Sets, Relations & Function

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

1. **Sol.** **Obvious**

2. **Sol.** **Obvious**

3. **Sol.** **Obvious**

4. **Sol.** **Obvious**

5. **Sol.** **Obvious**

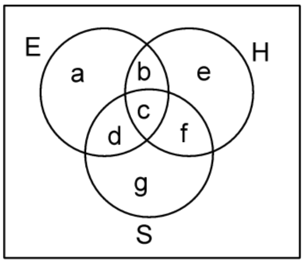
6. **Sol.** $n(A \cup B)= n(A)+n(B)–n(A \cap B)$
$0 \le n(A \cap B) \le 5$
$7 \le n(A \cup B) \le 12$

7. **Sol.** A = {2,3,5,7,11, 13, 17, 19,23}
B = {4,6,8,9,10,12,14,15,16,18}
$n(A \cup B) = 19$, $n(A \cap B) = 0$
$n(A \cup B') = 9$

8. **Sol.** $A = \varphi$
$P(A) = \{\varphi\} \Rightarrow n(P(A) = 1$
$n(P(P(\varphi))) = 2^1$
$n(P(P(P(\varphi)))) = 2^2 = 4$
$n(P(P(P(P(\varphi))))) = 2^4 = 16$

9. **Sol.** $A' \cup \{(A \cup B) \cap B'\}$
$A' \cup \{(A \cap B') \cup (B \cap B')\}$
$A' \cup (A \cap B') = (A' \cup A) \cap (A' \cup B') = A' \cup B' = (A \cap B)'$

10. **Sol.** a = 18 , a + d = 23, c + d = 8
c + f = 8, a + b + c + d = 26
c + d + f + g = 48
a + b + c + d + e + f + g = 100 – 24 = 76
a = 18, d = 5, c = 3, f = 5 , b = 0
g = 48 – (3 + 5 + 5) = 35
e = 76 – (18 + 0 + 3 + 5 + 5 + 35) = 10
Now b + c + e + f = 0 + 3 + 10 + 5 = 18



11. **Sol.** $n(2) = \left[\frac{1000}{2}\right] = 500$

$n(3) = \left[\frac{1000}{3}\right] = 333$

$n(5) = \left[\frac{1000}{5}\right] = 200$

$n(2 \cap 3) = 166$, $n(3 \cap 5) = 66$

$n(5 \cap 2) = 100$ $n(2 \cap 3 \cap 5) = 33$, $n(2 \cup 3 \cup C) = 734$

$n(2' \cap 3' \cap 5') = 1000 - 734 = 266$

12. **Sol.** $80 = 40 + 50 + 60 - 2(n(A \cap B) + n(B \cap C) + n(C \cap A)) + 30$

$\Rightarrow n(A \cap B) + n(B \cap C) + n(C \cap A) = 50$

Required number of members $T = n(A \cap B) + n(B \cap C) + n(C \cap A) - 2n(A \cap B \cap C)$

$= 50 - 2 \times 10 = 30$

13. **Sol.** $0 \le n(A \cap B) \le \min.\{n(A), n(B)\}$

$0 \le n(A \cap B) \le 12$

$n(A' \cap B) = n(B) - n(A \cap B)$

$3 \le n(\backslash A' \cap B) \le 1\,5$

$\Rightarrow x = 3, y = 15$

14. **Sol.** $x^2 + 6x - 27 = 0$

$(x + 9)(x - 3) = 0 \Rightarrow x = 3$ and $y = \pm 3\sqrt{3}$

15. **Sol.** $a + b = 5$

$b - 3 = 1 \Rightarrow b = 4, a = 1$

16. **Sol.** $X \cap (X \cup Y)' = X \cap (X' \cap Y') = (X \cap X') \cap Y'$

$= \varphi \cap Y' = \varphi$

17. **Sol.** $n(A \times B) = 4 \times 3 = 12$

$n(A \times A) = 4 \times 4 = 16$

$n(B \times B) = 3 \times 3 = 9$

18. **Sol.** $N_3 \cap N_5 = N_{15}$ [$\because$ 3 and 5 are relatively prime numbers)

19. **Sol.** $\therefore x^2 + 4y^2 = 45$

We can see that $x = \pm 3$ $y = \pm 3$

(3,3) (3,– 3), (–3,–3) (–3,3) are 4 elements

20. **Sol.** **Obvious**

21. **Sol.** Given = 3x

R = {(1,3) (2,6) (3,9) (4,12)}

22. **Sol.** $n(A) = 3$, $n(B) = 4$, $n(A \times B) = 3 \times 4 = 12$

number of subset having no element = 1

number of subset having exactly one element = 12

23. **Sol.** $2^m + 2^n = 144$

$2^n\{2^{m-n} + 1\} = 2^4 \times 3^2$

$n = 4$, $m - n = 3$

$n = 4$, $m = 7$

24. **Sol.** **Obvious**

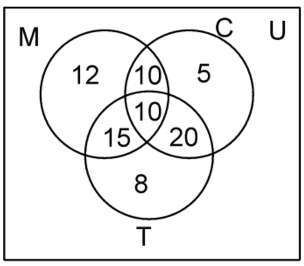
25. **Sol.** Reflexive x R x ⇒ |x – x| ≤ 1/3 ⇒ reflexive Symmetric : x R y ⇒ |x – y| ≤ 1/3
⇒ |y – x| ≤ 1/3, is also true ⇒ symmetric
Transitive : x R y ⇒ |x – y| ≤ 1/3
yRZ ⇒ |y – z| ≤ 1/3
but $x \not R z$ for all x,y,z ∈ R
for example, x = 1 y = 2/3 , z = 1/3
⇒ x R y, y R z but $x \not R z$.

26. **Sol.** Reflexive :x, y ∈ R, x R x ⇒ $x^2 \geq 0$ ∀ x ∈ R ⇒ Reflexive
symmetric : xy ∈ R, x R y ⇒ xy ≥ 0 ⇒ yx ≥ 0 ∀ x, y ∈ R ⇒ symmetric
Transitive : x, y z ∈ R
x R y ⇒ xy ≥ 0
y R z ⇒ yz ≥ 0
but $x \not R z$
⇒ it is not Transitive
for example x = 1, y = 0, z = – 3

27. **Sol.** Every element of X have 3 options
either in Y or in Z or none
o number of ordered pairs = 3^4

28. **Sol.** ∴ A ⊆ A ⇒ A R A, ∀ A ∈ P(s)
⇒ R is reflexive
Symmetric : A R B ⇒ A ⊆ B B ⊆ A
but not necessary so not symmetric
transitive A R B and B R C ⇒ A ⊆ B and B ⊆ C ⇒ A ⊆ C ⇒ R is transitive

29. **Sol.** n(U) = 100
n(M∩C∩T) = 10;n(M∩C) = 20;
n(C∩T) = 30; n(M∩T) = 25;
n(M only) = 12; n(only C C) = 5; n(only T T) = 8



∴ n(M∩C∪T) = 12+10+5+15+10+20+8 = 80
∴ n(M∩C∪T)' = 100 – 80 = 20

30. **Sol.** X = {1, 2, 3, 4, 5, 6, 7, 8, 9}
So , X – (A∩B) has 7 elements
A will has 7, 8 Rest elements can be assigned in 3 ways, either go to A or B or none so total pairs = $3^7 - 1 = 2186$

PART - II : PRACTICE QUESTIONS

1. **Sol.** Let C, H, F denote the sets of members who are on the cricket team, hockey team and football team respectively.

Then we are given n(C) = 21,n(H) = 26, n(F) = 29

$n(H \cap C) = 14$, $n(H \cap F) = 15$, $n(F \cap C) = 12$

$n'(C \cap H \cap F) = 8$

$n(C \cup H \cup F) = n(C) + n(H) + n(F)$

$- n(C \cap H) - n(H \cap F) - n(F \cap C) + n(C \cap H \cap F)$

Hence, $n(C \cup H \cup F) = (21 + 26 + 29) - (14 + 15 + 12) + 8 = 43$

2. **Sol.** Suppose that number of employee taking vacations is 100

Su – set of employee taking leave in summer

W – set of employee taking leave in winter

Sp – set of employee leave in spring

A– set of employee taking leave in Autumn

n(Su) = 90, n(W) = 65, n(Sp) = 10, n(A) = 7

$n(W \cap Su) = 55$, $n(Sp \cap Su) = 8$, $n(A \cap Su) = 6$

$n(W \cap Sp) = 4$, $n(W \cap A) = 4$, $n(Sp \cap A) = 3$, $n(Su \cap W \cap A) = 3$

$n(Su \cap W \cap Sp) = 3$, $n(Su \cap A \cap Sp) = 2$

$n(W \cap Sp \cap A) = 2$

$n(Su \cap Sp \cap W \cap A)$

$= n(Su) + n(Sp) + n(W) + n(A) - n(Su \cap Sp)$

$- n(Sp \cap W) - n(W \cap A) - n(Su \cap A) - n(Su \cap W)$

$- n(Sp \cap A) + n(Su \cap Sp \cap W) + n(Su \cap W \cap A)$

$+ n(W \cap A \cap Su) + n(Su \cap Sp \cap A) - n(Sp \cup Su \cup A \cup W)$

$- 90 + 65 + 10 + 7 - 55 - 8 - 6 - 4 - 4 - 3 + 3 + 3 + 2 + 2 - 100 = 2$

3. **Sol.** $(A \Delta B) \Delta C$ is disjoint union of $(A - B) - C$, $(B - C) - A$, $(C - A) - B$ and $A \cap B\ B \cap C$.

Therefore, number of elements is $(A\Delta B) \Delta C$ is 10 + 15 + 20 + 5 = 50

4. **Sol.** $N \cap Np = \{0\}$,

$I - N = \{ ....., -2, -1\}$

$N \Delta Np = \{N - Np) \cup (Np - N)$

$= \{1,2,3 ......\} \cup \{..... -3, -2, -1\}$

$= I \sim \{0\}$

and $E \cap P = \{2\}$

5.*_ **Sol.** $D_f = [-5, -1]$, $R_f = [0,2]$

domain of f(x) $\Rightarrow -5 - 6x - x^2 \geq 0$

$\Rightarrow (x + 5)(x + 1) \leq 0$

$-5 \leq x \leq -1$

range of $y = f(x) = \sqrt{-5 - 6x - x^2}$

$\Rightarrow x^2 + 6x + 5 + y^2 = 0$ and $y \geq 0$

$\Rightarrow y^2 \leq 4$ and $y \geq 0 \Rightarrow y \in [0,2]$

6.* **Sol.** Domain of $\frac{f(x)}{g(x)}$

$x^2 - 4 \geq 0$ and $x - 3 > 0 \Rightarrow (3,\infty)$

Domain of h(x)

$\frac{x^2-4}{x-3} \geq 0$ $\Rightarrow$ $[-2,2] \cup (3,\infty)$

7.*_ **Sol.** Since $m \neq 0$ and 0 is divisible by m, therefore,
a – a is divisible by m
$\Rightarrow a \equiv a \pmod{m}$ for all $a \in Z$
$\Rightarrow$ a R a for all $a \in Z$
Hence, R is reflexive.
Next, let aR b ; $a, b \in Z$
$\Rightarrow a \equiv b \pmod{m}$
$\Rightarrow$ a – b is divisible by m
$\Rightarrow$ a – b = mk for some integer k
$\Rightarrow b - a = -(mk) \Rightarrow b - a = m(-k)$
$\Rightarrow b \equiv a \pmod{m} \Rightarrow$ b R a
$\therefore$ R is symmetric.
Again let a R b and b R c ; $a, b, c \in Z$
$\Rightarrow a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$
$\Rightarrow$ a – b is divisible by m and also b – c is divisible by m.
$\Rightarrow a - b = mk_1$ and $b - c = mk_2$
where $k_1, k_2 \in Z$
$\Rightarrow (a - b) + (b - c) = mk_1 + mk_2$
$\Rightarrow a - c = m(k_1 + k_2)$
$\Rightarrow$ a – c is divisible by m
$\Rightarrow a \equiv c \pmod{m} \Rightarrow$ a R c
Hence, R is transitive.
Thus, we see that R is an equivalence relation.

8. **Sol.** Here . $A_1 \times A_1$
$= \{(x, y) : x, y \in A_1\}$
$= \{(1, 1), (1, 3), (1, 4), (3, 1), (3, 3), (3, 4), (4, 1), (4, 3), (4, 4)\}$
This relation is symmetric and transtive on A_1 and hence on A also but not reflexive

9. **Sol.** As $A_1 \cup A_2 \cup A_3 = A$ and $(A_1 \times A_1) \cap (A_2 \times A_2) \cap (A_3 \times A_3) = \varphi$,
therefore, $\bigcap_{i=1}^{3} (A_i \times A_i)$ defines an equivlence relation on A, where $x, y \in A$ are related iff they are in the same subset.

10. **Sol.** Total number of subsets of A is 2^7. Out of these only one set, namely A, is improper.
$\therefore$ Number of proper subsets is $2^7 - 1 = 128 - 1 = 127$.

11. **Sol.** Here , a relation R on A is a subset of A × A. Since A contains n elements, therefore A × A contains $n \times n = n^2$ elements.
Moreover, $A \times A = \{ x, y) : x, y \in A\}$
$= \{(x, x) : x \in A\} \cup \{(x, y) : x, y \in A, x \neq y\} = P \cup Q$,
where $P = \{(x, x) : x \in A\}$ and $Q = (x, y): x, y \in A, x \neq y\}$
Here, P contains n elements and Q contains $n \times n - n$ elements. We try to compute the number of possible subsets R of $A \times A = P \cup Q$ such that $(x, y) \in R \Leftrightarrow (y,x) \in R$. This number is infact, the required number of symmetric relations on A. An element of P can be dealt in two ways; it may or may not be included in R. So, all the elements of P can be dealt in 2^n ways. Also, Q contains $n^2 - n$ elements, i.e., $\frac{n(n-1)}{2}$ pairs of elements (x,y) and (y.x), $x \neq y$. Either, both (x, y) and (y, x) will be put in R or neither will be put in R. So, elements of Q may be dealt in $2^{n(n-1)/2}$ ways. Hence, the required number of symentric relations on A $= 2^n \times 2^{n(n-1)/2} = 2^{n(n+1)/2}$.

12. Sol. Let $A = \{a_1, a_2, a_3, ..., a_{99}, ...\}$
$B = \{a_1, a_2, a_3, ..., a_{99}, ...\}$
common ordered pairs of A × B and B × A
are $= \{a_1, a_2, ..., a_{99}\} \times \{a_1, a_2, ..., a_{99}\}$
⇒ $(99)^2$ elements are common

13. Sol. S = {1, 2, 3, 4}. Let A, B be two subsets such that $A \cap B = \varphi$ and (A, B), (B, A) are considered same since we require unordered pair of disjoint subsets

$$= \frac{({}^4C_0\, 2^4 + {}^4C_1\, 2^3 + {}^4C_2\, 2^2 + {}^4C_3\, 2^1 + {}^4C_4\, 2^0) + 1}{2} = 41$$

14. Sol. The number of required ordered pairs (x, y) where x > y $= {}^{100}C_2 = 4950$

15.*_ Sol. **for reflexive**
$(A, A) \in R \Rightarrow A = PAQ$
which is true for P = I = Q
∴ reflexive
for symmetry
As $(A, B) \in R$ for matrix P and Q (A, B) ∈ R P Q ,
$A = PBQ \Rightarrow B = P^{-1}AQ^{-1}$
∴ B, A)∈ R for matrix P_{-1}, Q^{-1} , ∴ R is symmetric
for transitivity ,
As (A, B) ∈ R for matrix P,Q and (B, C) ∈ R for matrix R and S
(A, B) ∈ R P,Q (B, C) ∈ R , R S
$A = PBQ$ and $B = RCS \Rightarrow A = P(RCS)Q$
⇒ $A = (PR)\, C\, (SQ)$ ∴ (A, C) ∈ R for matrix PR , SQ
∴ R is transitive and hence R is equivalence.