reflection, the least optical path difference introduced by the film should be $\lambda/2$. The optical path difference between the waves reflected from the surfaces of the film is $2\mu d$.

Thus, for strong reflection,

$2\mu d = \lambda/2$

$$d = \frac{\lambda}{4\mu} = \frac{589}{4\times1.25} = 118 \text{ nm.}$$

KEY CONCEPT

Wavefronts :

(i) Rays are perpendicular to wavefronts.

(ii) The time taken by light to travel from one wavefront to another is the same along any ray.

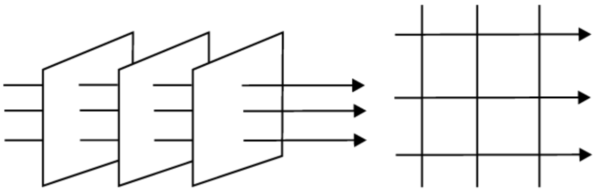
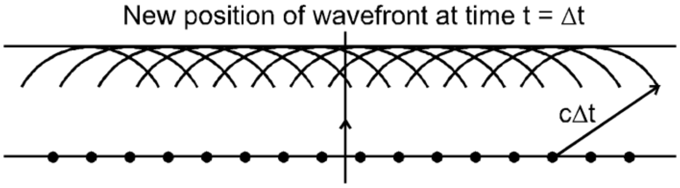


Figure : Wavefronts and the corresponding rays in case of plane wave.

Huygens' Principle :

All points on a wavefront serve as point sources of spherical secondary wavelets. After a time t, the new position of the wavefront will be that of a surface tangent to these secondary wavelets.



☞ **Interference of waves of intensity I_1 and I_2 :**

resultant intensity,

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\Delta\phi)$ where, $\Delta\phi$ = phase difference.

☞ **For Constructive Interference :**

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

☞ **For Destructive interference :**

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

If sources are incoherent

$I = I_1 + I_2$, at each point.

☞ **YDSE :**

Path difference, $\Delta p = S_2P - S_1P = d \sin\theta$

if $d << D$ $= \frac{dy}{D}$ if $y << D$

for maxima,

$\Delta p = n\lambda \Rightarrow y = n\beta \quad n = 0, \pm1, \pm2 \ldots\ldots$

for minima

$$\Delta p = \quad \Delta p = \begin{cases} (2n-1)\frac{\lambda}{2} & n = 1, 2, 3\ldots\ldots\ldots\ldots \\ (2n+1)\frac{\lambda}{2} & n = -1, -2, -3\ldots\ldots \end{cases}$$

$$\Rightarrow \quad y = \begin{cases} (2n-1)\frac{\beta}{2} & n = 1, 2, 3\ldots\ldots\ldots\ldots \\ (2n+1)\frac{\beta}{2} & n = -1, -2, -3\ldots\ldots \end{cases}$$

where, fringe width $\beta = \frac{\lambda D}{d}$

Here, λ = wavelength in medium.

☞ **Highest order maxima :** $n_{max} = \left[\frac{d}{\lambda}\right]$

total number of maxima = $2n_{max} + 1$

☞ **Highest order minima :** $n_{max} = \left[\frac{d}{\lambda} + \frac{1}{2}\right]$

total number of minima = $2n_{max}$.

☞ **Intensity on screen :**

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\Delta\phi)$ where, $\Delta\phi = \frac{2\pi}{\lambda}\Delta p$

If $I_1 = I_2$, $I = 4I_1 \cos^2\left(\frac{\Delta\phi}{2}\right)$

☞ **YDSE with two wavelength λ_1 & λ_2 :**

The nearest point to central maxima where the bright fringes coincide:

$y = n_1\beta_1 = n_2\beta_2$ = Lcm of β_1 and β_2

The nearest point to central maxima where the two dark fringes coincide,

$y = (n_1 - \frac{1}{2})\,\beta_1 = n_2 - \frac{1}{2})\,\beta_2$

☞ **Optical path difference**

$\Delta p_{opt} = \mu \Delta p$

Phase difference $\Delta\phi = \frac{2\pi}{\lambda}\Delta p = \frac{2\pi}{\lambda_{vacuum}}\Delta p_{opt.}$

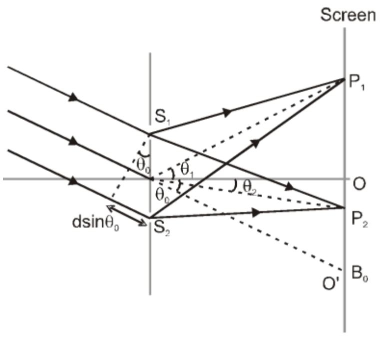
Displacement of fringe pattern on introduction of a glass-slab in the path of one of the slabs:

$$\Delta = (\mu - 1)\,t.\frac{D}{d} = (\mu - 1)t\frac{B}{\lambda}.$$

This shift is in the direction of the slit before which the glass slab is placed. If the glass slab is placed before the upper slit, the fringe pattern gets shifted upwards and if the glass slab is placed before the lower slit the fringe pattern gets shifted downwards.

☞ **YDSE with Oblique Incidence :**

In YDSE, ray is incident on the slit at an inclination of θ_0 to the axis of symmetry of the experimental set-up



We obtain central maxima at a point where, $\Delta p = 0$.

or $\theta_2 = \theta_0$.

This corresponds to the point O' in the diagram.

Hence we have or path difference.

$$\Delta p = \begin{cases} d(\sin\theta_0 + \sin\theta) - \text{for points above O} \\ d(\sin\theta_0 - \sin\theta) - \text{for points between O\&O'} \\ d(\sin\theta - \sin\theta_0) - \text{for points below O'} \end{cases} \quad \text{... (8.1)}$$

☞ **Thin-Film Interference :**

Case - 1: Medium on the two sides of the film are either both denser or both rarer

for interference in reflected light $2\mu d = \begin{cases} n\lambda & \text{for destructive interference} \\ (n+\frac{1}{2})\lambda & \text{for constructive interference} \end{cases}$

for interference in transmitted light $2\mu d = \begin{cases} n\lambda & \text{for constructive interference} \\ (n+\frac{1}{2})\lambda & \text{for destructive interference} \end{cases}$

Case - 2 : Medium on one side of the film is denser and that on the other side is rarer.

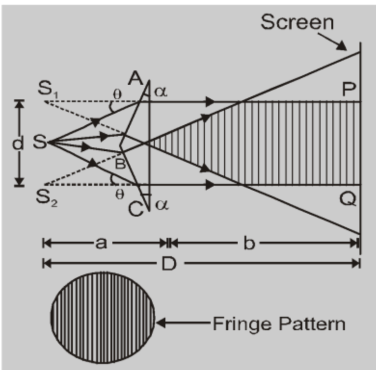
Here condition for interference in reflected light is same as the condition for interference in transmitted light of case 1, and vice versa.

☞ **Fresnel's Birpism Experiment :**

(1) It is optical device to obtain two coherent sources by refraction of lights.

(2) The angle of biprism is 179º & refracting angle is $\alpha = 1/2^{\circ}$.

(3) Distance between source & screen D = a + b.



Distance between two coherent source = d = 2a $(\mu - 1)\alpha$

Where a = distance between source & Biprism

b = distance between screen & Biprism

μ = refractive index of the material of prism.

$$\lambda = \frac{d\beta}{D} = \frac{2a(\mu - 1)\alpha\beta}{(a+b)}$$

☞ **Diffraction :**

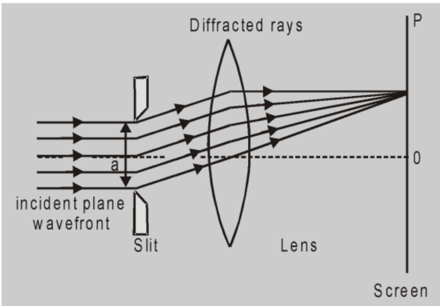
Definltion : The phenomenon of bending of light waves around the sharp edges of opaque obstacles or aperture and their encroachment in the geometrical shadow of obstacle or aperture is defined as diffraction of light.

Necessary Conditions of Diffraction of Waves

The size of the obstacle (a) must be of the order of the wavelength of the waves (λ).

$$\frac{a}{\lambda} \approx 1$$

☞ **fraunhofer diffraction for single slit :**



In this diffraction pattern central maxima is bright on the both side of it, maxima & minima occurs symmetrically

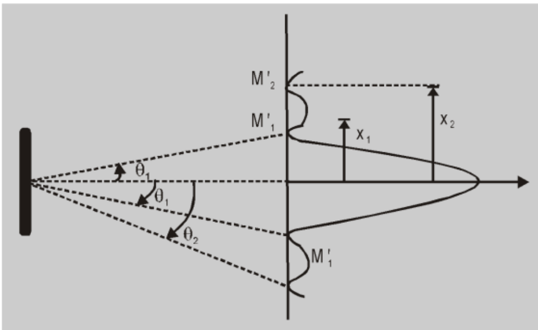
For Diffraction Maxima : $a \sin\theta = (2n + 1)\lambda/2$

For Diffraction Minima : $a \sin\theta = n\lambda$

The maxima or minima is observed due to the superposition of waves emerging from infinite secondary sources between A & B points of slit.

Fringe width :

The distance between two secondary minima formed on two sides of central maximum is known as the width of central maximum



$$W = \frac{2f\lambda}{a}$$

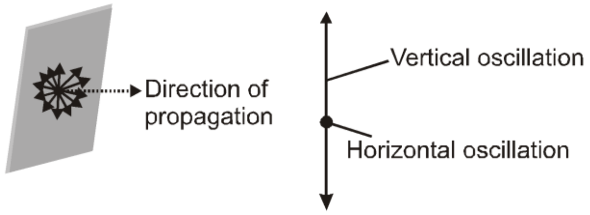
f = focal distance of convex lens

a = width of slit

Angular width = $W_\theta = \frac{2\lambda}{a}$

☞ **Polarisation of Light :**

(1) **Unpolarised light** : In ordinary light (light from sun, bulb etc.) the electric field vectors are distributed in all directions in **a** light is called unpolarised light.



(2) **Polarised light :** The phenomenon of limiting the vibrating of electric field vector in one direction in a plane perpendicular to the direction of propagation of light wave is called polarization of light

(i) The plane in which oscillation occurs in the polarised light is called plane of oscillation.

(ii) The plane perpendicular to the plane of oscillation is called plane of polarisation.

(iii) Light can be polarised by transmitting through certain crystals such as tourmaline or polaroids.

(3) **Brewster's Low :** For light incident at the Brewster angle θ_B, The reflected and refracted rays are perpendicular to each other. The refracted light has only perpendicular components. The reflected light is then fully polarized perpendicular to the plane of incidence.

Brewster angle = $\theta_B = \tan^{-1}\frac{n_2}{n_1}$