FLUID MECHANICS

DEFINITION OF FLUID

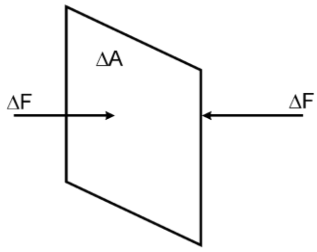
The term fluid refers to a substance that can flow and does not have a shape of its own. For example liquid and gases.

Fluid includes property → (A) Density (B) Viscosity (C) Bulk modulus of elasticity (D) pressure (E) specific gravity

PRESSURE IN A FLUID

The pressure p is defined as the magnitude of the normal force acting on a unit surface area.

$P = \frac{\Delta F}{\Delta A}$ ΔF = normal force on a surface area ΔA.



The pressure is a scalar quantity. This is because hydrostatic pressure is transmitted equally in all directions when force is applied, which shows that a definite direction is not associated with pressure.

Thrust. The total force exerted by a liquid on any surface in contact with it is called thrust of the liquid.

Note :

The normal force exerted by liquid at rest on a given surface in contact with it is called thrust of liquid on that surface.

The normal force (or thrust) exerted by liquid at rest per unit area of the surface in contact with it, is called pressure of liquid or hydrostatic pressure.

If F be the normal force acting on a surface of area A in contact with liquid, then pressure exerted by liquid on this surface is P = F/A

(1) Units : N/m^2 or Pascal (S.I) and $Dyne/cm^2$ (C.G.S)

(2) Dimension : (P) = $\frac{[F]}{[A]} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

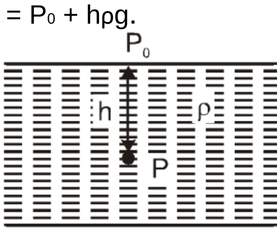
(3) At a point pressure acts in all directions and a definite direction is not associated with it. So pressure is a tensor quantity.

(4) Atmospheric pressure : The gaseous envelope surrounding the earth is called the earth's atmosphere and the pressure exerted by the atmosphere is called atmospheric pressure its value on the surface of the earth at sea level is nearly 1.013×10^5 N/m^2 or Pascal in S.I. other practical units of pressure are atmosphere, bar and torr (mm of Hg)

1 atm = 1.01×10^5 Pa = 1.01 bar = 760 torr

The atmospheric pressure is maximum at the surface of earth and goes on decreasing as we move up into the earth's

(5) If P_0 is the atmospheric pressure then for a point at depth h below the surface of a liquid of density ρ. hydrostatic pressure P is given by $P = P_0 + h\rho g$.



(6) Hydrostatic pressure depends on the depth of the point below the surface (h). nature of liquid (ρ) and acceleration due to gravity (g) while it is independent of the amount of liquid, shape of the container or cross-sectional area considered. So if a given liquid is filled in vessels of different shapes to same height, the pressure at the base in each vessel's will be the same, though the volume or weight of the liquid in different vessels will be different.

(A) (B) (C)

(7) In a liquid at same level, the pressure will be same at all points, if not, due to pressure difference the liquid cannot be at rest. This is why the height of liquid is the same in vessels of different shapes containing different amounts of the same liquid at rest when they are in communication with each other.

(8) Gauge pressure : The pressure difference between hydrostatic pressure P and atmospheric pressure P_0 is called gauge pressure.
$P - P_0 = h\rho g$

CONSEQUENCES OF PRESSURE

(i) Railway tracks are laid on large sized wooden or iron sleepers. This is because the weight (force) of the train is spread over a large area of the sleeper. This reduces the pressure acting on the ground and hence prevents the yielding of ground under the weight of the train.

(ii) A sharp knife is more effective in cutting the objects than a blunt knife.
The pressure exerted = Force/area. The sharp knife transmits force over a small area as compared to the blunt knife. Hence the pressure exerted in case of sharp knife is more than in case of blunt knife.

(iii) A camel walks easily on sand but a man cannot inspite of the fact that a camel is much heavier than man. This is because the area of camel's feet is large as compared to man's feet. So the pressure exerted by camel on the sand is very small as compared to the pressure exerted by man. Due to large pressure, sand under the feet of man yields and hence he cannot walk easily on sand.

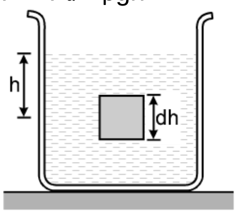
VARIATION OF PRESSURE WITH HEIGHT

Assumptions : (i) unaccelerated liquid (ii) uniform density of liquid (iii) uniform gravity

Weight of the small element dh is balanced by the excess pressure. It means $\frac{dp}{dh} = \rho g$.

$$\int_{P_a}^{P} dp = \rho g \int_{0}^{h} dh$$

$\Rightarrow$ $P = P_a + \rho gh$



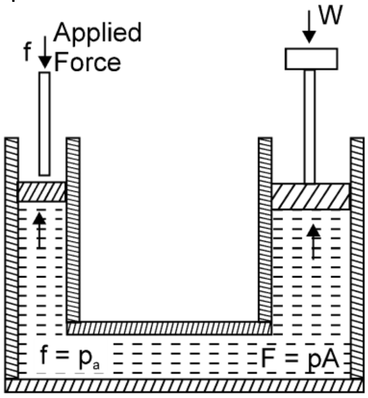
PASCAL'S LAW

if the pressure in a liquid is changed at a particular, point the change is transmitted to the entire liquid without being diminished in magnitude. In the above case if P_a is increased by some amount than P must increase to maintained the difference $(P - P_a) = h\rho g$. This is Pascal's Law which states that Hydraulic lift is common application of Pascal's Law.

1. Hydraulic press.

$p = \frac{f}{a} = \frac{W}{A}$ or $f = \frac{W}{A} \times a$ as A >> a then f << W..

This can be used to lift a heavy load placed on the platform of larger piston or to press the things placed between the piston and the heavy platform. The work done by applied force is equal to change in potential energy of the weight in hydraulic press.



Density

In a fluid, at a point, density ρ is defined as :

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$

(1) In case of homogenous isotropic substance, it has no directional properties, so is a scalar.

(2) It has dimensions (ML^{-3}) and S.I. unit kg/m^3 while C.G.S. unit g/cc with 1g /cc = 10^3 kg/m^3

(3) Density of substance means the ratio of mass of substance to the means the ratio of mass of a body to the volume of the body. So for a solid body.

Density of body = Density of substance

While for a hollow body, density of body is lesser than that of substance [As $V_{body} > V_{sub}$.]

(4) When immiscible liquids of different densities are poured in a container, the liquid of highest density will be at the bottom while that of lowest density at the top and interfaces will be plane.

(5) Sometimes instead of density we use the term relative density or specific gravity which is defined as :

$$RD = \frac{\text{Density of body}}{\text{Density of water}}$$

(6) If m_1 mass of liquid of density ρ_1 and m_2 mass of density ρ_2 are mixed. then as

$m = m_1 + m_2$ and $V = (m_1 / \rho_1) + (m_2 / \rho_2)$

[As $V = m/\rho$]

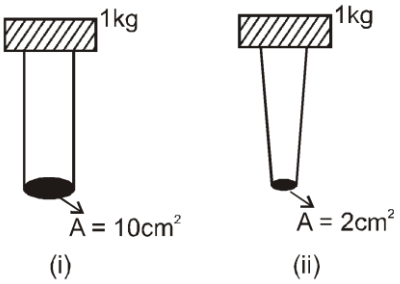
(7) If V_1 volume of liquid of density ρ_1 and V_2 volume of liquid of density ρ_2 are mixed, then as

$m = \rho_1 V_1 + \rho_2 V_2$ and $V = V_1 + V_2$ [As $\rho = m/V$]

If $V_1 = V_2 = V$ $\rho_1(\rho_1 + \rho_2)/2$ = Arithmetic Mean

Solved Examples

Example 1. A body of one kg placed on two object of negligible mass. Calculate pressure due to force on its bottom.



Solution : $P = \frac{F}{A} = \frac{mg}{A}$

(i) $P_1 = \frac{1 \times 10N}{10 \times 10^{-2} m^2} = 10^4 N/m^2$

(ii) $P_2 = \frac{1 \times 10N}{2 \times 10^{-4}}$ = 5 × 10^4 (= $5p_1$)

Example 2. For a hydraulic system A car of mass 2000 kg standing on the platform of Area $10m^2$ while the area other side platform 10 cm^2 find the mass required to balance the car

Solution : According to the Pascal Law

$P_1 = P_2 \Rightarrow \frac{m_{car} g}{A_{car}} = \frac{mg}{A}$

$\Rightarrow m = \left(\frac{A}{A_{car}}\right) \times m_{car} = \frac{10cm^2}{10m^2} \times 2000\ kg$ = 0.2 kg = 200 gm

Example 3. If two liquids of same masses but densities P_1 and P_2 respectively are mixed, then density of mixture is given by

(1) $\rho = \frac{\rho_1 + \rho_2}{2}$ (2) $\rho = \frac{\rho_1 + \rho_2}{2\rho_1\rho_2}$ (3) $\rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$ (4) $\rho = \frac{\rho_1\rho_2}{\rho_1 + \rho_2}$

Solution : $\rho = \frac{\text{Total mass}}{\text{Total volume}} = \frac{2m}{V_1 + V_2} = \frac{2m}{m\left(\frac{1}{\rho_1 + \rho_2}\right)}$ $\therefore \rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$

Example 4. If two liquids of same masses but different densities ρ_1 and ρ_2 are mixed, then density of mixture is given by

(1) $\rho = \frac{\rho_1 + \rho_2}{2}$ (2) $\rho = \frac{\rho_1 + \rho_2}{2\rho_1\rho_2}$ (3) $\rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$ (4) $\rho = \frac{\rho_1\rho_2}{\rho_1 + \rho_2}$

Solution : $\rho = \frac{\text{Total mass}}{\text{Total volume}} = \frac{m_1 + m_2}{2V} \frac{V(\rho_1 + \rho_2)}{2V} = \frac{\rho_1 + \rho_2}{2}$

Example 5. If pressure at the half depth of a lake equal to 2/3 pressure at the bottom of the lake, then the depth of the lake [$\rho_{water} = 10^3\ kgm^{-3}$, $P_0 = 10^5\ N/m^2$]

(1) 10 m (2) 20 m (3) 60 m (4) 30 m

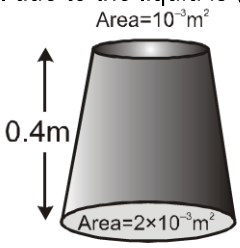
Solution : Pressure at bottom of the lake = $P_o + h\rho g$

Pressure at half the depth of a lake = $P_o + \frac{h}{2}\rho g$

According to given condition $P_o + \frac{1}{2}h\rho g = \frac{2}{3}(P_o + h\rho g)$

$\Rightarrow \frac{1}{3}P_o = \frac{1}{6}h\rho g \Rightarrow h = \frac{2P_o}{\rho g} = \frac{2 \times 10^5}{10^3 \times 10}$ = 20m.

Example 6. A uniformly tapering vessel is filled with a liquid of uniform density 900 kg/m^3. The force that acts on the base of the vessel due to the liquid is (g = 10 ms^{-2})

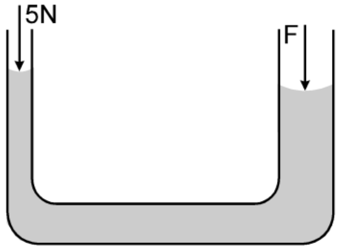


(1) 3.6 N (2) 7.2 N (3) 9.0 N (4) 14.4 N

Solution : Force acting on the base

F = P × A = hdgA = 0.4 × 900 × 10 × 2 × 10^{-3} = 7.2N

Example 7. The area of cross-section of the two arms of a hydraulic press are 1 cm^2 and 10 cm^2 respectively (figure). A force of 5 N is applied on the water in the thinner arm. What force should be applied on the water in the thicker arms so that the water may remain in equilibrium?



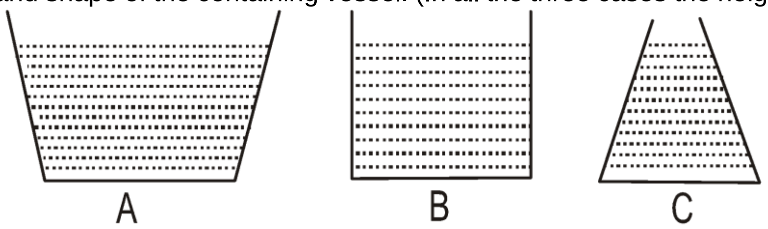
Solution : In equilibrium, the pressures at the two surfaces should be equal as they lie in the same horizontal level. If the atmospheric pressure is P and a force F is applied to maintain the equilibrium, the pressures are

$P_0 + \frac{5\,N}{1\,cm^2}$ and $P_0 + \frac{F}{10\,cm^2}$ respectively. This givens F = 50 N.

2. Hydraulic Brake.
Hydraulic brake system is used in automobiles to retard the motion.

HYDROSTATIC PARADOX

Pressure is directly proportional to depth and by applying pascal's law it can be seen that pressure is independent of the size and shape of the containing vessel. (In all the three cases the heights are same).



$P_A = P_B = P_C$

ATMOSPHERIC PRESSURE

Definition.
The atmospheric pressure at any point is numerically equal to the weight of a column of air of unit cross-sectional area extending from that point to the top of the atmosphere.
At 0ºC, density of mercury = 13.595 g cm^{-3}, and at sea level, g = 980.66 cm s^{-2}
Now P = hρg.
Atmospheric pressure = 76 × 13.595 × 980.66 dyne cm^{-2} = 1.013 × 10^{-5} N-m^2 (p_a)

Height of Atmosphere
The standard atmospheric pressure is 1.013 × 10^5 Pa (N m^{-2}). If the atmosphere of earth has a uniform density ρ = 1.30 kg m^{-3}, then the height h of the air column which exerts the standard atmospheric pressure is given by

⇒ $h\rho g = 1.013 \times 10^5$

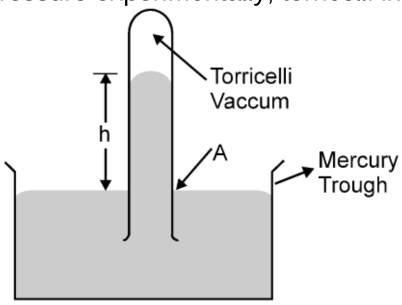
$h = \frac{1.013\times10^5}{\rho g} = \frac{1.013\times10^5}{1.13\times9.8}$ m = 7.95 × 10^3 m ~ 8 km.

In fact, density of air is not constant but decreases with height. The density becomes half at about 6 km high, $\frac{1}{4}$th at about 12 km and so on. Therefore, we can not draw a clear cut line above which there is no atmosphere. Anyhow the atmosphere extends upto 1200 km. This limit is considered for all practical purposes.

MEASUREMENT OF ATMOSPHERIC PRESSURE

1. Mercury Barometer.

To measure the atmospheric pressure experimentally, torricelli invented a mercury barometer in 1643.



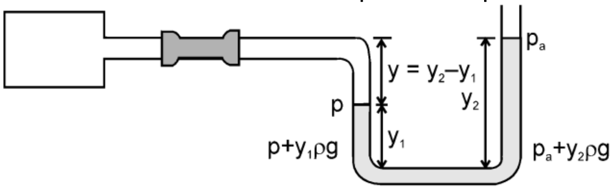
$p_a = h\rho g$

The pressure exerted by a mercury column of 1mm high is called 1 Torr.

1 Torr = 1 mm of mercury column

2. Open tube Manometer

Open-tube manometer is used to measure the pressure gauge. When equilibrium is reached, the pressure at the bottom of left limb is equal to the pressure at the bottom of right limb.



i.e. $p + y_1 \rho g = p_a + y_2 \rho g$

$p - p_a = \rho g (y_2 - y_1) = \rho g y$

$p - p_a = \rho g (y_2 - y_1) = \rho g y$

p = absolute pressure, $p - p_a$ = gauge pressure.

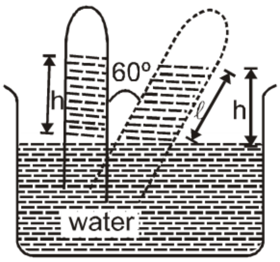
Thus, knowing y and ρ (density of liquid), we can measure the gauge pressure.

Solved Examples

Example 8. A barometer tube reads 76 cm of mercury. If the tube is gradually inclined at an angle of 60° with vertical, keeping the open end immersed in the mercury reservoir, the length of the mercury column will be

(1) 152 cm (2) 76 cm (3) 38 cm (4) $38\sqrt{3}$cm

Solution : $\cos 60^\circ = \frac{h}{\ell}$



$\Rightarrow \quad \ell = \frac{h}{\cos 60^\circ} = \frac{76}{1/2} \qquad \therefore \quad \ell = 152$ cm

Example 9. When a large bubble rises from the bottom of a lake to the surface. Its radius doubles. If atmospheric pressure is equal to that of column of water height H, then the depth of lake is

(1) H (2) 2H (3) 7H (4) 8H

Solution : $P_1V_1 = P_2V_2$

$\Rightarrow \quad (P_o + h\rho g) \times \frac{4}{3}\pi r^2 = P_o \times \frac{4}{3}\pi(2r)^3$

Example 10. A beaker containing liquid is kept inside a big closed jar If the air inside the jar is continuously pumped out, the pressure in the liquid near the bottom of the liquid will

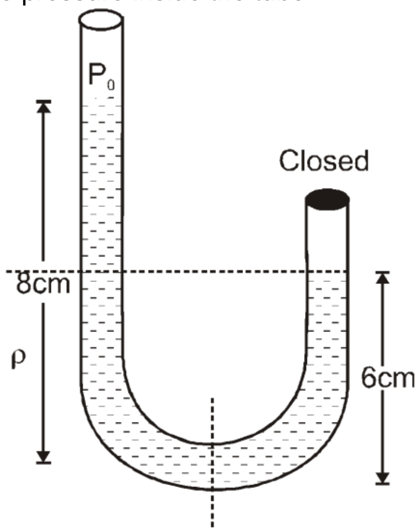
(1) Increase (2) Decreases
(3) Remain constant (4) First decrease and then increase

Solution : Total pressure at (near) bottom of the liquid

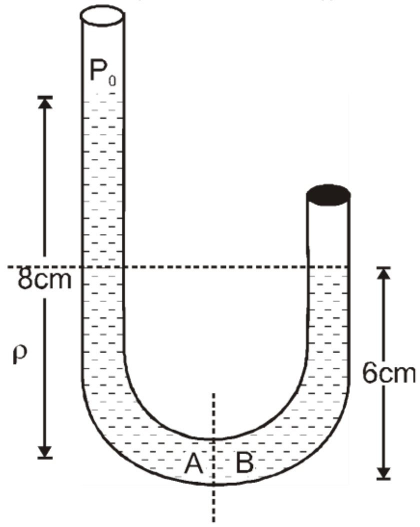
$P = P_0 + h\rho g$

As air is continuously pumped out from jar (container), P_0 decreases and hence P decreases.

Example 11. Write the pressure inside the tube



Solution : $P_A = P_0 + \rho g \frac{8}{100} = P_B$ ...(i)



$P_{tube} = P_B - \rho g \frac{6}{100}$ ...(ii)

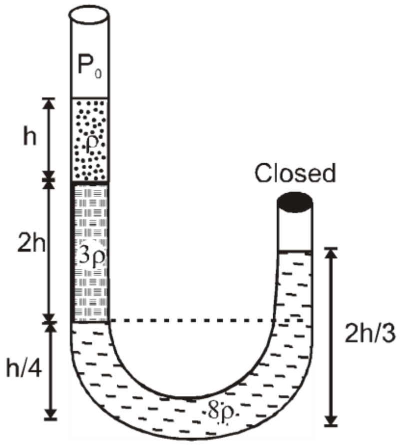
(i) & (ii)

$P_{tube} = \left(P_0 + \rho g \frac{8}{100}\right) - \rho g \frac{6}{100}$

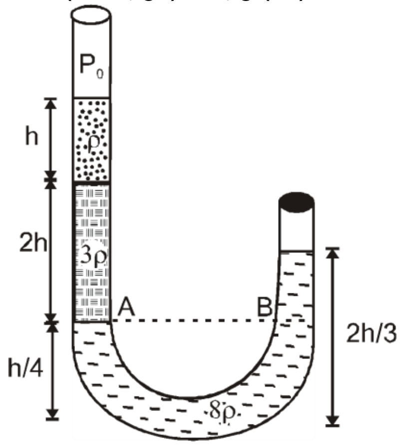
$P_{tube} = \left(P_0 + \frac{\rho g}{50}\right)$

Ans. $P_0 + \frac{\rho g}{50} = P$

Example 12. Find the pressure inside the tube



Solution : $P_A = (P_0 + \rho gh) + 3\rho g\,(2h) = P_B$



$P_{tube} = P_B - 8\rho g\left(\frac{2h}{3} - \frac{h}{4}\right)$

$= P_0 + \rho gh + 6\rho gh - 8\rho g\left(\frac{5}{12}h\right)$

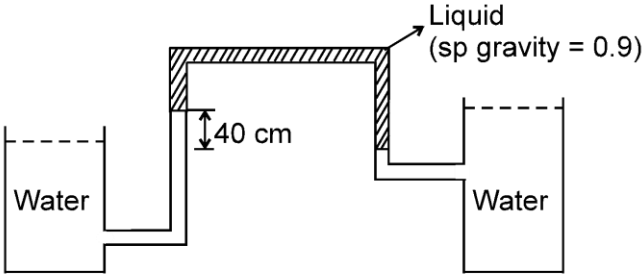
$= P_0 + \rho gh + 6\rho gh - \frac{10}{3}\rho gh$

$= P_0 + \frac{(21-10)\rho gh}{3} = P_0 + \frac{11}{3}\rho gh$

Ans. $P_0 + \frac{11}{3}\rho gh$

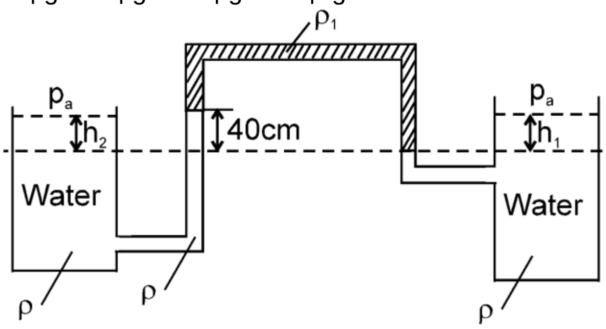
Solved Examples

Example 13. The manometer shown below is used to measure the difference in water level between the two tanks. Calculate this difference for the conditions indicated.



Solution : $p_a + h_1\,\rho g - 40\rho_1 g + 40\rho g = p_a + h_2\,\rho g$

$h_2 \rho g - h_1 \rho g = 40 \rho g - 40 \rho_1 g$



as $\rho_1 = 0.9\rho$
$(h_2 - h_1) \rho g = 40\rho g - 36\rho g$
$h_2 - h_1 = 4$ cm

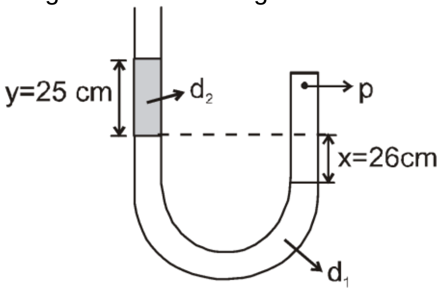
3. Water Barometer.
Let us suppose water is used in the barometer instead of mercury.

$h\rho g = 1.013 \times 10^5$ or $h = \frac{1.013 \times 10^5}{\rho g}$

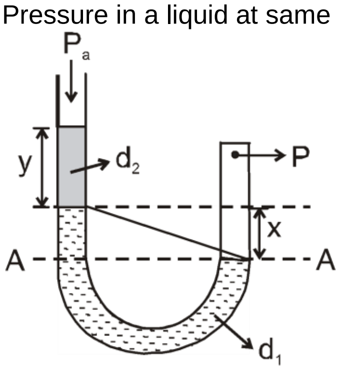
The height of the water column in the tube will be 10.3 m. Such a long tube cannot be managed easily, thus water barometer is not feasible.

Solved Examples

Example 14. In a given U-tube (open at one-end) find out relation between p and p_a.
Given $d_2 = 2 \times 13.6$ gm/cm^3 $d_1 = 13.6$ gm/cm^3

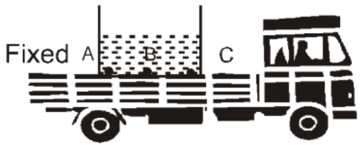


Solution : Pressure in a liquid at same level is same i.e. at A – A–, $p_a + d_2 y g + x d_1 g = p$



In C.G.S.
$p_a + 13.6 \times 2 \times 25 \times g + 13.6 \times 26 \times g = p$
$p_a + 13.6 \times g\,[50 + 26] = p$
$2p_a = p$ $[p_a = 13.6 \times g \times 76]$

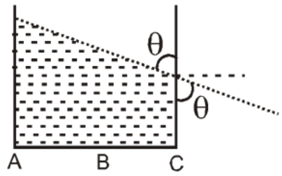
Example 15. Truck start from rest with acceleration 2.5 ms^{-2} then the angle (acute) between vertical and surface at the liquid, In equilibrium (assume that liquid is at with respect to truck)

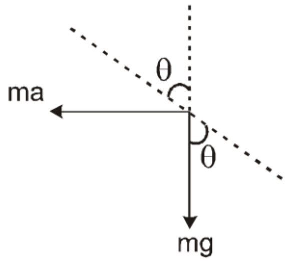


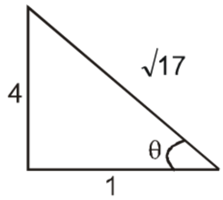
(1) $\sin\theta = \frac{4}{\sqrt{17}}$ (2) $\cos\theta = \frac{1}{\sqrt{17}}$

(3) $\tan\theta = 4$ (4) None of these

Solution :







Consider a particle on the liquid surface

mg cos θ = ma cos θ

gcosθ = a sinθ

$\tan\theta = \frac{g}{a} \Rightarrow \tan\theta = \frac{10}{2.5} = 4$ **Ans.** ABC

Example 16. In previous question pressure at the point A, B and C

(1) $P_A = P_B = P_C$ (2) $P_A > P_B > P_C$

(3) $P_A < P_B < P_C$ (4) Non of these

Example 17. In previous question, three different point, above the point A,B and C of an accelerated liquid surface in equilibrium called A' B', C' then pressure at the point A' B' and C'

(1) $P_{A'} = P_{B'} = P_{C'}$ (2) $P_{A'} > P_{B'} > P_{C'}$

(3) $P_{A'} < P_{B'} < P_{C'}$ (4) P_o = atmosphere pressure

Example 18. Highest pressure at the point inside the liquid :

(1) A

(2) C

(3) Pressure at A, B and C are equal and highest

(4) None of these

Example 19. Slope of the line on which pressure is same consider the direction of acceleration of truck as the X-axis

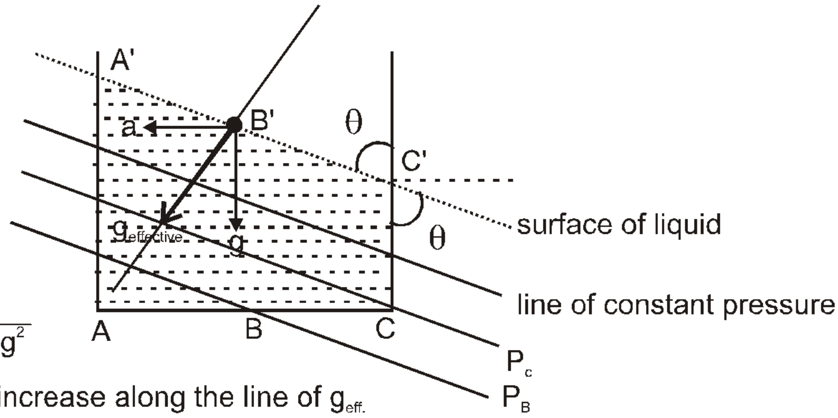
(1) –4 (2) –0.25 (3) –2.5 (4) $-\frac{1}{4}$

Solution : (16 to 19)

In the frame of truck liquid is in equilibrium, liquid surface is ⊥ to the g_{eff} and pressure increase along the line of $g_{(eff)}$ as $P = P_0 + \rho g_{eff}(h)$, where h is depth along the g_{eff} or ⊥ to the surface, all the line which is parallel to the liquid surface have same magnitude of pressure and magnitude increase as one move along the g_{eff} inside the liquid.

So $P_A > P_B > P_C$ & $P_{A'} = P_{B'} = P_{C'} = P_0$ (pressure is highest at A)

and slope $\tan(90 + \theta) = -\cot\theta = -\frac{1}{\tan\theta} = -\frac{1}{4}$



$g_{eff.} = \sqrt{a^2 + g^2}$

Pressure increase along the line of $g_{eff.}$

$P = P_0 + \rho g_{eff} h$

& $P_{A'} = P_{B'} = P_{C'} = P_0$

& $P_A > P_B > P_c$

ARCHIMEDES' PRINCIPLE

According to this principle, when a body is immersed wholly or partially in a fluid, it loses its weight which is equal to the weight of the fluid displaced by the body.

Up thrust = buoyancy = $V\rho_\ell g$

V = volume submerged

ρ_ℓ = density of liquid.

Relation between density of solid and liquid

weight of the floating solid = weight of the liquid displaced

$V_1 \rho_1 g = V_2 \rho_2 g \quad \Rightarrow \quad \frac{\rho_1}{\rho_2} = \frac{V_2}{V_1}$

or $\frac{\text{Density of solid}}{\text{Density of liquid}} = \frac{\text{Volume of the immeresed portion of the solid}}{\text{Total Volume of the solid}}$

This relationship is valid in accelerating fluid also. Thus, the force acting on the body are :

(i) its weight Mg which acts downward and

(ii) net upward thrust on the body or the buoyant force (mg)

Hence the apparent weight of the body = Mg – mg = weight of the body – weight of the displaced liquid.

Or Actual Weight of body – Apparent weight of body = weight of the liquid displaced.

The point through which the upward thrust or the buoyant force acts when the body is immersed in the liquid is called its centre of buoyancy. This will coincide with the centre of gravity if the solid body is homogeneous. On the other hand if the body is not homogeneous, then the centre of gravity may not lie on the line of the upward thrust and hence there may be a torque that causes rotation in the body.

If the centre of gravity of the body and the centre of buoyancy lie on the same straight line, the body is in equilibrium.

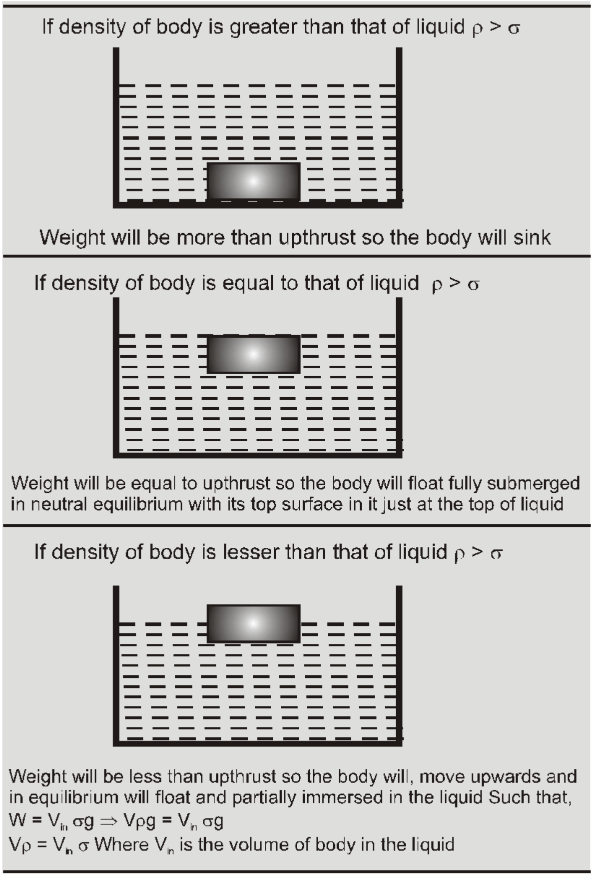
If the centre of gravity of the body does not coincide with the centre of buoyancy (i.e., the line of upthrust), then torque acts on the body. This torque causes the rotational motion of the body.

Floatation

1. **Translatory equilibrium :** When an body of density p and volume V is immersed in a liquid of density σ, the forces acting on the body are

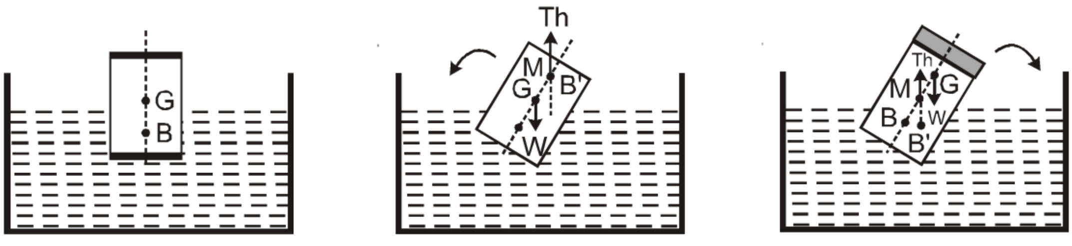
Weight of body W = mg = Vρg, acting vertically downwards through centre of gravity of the body.

Upthrust force = Vσg acting vertical upwards through the centre of gravity of the displaced liquid i.e., centre of buoyancy.



(i) A body will float in liquid only and only if $\rho \le \sigma$
(ii) In case of floating as weight of body = upthrust
So W_{App} = Actual weight – upthrust = 0
(iii) In case of floating $V\rho g = V_{in}\sigma g$
So the equilibrium of floating bodies is unaffected by variations in g though both thrust and weight depend on g.

(2) Rotatory Equilibrium : When a floating body is slightly tied from equilibrium position, the centre of buoyancy B shift. The vertical line passing through the new centre of buoyancy B' and initial vertical line meet at a point M called meta - centre. If the meta-centre M is above the centre of gravity the couple due to forces at G (weight of body W) and at B' (upthrust) tends to bring the body back body the meta-centre must always be higher then the centre of gravity of the body.



(A) (B) (C)

However, if meta-centre goes CG, the couple due to forces at G and B' tends to topple the floating body. That is why a wooden log cannot be made to float vertical in water or a boat is likely to capsize if the sitting passengers stand on it. In these situations CG becomes higher than MG and so the body will topple if slightly tilted.

Solved Examples

Example 20. A concrete sphere of radius R has cavity of radius r which is paked with sawdust. The specific gravities of concrete and sawdust are respectively 2.4 and 0.3 for this sphere to float with its entire volume submerged under water. Ration of mass of concrete to mass of sawdust will be

(1) 8 (2) 4 (3) 3 (4) Zero

Solution : Let specific gravities of consrete and saw dust are ρ_1 and ρ_2 respectively.

According to principle of floatation weight of whole sphere = upthrust on the sphere

$$\frac{4}{5}\pi(R^3 - r^3)\rho_1 g + \frac{4}{3}\pi r^3 \rho_2 g = \frac{4}{3}\pi R^3 \times 1 \times g$$

$\Rightarrow$ $R^3\rho_1 - r^3\rho_1 + r^3\rho_2 = R^3$ $\Rightarrow$ $R^3(\rho_1 - 1) = r^3(\rho_1 - \rho_2) =$ $\frac{R^3}{r^3} = \frac{\rho_1 - \rho_2}{\rho_1 - 1}$

$\Rightarrow$ $\frac{R^3 - r^3}{r^3} = \frac{\rho_1 - \rho_2 - \rho_1 + 1}{\rho_1 - 1}$ $\Rightarrow$ $\frac{(R^3 - r^3)\rho_1}{r^3\rho_2} = \left(\frac{1-\rho_2}{\rho_1 - 1}\right)\frac{\rho_1}{\rho_2}$

$\Rightarrow$ $\frac{\text{Mass of concrete}}{\text{Mass of saw dust}} = \left(\frac{1-0.3}{2.4-1}\right) \times \frac{2.4}{0.3}$ = 4

Example 21. A metallic block of density 5 gm cm^{-3} and having dimensions 5 cm × 5 cm × 5cm is weighed in water. Its apparent weight will be

(1) 5 × 5× 5 × 5 gf (2) 4 × 4 × 4 × 4 gf (3) 5 × 4× 4 × 4 gf (4) 4 × 5× 5 × 5 gf

Solution : Apparent weight

= V (ρ – σ) g = 1 × b × h × (5 – 1) × g

= 5 × 5 × 5 × 4 × g

Dyne = 4 × 5 × 5 × 5 gf

Example 22. A cobical block is floating in a liquid with half of its volume immersed in the liquid. When the whole system accelerates upwards with acceleration of g/3, the fraction of volume immersed in the liquid will be

(1) $\frac{1}{2}$ (2) $\frac{3}{8}$ (3) $\frac{2}{3}$ (4) $\frac{3}{4}$

Solution : Fraction of volume immersed in the liquid $V_{in} = \left(\frac{\rho}{\sigma}\right) V$

i.e. it depends upon the densities of the block and liquid. So there will be no change in it if system moves upward or downward with constant velocity or some acceleration.

Example 23. A silver ingot weighing 2.1 kg is held by a string so as to be completely immersed in a liquid of relative density 0.8. The relative density of silver is 10.5. The rension in the string in kg-wt is

(1) 1.6 (2) 1.94 (3) 3.1 (4) 5.25

Solution : T = Apparent weight = V(ρ – σ) g = $\frac{M}{\rho}$ (ρ – σ) g

$T = M\left(1 - \frac{\sigma}{\rho}\right)g = 2.1\left(1 - \frac{0.8}{10.5}\right)$ g = 1.94 gN

T = 1.94 Kg-wt

Example 24. A sample of metal weighs 210 gm in air, 180 gm in water and 120 gm in liquid. Then density (RD) of

(1) Metal is 3 (2) Metal is 7 (3) Liquid is 3 (4) Liquid is $\frac{1}{3}$

Solution : Density of metal = ρ. Density of liquid = σ

If V is the volume of sample then according to problem

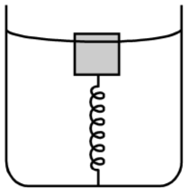
210 = Vρg ......(i)

180 = V (ρ – 1)g ......(ii)

120 = V (ρ – σ)g ......(iii)

By solving (i), (ii) and (iii) we get ρ = 7 and σ = 3.

Example 25. A cubical block of wood of edge 3 cm floats in water. The lower surface of the cube just touches the free end of a vertical spring fixed at the bottom of the pot. Find the maximum weight that can be put on the block without wetting it. Density of wood = 800 kg/m^3 and spring constant of the spring = 50 N/m. Take g = 10 m/s^2.



Solution : The specific gravity of the block = 0.8. Hence the height inside water = 3 cm × 0.8 = 2.4 cm. The height outside ater = 3 cm – 2.4 = 0.6 cm. Suppose the maximum weight that can be put without wetting it is W. The block in this case is completely immersed in the water. The volume of the displaced water

= volume of the block = $27 \times 10^{-6}\ m^3$.

Hence, the force of buoyancy

= $(27 \times 10^{-6}\ m^3) \times 1(1000\ kg/m3) \times (10\ m/s^2)$ = 0.27 N.

The spring is compressed by 0.6 cm and hence the upward force exerted by the spring

= 50 N/m × 0.6 cm = 0.3 N.

The force of buoyancy and the spring force taken together balance the weight of the block plus the weight W put on the block. The weight of the block is

$W' = (27 \times 10^{-6}\ m) \times (800\ kg/m^3) \times (10\ m/s^2)$ = 0.22 N.

Thus, W = 0.27 N + 0.3 N – 0.22 N = 0.35 N.

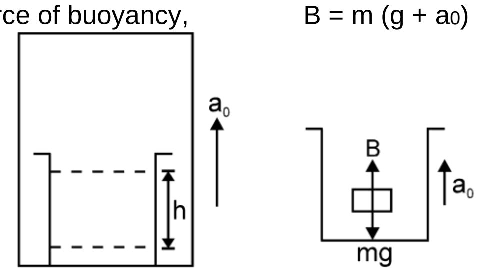
PRESSURE IN CASE OF ACCELERATING FLUID

(i) Liquid Placed in elevator :

When elevator accelerates upward with acceleration a_0 then pressure in the fluid, at depth ‘h’ may be given by,

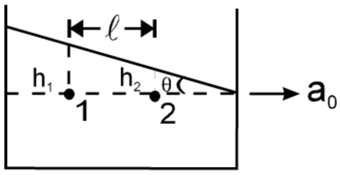
$p = h\rho\,[g + a_0]$

and force of buoyancy, $B = m\,(g + a_0)$



(ii) Free surface of liquid in horizontal acceleration :

$\tan\theta = \frac{a_0}{g}$

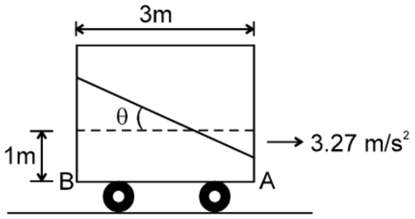


$p_1 - p_2 = \ell\rho\, a_0$ where p_1 and p_2 are pressures at point 1 & 2. Then $h_1 - h_2 = \frac{\ell a_0}{g}$

Solved Examples

Example 26. An open rectangular tank 1.5 m wide 2m deep and 2m long is half filled with water. It is accelerated horizontally at 3.27 m/sec^2 in the direction of its length. Determine the depth of water at each end of tank. [g = 9.81 m/sec^2]

Solution : $\tan\theta = \frac{a}{g} = \frac{1}{3}$



Depth at corner 'A'

$= 1 - 1.5\tan\theta$

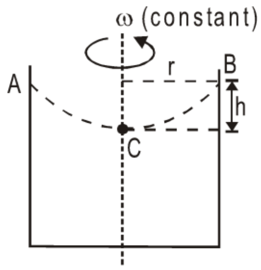
$= 0.5$ m **Ans.**

Depth at corner 'B'

$= 1 + 1.5\tan\theta = 1.5$ m **Ans.**

(iii) Free surface of liquid in case of rotating cylinder.

$$h = \frac{v^2}{2g} = \frac{\omega^2 r^2}{2g}$$



STREAMLINE FLOW

The path taken by a particle in flowing fluid is called its line of flow. In the case of steady flow all the particles passing through a given point follow the same path and hence we have a unique line of flow passing through a given point which is also called streamline.

CHARACTERISTICS OF STREAMLINE

1. A tangent at any point on the stream line gives the direction of the velocity of the fluid particle at that point.

2. Two steamlines never intersect each other.

Laminar flow : If the liquid flows over a horizontal surface in the form of layers of different velocities, then the flow of liquid is called Laminar flow. The particle of one layer do not go to another layer. In general, Laminar flow is a streamline flow.

Turbulent Flow : The flow of fluid in which velocity of all particles crossing a given point is not same and the motion of the fluid becomes disorderly or irregular is called turbulent flow.

REYNOLD'S NUMBER

According to Reynold, the critical velocity (v_c) of a liquid flowing through a long narrow tube is

(i) directly proportional to the coefficient of viscosity (η) of the liquid.

(ii) inversely proportional to the density ρ of the liquid and

(iii) inversely proportional to the diameter (D) of the tube.

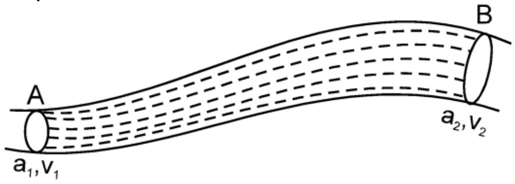
That is $v_c \propto \frac{\eta}{\rho D}$ or $v_c = \frac{R\eta}{\rho D}$ or $= \frac{v_c \rho D}{\eta}$ ....(1)

where R is the Reynold number.

If R < 2000, the flow of liquid is streamline or laminar. If R > 3000, the flow is turbulent. If R lies between 2000 and 3000, the flow is unstable and may change from streamline flow to turbulent flow.

EQUATION OF CONTINUITY

The equation of continuity expresses the law of conservation of mass in fluid dynamics.



$a_1v_1 = a_2v_2$

In general **av = constant** . This is called equation of continuity and states that as the area of cross section of the tube of flow becomes larger, the liquid's (fluid) speed becomes smaller and vice-versa.

Illustrations -

(i) Velocity of liquid is greater in the narrow tube as compared to the velocity of the liquid in a broader tube.

(ii) Deep waters run slow can be explained from the equation of continuity i.e., av = constant. Where water is deep the area of cross section increases hence velocity decreases.

ENERGY OF A LIQUID

A liquid can posses three types of energies :

(i) **Kinetic energy :**

The energy possessed by a liquid due to its motion is called kinetic energy. The kinetic energy of a liquid of mass m moving with speed v is $\frac{1}{2}mv^2$.

$\therefore$ K.E. per unit mass = $\frac{\frac{1}{2}mv^2}{m} = \frac{1}{2}v^2$.

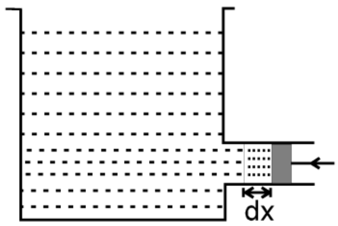
(ii) **Potential energy :**

The potential energy of a liquid of mass m at a height h is m g h.

$\therefore$ P.E. per unit mass = $\frac{mgh}{m}$ = gh

(iii) **Pressure energy :**

The energy possessed by a liquid by virtue of its pressure is called pressure energy.
Consider a vessel fitted with piston at one side (figure). Let this vessel is filled with a liquid. Let 'A' be the area of cross section of the piston and P be the pressure experienced by the liquid.
The force acting on the piston = PA
If dx be the distance moved by the piston, then work done by the force = PA dx = PdV
where dV = Adx, volume of the liquid swept.
This work done is equal to the pressure energy of the liquid.
$\therefore$ Pressure energy of liquid in volume dV = PdV.
The mass of the liquid having volume dV = ρdV,
ρ is the density of the liquid.

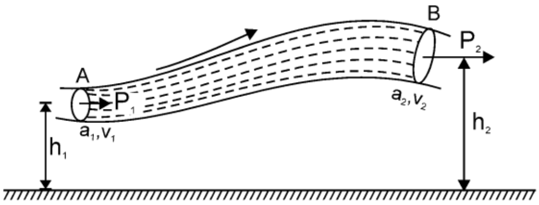


$\therefore$ Pressure energy per unit mass of the liquid = $\frac{PdV}{\rho dV} = \frac{P}{\rho}$.

BERNOULLI'S THEOREM

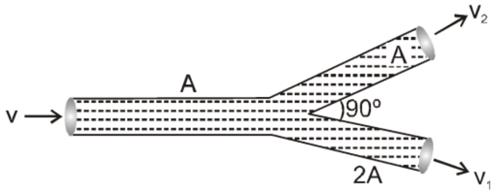
It states that the sum of pressure energy, kinetic energy and potential energy per unit mass or per unit volume or per unit weight is always constant for an ideal (i.e. incompressible and non-viscous) fluid having stream-line flow.

i.e. $\frac{P}{\rho} + \frac{1}{2} v^2 + gh$ = constant.



Solved Examples

Example 27. An incompressible liquid flows through a horizontal tube as shown in the following fig. Then the velocity υ of the fluid is



(1) $v = 2v_1 + v_2$ (2) $v = v_1 + v_2$ (3) $v = \frac{v_1 v_2}{v_1 + v_2}$ (4) $v = \sqrt{v_1^2 + v_2^2}$

Solution : $m = m_1 + m_2$

$\rho V = \rho V_1 + \rho V_2$

$\rho A v = \rho 2 A v_1 + \rho A v_2$

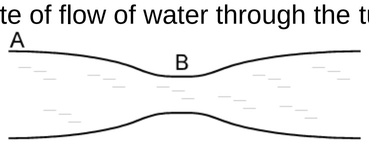
$v = 2v_1 + v_2$

Example 28. Water enters through end A with speed υ_1 and leaves through end B with speed υ_2 of a cylindrical tube AB. The tude is always completely filled with water. In case I tude is horizontal and is case II it is vertical with end A upwards and in case III it is vertical with end B upwards. We have $\upsilon_1 = \upsilon_2$

(1) Case I (2) Case II (3) Case III (4) Each case

Solution : This happens in accordance with equation of continuity and this equation was derived on the principle of conservation of mass and it is true in every case, either tube remain horizontal or vertical.

Example 29. Water flows in a horizontal tube as shown in figure. The pressure of water changes by 600 N/m^2 between A and B where the areas of cross-section are $30 cm^2$ and $15 cm^2$ respectively. Find the rate of flow of water through the tube.



Solution : Let the velocity at A = v_A and that at B = v_B.

By the equation of continuity, $\frac{v_B}{v_A} = \frac{30 cm^2}{15 cm^2} = 2$.

By Bernoulli's equation,

$P_A + \frac{1}{2} \rho v_A^2 = P_B + \frac{1}{2} \rho v_B^2$ or, $P_A - P_B = \frac{1}{2} \rho (2v_A)^2 - \frac{1}{2} \rho v_A^2 = \frac{3}{2} \rho v_A^2$

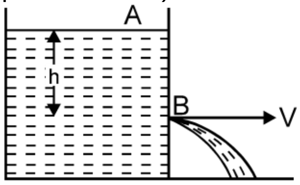
or, $600 \frac{N}{m^2} = \frac{3}{2}\left(1000 \frac{kg}{m^3}\right) v_A^2$ or, $v_A = \sqrt{0.4 m^2/s^2} = 0.63$ m/s.

The rate of flow = ($30 cm^2$) (0.63 m/s) = 1800 cm^3/s.

APPLICATION OF BERNOULLI'S THEOREM

(i) **Bunsen burner**
(ii) **Lift of an airfoil.**
(iii) **Spinning of a ball (Magnus effect)**
(iv) **The sprayer.**
(v) **A ping-pong ball in an air jet**
(vi) **Torricelli's theorem (speed of efflux)**

At point A, $P_1 = P$, $v_1 = 0$ and $h_1 = h$
At point B, $P_2 = P$, $v_2 = v$ (speed of efflux) and $h = 0$



Using Bernoulli's theorem $\frac{P_1}{\rho} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh_2 = \frac{1}{2}v_2^2$, we have

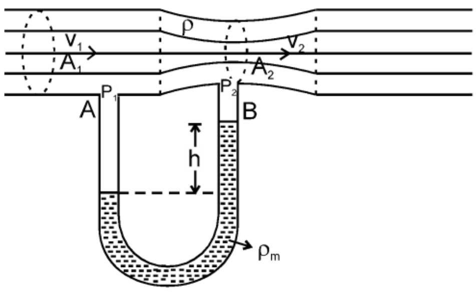
$\frac{P}{\rho} + gh + 0 = \frac{P}{\rho} + 0 + \frac{1}{2}v^2 \Rightarrow \frac{1}{2}v^2 = gh$ or $v = \sqrt{2gh}$

(VII) **Venturimeter.**

It is a gauge put on a flow pipe to measure the flow of speed of a liquid (Fig). Let the liquid of density ρ be flowing through a pipe of area of cross section A_1. Let A_2 be the area of cross section at the throat and a manometer is attached as shown in the figure. Let v_1 and P_1 be the velocity of the flow and pressure at point A, v_2 and P_2 be the corresponding quantities at point B.

Using Bernoulli's theorem :

$\frac{P_1}{\rho} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh_2 + \frac{1}{2}v_2^2$, we get



$\frac{P_1}{\rho} + gh + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh + \frac{1}{2}v_2^2$ (Since $h_1 = h_2 = h$)

or $(P_1 - P_2) = \frac{1}{2}\rho(v_2^2 - v_1^2)$ ....(1)

According to continuity equation, $A_1 v_1 = A_2 v_2$

or $v_2 = \left(\frac{A_1}{A_2}\right)v_1$

Substituting the value of v_2 in equation (1) we have

$(P_1 - P_2) = \frac{1}{2}\rho\left[\left(\frac{A_1}{A_2}\right)^2 v_1^2 - v_1^2\right] \frac{1}{2}\rho v_1^2 \left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$

Since $A_1 > A_2$, therefore, $P_1 > P_2$

or $$v_1^2 = \frac{2(P_1 - P_2)}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]} = \frac{2A_2^2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}$$

where $(P_1 - P_2) = \rho_m gh$ and h is the difference in heights of the liquid levels in the two tubes.

$$v_1 = \sqrt{\frac{2\rho_m gh}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}}$$

The flow rate (R) i.e., the volume of the liquid flowing per second is given by $R = v_1 A_1$.

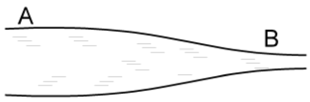
(viii) During wind storm,

The velocity of air just above the roof is large so according to Bernoulli's theorem, the pressure just above the roof is less than pressure below the roof. Due to this pressure difference an upward force acts on the roof which is blown of without damaging other parts of the house.

(ix) When a fast moving train cross a person standing near a railway track, the person has a tendency to fall towards the train. This is because a fast moving train produces large velocity in air between person and the train and hence pressure decreases according to Bernoulli's theorem. Thus the excess pressure on the other side pushes the person towards the train.

Solved Examples

Example 30. Water flows through a horizontal tube of variable cross-section (figure). The area of cross-section at A and B are 4 mm^2 and 2 mm^2 respectively. If 1 cc of water enters per second through A, find

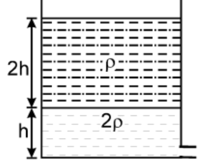


(i) the speed of water at A, (ii) the speed of water at B and (iii) the pressure difference $P_A - P_B$.

Solution : $A_1v_1 = A_2v_2$ ⇒ $\rho_1 + \frac{1}{2}\rho v_1^2 + 0 = r_2 + \frac{1}{2}\rho v_2^2 + \rho gh$.

Ans. (i) 25 cm/s, (ii) 50 cm/s (iii) 94 N/m^2

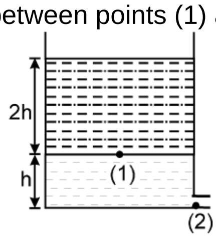
Example 31. The velocity of the liquid coming out of a small hole of a large vessel containing two different liquids of densities 2ρ and ρ as shown in figure is



(1) $\sqrt{6gh}$ (2) $2\sqrt{gh}$ (3) $2\sqrt{2gh}$ (4) $\sqrt{gh}$

Solution : (2) Pressure at (1) : $P_1 = P_{atm} + \rho g(2h)$

Applying Bernoulli's theorum between points (1) and (2)



$[P_{atm} + 2\rho gh] + \rho g(2h) + \frac{1}{2}(2\rho)(0)^2$

$= P_{atm} + (2\rho)g(0) + \frac{1}{2}(2\rho)v^2$ ⇒ $v = 2\sqrt{gh}$ **Ans.**

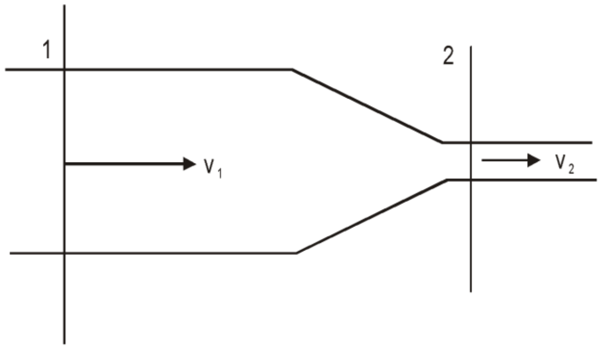
Example 32. A horizontal pipe line carries water in a streamline flow. At a point along the pipe where the cross-sectional area is 10 cm^2, the water velocity is 1 ms^{-1} and the pressure is 2000 Pa. The pressure of water at another point where the cross-sectional area is 5 cm^2 will be :.
[Density of water = 10^3 kg. m^{-3}]

Solution : From continuity equation

$$A_1 v_1 = A_2 v_2$$

$\therefore \quad v_2 = \left(\frac{A_1}{A_2}\right) v_1 = \left(\frac{10}{5}\right)(1) = 2$ m/s

Applying Bernoulli's theorem at 1 and 2

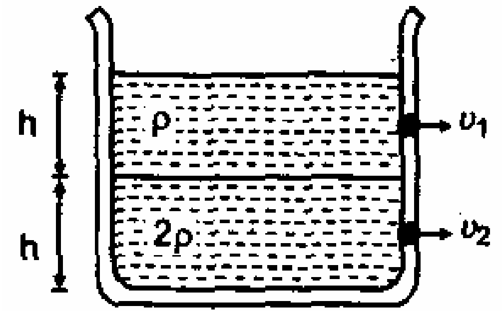


$$P_2 + \frac{1}{2}\rho v_2^2 = P_1 + \frac{1}{2}\rho v_1^2$$

$\therefore \quad P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2)$

$= \left(2000 + \frac{1}{2}\times 10^3(1-4)\right)$ **Ans.** 500 Pa

Example 33. Equal volumes of two immiscible liquids of densities ρ and 2ρ are filled in a vessel as shown in figure. Two small holes are punched at depth h/2 and 3h/2 from the surface of lighter liquid. If v_1 and v_2 are the velocities of efflux at these two holes, then v_1/v_2 is :



(1) $\frac{1}{2\sqrt{2}}$ (2) 0.5 (3) 0.25 (4) $\frac{1}{\sqrt{2}}$

Solution : for hole (i)

$P_0 + \rho\frac{V_1^2}{2} = P_0 + \rho g\frac{h}{2} \quad \Rightarrow \quad V_1 = \sqrt{gh}$

for hole (ii)

$P_0 + \rho\frac{V_2^2}{2} = P_0 + \rho gh + 2\rho h\left(\frac{h}{2}\right)$

$\frac{\rho V_2^2}{2} = 2\rho gh. \quad \Rightarrow \quad V_2 = 2\sqrt{gh}$

$\frac{V_1}{V_2} = \frac{1}{2} = 0.5$