Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

1. **Sol.** Let the number of men be x

$\frac{70x + 55(150 - x)}{150} = 60$

x = 50

2. **Sol.** $\overline{x}_{new} = \overline{x}_{old} - 5$

3._ **Sol.** We known that $y_i = \frac{100}{60}x_i = \frac{5}{3}x_i$ so h = $\frac{5}{3}$ Thus $\sigma_y = |h|\ \sigma_x = \frac{5}{3} \times 5 = \frac{25}{3}$

so new variance = $\left(\frac{25}{3}\right)^2 = \frac{625}{9}$

4. **Sol.** Range = 21 – 12 = 9

5. **Sol.** variance $(x_i) = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

$1^2 + 2^2 + 3^2 ....10^2 = \frac{n(n+1)(2n+1)}{6} = \frac{10 \times 11 \times 21}{6} = 385$

$1 + 2 + 3 ....10 = \frac{n(n+1)}{2} = \frac{(10)(11)}{2} = 55$

var(xi) = $\frac{385}{10} - \left(\frac{55}{10}\right)^2 = \frac{825}{100} = \frac{33}{4}$

6. **Sol.** 4, 8, 12, 17, 19, 23, 27, ⇒ Median = 17

7. **Sol.** Most frequent data = 3

8. **Sol.** $\frac{L - S}{L + S} = \frac{11 - 2}{11 + 2} = \frac{9}{13}$

9. **Sol.** Mode + 2 mean = 3 median

(mean –3) + 2 mean = 3 median

3(mean – median) = 3

10 **Sol.** Most frequent data

11. **Sol.** variance $(x_i - 4) = \text{var}(x_i) = \frac{44}{11} - \left(\frac{11}{11}\right)^2 = 3$

$(x_i - 4) = \text{var}(x_i) = \frac{44}{11} - \left(\frac{11}{11}\right)^2 = 3$

12 **Sol.** $S_n = \frac{n}{2}\ [2 + (n-1)3]$

$\bar{x} = \frac{S_n}{n} = \frac{1}{2}$ [3n–1]

13. Sol. variance $(ax_i + b) = a^2 \text{ var}(x_i)$

14. Sol. variance $(ax_i) = a^2 \text{var}(x_i)$

S.D. $(ax_i) = |a| \sqrt{\text{var}(x_i)}$

15. Sol. Variance $(x_i) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

16. Sol. $\frac{\sum x_i}{7} = 7$

2+ 4 + 7 + 11 + 10 + a + b = 49 ⇒ a + b = 15

$\frac{\sum(x_i - \bar{x})^2}{7} = \frac{100}{7}$

⇒ $25 + 9 + 0 + 16 + 99 + (7 - a)^2 + (7 - b)^2 = 100$

⇒ $(7 - a)^2 + (7 - b)^2 = 41$

⇒ a = 3

b = 12

17. Sol. $\frac{\sigma}{\bar{x}} \times 100$ = coefficient of variation σ = 3

18. Sol. Median = $\frac{24^{th}\text{ term} + 25^{th}\text{ term}}{2} = \frac{76+77}{2}$ = 76.5

mean deviation = $\frac{(23.5 + 22.5......0.5 + 0.5 + ...23.5)}{48}$ = 12

19. Sol. $\sum x_i$ = 20 × 11 = 220 ⇒ $\sum y_i$ = 10 × 8 = 80

variance $(x_i) = \frac{\sum x_i^2}{20} - (121)$ ⇒ $2500 = \sum x_i^2$

variance $(y_i) = \frac{\sum y_i^2}{10} - 64$ ⇒ $980 = \sum y_i^2$

$\sum x_i^2 + \sum y_i^2 = \left(x_i^2 + y_i^2\right)$ = 3480 ⇒ $\sum x_i + \sum y_i = \frac{\sum x_i^2 + y_i^2}{30} - \left(\frac{\sum x_i + y_i}{30}\right)^2$ = 116 – 100 = 16

20. Sol. Standard deviation is independent of change of origin bu not scale.

21. Sol. Actual data is more close to mean, therefore less variance

22 Sol. Variations) = $\frac{2^2 + 4^2 + 6^2 + 8^2 + 10^2}{5} - \left(\frac{2+4+6+8+10}{5}\right)^2$ = 44 – 36 = 8

23 Sol. $\text{Var}(3x_i+4) = 9\sigma^2$

24. Sol. Arranging the data in ascending order 34, 38, 42, 44, 46, 48, 54, 55, 63, 70 median = $\frac{46+48}{2} = 47$

$\sum|xi - 47|$ = 9 + 23 + 1 + 13 + 5 + 8 + 16 + 1 + 7 + 3 = 86.

Mean deviation = $\frac{\sum|xi-47|}{10} = 8.6$

25. **Sol.** $\bar{x} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$ ⇒ $\sigma_2 = \left(\frac{\sum_{i=1}^{n} x_i^2}{n}\right) - \left(\frac{\sum ni}{n}\right)^2$ $= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{(n+1)}{2}\right)^2 = \frac{n^2-1}{12}$

26. **Sol.** Arrange marks in ascending order

(................), 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29

8 boys failed

median = $\frac{18+19}{2}$ = 18.5

27. **Sol.** $\frac{\sigma_1}{\bar{x}_1}$ = 0.6 **;** $\frac{\sigma_2}{\bar{x}_2}$ = 0.7

$\frac{\sigma_1}{\sigma_2} = \frac{0.6 \times \bar{x}_1}{0.7 \times \bar{x}_2}$ ⇒ $\frac{21}{14} \times \frac{0.7}{0.6} = \frac{\bar{x}_1}{\bar{x}_2}$ ⇒ $\frac{7}{4} = \frac{\bar{x}_1}{\bar{x}_2}$

28. **Sol.** $\sum x_i = 63 \times 50 = 3150$; $\sum y_i = 40 \times 54 = 2160$

var(x_i) = 81 = $\frac{\sum x_i^2}{50} - (63)^2$, var(y_i) = 36 = $\frac{\sum y_i^2}{40} - (54)^2$

$\sum x_i^2 = 202500$

$\sum y_i^2 = 118080$

combined variance = $\frac{\left(\sum x_i^2 + \sum y_i^2\right)}{90} - \left(\frac{\sum x_i + \sum y_i}{90}\right)^2$

= $\frac{320580}{90} - (59)^2$

= 3562 – 3481

= 81

29. **Sol.** Mean and median both will be increased by 2

30. **Sol.** $\frac{(2\times1)+(14\times2)+(8\times5)+(32\times7)}{2+14+8+32} = \frac{294}{56}$ = 5.25

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Statistics

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

1._ **Sol.** We have : $\Sigma X = a\,\Sigma U + b\Sigma V$. Therefore ,,

$$\bar{X} = \frac{1}{n}\sum X = a.\frac{1}{n}\sum U + b.\frac{1}{n}\sum V = a\bar{U} + b\bar{V}$$

2._ **Sol.** Let the n-numbers be $x_1, x_2, \ldots, x_n$ then,

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\Rightarrow \bar{X} = \frac{x_1 + x_2 + \ldots x_{n-1} + x_n}{n}$$

$$\Rightarrow \bar{X} = \frac{k + x_n}{n} \qquad [\because x_1 + x_2 + \ldots\ldots x_{n-1} = k]$$

$\Rightarrow x_n = n - \bar{X} k$

3._ **Sol.** Let n_1 and n_2 be the number of observations in two groups having means $\bar{X}_1$ and $\bar{X}_2$ respectively. Then

$$\bar{X} = \frac{n_1\bar{X} + n_2\bar{X}_2}{n_1 + n_2}$$

Now , $\bar{X} - \bar{X}_1 = \dfrac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2} - \bar{X}$

$= \dfrac{n_2(\bar{X}_2 - \bar{X}_1)}{n_1 + n_2} > 0 \qquad [\because \bar{X}_2 - \bar{X}_1]$

$\Rightarrow \bar{X}_1 - \bar{X}_2$ .... (i)

And , $\bar{X} - \bar{X}_2 = \dfrac{n(\bar{X}_1 - \bar{X}_2)}{n_1 + n_2} < 0 \qquad [\because \bar{X}_2 > \bar{X}_1]$

$\Rightarrow \bar{X} < \bar{X}_2$ .... (ii)

From (i) and (ii) $\bar{X}_1 < \bar{X} < \bar{X}_2$.

4. **Sol.** Let σ_1 and σ_2 be the standard deviations of the two data, then

$\dfrac{50}{100} = \dfrac{\sigma_1}{30}$ and $\dfrac{60}{100} = \dfrac{\sigma_2}{25}$

$\Rightarrow \sigma_1 = 15$ and $\sigma_2 = 15$