Probability

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

1. **Sol.** S = {HH, HT, TH,TT}

Probability, p = $\frac{2}{4} = \frac{1}{2}$, q = $\frac{1}{4}$

2. **Sol.** Odds against foi{k esa la;ksxkuqikr = 4:11

Probability, = $\frac{11}{4+11} = \frac{11}{15}$

3. **Sol.** S= {1, 2, 3, 4, 5, 5, 6}

Probability, = $\frac{4}{6} = \frac{2}{3}$

4. **Sol.** P(A'∩B') = P(A') P(B') ∵ A,B independent , A ,oa B LorU=k gSA

= $\left(\frac{3}{4}\right)\left(\frac{2}{3}\right) = \frac{1}{2}$

5. **Sol.** P(Gargy to speak truth) = $\frac{60}{100} = \frac{3}{5}$

P(Ashmeet to seak truth) = $\frac{90}{100} = \frac{9}{10}$

required probability = $\frac{3}{5}\left(1-\frac{9}{10}\right)+\left(1-\frac{3}{5}\right)\left(\frac{9}{10}\right) = \frac{3}{50}+\frac{18}{50}=\frac{21}{50}$ $\therefore$ $\frac{21}{50}\times 100\% \Rightarrow$ 42%

6. **Sol.** Required probability = $\frac{1}{4}+\frac{1}{5}-\frac{1}{15} = \frac{15+12-4}{60}=\frac{23}{60}$

7. **Sol.** S= {2, 3, 5}

Probability, = $\frac{2}{3}$

8. **Sol.** S={(2,2), (2,3) (2,5), (3, 2) (3, 3) (3,5), (5,2) (5,3), (5,5)}

Probability, = $\frac{4}{9}$

9. **Sol.** Probability, = $\frac{{}^8C_1}{{}^9C_2} = \frac{8\times 2}{9\times 8} = \frac{2}{9}$

10. **Sol.** P(Thomas to be true, = $\frac{3}{4}$

P(Thomas to be false, = $\frac{1}{4}$

E→ Thomas said six

A→ Actually a six

$P\left(\frac{A}{E}\right) = \frac{P(A \cap E)}{P(E)} = \frac{1/6 \times 3/4}{1/6 \times 3/4 + 5/6 \times 1/4} = \frac{3/24}{3/24 + 5/24} = \frac{3}{8}$

11. **Sol.** Probability, $= \frac{^{4}C_2}{^{52}C_2}$

12. **Sol.** Probability, $= \frac{^{12}C_4}{^{52}C_4}$

13. **Sol.** P(all boys sit together) = $\frac{8! \times 7!}{14!}$

Probability, $1 - \frac{8! \times 7!}{14!}$

14. **Sol.** Probability, $= \frac{^{40}C_1 . ^{4}C_1}{^{52}C_2}$

15. **Sol.** A → First ticket is perfect square

B → Second ticket to be perfect square

$P\left(\frac{B}{A}\right) = \frac{\left(\frac{5}{25}\right)\left(\frac{4}{24}\right)}{\left(\frac{5}{25}\right)(1)} = \frac{4}{24} = \frac{1}{6}$

16. **Sol.** P (getting composite number) = $\frac{2}{6} = \frac{1}{3}$

Probability, $= {^{5}C_3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2 \Rightarrow 10 \times \frac{1}{27} \times \frac{4}{9} = \frac{40}{243}$

17. **Sol.** P (Red) = $\frac{3}{5}$

P (Black) = $\frac{2}{5}$

Probability, = ${^{4}C_2}\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^2 = \frac{3^3 2^3}{5^4}$

18. **Sol.** Sum, 10 → {(4, 6), (5, 5), (6, 4)}

P (getting sum, 10) = $\frac{3}{36} = \frac{1}{12}$

Probability, $= {^{5}C_3}\left(\frac{1}{12}\right)^3\left(\frac{11}{12}\right)^2$

19. **Sol.** Probability, $= {^{3}C_2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) = \frac{3}{8}$

20. **Sol.** (52 weeks) + (2 days extra)

⇒ {SM, MT, TW, WT, TF, FS, SS}

Probability, $= \frac{2}{7}$

21. **Sol.** $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= \frac{1}{8} + \frac{2}{5} - 0 = \frac{21}{40}$

22. **Sol.** $P(A/B) = \frac{P(A\cap B)}{P(B)} \Rightarrow P(A\cap B) = \frac{2}{3}\times\frac{3}{5} = \frac{2}{5}$

$P(\bar{A}\cap B) = P(B) - P(A\cap B)$

$= \frac{3}{5} - \frac{2}{5} = \frac{1}{5}$

23. **Sol.** $B_1 \rightarrow$ Ball drawn from urn A is red but ball returned to it is black, $P(B_1) = \frac{6}{10} \times \frac{6}{11}$

$D_1 \rightarrow$ Ball drawn from urn A is black and ball returned is red, $P(D_1) = \frac{4}{10} \times \frac{4}{11}$

Required probability $P(R) = P(B_1) \times P\left(\frac{R}{B_1}\right) + P(D_1) \times P\left(\frac{R}{D_1}\right)$

$= \frac{6}{10}\times\frac{6}{11} \times \frac{5}{10} + \frac{4}{10}\times\frac{4}{11} \times \frac{7}{10} = \frac{73}{275}$

24. **Sol.** n (S) = 6 × 6 × 6 = 216 also to get sum of '8' $x_1 + x_2 + x_3 = 7$ where $1 \le x_i \le 6$

$x_i = t_i + 1 \qquad 0 \le t_1 \le 4 \Rightarrow t_1 + t_2 + t_3 = 4$

by fictious partition method number of solution of this equation is ${}_6C_2$

n (E) = 15 $\Rightarrow$ $P = \frac{15}{216} = \frac{5}{72}$

25. **Sol.** Since ten places are vacant. Probability of finding vacant places = $\frac{{}^{21}C_{11}}{{}^{23}C_{11}} = \frac{6}{23}$

26. **Sol.** P(Product of digits) = 15

if 35, 53 $\Rightarrow$ $P(A) = \frac{2}{90} = \frac{1}{45}$ $\Rightarrow$ $P(\bar{A}) = \frac{44}{45}$

Probability $= 1 - \left(\frac{44}{45}\right)^3$

27. **Sol.** $\frac{np}{npq} = \frac{3}{2}$ $\Rightarrow$ $q = \frac{2}{3}$ $\Rightarrow$ $p = \frac{1}{3}$

Probability of getting exactly 5 success ${}^6C_5 \left(\frac{1}{3}\right)^5 = \frac{2}{81}$

28. **Sol.**

$$\left.\begin{aligned} p(x=4) &= {}^nc_4\left(\frac{1}{2}\right)^n \\ p(x=5) &= {}^nc_5\left(\frac{1}{2}\right)^n \\ p(x=6) &= {}^nc_6\left(\frac{1}{2}\right)^n \end{aligned}\right] \Rightarrow ({}^nC_5)^2 = {}^nC_4\ {}^nC_6 \Rightarrow n = -1 \text{ which is not possible}$$

29. **Sol.** $E_1 = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$

$E_2 = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$\therefore n(E_1) = 5, n(E_2) = 6$

Also $n(S) = 6 \times 6 = 36$

Now, $P(E_1) = \frac{5}{36}$ and $P(E_2) = \frac{6}{36} = \frac{1}{6}$

Let E = the event that A wins

Let F = the event that B wins

$P(E) = \frac{5}{36} + \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36} + \frac{31}{36} \times \frac{5}{6} \times \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36} + \ldots\ldots\ldots \frac{30}{61}$

$P(F) = 1 - P(E) = \frac{31}{61}$

30. **Sol.** Unit digit of 3^a = 3, 9, 7, 1

Unit digit of 2^b = 2, 4, 8, 6

3^a	2^b
3	8

3^a	2^b
9	2

3^a	2^b
7	4

$\Rightarrow \frac{25}{100} \times \frac{25}{100} + \frac{25}{100} \times \frac{25}{100} + \frac{25}{100} \times \frac{25}{100} = \frac{3}{16}$

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

1*. **Sol.** $P(x_1) = \frac{1}{2}$

$P(x_2) = \frac{1}{4}$

$P(x_3) = \frac{1}{4}$

$P(x) = P(E_1 E_2 E_3) + P(\bar{E}_1 E_2 E_3) + P(E_1 \bar{E}_2 E_3) + P(E_1 E_2 \bar{E}_3)$

$= \frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{3}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{3}{4}$

$P(x) = \frac{1}{4}$

(1) $P\left(\frac{x_1^c}{x}\right) = \frac{P(x_1^c \cap x)}{P(x)}$

$= \frac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4}}{\frac{1}{4}} = \frac{1}{8}$

(2) P(exactly two / x) = $\frac{P(\text{exactly two} \cap x)}{P(x)}$

$= \frac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{3}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{3}{4}}{\frac{1}{4}} = \frac{7}{8}$

(3) $P(x / x_2) = \frac{P(x \cap x_2)}{P(x_2)} = \frac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{3}{4}}{\frac{1}{4}} = \frac{5}{8}$

(4) $P(x / x_1) = \frac{P(x \cap x_1)}{P(x_1)} = \frac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{3}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{3}{4}}{\frac{1}{2}} = \frac{7}{16}$

2. **Sol.** $x + \frac{100}{x} > 50$

$\left.\begin{matrix} x = 1 \text{Satisfis} \\ = 2 \quad " \end{matrix}\right]$

= 3 does ' nt satsfies

= 47 doest satsfies

$\left.\begin{matrix} = 48 \ \text{Satifes} \\ = 100 \text{Safies} \end{matrix}\right]$ 53 number

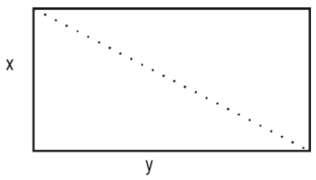
then no. of x which satisfy given inequation = 55

Pro. = $\frac{55}{100}$

3. **Sol.** 0 < x < 10 $\quad x^2 + y^2 < 100$

0 < y < 10

p = $\frac{\frac{1}{4}\Pi \times 10^2}{10 \times 10}$



4. **Sol.** P = P(1 person lies) + P(2 person lies) P(A lies first / 2 person lied) + P(3 person lied)

P(A 1 died first / 3 person lied)

$= {}^nC_1\, pq^{n-1} \times \frac{1}{n} + {}^nC_1 p^2 q^{n-2} \times \frac{1}{2} + {}^nC_2\, p^3 q^{n-3} \times \frac{1}{3} + \ldots\ldots$

$= pq^{n-1} + {}^{n-1}C_{r-1} p^2 q^{n-2} \frac{1}{2} + {}^nC_3\, p^3 q^{n-3} \frac{1}{3} + \ldots\ldots = pq^{n-1} + \sum_{r=2} {}^{n-1}C_{r-1} p^r q^{n-r} \frac{1}{r}$

As $\frac{{}^{n-1}C_{r-1}}{r} = \frac{{}^nC_1}{n}$

P = Pq_{n-1} + $\frac{1}{n}\sum_{r=2}^{n} {}^nC_r P^r q^{n-r}$ $\Rightarrow$ P = pq_{n-1} + $\frac{1}{n}(1 - {}^nC_0\, p^0 q^n - {}^nC_1 P^1 q^{n-1})$

P = Pq_{n-1} + $\frac{1}{n}(1 - q^n - nPq^{n-1}) = \frac{1-(1-p)^n}{n}$

5. **Sol.** Person is selected if either he passes all the tests or exactly two of the tests.

P(passing all the tests) = p.p.p = p_3

Prob. of passing exactly 2 of the tests = P(first two tests) + P(first and third tests) + P(second and third tests)

= p.p.(1 – p) + p.(1 – p). $\frac{p}{2}$ + (1 – p) $\frac{p}{2}$. p

= p_2(1 – p) + $\frac{1}{2}$ p_2 (1 – p) + p_2 (1 – p)

= 2p_2 (1 – p)

Thus, required probability = p_3 + 2p_2 (1 – p) = 2p_2 – p_3 .

6. **Sol.** Here,

P(A) = probability that A will hit B = $\frac{2}{3}$

P(B) = probability that B will hit A = $\frac{1}{2}$

P(C) = probability that C will hit A = $\frac{1}{3}$

P(E) = probability that A will be hit

$P(E) = 1 - P(\bar{B}).P(\bar{C}) = 1 - \frac{1}{2}.\frac{2}{3} = \frac{2}{3}$

Probability if A is hit by B and not by C.

⇒ $P(B \cap \bar{C}/E)$

⇒ $\frac{P(B).P(\bar{C})}{P(E)} = \frac{\frac{1}{2}.\frac{2}{3}}{\frac{2}{3}} = \frac{1}{2}$

7. **Sol.** Statement-1 If $P(H_i \cap E) = 0$ for some i, then

$P\left(\frac{H_i}{E}\right) = P\left(\frac{E}{H_1}\right) = 0$

If $P(H_i \cap E) \neq 0$ for $\forall\ i = 1, 2, ...n$, then

$P\left(\frac{H_i}{E}\right) = \frac{P(H_i \cap E)}{P(H_i)} \times \frac{P(H_i)}{P(E)} = \frac{P\left(\frac{H_i}{E}\right) \times P(H_i)}{P(E)} > P\left(\frac{E}{H_i}\right)$, $P(H_i)$ [as 0 < P(E) < 1]

Hence statement -1 may not always be true.

Statement-2 Clearly, $H_1 \cup H_2 \cup .... \cup H_n = S$ (sample space)

$P(H_1) + P(H_2) + ... + P(H_n) = 1$

8. **Sol.** Here, n(s) = 1 length of the interval [0, 5] = 5 ; n(E) = length of the interval ≤ [0, 5] in which P belongs such that the given equation has real roots.

Now $x^2 + Px + \frac{1}{4}(P + 2) = 0$ will have real roots

if $P^2 - 4.1.\frac{1}{4}(P + 2) \geq 0$ ⇒ $P^2 - P - 2 \geq 0$

⇒ (P + 1) (P – 2) ≥ 0 ⇒ P ≤ – 1 or P ≥ 2

But P ∈ [0, 5]. So, E = [2, 5]

∴ n(E) = length of the interval [2, 5] = 3

∴ Required Probability = $\frac{3}{5}$

9. **Sol.** Here, $P(A \cup B).P(A' \cap B')$

⇒ {P(A) + P(B) – $P(A \cap B)$ }{P(A').P(B')}

Since A, B are independent

⇒ A', B' are independent}

= P(A). P(A'). P(B') + P(B).P(A') . P(B') ....(1)

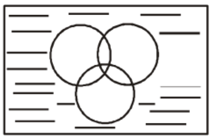
≤ P(A). P(B') + P(B). P(A')

{Since in (1), P(A') ≤ 1 and P(B') ≤ 1}

⇒ $P(A \cup B)$. $P(A' \cap B')$ ≤ P(A) . P(B') + P(B). P(A')

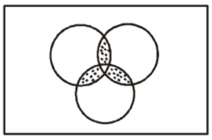
$\Rightarrow P(A \cup B) . P(A' \cap B') \leq P(C)$
{as P(C) = P(A). P(B') + P(B). P(A')}

10. **Sol.**

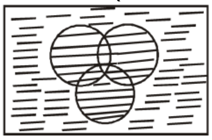


$P = 1 - P(A \cup B \cup C)$
$= 1 - P(A) - P(B) - P(C) + P(A \cap B) + P(B \cap C) + P(C \cap A) - P(A \cap B \cap C)$
$= P(A \cap B \cap C)_1 - P(A) - P(B) - P(C) + P(A \cap B) + P(B \cap C) + P(C \cap A)$
$= P(\bar{A} \cup \bar{B} \cup \bar{C}) - P(A) - P(B) - P(C) + P(A \cap B) + P(B \cap C) + P(C \cap A)$

11. **Sol.**



12.
Sol. $P = 1 - P(A \cap B \cap C)$



$= 1 - P(A \cup B \cup C) + P(A) + P(B) + P(C) - P(A \cup B)$
$= 1 - P(A \cup B \cup C)_c + P(A) + P(B) + P(C) - P(A \cup B) - P(B \cup C) - P(C \cap A)$

13. **Sol.** Total number of ways = M_n , number of favorable cases = n!

14. **Sol.** Total number of ways = ${}_{M+n-1}C_{M-1}$, number of favorable cases = 1

15. **Sol.** Total number of ways = ${}_{M}C_{n}$, number of favorable cases = 1

16. **Sol.** P(white) = P (H ∩ white) + P(T ∩ white)

$= \frac{1}{2}\cdot\left\{\frac{3}{5}\times 1+\frac{2}{5}\cdot\frac{1}{2}\right\}+\frac{1}{2}\cdot\left\{\frac{{}^3C_2}{{}^5C_2}\times 1+\frac{{}^2C_2}{{}^5C_2}\times\frac{1}{3}+\frac{{}^3C_1\cdot{}^2C_1}{{}^5C_2}\times\frac{2}{3}\right\}$

$= \frac{1}{2}\times\frac{8}{10}+\frac{1}{2}\times\left\{\frac{3}{10}+\frac{1}{30}+\frac{12}{30}\right\} = \frac{4}{10}+\frac{1}{2}\times\frac{22}{30} = \frac{23}{30}$

17. **Sol.** $P\left(\text{Head}/\text{White}\right) = \dfrac{P(\text{Head} \cap \text{white})}{P(\text{white})} = \dfrac{\frac{1}{2}\times\left\{\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{2}\right\}}{\frac{23}{30}} = \dfrac{\frac{4}{10}}{\frac{23}{30}} = \dfrac{12}{23}$

18. **Sol.** $ax + by = 0$ $\quad a, b, c, d \in \{0, 1\}$

$cx + dy = 0$

system has unique solution if and only if $ad - bc \neq 0$

For which $a = d = 1$ and $bc = 0$ $\Rightarrow$ 3 combination

Similarly if $bc = 1$, $ad = 0$ $\Rightarrow$ 3 combination

Total choice for a, b, c, d is 2^4

Hence probability of unique solution is $\frac{6}{16} = \frac{3}{8}$. Statement-2 is also true since (0, 0) is a solution

Thus 'B' is correct

Aliter : $ad - bc \neq 0$

If (i) $ad = 1, bc = 0$

(ii) $ad = 0, bc = 1$

$P(ad = 1) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$; $P(bc = 0) = 1 - P(bc = 1) = 1 - \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}$

$\therefore$ $P(ad = 1 \text{ and } bc = 0) = \frac{3}{16}$; $P(ad = 0 \text{ and } bc = 1) = \frac{3}{16}$

$\therefore$ required probability $= \frac{3}{8}$

19. **Sol.** Let x, y, z be probability of E_1, E_2, E_3 respectively

$\Rightarrow$ $x(1 - y)(1 - z) = \alpha$ $\Rightarrow$ $y(1 - x)(1 - z) = \beta$

$\Rightarrow$ $z(1 - x)(1 - y) = \gamma$ $\Rightarrow$ $(1 - x)(1 - y)(1 - z) = P$

Putting in the given relation we get $x = 2y$ and $y = 3z$ $\Rightarrow$ $x = 6z$ $\Rightarrow$ $\frac{x}{z} = 6$

20. **Sol.** Three squares are shown as below

I^{st} row	1								
II^{nd} row		2							
III^{rd} row				3					

$\frac{1}{3}\times\frac{2}{7}\times\frac{3}{6}+\frac{1}{3}\times\frac{2}{7}\times\frac{3}{6}=\frac{2}{21}$

digit 3 may come only in I_{st} and II_{nd} rows. In second square if ? is replaced by 3 then probability is 1/3.

Case-1 : We assume that first square contains digit 3 in first row $\therefore$ probability is 2/7

and corresponding to it in third square digit 3 may come in II_{nd} row $\therefore$ probability is 3/6

Case-2 : We assume that first square contains digit 3 in second row $\therefore$ probability is 2/7

and corresponding to it in third square digit 3 may come in I_{st} row $\therefore$ probability is 3/6

Hence probability = $\frac{1}{3}\times\frac{2}{7}\times\frac{3}{6}+\frac{1}{3}\times\frac{2}{7}\times\frac{3}{6}=\frac{2}{21}$