Additional Problems For Self Practice (APSP)

PART -: PRACTICE TEST PAPER

1. **Sol.** S = {HH, HT, TH,TT}

Probability, p = $\frac{2}{4}=\frac{1}{2}$, q = $\frac{1}{4}$

2. **Sol.** Odds against = 4:11

Probability, = $\frac{11}{4+11}=\frac{11}{15}$

3. **Sol.** S= {1, 2, 3, 4, 5, 5, 6}

Probability, = $\frac{4}{6}$ = $\frac{2}{3}$

4. **Sol.** P(A'∩B') = P(A') P(B') ∵ A,B independent

= $\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)$ = $\frac{1}{2}$

5. **Sol.** P(Gargy to speak truth) = $\frac{60}{100}=\frac{3}{5}$

P(Ashmeet to seak truth) = $\frac{90}{100}=\frac{9}{10}$

required probability = $\frac{3}{5}\left(1-\frac{9}{10}\right)+\left(1-\frac{3}{5}\right)\left(\frac{9}{10}\right)$ = $\frac{3}{50}+\frac{18}{50}=\frac{21}{50}$ ∴ $\frac{21}{50}\times 100\% \Rightarrow$ 42%

6. **Sol.** Required probability = $\frac{1}{4}+\frac{1}{5}-\frac{1}{15}$ = $\frac{15+12-4}{60}=\frac{23}{60}$

7. **Sol.** S= {2, 3, 5}

Probability,= $\frac{2}{3}$

8. **Sol.** S={(2,2), (2,3) (2,5), (3, 2) (3, 3) (3,5), (5,2) (5,3), (5,5)}

Probability,= $\frac{4}{9}$

9. **Sol.** Probability,= $\frac{{}^8C_1}{{}^9C_2}=\frac{8\times 2}{9\times 8}=\frac{2}{9}$

10. **Sol.** P(Thomas to be true, = $\frac{3}{4}$

P(Thomas to be false, = $\frac{1}{4}$

E→ Thomas said six

A→ Actually a six

$P\left(\frac{A}{E}\right)=\frac{P(A\cap E)}{P(E)}$ = $\frac{1/6\times 3/4}{1/6\times 3/4+5/6\times 1/4}$ = $\frac{3/24}{3/24+5/24}$ = $\frac{3}{8}$

11. Sol. Probability,= $\frac{^{4}C_2}{^{52}C_2}$

12. Sol. Probability,= $\frac{^{12}C_4}{^{52}C_4}$

13. Sol. P(all boys sit together) = $\frac{8! \times 7!}{14!}$

Probability,1– $\frac{8! \times 7!}{14!}$

14. Sol. Probability,= $\frac{^{40}C_1 . ^{4}C_1}{^{52}C_2}$

15. Sol. A → First ticket is perfect square

B → Second ticket to be perfect square

$P\left(\frac{B}{A}\right) = \frac{\left(\frac{5}{25}\right)\left(\frac{4}{24}\right)}{\left(\frac{5}{25}\right)(1)} = \frac{4}{24} = \frac{1}{6}$

16. Sol. P (getting composite number) = $\frac{2}{6} = \frac{1}{3}$

Probability,= $^{5}C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2 \Rightarrow 10 \times \frac{1}{27} \times \frac{4}{9} = \frac{40}{243}$

17. Sol. P (Red) = $\frac{3}{5}$

P (Black) = $\frac{2}{5}$

Probability,= $^{4}C_2\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^2 = \frac{3^3 2^3}{5^4}$

18. Sol. Sum, 10 → {(4, 6), (5, 5), (6, 4)}

P (getting sum, 10) = $\frac{3}{36} = \frac{1}{12}$

Probability,= $^{5}C_3\left(\frac{1}{12}\right)^3\left(\frac{11}{12}\right)^2$

19. Sol. Probability, = $^{3}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) = \frac{3}{8}$

20. Sol. (52 weeks) + (2 days extra)

⇒ {SM, MT, TW, WT, TF, FS, SS}

Probability,= $\frac{2}{7}$

21. Sol. P(A∪B) = P(A) + P(B) – P(A∩B)

$= \frac{1}{8} + \frac{2}{5} - 0 = \frac{21}{40}$

22. Sol. $P(A/B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = \frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$= \frac{3}{5} - \frac{2}{5} = \frac{1}{5}$

23. Sol. $B_1 \rightarrow$ Ball drawn from urn A is red but ball returned to it is black, $P(B_1) = \frac{6}{10} \times \frac{6}{11}$

$D_1 \rightarrow$ Ball drawn from urn A is black and ball returned is red, $P(D_1) = \frac{4}{10} \times \frac{4}{11}$

Required probability $P(R) = P(B_1) \times P\left(\frac{R}{B_1}\right) + P(D_1) \times P\left(\frac{R}{D_1}\right)$

$= \frac{6}{10} \times \frac{6}{11} \times \frac{5}{10} + \frac{4}{10} \times \frac{4}{11} \times \frac{7}{10} = \frac{73}{275}$

24. Sol. $n(S) = 6 \times 6 \times 6 = 216$ also to get sum of '8' $x_1 + x_2 + x_3 = 7$ where $1 \le x_i \le 6$

$x_i = t_i + 1$ $\quad 0 \le t_1 \le 4 \Rightarrow t_1 + t_2 + t_3 = 4$

by fictious partition method number of solution of this equation is ${}_6C_2$

$n(E) = 15 \quad \Rightarrow \quad P = \frac{15}{216} = \frac{5}{72}$

25. Sol. Since ten places are vacant. Probability of finding vacant places = $\frac{{}^{21}C_{11}}{{}^{23}C_{11}} = \frac{6}{23}$

26. Sol. P(Product of digits) = 15

if 35, 53 $\Rightarrow$ $P(A) = \frac{2}{90} = \frac{1}{45}$ $\Rightarrow$ $P(\bar{A}) = \frac{44}{45}$

Probability= $1 - \left(\frac{44}{45}\right)^3$

27. Sol. $\frac{np}{npq} = \frac{3}{2} \quad \Rightarrow \quad q = \frac{2}{3} \quad \Rightarrow \quad p = \frac{1}{3}$

Probability of getting exactly 5 success ${}^6C_5 \left(\frac{1}{3}\right)^5 = \frac{2}{81}$

28. Sol.

$$\left.\begin{aligned} p(x=4) &= {}^n c_4 \left(\frac{1}{2}\right)^n \\ p(x=5) &= {}^n c_5 \left(\frac{1}{2}\right)^n \\ p(x=6) &= {}^n c_6 \left(\frac{1}{2}\right)^n \end{aligned}\right]$$

$\Rightarrow ({}_nC_5)^2 = {}_nC_4 \; {}_nC_6 \Rightarrow n = -1$ which is not possible

29. Sol. $E_1 = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$

$E_2 = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$\therefore$ $n(E_1) = 5$, $n(E_2) = 6$

Also $n(S) = 6 \times 6 = 36$

Now, $P(E_1) = \frac{5}{36}$ and $P(E_2) = \frac{6}{36} = \frac{1}{6}$

Let E = the event that A wins

Let F = the event that B wins

$$P(E) = \frac{5}{36} + \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36} + \frac{31}{36} \times \frac{5}{6} \times \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36} + \dots\dots\dots\ \frac{30}{61}$$

$$P(F) = 1 - P(E) = \frac{31}{61}$$

30. **Sol.** Unit digit of 3^a = 3, 9, 7, 1

Unit digit of 2^b = 2, 4, 8, 6

3^a	2^b
3	8

3^a	2^b
9	2

3^a	2^b
7	4

$$\Rightarrow \frac{25}{100} \times \frac{25}{100} + \frac{25}{100} \times \frac{25}{100} + \frac{25}{100} \times \frac{25}{100} = \frac{3}{16}$$

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

1*. **Sol.** $P(x_1) = \frac{1}{2}$

$P(x_2) = \frac{1}{4}$

$P(x_3) = \frac{1}{4}$

$P(x) = P(E_1 E_2 E_3) + P(\bar{E}_1 E_2 E_3) + P(E_1 \bar{E}_2 E_3) + P(E_1 E_2 \bar{E}_3)$

$= \frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{3}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{3}{4}$

$P(x) = \frac{1}{4}$

(1) $P\left(\frac{x_1^c}{x}\right) = \frac{P(x_1^c \cap x)}{P(x)}$

$= \frac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4}}{\frac{1}{4}} = \frac{1}{8}$

(2) P(exactly two / x) = $\frac{P(\text{exactly two} \cap x)}{P(x)}$

$= \frac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{3}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{3}{4}}{\frac{1}{4}} = \frac{7}{8}$

(3) $P(x / x_2) = \frac{P(x \cap x_2)}{P(x_2)} = \frac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{3}{4}}{\frac{1}{4}} = \frac{5}{8}$

(4) $P(x / x_1) = \frac{P(x \cap x_1)}{P(x_1)} = \frac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{3}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{3}{4}}{\frac{1}{2}} = \frac{7}{16}$

2. **Sol.** $x + \frac{100}{x} > 50$

$\left.\begin{array}{l} x = 1 \text{Satisfis} \\ \;\;= 2 \quad " \end{array}\right]$

= 3 does ' nt satsfies

= 47 doest satsfies

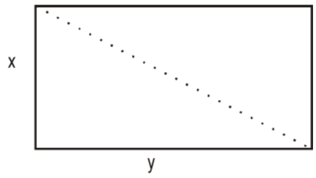
$\left.\begin{array}{l} = 48 \;\; \text{Satifes} \\ = 100 \,\text{Safies} \end{array}\right]$ 53 number

then no. of x which satisfy given inequation = 55

Pro. = $\frac{55}{100}$

3. **Sol.** $0 < x < 10$ $x^2 + y^2 < 100$
$0 < y < 10$

p = $\dfrac{\frac{1}{4}\Pi\times 10^2}{10\times 10}$



4. **Sol.** P = P(1 person lies) + P(2 person lies) P(A lies first / 2 person lied) + P(3 person lied)
P(A 1 died first / 3 person lied)

$= {}^nC_1\, pq^{n-1}\times\frac{1}{n} + {}^nC_1 p^2q^{n-2}\times\frac{1}{2} + {}^nC_2\, p^3q^{n-3}\times\frac{1}{3} + ...$

$= pq^{n-1} + {}^{n-1}C_{r-1}p^2q^{n-2}\,\frac{1}{2} + {}^nC_3\, p^3q^{n-3}\,\frac{1}{3} + ...$ $= pq^{n-1} + \sum_{r=2} {}^{n-1}C_{r-1}p^r\, q^{n-r}\,\frac{1}{r}$

As $\dfrac{{}^{n-1}C_{r-1}}{r} = \dfrac{{}^nC_1}{n}$

$P = Pq_{n-1} + \frac{1}{n}\sum_{r=2}^{n} {}^nC_r\, P^rq^{n-r}$ $\Rightarrow$ $P = pq_{n-1} + \frac{1}{n}(1 - {}^nC_0\, p^0q^n - {}^nC_1P^1q^{n-1})$

$P = Pq_{n-1} + \frac{1}{n}(1 - q^n - nPq^{n-1}) = \dfrac{1-(1-p)^n}{n}$

5. **Sol.** Person is selected if either he passes all the tests or exactly two of the tests.
P(passing all the tests) = p.p.p = p_3
Prob. of passing exactly 2 of the tests = P(first two tests) + P(first and third tests) + P(second and third tests)

$= p.p.(1 - p) + p.(1 - p).\frac{p}{2} + (1 - p)\,\frac{p}{2}.\,p$

$= p_2(1 - p) + \frac{1}{2}\,p_2\,(1 - p) + p_2\,(1 - p)$
$= 2p_2\,(1 - p)$

Thus, required probability = $p_3 + 2p_2\,(1 - p) = 2p_2 - p_3$.

6. **Sol.** Here,

P(A) = probability that A will hit B = $\frac{2}{3}$

P(B) = probability that B will hit A = $\frac{1}{2}$

P(C) = probability that C will hit A = $\frac{1}{3}$

P(E) = probability that A will be hit

$P(E) = 1 - P\,(\bar{B}).\,P(\bar{C}) = 1 - \frac{1}{2}.\frac{2}{3} = \frac{2}{3}$

Probability if A is hit by B and not by C.

⇒ $P(B \cap \overline{C}/E)$

⇒ $\frac{P(B).P(\overline{C})}{P(E)} = \frac{\frac{1}{2}.\frac{2}{3}}{\frac{2}{3}} = \frac{1}{2}$

7. **Sol.** Statement-1 If $P(H_i \cap E) = 0$ for some i, then

$P\left(\frac{H_i}{E}\right) = P\left(\frac{E}{H_1}\right) = 0$

If $P(H_i \cap E) \neq 0$ for $\forall\, i = 1, 2, ...n,$ then

$P\left(\frac{H_i}{E}\right) = \frac{P(H_i \cap E)}{P(H_i)} \times \frac{P(H_i)}{P(E)} = \frac{P\left(\frac{H_i}{E}\right) \times P(H_i)}{P(E)} > P\left(\frac{E}{H_i}\right)$, $P(H_i)$ [as 0 < P(E) < 1]

Hence statement -1 may not always be true.

Statement-2 Clearly, $H_1 \cup H_2 \cup .... \cup H_n = S$ (sample space)

$P(H_1) + P(H_2) + ...+ P(H_n) = 1$

8. **Sol.** Here, n(s) = 1 length of the interval [0, 5] = 5 ; n(E) = length of the interval ≤ [0, 5] in which P belongs such that the given equation has real roots.

Now $x^2 + Px + \frac{1}{4}(P + 2) = 0$ will have real roots

if $P^2 - 4.1.\frac{1}{4}(P + 2) \geq 0$ ⇒ $P^2 - P - 2 \geq 0$

⇒ (P + 1) (P – 2) ≥ 0 ⇒ P ≤ – 1 or P ≥ 2

But P ∈ [0, 5]. So, E = [2, 5]

∴ n(E) = length of the interval [2, 5] = 3

∴ Required Probability = $\frac{3}{5}$

9. **Sol.** Here, $P(A \cup B).P(A' \cap B')$

⇒ {P(A) + P(B) – $P(A \cap B)$ }{P(A').P(B')}

Since A, B are independent

⇒ A', B' are independent}

= P(A). P(A'). P(B') + P(B).P(A') . P(B') ....(1)

≤ P(A). P(B') + P(B). P(A')

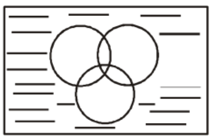
{Since in (1), P(A') ≤ 1 and P(B') ≤ 1}

⇒ $P(A \cup B) . P(A' \cap B')$ ≤ P(A) . P(B') + P(B). P(A')

⇒ $P(A \cup B) . P(A' \cap B')$ ≤ P(C)

{as P(C) = P(A). P(B') + P(B). P(A')}

10. **Sol.**

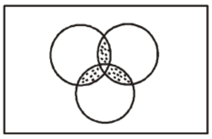


P = 1 – P(A ∪ B ∪ C)

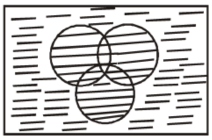
= 1 – P(A) – P(B) – P(C) + P(A ∩ B) + P(B ∩ C) +

P(C ∩ A) – P(A ∩ B ∩ C)
= $P(A \cap B \cap C)_1$ – P(A) – P(B) – P(C) + P(A ∩ B) +
P(B ∩ C) + P(C ∩ A)
= $P(\bar{A} \cup \bar{B} \cup \bar{C})$ – P(A) – P(B) – P(C) + P (A ∩ B) +
P(B ∩ C) + P(C ∩ A)

11. **Sol.**



12. **Sol.** P = 1 – P(A ∩ B ∩ C)



= 1 – P(A ∪ B ∪ C) + P(A) + P(B) + P(C)
– P(A ∪ B)
= 1 – $P(A \cup B \cup C)_c$ + P(A) + P(B) + P(C)
– P (A ∪ B) – P(B ∪ C) – P(C ∩ A)

13. **Sol.** Total number of ways = M_n , number of favorable cases = n!

14. **Sol.** Total number of ways = ${}_{M+n-1}C_{M-1}$, number of favorable cases = 1

15. **Sol.** Total number of ways = ${}_MC_n$, number of favorable cases = 1

16. **Sol.** P(white) = P (H ∩ white) + P(T ∩ white)

$= \frac{1}{2}\cdot\left\{\frac{3}{5}\times 1+\frac{2}{5}\cdot\frac{1}{2}\right\}+\frac{1}{2}\cdot\left\{\frac{{}^3C_2}{{}^5C_2}\times 1+\frac{{}^2C_2}{{}^5C_2}\times\frac{1}{3}+\frac{{}^3C_1\cdot{}^2C_1}{{}^5C_2}\times\frac{2}{3}\right\}$

$= \frac{1}{2}\times\frac{8}{10}+\frac{1}{2}\times\left\{\frac{3}{10}+\frac{1}{30}+\frac{12}{30}\right\} = \frac{4}{10}+\frac{1}{2}\times\frac{22}{30} = \frac{23}{30}$

17. **Sol.** $P\left(\text{Head}/\text{White}\right) = \frac{P(\text{Head}\cap\text{white})}{P(\text{white})} = \frac{\frac{1}{2}\times\left\{\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{2}\right\}}{\frac{23}{30}} = \frac{\frac{4}{10}}{\frac{23}{30}} = \frac{12}{23}$

18. **Sol.** ax + by = 0 a, b, c, d ∈ {0, 1}
cx + dy = 0
system has unique solution if and only if ad – bc ≠ 0
For which a = d = 1 and bc = 0 ⇒ 3 combination
Similarly if bc = 1, ad = 0 ⇒ 3 combination
Total choice for a, b, c, d is 2_4

Hence probability of unique solution is $\frac{6}{16} = \frac{3}{8}$. Statement-2 is also true since (0, 0) is a solution
Thus 'B' is correct

Aliter : ad – bc ≠ 0
If (i) ad = 1, bc = 0
(ii) ad = 0, bc = 1

$P(ad = 1) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$; $P(bc = 0) = 1 - P(bc = 1) = 1 - \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}$

$\therefore$ $P(ad = 1 \text{ and } bc = 0) = \frac{3}{16}$; $P(ad = 0 \text{ and } bc = 1) = \frac{3}{16}$

$\therefore$ required probability $= \frac{3}{8}$

19. **Sol.** Let x, y, z be probability of E_1, E_2, E_3 respectively

$\Rightarrow$ $x(1 - y)(1 - z) = \alpha$ $\Rightarrow$ $y(1 - x)(1 - z) = \beta$

$\Rightarrow$ $z(1 - x)(1 - y) = \gamma$ $\Rightarrow$ $(1 - x)(1 - y)(1 - z) = P$

Putting in the given relation we get $x = 2y$ and $y = 3z$ $\Rightarrow$ $x = 6z$ $\Rightarrow$ $\frac{x}{z} = 6$

20. **Sol.** Three squares are shown as below

I^{st} row	1								
II^{nd} row		2							
III^{rd} row				3					

$\frac{1}{3} \times \frac{2}{7} \times \frac{3}{6} + \frac{1}{3} \times \frac{2}{7} \times \frac{3}{6} = \frac{2}{21}$

digit 3 may come only in I_{st} and II_{nd} rows. In second square if ? is replaced by 3 then probability is 1/3.

Case-1 : We assume that first square contains digit 3 in first row $\therefore$ probability is 2/7

and corresponding to it in third square digit 3 may come in II_{nd} row $\therefore$ probability is 3/6

Case-2 : We assume that first square contains digit 3 in second row $\therefore$ probability is 2/7

and corresponding to it in third square digit 3 may come in I_{st} row $\therefore$ probability is 3/6

Hence probability $= \frac{1}{3} \times \frac{2}{7} \times \frac{3}{6} + \frac{1}{3} \times \frac{2}{7} \times \frac{3}{6} = \frac{2}{21}$