Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

1. **Sol.** $4 \times 5 = 20$ ways

2. **Sol.** $1! + 2! + 3! + 4! + 5! + 6! = 1 + 2 + 6 + 24 + 120 + 720 = 873$

3. **Sol.** ${}_{10}C_4 = \dfrac{10 \times 9 \times 8 \times 7}{24} = 210$

4. **Sol.** Remaining = 16
selection of 9 = ${}_{16}C_9$

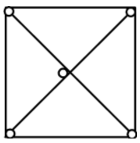
5. **Sol.** Total ways = $2^6 = 64$
ways to get fail = 64 – 1 = 63

6. **Sol.** No of ways = ${}_{10}C_3 + {}_8C_5 = 120 + 56 = 176$

7. **Sol.** No. of gifts = $({}_{10}C_2) \times 2$

8. **Sol.** ${}_8C_2 - 2 \,.\, {}_4C_2 + 2 = 28 - 12 + 2 = 18$

9. **Sol.**



${}_5C_3 - ({}_3C_3) \times 2 = 10 - 2 = 8$

10. **Sol.** ${}_nC_2 - n = 54 \Rightarrow \dfrac{n(n-1)}{2} - n = 54 \Rightarrow n(n - 3) = 108$
$n^2 - 3n - 108 = 0$
$(n - 12)(n + 9) = 0 \Rightarrow n = 12$

11. **Sol.** No of ways = 1

12. **Sol.** Total ways = ${}_{10}C_7 \cdot 7! = {}_{10}P_7$

13. **Sol.** Total ways $= {}_{20}C_2 \times 1 \times 18! \Rightarrow \dfrac{20 \times 19}{2} \times 18! \Rightarrow 10(19!)$

14. **Sol.** Ways $= ({}_2C_1 \times {}_{13}C_{10}) \times (11!)$

15. **Sol.** Number of ways = $1 \times 4! = 24$

16. **Sol.** Total ways $= 9!$

17. **Sol.** Total No. of ways $= ({}_1C_1) \times 1 \times 10!$

18. **Sol.** beads are identical
There is only one way to arrange

19. **Sol.** Total ways $= 7!$

20. **Sol.** C , R, E, A , 2T , 2I , V, Y

Total words $= \frac{10!}{2!2!}$, words starting with c $\Rightarrow \frac{9!}{2!\,2!}$

Required words $= \frac{10!}{2!\,2!} - \frac{9!}{2!\,2!} = \frac{9(9!)}{2!\,2!}$

21. **Sol.** No. of ways $= 3 \times 5 \times 4 = 60$

22. **Sol.** Total numbers = - - - - -0 + - - - - 2 or 4
$= 6 \times 5 \times 4 \times 1 + 5 \times 5 \times 4 \times 2$
$= 120 + 200 = 320$

23. **Sol.** C , C , E , I , L , R
C C _ _ _ _ _ _ _ _ _ _ = 4! = 24
C E _ _ _ _ _ _ _ _ _ _ = 4! = 24
CIC _ _ _ _ _ _ _ _ _ _ = 3! = 6
CIE _ _ _ _ _ _ _ _ _ _ = 3! = 6
CIL _ _ _ _ _ _ _ _ _ _ = 3! = 6
CIRCEL = 1
CIRCLE = 1
Total = 68

24. **Sol.** Total ways $= 4 \times 4 \times 4 \times 4 \times 4 = 4_5$

25. **Sol.** $x_1 + x_2 + x_3 + x_4 + x_5 = 20$
ways $= {}_{20+4}C_4 = {}_{24}C_4$

26. **Sol.** Total ways $= \frac{15!}{(5!)^3\,3!} \times 3!$

27. **Sol.** No of ways $= {}_{10+3}C_3 = {}_{13}C_3$

28. **Sol.** Now of ways = 1 correct + 3 incorrect
$= {}_4C_1 \times 1$ (dearrangement of 3)
$= 4 \times 1 \times D_3 = 4 \times 1 \times (2) = 8$

29. **Sol.** Ways $= 5 \times 3 \times 6 = 90$

30. **Sol.** $1000 = 2_3 \times 5_3$
Total divisors $= 4 \times 4 = 16$

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

1. **Sol.** S = {1, 2, 3, 4}

Each element can be put in 3 ways either in subsets or we don't put in any subset.

So total number of unordered pairs = $\frac{3\times 3\times 3\times 3-1}{2}$ + 1 = 41. [Both subsets can be empty also]

2. **Sol.** $T_k = k.2^{n+1-k}$ and $S_n = \left(\frac{n+1}{4}\right)\left(2^{n+1}-n-2\right)$

Now, $S_n = \sum_{k=1}^{n} k.2^{n+1-k} = 2^{n+1}\sum_{k=1}^{n} k.2^{-k}$

$S_n = 2^{n+1}.2.\left[1-\frac{1}{2^n}-\frac{n}{2^{n+1}}\right]$ (sum of A.G.P.)

$\left(\frac{n+1}{4}\right)\left(2^{n+1}-n-2\right) = 2.\left[2^{n+1}-2-n\right]$

$\Rightarrow \frac{n+1}{4} = 2 \Rightarrow n = 7$

3. **Sol.** maximum size of square can be 8 x 8

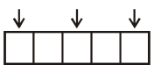
so required no. of square be $= \sum_{r=1}^{8}(16-r)(9-r) = \sum_{r=1}^{8}\left(r^2-25r+144\right) = 456$

4. **Sol.** First fill 3 places by 1,2,3 in ${}_5P_3$ ways and then remaining one in 7 x 7 ways so total no. of ways = ${}_5P_3$ x 7 x 7 = 2940

5. **Sol.** First we select 3 length from the given 6 length so the no. of ways = ${}_6C_3$

But these some pair i.e. (2, 3, 7), (2, 3, 6), (2, 3, 5) (2, 4, 6), (2, 4, 7), (2, 5, 7), (3, 4, 7) are not form a triangle so that total no. of ways is

${}_6C_3$ – 7 ways

6. **Sol.** 

There are 2M, 2T, 2A and 1 H, E, I, C, S

First find the number of ways if odd's no. position place be filled is ${}_5P_3 = 60$

Now Case I   If even place words is same i.e no. of ways = 3

Case II   If even place words is different i.e no. of ways = ${}_3C_2 \times 2! = 6$

Hence total no. of arragment is

$60 \times (3 + 6) = 540$

7. **Sol.** First we select n grand children from 2n grand children is ${}_{2n}C_n$

Now arrangement of both group is $n! \times n!$

Now Rest all (m + 1) place where we occupy the grandfather and m sons but grandfather refuse the sit to either side of grand children so the out of m – 1 seat one seat can be selected

Now required number of sitting in

${}_{2n}C_n \times n! \times n! \times {}_{(m-1)}C_1 \,.\, m!$

$= \frac{(2n)!}{n! \times n!} \times n! \times n! \times {}_{(m-1)}C_1 \,.\, m!$

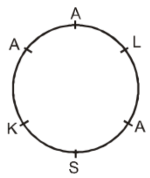
$= 2n! \,.\, m! \,.\, (m-1)$

8. **Sol.** $2!\ \frac{4!}{2!}\ \frac{4!}{2!2!} = 144$

9. **Sol.** Number of words = ${}_8C_4 \frac{4!}{2!} = 840$

10. **Sol.** Number of words = $\frac{5!}{2!}\,{}_6C_2 = 900$

11. **Sol.** A L A S K A



Total number of arrangement is $\frac{5!}{3!} = \frac{120}{6} = 20$

12. **Sol.** Required sum $= 3! \times 20 \times (1111)$

$= 133320$

COMPREHENSION

13. **Sol.** $\underline{1}$----------------- $\underline{1}$ # a_{n-1}

----------------- $\underline{1}$ $\underline{0}$ # a_{n-2}

So (1) choice is correct

consider (2) choice $c_{17} \neq c_{16} + c_{15}$

$c_{15} \neq c_{14} + c_{13}$ is not true

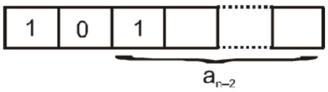
consider (3) choice $b_{17} \neq b_{16} + c_{16}$

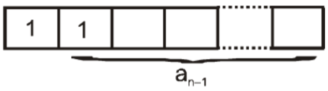
$a_{16} \neq a_{15} + a_{14}$ is not true

consider (4) choice $a_{17} = c_{17} + b_{16}$

$a_{17} = a_{15} + a_{15}$ which is not true

Aliter





using the Recursion formula

$a_n = a_{n-1} + a_{n-2}$

Similarly $b_n = b_{n-1} + b_{n-2}$ and $c_n = c_{n-1} + c_{n-2}$ $\quad \forall\ n \geq 3$

and $\quad a_n = b_n + c_n \quad \forall\ n \geq 1$

so $a_1 = 1$, $a_2 = 2$, $a_3 = 3$, $a_4 = 5$, $a_5 = 8$..........

$b_1 = 1$, $b_2 = 1$, $b_3 = 2$, $b_4 = 3$, $b_5 = 5$, $b_6 = 8$ ....

$c_1 = 0$, $c_2 = 1$, $c_3 = 1$, $c_4 = 2$, $c_5 = 3$, $c_6 = 5$ ....

using this $b_{n-1} = c_n \ \forall n \geq 2$

14. Sol. $b_6 = a_5$

$a_5 = \underline{1} - - - \underline{1} \qquad \underline{1} - - - \underline{0}$

${}_3C_0 + {}_3C_1 + 1 + {}_2C_1 + 1$

$1 + 3 + 1 + 2 + 1$

$4 + 4 = 8$

15. Sol. 3 official out of 8 can be selected by ${}_8C_3 = 56$ ways

2 non-official out of 4 can be selected in ${}_4C_2 = 6$ ways

$\therefore$ required number of committees are $56 \times 6 = 336$.

16. Sol. Two non-officials and 3 officials i.e.

${}_4C_2 \times {}_8C_3 = 6 \times 56 = 336$.

Three non-official and 2 officials

${}_4C_3 \times {}_8C_2 = 4 \times 28 = 112$.

Four non-officials and 1 official

${}_4C_4 \times {}_8C_1 = 1 \times 8 = 8$

Total $336 + 112 + 8 = 456$.

17. Sol. Required no. of ways

$= {}_{12-1}C_5 = {}_{11}C_5 = 462$

18. Sol. In the word RESONANCE there are 9 letters.

Consonants (5), 1R, 1S, 1C and 2N

Vowels (4), 2E, 1O, 1A

total even places 4 ;

No. of ways arranging vowels in even places is $\frac{4!}{2!} = 12$

No. of ways arranging consonants in remaining odd places is $\frac{5!}{2!} = 60$

required number of arrangement $= 12 \times 60 = 720 = n$

19. Sol. Required number of arrangements are $\frac{9!}{2!\,2!\,4!} = 3780$

20. **Sol.** Now exponents of 5 in 720

$= \left[\frac{720}{5}\right] + \left[\frac{720}{5^2}\right] + \left[\frac{720}{5^3}\right] + \left[\frac{720}{5^4}\right]$

= 144 + 28 + 5 + 1 = 178

ASSERTION / REASONING

21. **Ans. (1)**

Sol. Here n^2 objects are distributed in n groups, each group containing n identical objects.

∴ Number of ways of arrangements

$= {}^{n^2}C_n . {}^{n^2-n}C_n . {}^{n^2-2n}C_n . {}^{n^2-3n}C_n . {}^{n^2-2n}C_n .... {}^{n}C_n$

$= \frac{(n^2)!}{n!(n^2-n)!} \cdot \frac{n^2-n!}{n!(n^2-2n)!} ... \frac{n!}{n!.1} = \frac{(n^2)!}{(n!)^n} =$ Integer

(as number of arrangements has to be integer) ($n \in I_+$)