ELECTROMAGNETIC WAVES

INTRODUCTION

Maxwell formulated a set of equations involving electric and magnetic fields, and their sources, the charge and current densities. These equations are known as Maxwell's equations. Together with the Lorentz force formula, they mathematically express all the basic laws of electromagnetism.

The most important prediction to emerge from Maxwell's equations is the existence of electromagnetic waves, which are (coupled) time varying electric and magnetic fields that propagate in space. The speed of the waves, according to these equations, turned out to be very close to the speed of light (3×10^8 m/s), obtained from optical measurements. This led to the **remarkable conclusion that light is an electromagnetic wave**. Maxwell's work thus unified the domain of electricity, magnetism and light, Hertz, in 1885, experimentally demonstrated the existence of electromagnetic waves. Its technological use by marconi and others led in due course to the revolution in communication that we are witnessing today.

In this unit, we first discuss the need for displacement current and its consequences. Then we present a descriptive account of electromagnetic waves. The broad spectrum electromagnetic waves, is stretching from γ rays (wavelength ~ 10^{-12} m) to long radio waves (wavelength ~ 10^6 m)

DISPLACEMENT CURRENT

We have seen that an electrical current produces a magnetic field around it. Maxwell showed that for logical consistency, a changing electric field must also produce a magnetic field. This effect is of great importance because it explains the existence of radio waves, gamma rays and visible light, as well as all other forms of electromagnetic waves.

To see how, a changing electric field gives rise to a magnetic field, let us consider the process of charging of a capacitor and apply Ampere's circuital law given by

$$\oint B.dl = \mu_0 i \qquad \text{........ (1)}$$

Figure 1 Origin of displacement current

Figure 1 (a) shows a parallel plate capacitor C which is a part of circuit through which a time-dependent current i (t) flows. Let us find the magnetic field at a point such as P, in a region outside the parallel plate capacitor. For this, we consider a plane circular loop of radius r whose plane is perpendicular to the direction of the current - carrying wire, and which is centred symmetrically with respect to the wire. From symmetry, the magnetic field is directed along the circumference of the circular loop and is the same in magnitude at all points on the loop so if B is the magnitude of the field, the left side of equation. (1) is B (2πr). So we have

$$B(2\pi r) = \mu_0 i(t) \qquad \text{......... (2)}$$

Now, consider a different surface, which has the same boundary. This is a pot like surface (Fig.1 (b)] which nowhere touches the current, but has its bottom between the capacitor plates; its mouth is the circular loop and is shaped like a tiffin box (without the lid) [Fig. 1 (b)]. On applying Ampere's circuital law to such surface with the same perimeter, we find that the left hand side of Eq. (1) has not changed but the right hand side is zero and not μ_0 i, since no current passes through the surface of Fig 1 (b). So we have a contradiction; calculated one way, there is a magnetic field at a point P; calculated another way, the magnetic field at P is zero. Since the contradiction arises from our use of Ampere's circuital law, this

law must be missing something. The missing term must be such that one gets the same magnetic field at point P, no matter what surface is used.
We can actually guess the missing term by looking carefully at Fig. 1 (b). Is there anything passing through the surface S between the plates of the capacitor ? Yes, of course, the electric flux. If the plates of the capacitor have an area A, and a total charge Q, the magnitude of the electric field **E** between the plates is (Q/A)/ ε_0. The field is perpendicular to the surface S of Fig.1 (b). It has the same magnitude over the area A of the capacitor plates, and vanishes outside it. So the electric flux Φ_E through the surface S is found using Gauss's law, is given by,

$$\Phi_E = |E| A = \frac{1}{\varepsilon_0}\frac{Q}{A} A = \frac{Q}{\varepsilon_0} \qquad \text{........... (3)}$$

Now if the charge Q on the capacitor plates changes with time, there is a current i = (dQ / dt), so that using Eq. (3), we have

$$\frac{d\Phi_E}{dt} = \frac{d}{dt}\left(\frac{Q}{\varepsilon_0}\right) = \frac{1}{\varepsilon_0}\frac{dQ}{dt}$$

This implies that for consistency, the additional current should be

$$\varepsilon_0 \left(\frac{d\Phi_E}{dt}\right) = i_d \qquad \text{............. (4)}$$

This is the missing term in Ampere's circuital law. If we generalise amperes law by adding to the current carried by conductors through the surface, another term which is ε_0 times the rate of change of electric flux through the same surface, the total current has the same value for all surfaces. If this is done, there is no contradiction in the value of B obtained anywhere using the generalized Amper's law. B at the point P is non-zero no matter which surface is used for calculating it. B at a point P outside the plates [Fig.1 (a)] is the same as at a point M just inside, as it should be. The current carried by conductors due to flow of charges is called conduction current. The current, given by Eq. (4), is a new term, and is due to time changing electric field (or electric displacement, $\varepsilon_0 E$). It is therefore called *displacement current or Maxwell's displacement current. Figure 2 shows the electric and magnetic fields inside the parallel plates capacitor discussed above. The generalisation made by Maxwell then is the following. The source of a magnetic field is not just the conduction electric current due to flowing charges, but also the time rate of change of electric field. More precisely, the total current i is the sum of the conduction current denoted by* i_C, *and the displacement current denoted by* i_d *(* $= \varepsilon_0 (d\Phi_E)/ dt$*). So we have*

$$i = i_c + i_d = i_c + \varepsilon_0 \frac{d\Phi_E}{dt} \qquad \text{............ (5)}$$

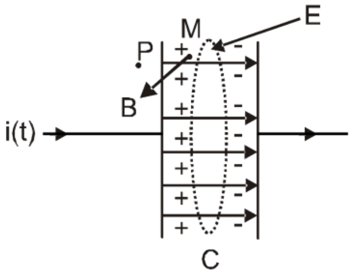


Figure 2 continuity of electric current $i = i_c + I_a$ outside condenser it only i_c and inside it is only i_d

In explicit terms, this means that outside the capacitor plates, we have only conduction current i_c = i, and no displacement current, i.e., i_d = 0. On the other hand, inside the capacitor, there is no conduction current, i.e., i_c = 0, and there is only displacement current, so that i_d = i.
The generalised (and correct) Ampere's circuital law has the same form as Eq, (1), with one difference: " the total current passing through any surface of which the closed loop is the perimeter" is the sum of the conduction current and the displacement current The generalised law is

$$\oint B \bullet d\ell = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \qquad \text{........... (6)}$$

and is known as Ampere-Maxwell's law.

In all respects, the displacement current has the same physical effects as the conduction current. In some cases, for example, steady electric fields in a conducting wire, the displacement current may be zero since the electric field E does not change with time. In other cases, for example, the charging capacitor above, both conduction and displacement currents may be present in different regions of space. *In most of the cases, they both may be present in the same region of space, as there exist no perfectly conducting or perfectly insulating medium. Most interestingly, there may be large regions of space where there is no conduction current, but there is only a displacement current due to time-varying electric fields.* In such a region, we expect a magnetic field, though there is no (conduction) current source nearby. The prediction of such a displacement current can be verified experimentally. For example, a magnetic field (say at point M) between the plates of the capacitor in Fig. 3 can be measured and is seen to be the same as that just outside (at P).

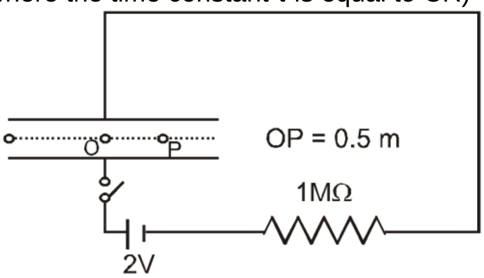
The displacement current has (literally) far reaching consequences. One thing we immediately notice is that the laws of electricity and magnetism are now more symmetrical.
The time-dependent electric and magnetic field give rise to each other. Faraday's law of electromagnetic induction and Ampere-Maxwell law give a quantitative expression of this statement, with the current being the total current as in Eq. (5). One very important consequence of this symmetry is the existence of electromagnetic waves, which we discuss qualitatively in the next section.

Maxwell's Equations

1. $\oint \vec{E} \bullet \vec{dA} = Q/\varepsilon_0$ ………7(a) (Gauss's Law for electricity)
2. $\oint \vec{B} \bullet \vec{dA} = 0$ ………7(b) (Gauss's Law for magnetism)
3. $\oint \vec{E} \bullet \vec{d\ell} = \frac{-d\Phi_B}{dt}$ ………7(c) (Faraday's Laws of electromagnetic induction)
4. $\oint \vec{B} \bullet \vec{d\ell} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$ ………7(d) (Ampere - Maxwell Law)

Solved Example

Example 1. A parallel plate capacitor with circular plates of radius 1 m has a capacitance of 1 nF. At t = 0, it is connected for charging in series with a resistor R = 1 M Ω across a 2 V battery (as shown). Calculate the magnetic field at a point P. halfway between the centre and the periphery of the plates, after $t = 10^{-3}$ s. (The charge on the capacitor at time t is q (t) = CV [1 – exp (–t / τ)], where the time constant τ is equal to CR)



Solution. The time constant of the CR circuit is $\tau = CR = 10^{-3}$ s. Then we have
$q(t) = CV [1 - \exp(-t/\tau)]$
$= 2 \times 10^{-9} [1 - \exp(-t/10^{-3})]$
The electric field in between the plates at time t is
$E = \frac{q(t)}{\varepsilon_0 A} = \frac{q}{\pi \varepsilon_0}$; $A = \pi (1)^2\ m^2$ = area of the plates. …..(i)
Consider now a circular loop of radius (1 / 2) m parallel to the plates passing through P. The magnetic field **B** at all points on the loop is along the loop and of the same value. The flux Φ_E through this loop is

Φ_E = E × area of the loop

$= E \times \pi \times \left(\frac{1}{2}\right)^2 = \frac{\pi E}{4} = \frac{q}{4\varepsilon_0}$ ….(ii) follows from (i)

The displacement current

$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{1}{4}\frac{dq}{dt} = 0.5 \times 10^{-6} \exp(-1)$

at $t = 10^{-3}$ s. Now, applying Ampere-Maxwell law to the loop, we get

$B \times 2\pi \times \left(\frac{1}{2}\right) = \mu_0 (i_c + i_d) = \mu_0 (0 + i_d) = 0.5 \times 10^{-6}\ \mu_0 \exp(-1)$

or, $B = 0.74 \times 10^{-3}$ T

(A) ELECTROMAGNETIC WAVES

From equation 7(c) it follows that time varying magnetic field produces electric field. Where as equation 7(d) it follows that time varying electric field produces magnetic field. In case of oscillating charge both electric and magnetic fields are oscillating. Consider a loop of wire carrying alternating current. This will generate circulating time varying (sinusoidal) magnetic filed normal to current loop as shown in figure 3. This time varying sinusoidal magnetic field in turn shall give ruse to circulating electric field. The electric field lines will be perpendicular to circulating magnetic field lines. One field generates the other. Consequently continuous induction and speeding electric and magnetic fields occur.

Sources of electromagnetic waves

How are electromagnetic waves produced ? Neither stationary charges Nor charges in uniform motion (steady currents) can be sources of electromagnetic waves. The former produces only electrostatic field, while the latter produces magnetic fields that, however, do not vary with time. It is an important result of Maxwell's theory that accelerated charges radiate electromagnetic waves.

An oscillating charge generates harmonic electric and magnetic fields. Hence it is a source of electromagnetic waves. Oscillating charge radiates electromagnetic energy in the form of EM waves. As electron transiting from higher energy state to a lower energy state in an atom generates time varying pervades electric and magnetic fields and hence radiate energy in the form of electromagnetic waves (light)

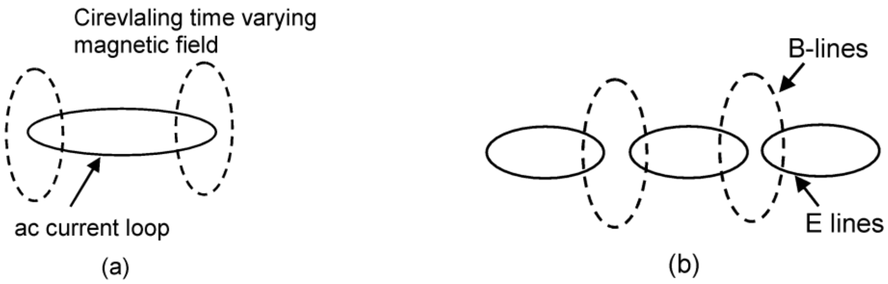


Figure (3) origin of EM waves

(a) ac current loop generating time varying magnetic field.

(b) Generation of E-M waves

The proof of this basic result is beyond the scope of this text, but we can accept it on the basis of rough qualitative reasoning. Consider a charge oscillating with some frequency. (An oscillating charge is an example of accelerating charge.) This produces an oscillating electric field in space, which produces an oscillating magnetic field, which in turn, is a source of oscillating electric field, and so on. The oscillating electric and magnetic fields thus regenerate each other thus the waves propagates through the space. The frequency of electromagnetic wave naturally equals the frequency of oscillation of the charge. The energy associated with the propagating wave comes at the expense of the energy of the source-the accelerated charge. When are electron transits from higher energy state to tower energy state in are atom. The transiting electron generates oscillating electric and magnetic fields. The difference in energy of two levels comes out an electro magnetic wave. Light is thus an e.m.
wave

Nature of electromagnetic wave

It can be shown from Maxwell's equations that electric and magnetic field in an electromagnetic wave are perpendicular to each other and to the direction of propagation. It appears reasonable.
In Fig 4, we show a typical example of a plane electromagnetic wave propagating along the z direction (the fields are shown as a function of the z coordinate, at a given time t). The electric field E_x is along the x-axis, and varies Sinusoidal with z, at a given time. The magnetic field B_y is along the y-axis and again varies Sinusoidal with z. The electric and magnetic fields E_x and B_y are perpendicular to each other, and to the direction z of propagation. *EM waves are transverse in nature*. We can write E_x and B_y as follows :

$E_x = E_0 \sin (kz - \omega t)$ .... 8(a)

$B_y = B_0 \sin (kz - \omega t)$ .... 8 (b)

Here k is related to the wave length λ of the wave by the usual equation

$k = \frac{2\pi}{\lambda}$ .... (9)

and ω is the angular frequency. k is the magnitude of the wave vector (or propagation vector) $\vec{k}$ and its direction describes the direction of propagation of the wave.
The speed of propagation of the wave is (ω/k). Using Eqs. [8 (a) and (b)] for E_x and B_y and Maxwells equation we finds that

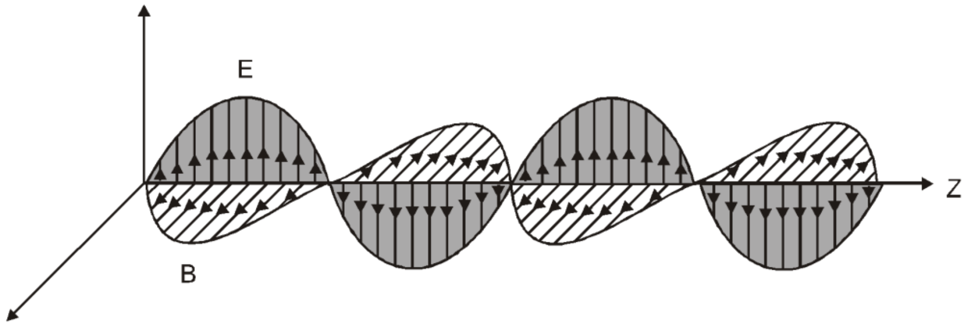


Figure 4 Plane EM wave propagating in z direction

Transverse Nature of EM waves. E and B are mutually perpendicular and perpendicular to direction of propagation. $\left(\vec{E} \times \vec{B} = \text{const}\,\vec{k}\right)$

$\omega = ck$, where, $c = 1 / \sqrt{\mu_0 \varepsilon_0}$ .... (10)

The relation ω = ck is the standard one for waves. This relation is often written in terms of frequency.
ν (=ω/ 2π) and wavelength. λ (= 2π / k) as

$2\pi\nu = c\left(\frac{2\pi}{\lambda}\right)$ or

$\nu\lambda = c$ .... (11)

It follows from Maxwell's equations that the magnitude of the electric and the magnetic fields in an electromagnetic waves are related as

$\frac{E_0}{B_0} = c$ ……….(12)

In a material medium, the total electric and magnetic fields inside a medium are described in terms of a permittivity ε and a magnetic permeability μ (These describe the factors by which the total fields differ from the external fields). These replace ε_0 and μ_0 in the description to electric and magnetic fields in maxwell's equation in free space with the result that in a material medium of permittivity ε and magnetic permeability μ, the velocity of light becomes,

$\upsilon = \frac{1}{\sqrt{\mu\varepsilon}}$ .... (13)

In a medium and consequently $\frac{E}{B} = \upsilon$ .... (14)

(B) **Refractive index :** The velocity of light depends on electric and magnetic properties of the medium. The velocity of electromagnetic waves in free space or vacuum is an important fundamental constant. It has been shown by experiments on electromagnetic waves of different wavelength that this velocity in free space is the same (independent of wavelength)

Refractive index is defined as ratio of speed of light is vacuum to its velocity in medium therefore,

Refractive index $$n = \frac{c}{v} = \sqrt{\frac{\mu}{\mu_0}\frac{\varepsilon}{\varepsilon_0}} = \sqrt{\varepsilon_r}$$ ....(15)

ε_r is relative permittivity or dielectric constant of the medium

(C) **Energy and Momentum :** Do electromagnetic waves carry energy and momentum like other waves ? Yes, they do. In a region of free space with electric field E, there is an electric energy density

$$U_E = \frac{\varepsilon_0 E^2}{2} (J/m^3)$$ ....16(a)

Similarly, as seen associated with a magnetic field B is a magnetic energy density is given by

$$U_B = \frac{B^2}{2\mu_0} (J/m^3)$$ ....16(b)

As electromagnetic wave contains both electric and magnetic fields, there is a non-zero energy density associated with it. Now consider a plane perpendicular to the direction of propagation of the electromagnetic wave (Fig. 4). If there are, on this plane, electric charges these will be set to sustained motion by electric and magnetic fields of the electromagnetic wave. The charges thus acquire energy and momentum from the waves. *This just illustrates the fact that an electromagnetic wave (like other waves.) carry energy and momentum light carries energy from the sun to the earth, thus making life possible on the earth.*

(D) **Radiation pressure :** Since it carries momentum, an electromagnetic wave also exerts pressure called radiation pressure. If the total energy transferred to a surface in time t is U. It can be shown that the magnitude of the total momentum delivered to this surface (for complete absorption) is,

$p = \frac{U}{c}$, and ....17(a)

$p = \frac{2U}{c}$ for perfectly reflecting surface. ....17(b)

(E) **Poynting vector :**
It is vector directed along the line of propagation of electromagnetic wave. Its magnitude is equal to amount of energy flowing per unit time, per unit area perpendicular to electromagnetic wave.

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$ 18

This is also the intensity of propagating electromagnetic wave

Solved Examples

Example 2. A plane electromagnetic wave of frequency 25 MHz travels in free space along the x-direction. At a particular point in space and time, **E** = 6.3 $\hat{j}$ V/m. What is B at this point ?

Solution. Using Eq. (8.10), the magnitude of B is

$B = \frac{E}{c} = \frac{6.3 V/m}{3 \times 10^8 m/s} = 2.1 \times 10^{-8}$ T

To find the direction, we note that E is along y-direction and the wave propagate along x-axis. Therefore, **B** should be in a direction perpendicular to both x- and y-axes. Using vector algebra, **E × B** should be along x-direction. Since, $(+\hat{j}) \times (+\hat{k}) = \hat{i}$, Thus, B is along the z-direction

Thus, **B** = $2.1 \times 10^{-8}\ \hat{k}$ T

Example 3. The magnetic field in a plane electromagnetic wave is given by $B_y = 2 \times 10^{-7} \sin (0.5 \times 10^3 x + 1.5 \times 10^{11} t)$ T.
(a) What is the wavelength and frequency of the wave ?
(b) Write an expression for the electric field.

Solution. (a) Comparing the given equation with

$B_y = B_0 \sin \left[2\pi\left(\frac{x}{y} + \frac{t}{T}\right)\right]$

we get, $\lambda = \frac{2\pi}{0.5 \times 10^3}$ m = 1.26 cm,

and $\frac{1}{T} = \nu = (1.5 \times 10^{11}) / 2\pi = 23.9$ GHz

(b) $E_0 = B_0 c = 2 \times 10^{-7}$ T $\times 3 \times 10^8$ m/s = 60 V/m

The electric field is perpendicular to the direction of propagation and the direction of magnetic field. Therefore, the electric field along the z-axis is obtained as

$E_z = 60 \sin (0.5 \times 10^3 x + 1.5 \times 10^{11} t)$ V/m

Example 4. Light with an energy flux of 18 W/cm^2 falls on a nonreflecting surface at normal incidence. If the surface has an area of 20cm^2, find the average force exerted on the surface during a 30 minute time span.

Solution. The total energy falling on the surface is

$U = (18\ \text{W/cm}^2) \times (20\text{cm}^2) \times (30 \times 60)$

$= 6.48 \times 10^5$ J

Therefore, the total momentum delivered (for complete absorption) is

$p = \frac{U}{c} = \frac{6.48 \times 10^5\ \text{J}}{3 \times 10^8\ \text{m/s}} = 2.16 \times 10^{-3}$ kg m/s

The average force exerted on the surface is

$F = \frac{p}{t} = \frac{2.16 \times 10^{-3}}{0.18 \times 14^4} = 1.2 \times 10^{-6}$ N

Example 5. Calculate the electric and magnetic fields produced by the radiation coming from a 100 W bulb at a distance of 3 m. Assume that the efficiency of the bulb is 2.5% and it is a point source.

Solution. The bulb, as a point source, radiates light in all directions uniformly. At a distance of 3 m, the surface area of the surrounding sphere is

$A = 4\pi r^2 = 4\pi(3)^2 = 113\ \text{m}^2$

The intensity at this distance is

$I = \frac{\text{Power}}{\text{Area}} = \frac{100\text{W} \times 2.5\%}{113\text{m}^2} = 0.022\ \text{W/m}^2$

On an average half of this intensity is provided by the electric field and half by the magnetic field.

$\frac{1}{2}I = \frac{1}{2}\left(\varepsilon_0 E_{rms}^2 c\right) = \frac{1}{2}(0.022\ \text{W/m}^2)$

$E_{rms} = \sqrt{\frac{0.022}{(8.85 \times 10^{-12})(3 \times 10^8)}}\ \text{V/m} = 2.9$ V/m

The value of E found above is the root mean square value of the electric field. Since the electric field in a light beam is sinusoidal, the peak electric field, E_0 is

$E_0 = \sqrt{2} E_{rms} = \sqrt{2} \times 2.9$ V/m = 4.07 V/m

Electric field strength of light is fairly large

$$B_{rms} = \frac{E_{rms}}{c} = \frac{2.9\,Vm^{-1}}{3 \times 10^{8}\,ms^{-1}} = 9.6 \times 10^{-9}\,T$$

Again, since the field in the light beam is sinusoidal, the peak magnetic field is $B_0 = \sqrt{2}\,B_{rms} = 1.4 \times 10^{-8}$ T. Note that although the energy in the magnetic field is equal to the energy in the electric field, the magnetic field is evidently very weak.

ELECTROMAGNETIC SPECTRUM

At the time Maxwell predicted the existence of electromagnetic waves, the only familiar electromagnetic waves were the visible light waves. The existence of ultraviolet and infrared waves was barely established. By the end of the nineteenth century, X -rays and gamma rays had also been discovered. We now know that, electromagnetic waves include visible light waves, X-rays gamma rays, radio waves, microwaves, ultraviolet and infrared waves. The classification of electromagnetic waves according to frequency is the electromagnetic spectrum is shown in table 1 along with applications. There is no sharp division between one kind of wave and the next. The classification is based roughly on how the waves are produced and / or detected.

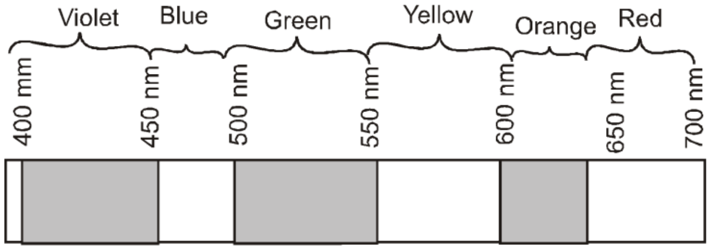


Figure - spectrum of visible light

Table-1 : Summaries various bands of the electromagnetic spectrum, there origin and detection

Type	Wavelength range	Production	Detection
Radio	> 0. 1 m	Rapid acceleration and decelerations of electrons in aerials (LCR)	Receiver's aerials
Microwave	0.1 m to 1 mm	Klystron valve or magnetron Valve electromagnetic oscillations in closed metallic cavity	Point contact diodes
Infra-red	1 mm to 700 nm	Vibration of atoms in molecules and theie rotations	Thermopiles Bolometer, Infrared photodiodes
Light	700 nm to 400 nm	Electrons in atoms emit light when they move from one energy level to a lower energy	The eye, photocells, Photographic film
Ultraviolet	400 nm to 1 nm	Atomic transitions in high atom electrons moving from higher energy level to a lower level	Photocells Photographic film
X-rays	1 nm to 10^{-3} nm	X-ray transitions of or inner shell electrons	Photograpic film, Geiger tubes, Ionisation chamber
Gamma rays	< 10^{-3} nm	Radioactive decay of the nucleus Nuclear reactions	Nuclear detectors

Table-2 : Describes various applications of the frequency bands

Radio (RF)	AM, FM radio cell phone
Micro waves	Microwave oven, Radar satellite communication
Infrared (IR)	Remote sensing, Remote, control study molecular dynamics night photography

Visible light	Human vision snaker have IR vision.
Ultraviolet (UV)	Photography there, production g melanin on human skin water purefrer
X-Rays	Diagnosis of one fractures, radiation therapy
γ-rays	Sterilization of medical equipments material radiation therapy cancer cells proscing step nuclear processes