Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

1. **Sol.** Let $s_n = 2 + 3 + 6 + 11 + 18 + ...+ t_n$ ....(1)

$s_n = 2 + 3 + 6 + 11 + ... + t_n$ ....(2)

by (1) - (2) ls

$0 = 2 + [1+3+5+7+....$ (n–1) terms in$] – t_n$

$t_n = 2 + \frac{n-1}{2}(2+(n–1)2)$

$\Rightarrow t_n = 2 + (n–1)^2$

$\Rightarrow t_n = n^2 – 2n+3$

$t_{50} = 2403$

2. **Sol.** Common terms are 31, 41, 51, 61, ...

the largest termin the sequence 1,11,21,31,...... is 991

the largest term in the sequence 31,36,41,46,..... is 526

Hence the largest term common in both is 521

3. **Sol.** $\frac{S_{3r}-S_{r-1}}{S_{2r}-S_{2r-1}} = \frac{\frac{3r}{2}[2a+(3r-1)d]-\frac{r-1}{2}[2a+(r-2)d]}{t_{2r}}$

$= \frac{a[2r+1]+\frac{d}{2}[3r(3r-1)-(r-1)(r-2)]}{a+(2r-1)d}$

$= \frac{a(2r+1)+\frac{d}{2}(8r^2-2)}{a+(2r-1)d} = \frac{a(2r+1)+d(2r+1)(2r-1)}{a+(2r-1)d}$

$= (2r+1)\frac{(a+(2r-1)d)}{(a+(2r-1)d)} = 2r+1$

4. **Sol.** $\frac{S_{2n}}{S_n} = \frac{3}{1} \Rightarrow \frac{\frac{2n}{2}[2a+(2n-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{3}{1}$

$\Rightarrow 4a + 2(2n–1)d = 6a + 3(n–1)d$

$\Rightarrow (n+1)d = 2a$ ...(1)

Now $\frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}[2a+(3n-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{3[(n+1)d+(3n-1)d]}{[(n+1)d+(n-1)d]} = \frac{3[4nd]}{2nd} = 6$

5. **Sol.** $\therefore$ $(2n + r) r = (n + r)^2 – n^2$

= [1 + 3 + 5 + ...(n+r) terms in]

–[1 + 3 + 5 +....n terms in]

= sum of r consecutive odd natural number $\Rightarrow k = r$

6. Sol. $\therefore$ $\sum ab = \frac{1}{2}\left[(\sum a)^2 - \sum a^2\right]$

$= \frac{1}{2}\ [(1 - 1 + 2 - 2 + ... + 5 - 5)^2 - 2\ (1^2 + 2^2 + 3^2 + 4^2 + 5^2)]$

$= \frac{1}{2}\ [0 - 110] = - 55$

7. **Sol.** $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + ... = \left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + ...\infty\right) - \left(\frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + ...\infty\right)$

$= \frac{\pi^4}{90} - \frac{1}{16} . \frac{\pi^4}{90} = \frac{15\pi^4}{16.90} = \frac{\pi^4}{96}$

8. **Sol.** $a - 2b + c = 0$

$\Rightarrow$ $a^3 - 8b^3 + c^3 = + 3\ (a)\ (- 2b)\ (c)$

$= - 6\ abc$

9. **Sol.** $\frac{3 + 5 + 7 + ...n\ in\ rd}{5 + 8 + 11 + ...10\ in\ rd} = 7$

$\Rightarrow$ $\frac{n(n+2)}{185} = 7$

$\Rightarrow$ $n^2 + 2n - 1295 = 0$

$\Rightarrow$ $(n + 37)\ (n - 35) = 0$

$\Rightarrow$ $n = 35$

10. **Sol.** $(f(2x))^2 = f(x)\ .\ f(4x)$

$\Rightarrow$ $(4x+1)^2 = (2x+1)\ (8x+1)$ $\Rightarrow$ $16x^2 + 1 + 8x = 16x^2 + 10x + 1$

$\Rightarrow$ $2x = 0$ $\Rightarrow$ $x = 0$

11. **Sol.** $2 \log\left(\frac{3b}{5c}\right) = \log\left(\frac{5c}{a}\right) + \log\left(\frac{a}{3b}\right)$

$\Rightarrow$ $\left(\frac{3b}{5c}\right)^2 = \frac{5c}{a} . \frac{a}{3b}$

$\Rightarrow$ $3b = 5c$

$\Rightarrow$ $\frac{b}{5} = \frac{c}{3}$ ....(1)

also $b^2 = ac$ $\Rightarrow$ $\frac{a}{25} = \frac{c}{9}$ ....(2)

by (1) & (2) $\frac{a}{25} = \frac{b}{15} = \frac{c}{9}$

Now $b + c < a$

Hence A is not formed.

12. **Sol.** x, 2y, 3z in AP A

$\Rightarrow$ $4y = x + 3z$ ....(1)

also x, y, z in GP $\Rightarrow$ $y^2 = xz$ ....(2)

by (1) $16y^2 = x^2 + 9z^2 + 6\ xz$

$16xz = x^2 + 9z^2 + 6\ xz$

$10\ xz = x^2 + 9z^2$

$\Rightarrow$ $(x - z)\ (x - 9z) = 0$

$x \neq z$ $\frac{z}{x} = \frac{1}{9}$ $\Rightarrow$ $\Rightarrow$ common ratio $= \frac{1}{3}$

13. **Sol.** Let 1025_{th} term is = 2 ¼ 1025 ok; in = 2)

⇒ $1 + 2 + 4 + 8 + ...+ 2_{n-1} < 1025 < 1 + 2 + 4 + 8+ ...+2_n$

⇒ $2_n - 1 < 1025 < 2_{n+1} - 1$

⇒ n = 10

14. **Sol.** $(1 + x)(1 + x_2)(1 + x_4) \dots (1 + x_{128}) = \left(\frac{1-x^{256}}{1-x}\right)$ ...(1)

also $\sum_{r=0}^{n} x^r = 1 + x + x_2 + ... + x_n = \frac{1-x^{n+1}}{1-x}$

Hence n = 255

15. **Sol.** $\frac{t_{10}}{t_1} = r_9$

⇒ $= -\frac{1536}{3} = -512 = -2_9$

⇒ = – 2

16. Sol. $2.357 = \frac{2357-2}{999}$

17. Sol. Let no. of terms = 2n

According to the question.

sum of all terms = 5 (sum of terms at odd places)

⇒ $\frac{a(r^{2n}-1)}{r-1} = 5.\frac{a(r^{2n}-1)}{r^2-1}$

⇒ r + 1 = 5 ⇒ r = 4

18. Sol. By AM ≥ GM

$\frac{4^x + 4^{1-x}}{2} \ge \sqrt{4^x.4^{1-x}}$

⇒ $4_x + 4_{1-x} \ge 4$

19. **Sol.** By AM ≥ GM

$\frac{4^x + 4^{1-x}}{2} \ge \sqrt{4^x.4^{1-x}}$

$x_{\log y-\log z}+y_{\log z-\log x}+z_{\log x-\log y} \ge 3(x_{\log y-\log z}.y_{\log z-\log x}+z_{\log x-\log y})_{1/3}$

⇒ $x_{\log y-\log z}+y_{\log z-\log x}+z_{\log x-\log y} \ge 3$

as $\log(x_{\log y-\log z}.y_{\log z-\log x}.z_{\log x-\log y}) = 0$

20. **Sol.** $\frac{a_1+a_4}{a_1+a_4} = \frac{a_2+a_3}{a_2a_3}$ ⇒ $\frac{1}{a_1}+\frac{1}{a_4} = \frac{1}{a_2}+\frac{1}{a_3}$ ...(1)

also $\frac{a_1-a_4}{a_1a_4} = \frac{3(a_2-a_3)}{a_2a_3}$

⇒ $\frac{1}{a_4} - \frac{1}{a_1} = 3\left(\frac{1}{a_3} - \frac{1}{a_2}\right)$ ...(2)

by (1) & (2) $\frac{1}{a_1},\frac{1}{a_2},\frac{1}{a_3},\frac{1}{a_4}$

Hence $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \frac{1}{a_4}$ are in AP

⇒ a_1, a_2, a_3, a_4 are in HP

21. Sol. 4(GM) = 5(HM)

⇒ $4\sqrt{ab} = 5\left(\frac{2ab}{a+b}\right)$

⇒ $4(a+b)^2 = 25\ ab$

⇒ $4a^2 - 17\ ab + 4b^2 = 0$

⇒ $(4a - b)(a - 4b) = 0$

⇒ $a = 4b$ ($a = \frac{b}{4}$ neglecting)

22. Sol. Put x = (1–1/n) in given series (x = (1–1/n))

⇒ $S = 1 + 2x + 3x^2 + 4x^3 + ...$ (1)

$Sx = x + 2x^2 + 3x^3 + ...$ (2)

by (1) - (2) $S(1-x) = (1 + x + x^2 + x^3 + ...\infty)$

$S = \frac{1}{(1-x)^2} = n^2$

23. Sol. $\sum_{r=1}^{n} r^2 - \sum_{r=1}^{n}\sum_{r=1}^{m} r = \frac{n(n+1)(2n+1)}{6} - \sum_{m=1}^{n} \frac{m(m+1)}{2}$

$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{12} - \frac{n(n+1)}{4}$

$= \frac{n(n+1)(2n+1)}{12} - \frac{n(n+1)}{4}$

$= \frac{1}{2}\left(\sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r\right)$

24. Sol. Let the edges are $\frac{a}{r}$, a, ar, where r > 1 from the question

$\frac{a}{r}$. a. ar = 216 ⇒ a = 6

and $2\left(\frac{a}{r}.a + a.ar + ar.\frac{a}{r}\right) = 252$

⇒ $72(1 + r^2 + r) = 252\ r$

⇒ $2r^2 - 5r + 2 = 0$

⇒ $r = 2$

25. Sol. AM ≥ GM

⇒ $\frac{(2+\sqrt{2})^{x/2} + (2-\sqrt{2})^{x/2}}{2} \geq \left((2+\sqrt{2})^{x/2}.(2-\sqrt{2})^{x/2}\right)^{1/2}$

⇒ $(2+\sqrt{2})^{x/2} + (2-\sqrt{2})^{x/2} \geq 2\,(2)^{x/4}$

Equality holds only if $(2+\sqrt{2})^{x/2} = (2-\sqrt{2})^{x/2}$

⇒ $x = 0$

26. **Sol.** Average rate x kg per rupee $= \dfrac{4}{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}$

$= \dfrac{4\times 12}{12+6+4+3} = \dfrac{48}{25}$

= 1.92 kg per rupee

27. **Sol.** Let roots are a –d , a, a+ d with common diff. = d ¼ a –d , a, a+ d

⇒ a + d + a + a – d = 12 ⇒ a =4

also a ($a^2 - d^2$) = 28

⇒ 16 – d^2 =7 ⇒ d^2 =9

⇒ d = ± 3

28. **Sol.** $S_{n+3} - S_{n+2} = T_{n+2}$

& $T_{n+2} - T_{n+1}$ = d (difference)

29. **Sol.** $(x^2 + y^2 + z^2)(y^2 + z^2 + w^2) \le (xy + yz + zw)^2$

⇒ $x^2(z^2 + w^2) + y^4 + y^2w^2 + y^2z^2 + z^4 \le 2xy^2z + 2yz^2w + 2xywz$

⇒ $(xz - y^2)^2 + (yw - z^2)^2 + (xw - yz)^2 \le 0$

⇒ $xz = y^2$, $yw = z^2$ and $xw = yz$

⇒ $\dfrac{x}{y} = \dfrac{y}{z} = \dfrac{z}{w}$ ⇒ x, y, z, w are in G.P. (x, y, z, w)

30. **Sol.** $T_n = \dfrac{n^2}{500+3n^3} = \dfrac{1}{\frac{500}{n^2}+3n}$

Now $\dfrac{500}{n^2} + \dfrac{3n}{2} + \dfrac{3n}{2} \ge 3\left(\dfrac{500.3.3}{4}\right)^{1/3}$

$\dfrac{500}{n^2} = \dfrac{3n}{2} \Rightarrow n^3 = \left(\dfrac{1000}{3}\right)$

⇒ $n^3 = 333\dfrac{1}{3}$

but n∈N ⇒ n =6 or 7

$T_6 = \dfrac{36}{500+648} = \dfrac{36}{1148} = \dfrac{9}{287}$

$T_7 = \dfrac{49}{1529}$ Hence $T_7 > T_6$

PART - II : PRACTICE QUESTIONS

1. **Sol.** First number = 1
Last number = 100
Sum of integer 1 to 100

$S = \frac{100}{2}[101]$
$= 5050$
numbers which are divisible by 3 are 3, 6, 9 99

$S_1 = \frac{33}{2}[3 + 99] = 33 \times 51 = 1683$
numbers which are divisible by 5 are 5, 10 .... , 100

$S_2 = \frac{20}{2}[105] = 1050$
numbers which are divisible by 3 and 5 both are 15, 30 90

$S_3 = \frac{6}{2}[15 + 90] = 3(105) \quad = 315$
Now sum of integers which are not divisible by 3 or 5
$= S - S_1 - S_2 + S_3$
$= 5050 - 1683 - 1050 + 315 = 2632$

2. **Sol.** $a_1 = 15$

$\frac{a_k + a_{k-2}}{2} = a_{k-1}$ for k = 3, 4, ..., 11
$\Rightarrow$ $a_1, a_2, ..., a_{11}$ are in AP
$a_1 = a = 15$

$\frac{a_1^2 + a_2^2 + ... + a_{11}^2}{11} = 90 \quad \Rightarrow \quad \frac{(15)^2 + (15+d)^2 + ... + (15+10d)^2}{11} = 90$

$\Rightarrow \quad 7d^2 + 30d + 27 = 0 \quad \Rightarrow \quad d = -3$ or $-\frac{9}{7}$
Since $27 - 2a_2 > 0 \quad \Rightarrow \quad a_2 < \quad \Rightarrow \quad d = -3$

$\therefore \quad \frac{a_1 + a_2 + ... + a_{11}}{11} = \frac{11}{2}\frac{[30 + 10(-3)]}{11} = 0$

3*. **Sol.** $2b = a + c$ and $\frac{2}{b^2} = \frac{1}{a^2} + \frac{1}{c^2}$

$b^2 = \frac{2a^2c^2}{a^2 + c^2} \quad \Rightarrow \quad (a^2 + c^2)(a + c)^2 = 8a^2c^2$
$(a^2 + c^2)(a^2 + c^2 + 2ac) = 8a^2c^2$.
$a^4 + a^2c^2 + 2a^3c + a^2c^2 + c^4 + 2ac^3 = 8a^2c^2$
$a^4 + c^4 + 2ac^3 = 6a^2c^2 - 2a^3c$
$(a^4 + c^4 - 2a^2c^2) = 2ac(2ac - a^2 - c^2)$
$(a^2 - c^2)^2 = -2ac(a - c)^2$
$(a - c)^2[(a + c)^2 + 2ac] = 0$
$(a - c)^2[2b^2 + ac] = 0$
either $a = c$ or $2b^2 = -ac$
$\because \quad 2b = a + c$

either $a = b = c$ or $a, b, -\frac{c}{2}$ are in G.P.

4. Sol. a, b, c, d are four different real numbers in A.P. and are in decreasing A.P. and $a - b = m$,$(b - c)^2 = m^2$, $(c - a)^3 = -8m^3$

$a - d = 3m$, $(b - d) = 4m^2$, $(c - d)^3 = m^3$

$\therefore$ Given expression is

$2m + xm^2 - 8m^3 = 6m + 4m^2 + m^3$

$xm^2 = 9m^3 + 4m + 4m^2$

$x = \dfrac{9m^2 + 4m + 4}{m}$

$\therefore$ $9m^2 + (4 - x)\,m + 4 = 0$

For real m, $(x - 4)^2 - 144 \geq 0$

$\Rightarrow$ $(x + 8)(x - 16) \geq 0$

$\Rightarrow$ $x \leq -8$, $x \geq 16$

5. Sol. Let a and b are two numbers

$\dfrac{2ab}{a+b} = \dfrac{16}{5}$ ..... (1)

$\dfrac{a+b}{2} = A$ and $\sqrt{ab} = G$

$\because$ $2A + G^2 = 26$

$\Rightarrow$ $(a + b) + ab = 26$ ..... (2)

$\Rightarrow$ $\dfrac{10ab}{16} + ab = 26$

$\Rightarrow$ $26\,ab = 26 \times 16 \Rightarrow$ $ab = 16$ $\therefore$ from (1), we get $a + b = 10$

So a, b are (2, 8)

6. Sol. $a = 5^{1+x} + 5^{1-x} + 25^x + 25^{-x}$

$a = 5(5^x + 5^{-x}) + (25^x + 25^{-x})$

$a \geq 12$ $\{\therefore t^x + t^{-x} \geq 2\}$

7. Sol. If $a, b, c, x \in R$ and $(a^2 + b^2)\,x^2 - 2b\,(a + c)\,x + (b^2 + c^2) = 0$

This is quadratic in x, for equal root D = 0

$\Rightarrow$ $4b^2 (a + c)^2 = 4(a^2 + b^2)(b^2 + c^2)$

$\Rightarrow$ $b^2a^2 + b^2 c^2 + 2acb^2 = a^2b^2 + a^2c^2 + b^4 + b^2c^2$

$\Rightarrow$ $b^4 - 2acb^2 + a^2c^2 = 0$

$\Rightarrow$ $(b^2 - ac)^2 = 0$

$\Rightarrow$ $b^2 = ac$

$\therefore$ a, b, c are in G.P.

8. Sol. $a_n = \dfrac{3}{4} - \left(\dfrac{3}{4}\right)^2 + \left(\dfrac{3}{4}\right)^3 - .... + (-1)^{n-1}\left(\dfrac{3}{4}\right)^n$, $b_n = 1 - a_n$

$= \left[\dfrac{1-\left(-\dfrac{3}{4}\right)^n}{\left(1+\dfrac{3}{4}\right)}\right] = \dfrac{3}{7}\left[1-\left(-\dfrac{3}{4}\right)^n\right]$

$b_n > a_n$ $\Rightarrow$ $1 - a_n > a_n$

$2a_n < 1$ $\Rightarrow$ $\dfrac{6}{7} - \dfrac{6}{7}\left(-\dfrac{3}{4}\right)^n < 1$

$-\frac{6}{7}\left(-\frac{3}{4}\right)^n < \frac{1}{7}$ $\Rightarrow$ $-3(-3)^n < 2^{2n-1}$

$(-1)^{n+1}3^{n+1} < 2^{2n-1}$

For n even inequality is always valid

For n, odd

$n + 1 \rightarrow$ even

$3^{n+1} < 2^{2n-1}$

$n = 1$, $3^2 < 2^1$, (in valid)

$n = 3$, $3^4 < 2^5$, $81 < 32$ (in valid)

$n = 5$, $3^6 < 2^9$ (in valid)

$n = 7$, $3^8 < 2^{13}$, $6561 < 8192$ (valid)

min. natural number $n_0 = 5$

9. **Sol.** In the given series first term is 1 and common ratio is r.

$\therefore$ $S = \frac{1}{1-r} = 2 \Rightarrow r = \frac{1}{2}$

S_n is sum of n terms

$\therefore$ $S_n = \frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}} = 2 - \frac{1}{2^{n-1}}$

$S - S_n = \frac{1}{2^{n-1}}$

$S - S_n < \frac{1}{1000}$ (Given)

$\therefore$ $\frac{1}{2^{n-1}} < \frac{1}{1000} \Rightarrow n = 11$

10. **Sol.** **(1,4)**

$S_n = \sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}k^2$

$= -1^2 - 2^2 + 3^2 + 4^2 - 5^2 - 6^2 + 7^2 + 8^2 + \ldots\ldots\ldots$

$= (3^2 - 1) + (4^2 - 2^2) + (7^2 - 5^2) + (8^2 - 6^2) \ldots\ldots$

$= 2[\underbrace{4 + 6 + 12 + 14 + 20 + 22 + \ldots\ldots}_{2n \text{ terms}}]$

$= 2[(4 + 12 + 20 \ldots\ldots) + (6 + 14 + 22 \ldots\ldots\ldots)]$

n terms n terms

$= 2\left[\frac{n}{2}(4\times 2 + (n-1)8) + \frac{n}{2}(2\times 6 + (n-1)8)\right]$

$= 2[n(4 + 4n - 4) + n(6 + 4n - 4)]$

$= 2(4n^2 + (4n + 2)n)$

$= 2(8n^2 + 2n)$

$= 4n(4n + 1)$

(1)	1056	= 32 × 33	n = 8
(2)	1088	= 32 × 34	
(3)	1120	= 32 × 35	
(4)	1332	= 36 × 37	n = 9

11. **Sol.** $\frac{S_7}{S_{11}} = \frac{6}{11}$

$$\frac{\frac{7}{2}[2a+6d]}{\frac{11}{2}[2a+10d]} = \frac{6}{11}$$

Given 130 < a + 6d < 140

$$\frac{7(a+3d)}{11(a+5d)} = \frac{6}{11}$$

7a + 21d = 6a + 30d ⇒ 130 < 15d < 140

a = 9d

Hence d = 9

a = 81

Hence d = 9

12. **Sol.** $\left(x+\frac{1}{x}\right)^2 + \left(x^2+\frac{1}{x^2}\right)^2 + \left(x^3+\frac{1}{x^3}\right)^2 +$...... upto 10 terms

$= \left(x^2+\frac{1}{x^2}+2\right) + \left(x^4+\frac{1}{x^4}+2\right) + \left(x^6+\frac{1}{x^6}+2\right) +$ ......upto 10 terms

$= (x^2 + x^4 + x^6 +$.......upto 10 terms) $+ \left(\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{x^6}\text{.........upto 10 terms}\right) + 2 \times 10$

$$= \frac{x^2(x^{20}-1)}{x^2-1} + \frac{1}{x^2}\cdot\frac{\frac{1}{x^{20}}-1}{\frac{1}{x^2}-1} + 20$$

$$= \frac{x^2(x^{20}-1)}{x^2-1} + \frac{1}{x^{20}}\cdot\frac{x^{20}-1}{x^2-1} + 20 = \frac{x^{20}-1}{x^2-1}\left(x^2+\frac{1}{x^{20}}\right) + 20$$

$$= \left(\frac{x^{20}-1}{x^2-1}\right)\left(\frac{x^{22}+1}{x^{20}}\right) + 20$$

13. **Sol.** $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 +$ ......upto n terms

$= \frac{n(n+1)^2}{2}$, when n is even

$1^2 + 2 . 2^2 + 3^2 + ... 2 . n^2 = n\frac{(n+1)^2}{2}$

when n is odd n + 1 is even

$1^2 + 2 . 2^2 + 3^2 + ... n^2 + 2 . (n + 1)^2$

$= (n + 1)\frac{(n+2)^2}{2}$

$1^2 + 2 . 2^2 + 3^2 + ... n^2 = (n + 1)\left[\frac{(n+2)^2}{2} - 2(n+1)\right] = \frac{(n+1)\,n^2}{2}$

14. **Sol.** $a, a_1, a_2, a_3 ..... a_{2n-1}$ b are in A.P.

$a, b_1, b_2, b_3 ...... b_{2n-1}$, b are in G.P.

$a, c_1, c_2, c_3 ...... c_{2n-1}$, b are in H.P.

There are 2n + 1 terms in A.P., G.P. and H.P. If common difference is d for A.P. then $d = \frac{b-a}{2n}$

∴ a_n is $(n + 1)^{th}$ term;

∴ $a_n = a + \frac{n(b-a)}{2n} = \frac{a+b}{2}$

If r is common ratio for G.P. $b = a(r)^{2n}$

$\therefore \quad r = \left(\frac{b}{a}\right)^{\frac{1}{2n}}$

$\therefore b_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{1}{2n}.n}$

$\therefore \quad b_n = \sqrt{ab}$

similarly $c_n = \frac{2ab}{a+b}$

in equation $a_n x^2 - b_n x + c_n = 0$

$D = b_n^2 - 4a_n c_n = ab - 4\left(\frac{a+b}{2}\right).\frac{2ab}{a+b} = ab - 4ab = -3ab$

15. **Sol.** $g(n) - f(n) = \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} = \frac{n(n+1)}{2}\left(\frac{2n+1}{3} - 1\right) = \frac{n(n+1)}{2}\frac{2n-2}{3}$

$= \frac{n(n+1)(n-1)}{3} = \frac{(n-1)\,n\,(n+1)}{3}$

for n = 2 $\quad \frac{(n-1)\,n\,(n+1)}{3} = \frac{1.2.3}{3}$ which is divisible by 2 but not by 2^2

$\therefore$ greatest even integer which divides $\frac{(n-1)\,n\,(n+1)}{3}$, for every $n \in N$, $n \geq 2$, is 2

16. **Sol.** Let 1^{st} term be a . and common difference is 2

$T_{2n+1} = a + 4n = A$ (say) $\quad r = \frac{1}{2}$

Middle term of AP = T_{n+1}

Middle term of GP = T_{3n+1}

$T_{n+1} = a + 2n \quad \Rightarrow \quad T_{3n+1} = A . r^n = \frac{(a+4n)}{2^n}$

$(a + 2n) = \frac{a+4n}{2^n} \quad \Rightarrow \quad 2^n a + 2n2^n = a + 4n$

$a = \frac{4n - 2n.2^n}{2^n - 1}$