HEAT TRANSFER

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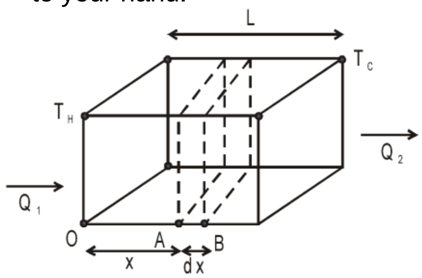
1. INTRODUCTION

Heat is energy in transit which flows due to temperature difference; from a body at higher temperature to a body at lower temperature. This transfer of heat from one body to the other takes place through three processes.

(i) Conduction (ii) Convection (iii) Radiation

2. CONDUCTION

The process of transmission of heat energy in which heat is transferred from one particle of the medium to the other, but each particle of the medium stays at its own position is called conduction, for example if you hold an iron rod with one of its end on a fire for some time, the handle will get heated. The heat is transferred from the fire to the handle by conduction along the length of iron rod. The vibrational amplitude of atoms and electrons of the iron rod at the hot end takes on relatively higher values due to the higher temperature of their environment. These increased vibrational amplitudes are transferred along the rod, from atom to atom during collision between adjacent atoms. In this way a region of rising temperature extends itself along the rod to your hand.



Consider a slab of face area A, Lateral thickness L, whose faces have temperatures T_H and T_C($T_H > T_C$).

Now consider two cross sections in the slab at positions A and B separated by a lateral distance of dx. Let temperature of face A be T and that of face B be T + ΔT. Then experiments show that Q, the amount of heat crossing the area A of the slab at position x in time t is given by

$$\boxed{\frac{Q}{t} = -KA\frac{dT}{dx}} \quad \text{... (2.1)}$$

Here K is a constant depending on the material of the slab and is named thermal conductivity of the material, and the quantity $\left(\frac{dT}{dx}\right)$ is called temperature gradiant. The (–) sign in equation (2.1) shows heat flows from high to low temperature (ΔT is a –ve quantity)

3. STEADY STATE

If the temperature of a cross-section at any position x in the above slab remains constant with time (remember, it does vary with position x), the slab is said to be in steady state.

Remember steady-state is distinct from thermal equilibrium for which temperature at any position (x) in the slab must be same.

For a conductor in steady state there is no absorption or emission of heat at any cross-section. (as temperature at each point remains constant with time). The left and right face are maintained at constant temperatures T_H and T_C respectively, and all other faces must be covered with adiabatic walls so that no heat escapes through them and same amount of heat flows through each cross-section in a given Interval of time. Hence $Q_1 = Q = Q_2$. Consequently the temperature gradient is constant throughout the slab.

Hence, $\frac{dT}{dx} = \frac{\Delta T}{L} = \frac{T_f - T_i}{L} = \frac{T_C - T_H}{L}$ and $\frac{Q}{t} = -KA\frac{\Delta T}{L} \Rightarrow \frac{Q}{t} = KA\,Q\left(\frac{T_H - T_C}{L}\right)$ .... (3.1)

Here Q is the amount of heat flowing through a cross-section of slab at any position in a time interval of t.
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Solved Examples

Example 1 One face of an aluminium cube of edge 2 metre is maintained at 100ºC and the other end is maintained at 0ºC. All other surfaces are covered by adiabatic walls. Find the amount of heat flowing through the cube in 5 seconds. (thermal conductivity of aluminium is 209 W/m–ºC)

Solution : Heat will flow from the end at 100ºC to the end at 0ºC.

Area of cross-section perpendicular to direction of heat flow,

$A = 4m^2$

then $\frac{Q}{t} = KA \frac{(T_H - T_C)}{L}$ $\quad Q = \frac{(209W/m^oC)(4m^2)(100^oC - 0^oC)(5\sec)}{2\ m} = 209$ KJ

4. THERMAL RESISTANCE TO CONDUCTION

If you are interested in insulating your house from cold weather or for that matter keeping the meal hot in your tiffin-box, you are more interested in poor heat conductors, rather than good conductors. For this reason, the concept of thermal resistance R has been introduced.

For a slab of cross-section A, Lateral thickness L and thermal conductivity K,

Resistance $R = \frac{L}{KA}$ ... (4.1)

In terms of R, the amount of heat flowing though a slab in steady-state (in time t)

$$\frac{Q}{t} = \frac{(T_H - T_L)}{R}$$

If we name as thermal current i_T

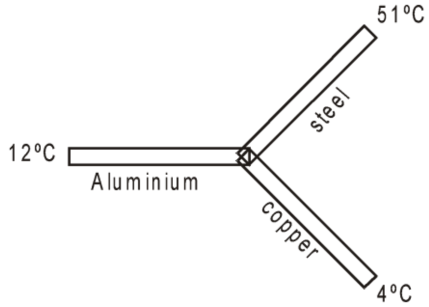
then, $i_T = \frac{T_H - T_L}{R}$ ....(4.2)

This is mathematically equivalent to OHM's law, with temperature donning the role of electric potential. Hence results derived from OHM's law are also valid for thermal conduction.

More over, for a slab in steady state we have seen earlier that the thermal current i_L remains same at each cross-section. This is analogous to kirchoff's current law in electricity, which can now be very conveniently applied to thermal conduction.

Solved Examples

Example 2. Three identical rods of length 1m each, having cross-section area of $1cm^2$ each and made of Aluminium, copper and steel respectively are maintained at temperatures of 12ºC, 4ºC and 50ºC respectively at their separate ends. Find the temperature of their common junction.



[K_{Cu} = 400 W/m-K , K_{Al} = 200 W/m-K , K_{steel} = 50 W/m-K]

Solution : $R_{Al} = \frac{L}{KA} = \frac{1}{10^{-4} \times 209} = \frac{10^4}{209}$

Similarly $R_{steel} = \frac{10^4}{46}$ and $R_{copper} = \frac{10^4}{385}$

Let temperature of common junction = T
then from Kirchoff;s Junction law.

$i_{Al} + i_{steel} + i_{Cu} = 0$

⇒ $\frac{T-12}{R_{Al}} + \frac{T-51}{R_{steel}} + \frac{T-u}{R_{Cu}} = 0$

⇒ (T – 12) 200 + (T – 50) 50 + (T – 4) 400 = 0

⇒ 4(T – 12) + (T – 50) + 8 (T – 4) = 0

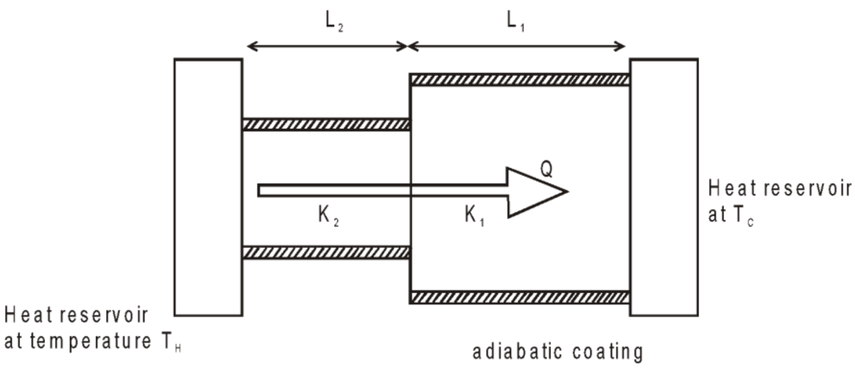
⇒ 13T = 48 + 50 + 32 = 130

⇒ T = 10ºC **Ans.**

5. SLABS IN PARALLEL AND SERIES

5.1 Slabs in series (in steady state)

Consider a composite slab consisting of two materials having different thickness L_1 and L_2, different cross-sectional areas A_1 and A_2 and different thermal conductivities K_1 and K_2. The temperature at the outer surface at the ends are maintained at T_H and T_C, and all lateral surfaces are covered by an adiabatic coating.



Let temperature at the junction be T, since steady state has been achieved thermal current through each slab will be equal. Then thermal current through the first slab.

$i = \frac{Q}{t} = \frac{T_H - T}{R_1}$ or $T_H - T = iR_1$ ... (5.1)

and that through the second slab,

$i = \frac{Q}{t} = \frac{T - T_C}{R_2}$

or $T - T_C = iR_2$ ....(5.2)

adding eqn. 5.1 and eqn 5.2

$T_H - T_L = (R_1 + R_2)\, i$

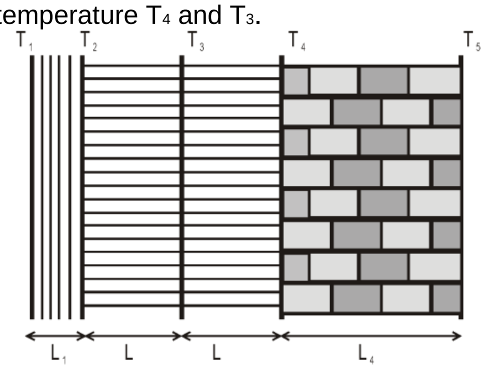
or $i = \frac{T_H - T_C}{R_1 + R_2}$

Thus these two slabs are equivalent to a single slab of thermal resistance $R_1 + R_2$. If more than two slabs are joined in series and are allowed to attain steady state, then equivalent thermal resistance is given by

$R = R_1 + R_2 + R_3 + ...$ .....(5.3)

Solved Examples

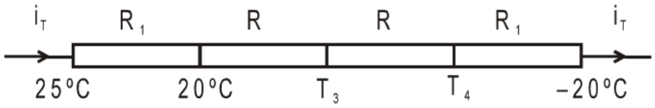
Example 3 The figure shows the cross-section of the outer wall of a house built in a hill-resort to keep the house insulated from the freezing temperature of outside. The wall consists of teak wood of thickness L_1 and brick of thickness ($L_2 = 5L_1$), sandwitching two layers of an unknown material with identical thermal conductivities and thickness. The thermal conductivity of teak wood is K_1 and that of brick is ($K_2 = 5K$). Heat conduction through the wall has reached a steady state with the temperature of three surfaces being known. ($T_1 = 25^oC$, $T_2 = 20^oC$ and $T_5 = -20^oC$). Find the interface temperature T_4 and T_3.



Solution : Let interface area be A. then thermal resistance of wood, $R_1 = \frac{L_1}{K_1 A}$

and that of brick wall $R_2 = \frac{L_2}{K_2 A} = \frac{5L_1}{5K_1 A} = R_1$

Let thermal resistance of the each sandwitched layer = R. Then the above wall can be visualised as a circuit



thermal current through each wall is same.

Hence $\frac{25-20}{R_1} = \frac{20-T_3}{R} = \frac{T_3-T_4}{R} = \frac{T_4+20}{R_1}$

$\Rightarrow$ $25 - 20 = T_4 + 20$ $\Rightarrow$ $T_4 = -15^oC$ **Ans.**

also, $20 - T_3 = T_3 - T_4$ $\Rightarrow$ $T_3 = \frac{20+T_4}{2} = 2.5^oC$ **Ans.**

Example 4 In example 3, $K_1 = 0.125$ W/m–ºC, $K_2 = 5K_1 = 0.625$ W/m–ºC and thermal conductivity of the unknown material is K = 0.25 W/mºC. L_1 = 4cm, $L_2 = 5L_1$ = 20cm and L = 10cm. If the house consists of a single room of total wall area of 100 m_2, then find the power of the electric heater being used in the room.

Solution : $R_1 = R_2 = \frac{(4\times10^{-2}m)}{(0.125w/m-^oC)(100m^2)} = 32 \times 10_{-4}$ ºC/w

$R = \frac{(10\times10^{-2}m)}{(0.25W/m-^oC(100m^2)} = 40 \times 10_{-4}$ ºC/w

the equivalent thermal resistance of the entire wall = $R_1 + R_2 + 2R = 144 \times 10_{-4}$ ºC/W

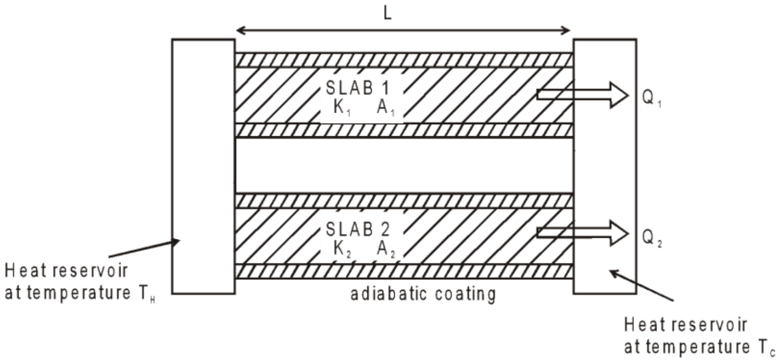
$\therefore$ Net heat current, i.e. amount of heat flowing out of the house per second =

$= \frac{T_H - T_C}{R} = \frac{25^oC-(-20^oC)}{144\times10^{-4}\,^oC/w} = \frac{45\times10^4}{144}$ watt = 3.12 Kwatt

Hence the heater must supply 3.12 kW to compensate for the outflow of heat. **Ans.**

5.2 Slabs in parallel :

Consider two slabs held between the same heat reservoirs, their thermal conductivities K_1 and K_2 and cross-sectional areas A_1 and A_2



then $R_1 = \frac{L}{K_1A_1}$, $R_2 = \frac{L}{K_2A_2}$

thermal current through slab 1

and that through slab 2

Net heat current from the hot to cold reservoir

$$i_1 = \frac{T_H - T_C}{R_1}$$

$$i_1 + i_2 = (T_H - T_C)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Comparing with i =

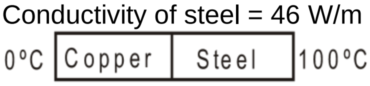
i = $\frac{T_H - T_C}{R_{eq}}$, we get, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

If more than two rods are joined in parallel, the equivalent thermal resistance is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \text{.....} \qquad \text{.... (5.4)}$$

Solved Examples

Example 5. Figure shows a copper rod joined to a steel rod. The rods have equal length and equal cross-sectional area. The free end of the copper rod is kept at 0ºC and that of steel rod is kept at 100ºC. Find the temperature of the junction of the rod. Conductivity of copper = 390 W/mºC Conductivity of steel = 46 W/m ºC



Solution : Heat current in first rod (copper) = $\frac{390 \times A(A - \theta)}{\ell}$

Here θ is temperature of the junction and A & ℓ are area and length of copper rod.

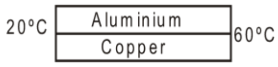
Heat current in second rod (steel) = $\frac{46 \times A(\theta - 100)}{\ell}$

In series combination. heat current remains same. So,

$\frac{390 \times A(0 - \theta)}{\ell} = \frac{46 \times A(\theta - 100)}{\ell}$ – 390 θ = = 46 θ – 4600

436 θ = 4600 θ = 10.6ºC

Example 6. An aluminium rod and a copper rod of equal length 1m and cross-sectional area 1cm2 are welded together as shown in the figure. One end is kept at a temperature of 20ºC and other at 60ºC. Calculate the amount of heat taken out per second from the hot end. Thermal conductivity of aluminum is 200 W/mºC and of copper is 390 W/mºC.

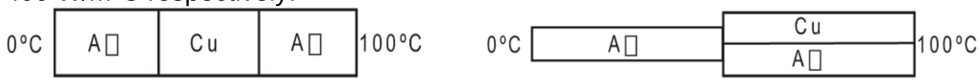


Solution : Heat current through the $\frac{200\times(1\times10^{-4})}{1}$ = .(60 – 20)

Heat current through the copper rod = $\frac{390\times(1\times10^{-4})}{1}$.(60 – 20)

Total heat = 200 × 10^{-4} × 40 + 390 × 10^{-4} × 40 = 590 × 40 × 10^{-4} = 2.36 Joule

Example 7. The three rods shown in the figure (1) have identical geometrical dimensions. Heat flows from the hot end at the rate of 40W in arrangement (1). Find the rate of heat flow when the rods are joined in arrangement (2). Thermal conductivity of aluminum and copper are 200 W/mºC and 400 W/mºC respectively.



(1) (2)

Solution : (a) In the arrangement (1), the three rods are joined in series. The rate of flow of heat,

$\frac{d\theta}{dt} = \frac{KA(\theta_1-\theta_2)}{\ell} = \frac{\theta_1-\theta_2}{R}$ But, R = $R_1 + R_2 + R_3$ [In series]

$\therefore$ $40 = \frac{100-0}{R_1+R_2+R_1}$ $\Rightarrow$ $40 = \frac{100}{\frac{\ell}{K_1A}+\frac{\ell}{K_2A}+\frac{\ell}{KA}}$ $\Rightarrow 40 = \frac{100}{\frac{\ell}{A}\left[\frac{2}{K_1}+\frac{1}{K_2}\right]}$

$\Rightarrow$ $\frac{\ell}{A}\left[\frac{2}{200}+\frac{1}{400}\right] = \frac{100}{40}$ $\Rightarrow$ $\frac{\ell}{A}$ = 200 per m

(b) In figure (2) two rods all in parallel and resultant of both is in series with the first rod

$\therefore$ $\frac{dQ}{dt} = \frac{\theta_1-\theta_2}{R}$ But R = $R_1 + \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$

$$\frac{dQ}{dt} = \frac{100-0}{\frac{\ell}{K_1A}+\frac{1}{\frac{K_1A}{\ell}+\frac{K_2A}{\ell}}} = \frac{100-0}{\frac{\ell}{A}\left[\frac{1}{K_1}+\frac{1}{K_1+K_2}\right]} = \frac{600\times100}{200\times4} = 75\text{W}$$

Example 8. A metal rod of length 20cm and diameter 2 cm is covered with a non conducting substance. One of its ends is maintained at 100ºC while the other end is put at 0ºC. It is found that 25 g of ice melts in 5 min. Calculate the coefficient of thermal conductivity of the metal. Latent heat of ice = 80 cal gram^{-1}

Solution : Here, length of the rod, Δx = 20 cm = 20 × 10^{-2} m

Diameter = 2cm, Radius = r = 1 cm = 10^{-2} m

Area of cross section a = $\pi r^2 = \pi(10^{-2})^2 = \pi \times 10^{-4}$ sq. m

ΔT = 100 – 0 = 100ºC Mass of ice melted, m = 25g

As L = 80 cal/ g

$\therefore$ Heat conducted, ΔQ = mL = 25 × 80 = 2000 cal = 2000 × 4.2 J

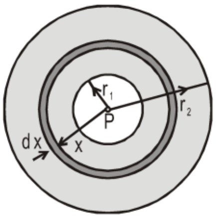
Δt = 5 min = 300 s

From $\frac{\Delta Q}{\Delta t} = KA\frac{\Delta T}{\Delta x}$ $\Rightarrow$ $K = \frac{2000\times4.2\times20\times10^{-2}}{300\times10^{-4}\pi\times100}$ = 1.78$Js^{-1}m^{-1}$ºC^{-1}

Example 9. Two thin concentric shells made from copper with radius r_1 and r_2 ($r_2 > r_1$) have a material of thermal conductivity K filled between them. The inner and outer spheres are maintained at temperatures T_H and T_C respectively by keeping a heater of power P at the centre of the two spheres. Find the value of P.

Solution : Heat flowing per second through each cross-section of the sphere = P = i.

Thermal resistance of the spherical shell of radius x and thickness dx,



dR = $\frac{dx}{K.4\pi x^2}$ $\Rightarrow$ R = $\int_{r_1}^{r_2}\frac{dx}{4\pi x^2.K}$ = $\frac{1}{4\pi K}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$

thermal current i = P = $\frac{T_H - T_C}{R}$ = $\frac{4\pi K(T_H - T_C)\ r_1\ r_2}{(r_2 - r_1)}$.

GROWTH OF ICE ON PONDS

When atmospheric temperature falls below 0°C the water in the lake will start freezing. Let at any time t, the thickness of ice in the lake be y and atmospheric temperature is –θ°C. The temperature of water in contact with the lower surface of ice will be 0ºC.

the area of the lake = A

heat escaping through ice in time dt is

Now due to escaping of this heat if dy thickness of water in contact with lower surface of ice freezes,

$$dQ_1 = KA\frac{[0-(-\theta)]}{y}dt$$

$dQ_2 = mL = \rho(dy\ A)L$ [as $m = \rho V = \rho A\ dy$]

But as $dQ_1 = dQ_2$, the rate of growth of ice will be $\frac{dy}{dt} = \frac{K\theta}{\rho L}\times\frac{1}{y}$

and so time taken by ice to grow a thickness y, $t = \frac{\rho L}{K\theta}\int_0^y y\ \ dy = \frac{1}{2}\frac{\rho L}{K\theta}\ \ y^2$

It is clear that time taken to double and triple the thickness will be in the ratio $t_1 : t_2 : t_3 : : 1^2 : 2^2 : 3^2$, i.e., $t_1 : t_2 : t_3 : : 1 : 4 : 9$ and the time intervals to change thickness from 0 to y, from y to 2y and so on will be in the ratio $\Delta t_1 : \Delta t_2 : \Delta t_3 : : (1_2 – 0_2) : (2_2 – 1_2) : (3_2 – 2_2)$, i.e., $\Delta t_1 : \Delta t_2 : \Delta t_3 : : 1 : 3 : 5$.

Can you now see how the following facts can be explained by thermal conduction ?

(a) In winter, iron chairs appears to be colder than the wooden chairs.

(b) Ice is covered in gunny bags to prevent melting.

(c) Woolen clothes are warmer.

(d) We feel warmer in a fur coat.

(e) Two thin blankets are warmer than a single blanket of double the thickness.

(f) Birds often swell their feathers in winter.

(g) A new quilt is warmer than old one.

(h) Kettles are provided with wooden handles.

(i) Eskimo's make double walled ice houses.

(j) Thermos flask is made double walled.

6. CONVECTION

When heat is transferred from one point to the other through actual movement of heated particles, the process of heat transfer is called convection. In liquids and gases, some heat may be transported through conduction. But most of the transfer of heat in them occurs through the process of convection. Convection occurs through the aid of earth's gravity. Normally the portion of fluid at greater temperature is less dense, while that at lower temperature is denser. Hence hot fluids rises up white colder fluid sink down, accounting to convection. In the absence of gravity convection would not be possible.
Also, the anamalous behaviour of water (its density increases with temperature in the range 0-4ºC) give rise to interesting consequences vis-a-vis the process of convection. One of these interesting consequences is the presence of aquatic life in temperate and polar waters. The other is the rain cycle.

Can you now see how the following facts can be explained by thermal convection ?

(a) Oceans freeze top-down and not bottom up. (this fact is singularly responsible for presence of acquatic life is temperate and polar waters.)

(b) The temperature in the bottom of deep oceans is invariably 4ºC, whether it is winter or summer.

(c) You cannot illuminate the interior of a lift in free fall or an artificial satellite of earth with a candle.

(d) You can Illuminate your room with a candle.

7. RADIATION :

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation. The term radiation used here is another word for electromagnetic waves. These waves are formed due to the superposition of electric and magnetic fields perpendicular to each other and carry energy.

Properties of Radiation:

(a) All objects emit radiations simply because their temperature is above alsolute zero, and all objects absorb some of the radiation that falls on them from other objects.

(b) Maxwell on the basis of his electromagnetic theory proved that all radiations are electromagnetic waves and their sources are vibrations of charged particles in atoms and molecules.

(c) More radiations are emitted at higher temperature of a body and lesser at lower temperature.

(d) The wavelength corresponding to maximum emission of radiations shifts from longer wavelength to shorter wavelength as the temperature increases. Due to this the colour of a body appears to be changing. Radiations from a body at NTP has predominantly infrared waves.

(e) Thermal radiations travels with the speed of light and move in a straight line.

(f) Radiations are electromagnetic waves and can also travel through vacuum.

(g) Similar to light, thermal radiations can be reflected, refracted, diffracted and polarized.

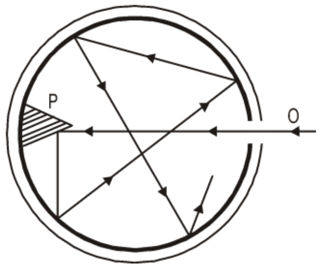
(h) Radiation from a point source obeys inverse square law (intensity α).

8. PREVOST THEORY OF HEAT EXCHANGE :

According to this theory, all bodies radiate thermal radiation at all temperatures. The amount of thermal radiation radiated per unit time depends on the nature of the emitting surface, its area and its temperature. The rate is faster at higher temperatures. Besides, a body also absorbs part of the thermal radiation emitted by the surrounding bodies when this radiation falls on it. If a body radiates more than what it absorbs, its temperature falls. If a body radiates less than what it absorbs, its temperature rises. And if the temperature of a body is equal to the temperature of its surroundings it radiates at the same rate as it absorbs.

9. PERFECTLY BLACK BODY AND BLACK BODY RADIATION (FERY'S BLACK BODY)

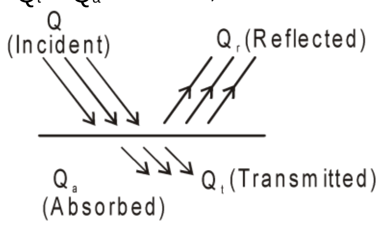
A perfectly black body is one which absorbs all the heat radiations of whatever wavelength, incident on it. It neither reflects nor transmits any of the incident radiation and therefore appears black whatever be the colour of the incident radiation.



In actual practice, no natural object possesses strictly the properties of a perfectly black body. But the lamp-black and platinum black are good approximation of black body. They absorb about 99 % of the incident radiation. The most simple and commonly used black body was designed by Fery. It consists of an enclosure with a small opening which is painted black from inside. The opeining acts as a perfect black body. Any radiation that falls on the opening goes inside and has very little chance of escaping the enclosure before getting absorbed through multiple reflections. The cone opposite to the opening ensures that no radiation is reflected back directly.

10. ABSORPTION, REFLECTION AND EMISSION OF RADIATIONS

$Q = Q_r + Q_t + Q_a \quad \Rightarrow \quad 1 = \frac{Q_r}{Q} + \frac{Q_t}{Q} + \frac{Q_a}{Q}$



$1 = r + t + a$

where r = reflecting power , a = absorptive power
and t = transmission power.

(i) $r = 0, t = 0, a = 1$, perfect black body
(ii) $r = 1, t = 0, a = 0$, perfect reflector
(iii) $r = 0, t = 1, a = 0$, perfect transmitter

10.1 Absorptive power :

In particular, absorptive power of a body can be defined as the fraction of incident radiation that is absorbed by the body.

$$a = \frac{Energy \quad absorbed}{Energy \quad incident}$$

As all the raditions incident on a black body are absorbed, a = 1 for a black body.

10.2 Emissive power:

Energy radiated per unit time per unit area along the normal to the area is known as emissive power.

$$E = \frac{Q}{\Delta A \quad \Delta t}$$

(Notice that unlike absorptive power, emissive power is not a dimensionless quantity).

10.3 Spectral Emissive power (E_λ) :

Emissive power per unit wavelength range at wavelength λ is known as spectral emissive power, E_λ. If E is the total emissive power and E_λ is spectral emissive power, they are related as follows,

$$E = \int_0^\infty E_\lambda \, d\lambda \quad \text{and} \quad \frac{dE}{d\lambda} = E_\lambda$$

10.4 Emissivity:

$$e = \frac{\text{Emissive power of a body at temperature T}}{\text{Emissive power of a black body at same temperature T}} = \frac{E}{E_0}$$

11. KIRCHOFF'S LAW:

The ratio of the emissive power to the absorptive power for the radiation of a given wavelength is same for all substances at the same temperature and is equal to the emissive power of a perfectly black body for the same wavelength and temperature.

$$\frac{E \quad (body)}{a \quad (body)} = E \quad (black \quad body)$$

Hence we can conclude that good emitters are also good absorbers.

APPLICATIONS OF KIRCHHOFF'S LAW

If a body emits strongly the radiation of a particular wavelength, it must also absorb the same radiation strongly.

(1) Let a piece of china with some dark painting on it be first heated to nerly 1300 K and then examined in dark room. It will be observed that the dark paintings appear much brighter than the white portion. This is because the paintings being better absorbers also emit much more light.

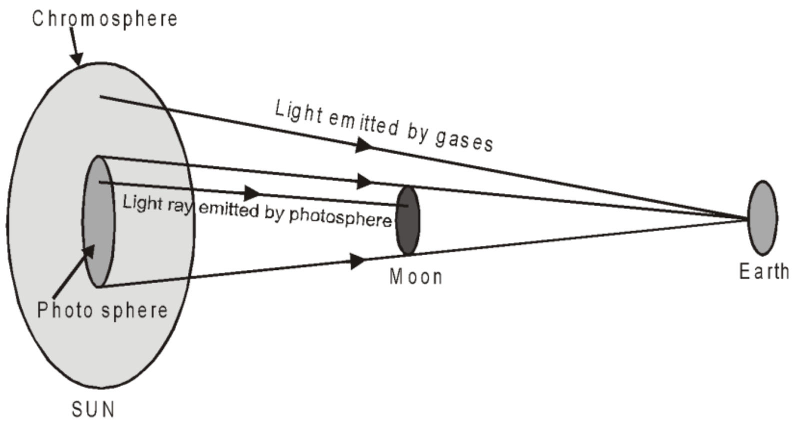
(2) The silvered surface of a thermos flask does not absorb much heat from outside. This stops ice from melting quickly. Also, the silvered surface does not radiate much heat from inside. This prevents hot liquids from becoming cold quickly.

(3) A red glass appears red at room temperature. This is because it absorbs green light strongly. However, if it is heated in a furnace, it glows with green light. This is because it emits green light strongly at a higher temperature.

(4) When white light is passed through sodium vapours and the spectrum of transmitted light is seen, we find two dark lines in the yellow region. These dark lines are due to absorption of radiation by sodium vapours which it emits when heated.

Fraunhofer lines are dark lines in the spectrum of the sun. When white light emitted from the central core of the sun (photosphere) passes through its atmosphere (chromosphere) radiations of those wavelengths will be absorbed by the gases present there which they usually emit (as a good emitter is a good absorber) resulting in dark lines in the spectrum of sun.

At the time of solar eclipse direct light rays emitted from photosphere cannot reach on the earth and only rays from chromosphere are able to reach on the earth surface. At that time we observe bright fraunhofer lines.



12. NATURE OF THERMAL RADIATIONS : (WIEN'S DISPLACEMENT LAW)

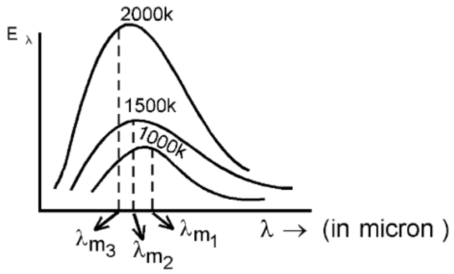
From the energy distribution curve of black body radiation, the following conclusions can be drawn :

(a) Higher the temperature of a body, higher is the area under the curve i.e. more amount of energy is emitted by the body at higher temperature.

(b) The energy emitted by the body at different temperatures is not uniform. For both long and short wavelengths, the energy emitted is very small.

(c) For a given temperature, there is a particular wavelength (λ_m) for which the energy emitted (E_λ) is maximum.

(d) With an increase in the temperature of the black body, the maxima of the curves shift towards shorter wavelengths.



From the study of energy distribution of black body radiation discussed as above, it was established experimentally that the wavelength (λ_m) corresponding to maximum intensity of emission decreases inversely with increase in the temperature of the black body. i.e.

$\lambda_m \propto$ or $\lambda_m T = b$

This is called Wien's displacement law.

Here b = 0.282 cm-K, is the Wien's constant.

Solved Examples

Example 10. Solar radiation is found to have an intensity maximum near the wavelength range of 470 nm. Assuming the surface of sun to be perfectly absorbing (a = 1), calculate the temperature of solar surface.

Solution : Since a =1, sun can be assumed to be emitting as a black body from Wien's law for a black body

$\lambda_m . T = b \Rightarrow T = \dfrac{0.282 \ (cm-K)}{(470\times10^{-7} cm)} = \simeq 6125$ K. **Ans.**

13. STEFAN-BOLTZMANN'S LAW :

According to this law, the amount of radiation emitted per unit time from an area A of a black body at absolute temperature T is directly proportional to the fourth power of the temperature.

$$u = \sigma A T^4 \qquad ..... (13.1)$$

where σ is Stefan's constant = 5.67×10^{-8} W/m^2 k^4

A body which is not a black body absorbs and hence emit less radiation then that given by equation (3.1).

For such a body, $$u = e \ \sigma \ A \ T^4 \qquad .....(13.2)$$

where e = emissivity (which is equal to absorptive power) which lies between 0 to 1

With the surroundings of temperature T_0 , net energy radiated by an area A per unit time..

$$\Delta u = u - u_0 = e \ \sigma \ A \ (T^4 - T_0^4) \qquad ....(13.3)$$

Solved Examples

Example 11. A black body at 2000K emits radiation with λ_m = 1250 nm. If for the radiation coming from the star SIRUS λ_m is 71 nm, then the temperature of this star is ....

Solution : Using wien's displacement law

$\dfrac{T_2}{T_1} = \dfrac{(\lambda_m)_1}{(\lambda_m)_2} \quad \therefore \quad T_2 = \dfrac{2000\times1250\times10^{-3}}{71\times10^{-9}} \quad \Rightarrow T_2 = 35.211$ K

Example 12. At 1600 K maximum radiation is emitted at a wavelength of 2μm. Then the corresponding wavelength at 2000 K will be -

Solution : Using $T_1 = T_2$ ∴ $= \frac{\lambda_{m_2} T_1}{T_2}$ ∴ $= \frac{2\times10^{-6}\times1600}{2000}$ ⇒ = 1.6 µm

Example 13. If the temperature of a body is increased by 50%, then the increase in the amount of radiation emitted by it will be

Solution : Percentage increase in the amount of radiations emitted

∴ $\frac{E_2 - E_1}{E_1}$ × 100 = $\frac{(1.5T_1)^4 - T_1^4}{T_1^4}$ × 100

⇒ $\frac{E_2 - E_1}{E_1}$ × 100 = $[(1.5)^4 - 1] \times 10$ $\frac{E_2 - E_1}{E_1}$ × 100 = 400%

Example 14. A blackened platinum wire of length 5cm and perimeter 0.02 cm is maintained at a temperature of 300K. Then at what rate the wire is losing its energy ? (Take $\sigma = 57 \times 10^{-8}$ units)

Solution : The rate of radiation heat loss is

$\frac{dQ}{dt} = eA\sigma T^4$ (watts)

for blackened surface e = 1

and A = (2πr)ℓ = $P_{erimeter}$ × length

∴ $A = 0.02 \times 5 \times 10^{-4}$ Thus

∴ $\frac{dQ}{dt} = 0.02 \times 5 \times 10^{-4} \times 5.7 \times 10^{-8} \times (300)^4$ ⇒ $\frac{dQ}{dt}$ = 46.2W

Example 15. A body of emissivity (e = 0.75), surface area of 300 cm^2 and temperature 227ºC is kept in a room at temperature 27ºC. Calculate the initial value of net power emitted by the body.

Using equation. (13.3) $P = e\sigma A (T^4 - T_0^4)$

$= (0.75)(5.67 \times 10^{-8}\ W/m^2 - k^4)(300 \times 10^{-4}\ m^2) \times \{(500\ K)^4 - (300\ K)^4\}$

= 69.4 Watt. **Ans.**

Example 16. A hot black body emits the energy at the rate of 16 J m^{-2} s^{-1} and its most intense radiation corresponds to 20,000 Å. When the temperature of this body is further increased and its most intense radiation corresponds to 10,000 Å, then find the value of energy radiated in Jm^{-2} s^{-1} .

Solution : Wein's displacement law is :

$\lambda_m . T = b$ i.e. $T \propto \frac{1}{\lambda_m}$

Here, λ_m becomes half.

∴ Temperature doubles.

Also $E = \sigma T^4$

⇒ $\frac{e_1}{e_2} = \left(\frac{T_1}{T_2}\right)^4$ ⇒ $E_2 = .E_1$ $\frac{e_1}{e_2} = \left(\frac{T_1}{T_2}\right)^4$ $e_1 = (2)^4 . 16$

= 16.16 = 256 J m^{-2} s^{-1} **Ans.**

14. NEWTON'S LAW OF COOLING :

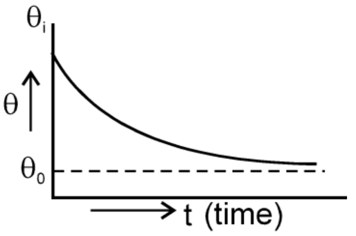
For small temperature difference between a body and its surrounding, the rate of cooling of the body is directly proportional to the temperature difference and the surface area exposed.

$\frac{dQ}{dt} \propto (\theta - \theta_0)$, where θ and θ_0 are temperature corresponding to object and surroundings.

From above expression $\frac{d\theta}{dt} = -k(\theta - \theta_0)$, ...(14.1)

This expression represents Newton's law of cooling. It can be derived directly from stefan's law, which gives,

$$k = \frac{4 e \sigma \theta_0^3}{mc} A \qquad \text{... (14.2)}$$



Now $\frac{d\theta}{dt} = -k \ [\theta - \theta_0]$ $\Rightarrow \int_{\theta_i}^{\theta_f} \frac{d\theta}{(\theta - \theta_0)} = \int_0^t -k \ dt$

$\ell n \frac{(\theta_f - \theta_0)}{(\theta_i - \theta_0)}$

where θ_i = initial temperature of object and

θ_f = final temperature of object.

$\Rightarrow$ = –kt $\Rightarrow (\theta_f - \theta_0) = (\theta_i - \theta_0)\, e^{-kt}$

$\Rightarrow$ $\theta_f = \theta_0 + (\theta_i - \theta_0)\, e^{-kt}$ ... (14.3)

14.1 Limitations of Newton's Law of Cooling:

(a) The difference in temperature between the body and surroundings must be small

(b) The loss of heat from the body should be by radiation only.

(c) The temperature of surroundings must remain constant during the cooling of the body.

14.2 Approximate method for applying Newton's law of cooling

Sometime when we need only approximate values from Newton's law, we can assume a constant rate of cooling, which is equal to the rate of cooling corresponding to the average temperature of the body during the interval.

$$\left\langle \frac{d\theta}{dt} \right\rangle = -k(\langle\theta\rangle - \theta_0) \qquad \text{...(14.4)}$$

If θ_i & θ_f be initial and final temperature of the body then,

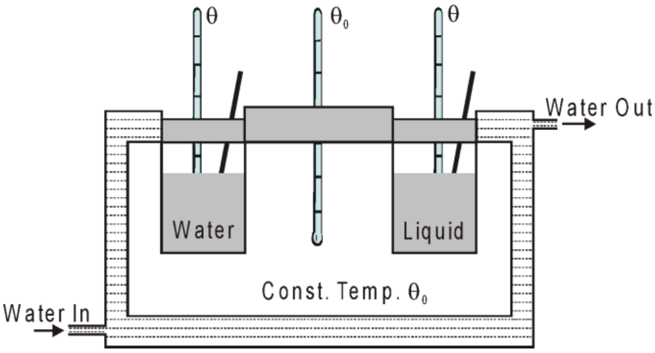
$$\langle\theta\rangle = \frac{\theta_i + \theta_f}{2} \qquad \text{...(14.5)}$$

Remember equation (14.5) is only an approximation and equation (14.1) must be used for exact values.

Comparision of specific heat of two liquids using Newton's law of cooling :

If equal volume of two liquids of densities and specific heats ρ_1 , s_1 and ρ_2 , s_2 respectively are filled in calorimeters having same surface area and finish, cool from same initial temperature θ_1 to same final temperature θ_2 with same temperature of surroundings, i.e., θ_0, in time intervals t_1 and t_2 respectively and water equivalent of calorimeter is w. According to Newton's law of cooling

$$\left(\frac{dQ}{dt}\right)_{liq.} = \left(\frac{dQ}{dt}\right)_{water}$$



$$\frac{(w+m_1s_1)\ \ (\theta_1-\theta_2)}{t_1}=\frac{(w+m_2s_2)\ \ (\theta_1-\theta_2)}{t_2} \quad \text{or} \quad \frac{w+m_1s_1}{t_1}=\frac{w+m_2s_2}{t_2}$$

If water equivalent of calorimeter w is negligible then $\frac{m_1s_1}{t_1}=\frac{m_2s_2}{t_2}$

So, $\frac{m_1s_1}{m_2s_2}=\frac{t_1}{t_2}$ or $\frac{\rho_1s_1}{\rho_2s_2}=\frac{t_1}{t_2}$ ($v_1 = v_2$, volume are equal) with the help of this eq^n. we can find specific heat of liquid.

Solved Examples

Example 17. A body at temperature 40ºC is kept in a surrounding of constant temperature 20ºC. It is observed that its temperature falls to 35ºC in 10 minutes. Find how much more time will it take for the body to attain a temperature of 30ºC.

Solution : from equation (14.3.)

$\Delta\theta_f = \Delta\theta_i\ e^{-kt}$

for the interval in which temperature falls from 40 to 35º C.

$(35 - 20) = (40 - 20)\ e^{-k.\,10}$

$\Rightarrow \quad e^{-10\,k} = \frac{3}{4} \quad \Rightarrow \quad K = \frac{\ell n\frac{4}{3}}{10}$

for the next interval

$(30 - 20) = (35 - 20)e^{-kt}$

$\Rightarrow \quad e^{-10k} = \frac{2}{3} \quad \Rightarrow \quad kt = \ell n\ \frac{3}{2}$

$\Rightarrow \quad \frac{\left(\ell n\frac{4}{3}\right)\ t}{10} = \ell n \quad \Rightarrow \quad t = 10\ \frac{\left(\ell n\frac{3}{2}\right)}{\left(\ell n\frac{4}{3}\right)}$ minute = 14.096 min **Ans.**

Alternative : (by approximate method)

for the interval in which temperature falls from 40 to 35ºC

$\langle\theta\rangle = \frac{40+35}{2} = 37.5^\circ C$

from equation (14.4) $\left\langle\frac{d\theta}{dt}\right\rangle = -k(\langle\theta\rangle - \theta_0)$

$\Rightarrow \quad \frac{(35^\circ C-40^\circ C)}{10(\text{min})} = -K(37.5^\circ C - 20^\circ C)$

$\Rightarrow$ K = $\frac{1}{35}(\min^{-1})$

for the interval in which temperature falls from 35ºC to 30ºC

<θ> = $\frac{(30^{o}C - 35^{o}C)}{t}$ = 32.5ºC

from equation (14.4)

= – K(32.5ºC – 20ºC)

$\Rightarrow$ required time, t = $\frac{5}{12.5} \times 35\min$ = 14 min **Ans.**

Example 18. Two liquids of same volume are cooled under same conditions from 65ºC to 50ºC. Time taken are 200sec and 480 sec. If ratio of their specific heats is 2 : 3 then find the ratio of their densities. (neglect the water equivalent of calorimeter)

Solution : From Newton's law of cooling

$$\left(\frac{m_1 s_1 + w_1}{t_1}\right)(\theta_1 - \theta_2) = \left(\frac{m_2 s_2 + w_2}{t_2}\right)(\theta_1 - \theta_2)$$

here $w_1 = w_2 = 0$ = $\frac{m_1 s_1}{t_1} = \frac{m_2 s_2}{t_2} \Rightarrow \frac{V d_1 s_1}{t_1} = \frac{V d_2 s_2}{t_2} \Rightarrow \frac{d_1}{d_2} = \frac{t_1 s_2}{t_2 s_1} = \frac{200}{480} \times \frac{3}{2} = \frac{5}{8}$

Example 19. A calorimeter of water equivalent 5 × 10^{-3} kg contains 25 × 10^{-3} kg of water. It takes 3 minutes to cool from 28°C to 21°C. When the same calorimeter is filled with 30 × 10^{-3} kg of turpentine oil then it takes 2 minutes to cool from 28°C to 21°C. Find out the specific heat of turpentine oil.

Solution : $R_{water} = R_{turpentine}$ $\frac{(m_1 + w)}{t_1} = \frac{(m_2 s_2 + w)}{t_2}$ or $\frac{(25\times10^{-3} + 5\times10^{-3})}{3} = \frac{30\times10^{-3} s_2 + 5\times10^{-3}}{2}$

$10 = \frac{30 \quad s_2 + 5}{2}$, 20 = 30 s_2 + 5

$\therefore$ specific heat of turpentine s_2 = 1/2 = 0.5 kcal/kg/°C

Example 20. A man, the surface area of whose skin is 2m², is sitting in a room where the air temperature is 20°C. If his skin temperature is 28°C, find the rate at which his body loses heat. The emissivity of his skin = 0.97.

Solution : Absolute room temperature (T_0) = 20 + 273 = 293 K

Absolute skin temperature (T) = 28 + 273 = 301 K

Rate of heat loss = σ e A ($T^4 - T_0^4$)

= 5.67 × 10^{-8} × 0.97 × 2 × {$(301)^4 - (293)^4$} = 92.2 W

Example 21. Compare the rate of loss of heat from a metal sphere of the temperature 827°C, with the rate of loss of heat from the same sphere at 427 °C, if the temperature of surroundings is 27°C.

Solution : Given : T_1 = 827 °C = 1100 K, T_2 = 427 °C = 700 K and T_0 = 27 °C = 300 K

According to Steafan's law of radiation, $\frac{dQ}{dt}$ = σ Ae ($T^4 - T_0^4$)

$\therefore \frac{(\frac{dQ}{dt})_1}{(\frac{dQ}{dt})_2} = \frac{(T_1^4 - T_0^4)}{(T_2^4 - T_0^4)} = \frac{[(1100)^4 - (300)^4]}{[(700)^4 - (300)^4]}$ = 6.276 ;k $(\frac{dQ}{dt})_1 : (\frac{dQ}{dt})_2 = 6.276 : 1$

Example 22. Two spheres of the same material have radii 6 cm and 9 cm respectively. They are heated to the same temperature and allowed to cool in the same enclosure. Compare their initial rates of emission of heat and initial rates of fall of temperature.

Solution : Given : r_1 = 6 cm r_2 = 9 cm, $\therefore \frac{r_1}{r_2} = \frac{2}{3}$

According to Stefan's law of radiation, the rate of emission of heat by an ordinary body,

$R_h = \left(\frac{dQ}{dt}\right) = \sigma AeT^4$ or $R_h \propto r^2$ $(A = 4\pi r^2)$

$\therefore \frac{R_{h1}}{R_{h2}} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$ $\frac{R_{FT1}}{R_{FT2}} = \frac{r_2}{r_1} = \frac{3}{2}$

[Rate of fall of temp. R_{FT}, $\frac{d\theta}{dt}$, $\frac{\sigma Ae(T^4 - T_0^4)}{msJ}$ = or $R_{FT} \propto \frac{A}{m}$ or $R_{FT} \propto \frac{1}{r}$]

∴ Initial rates of emission of heat are in the ratio 4 : 9 and initial rates of fall of temperature are in the ratio 3 : 2.

Example 23. The filament of an evacuated light bulb has a length 10 cm, diameter 0.2 mm and emissivity 0.2, calculate the power it radiates at 2000 K. ($\sigma = 5.67 \times 10^{-8}$ W/m² K^4)

Solution : ℓ = 10 cm = 0.1 m, d = 0.2 mm, r = 0.1 mm = 1×10^{-4} m,
e = 0.2, T = 2000 K, $\sigma = 5.67 \times 10^{-8}$ W/m² K^4

According to stefan's law of radiation, rate of emission of heat for an ordinary body (filament),

$E = \sigma AeT^4 = \sigma(2\pi r \ell) eT^4 = 5.67 \times 10^{-8} \times 2 \times 3.14 \times 1 \times 10^{-4} \times 0.1 \times 0.2 \times (2000)^4$

= 11.4 W

∴ Power radiated by the filament = 11.4 W [$A = 2\pi r\ell$]

Example 24. The energy radiated from a black body at a temperature of 727°C is E. By what factor the radiated energy shall increase if the temperature is raised to 2227°C?

Solution :

$$\frac{E_2}{E_1} = \left[\frac{T_2}{T_1}\right]^4 = \left[\frac{2227+273}{727+273}\right]^4 = \left[\frac{2500}{1000}\right]^4 = 39$$

Solved Miscellaneous Problems

Problem 1. An ice box made of 1.5 cm thick styrofoam has dimensions 60 cm × 30cm. It contains ice at 0ºC and is kept in a room at 40ºC. Find the rate at which the ice is melting. Latent heat of fusion of ice = 3.36×10^5 J/kg. and thermal conductivity of stryrofoam = 0.4 W/m-ºC.

Solution : The total surface area of the walls

= 2(60 cm × 60 cm + 60 cm × 30 cm + 60 cm × 30 cm)

= 1.44 m^2

The thickness of the wall = 1.5 cm = 0.015m

The rate of heat flow into the box is

$$\frac{\Delta Q}{\Delta t} = \frac{KA(\theta_1 - \theta_2)}{x} = \frac{(0.04W/m-^oC)(1.44m^2)(40^oC)}{0.015\ \ m} = 154 \text{ W.}$$

The rate at which the ice melts is = 0.46 g/s

Problem 2. A black body emits 10 watts per cm^2 at 327ºC. The sun radiates 10^5 watt per cm^2. Then what is the temperature of the sun ?

Solution : $\therefore \frac{E_{sun}}{E_{body}} = \left(\frac{E_{sun}}{E_{body}}\right)^4$ $\therefore \frac{T_{sun}}{T_{body}} = \left(\frac{10^5}{10}\right)^{1/4}$ T_{sun} = 6000 K

∴ T_{sun} = 6000 K

Problem 3. A bulb made of tungsten filament of surface are a 0.5 cm^2 is heated to a temperature 3000k when operated at 220V. The emissivity of the filament is e = 0.35 and take $\sigma = 5.7 \times 10^{-8}$ mks units. Then the wattage of the bulb is .....(calculate)

Solution : The emissive power watt/m^2 is

$E = e\sigma T^4$

Therefore the power of the bulb is

P = E x area (Watts)

∴ $P = e\sigma T^4 A$

∴ $P = 0.35 \times 0.5 \times 10^{-4} \times 5, 7 \times 10^{-8} \times (3000)^4$ ⇒ P = 80.8 W

Problem 4. In the above example, if the temperature of the filament falls to 2000k due to a drop of mains voltage, then what will be the wattage of the bulb ?

Solution : Now the power of the bulb will be such that

$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^4$ Thus $P_2 = P_1 \times \left(\frac{2}{3}\right)^4$ $\therefore P_2 = 80.8 \times \frac{16}{81}$ Thus $P_2 = P_1 \times$

$\therefore P_2 = 80.8 \times$ $\Rightarrow P_2 = 15.96$

Problem 5. A liquid takes 30 seconds to cool from 95ºC to 90ºC and 70 seconds to cool from 55 to 50ºC. Find the room temperature and the time it will take to cool from 50ºC to 45ºC

Solution : From the first data $\frac{95-90}{30}$ = k $\left(\frac{95\times 90}{2}-\theta_0\right)$ = k ....(1)

From the second data $\frac{55-50}{70}$ = k $\left(\frac{55-50}{0}-\theta_0\right)$ = k ....(2)

Dividing (1) and (2) we get $\frac{7}{3}$ = $\frac{92.5-\theta_0}{52.5-\theta_0}$ ⇒ θ_0 = 22.5º ....(3)

Let the time taken in cooling from 50ºC to 45ºC is t, then

$\frac{50-45}{t}$ = k $\left(\frac{50-45}{2}-\theta_0\right)$ ....(4)

Using θ_0 = 22.5ºC, and dividing (1) by (2) we get

$\frac{t}{30}$ = $\left(\frac{92.5-22.5}{47.5-22.5}\right)$ ⇒ t = 84 sec

Problem 6. A blackened metal disc is held normal to the sun rays, Both of its surfaces are exposed to atmosphere if the distance of earth from sun is 216 times the radius of sun and the temperature of sun is 6000K, the temperature of the disc in steady state will be

Solution : In the steady state the heat received from sun will be equal to heat radiated out. Heat received from sun will be on one side only and it will radiate from both sides.

∴ A σ $\left(\frac{Rs}{d}\right)^2$ $T^4 = 2A\sigma T^4$, $\frac{Rs}{d}$ = $\frac{1}{216}$ ∴ T′ = $\frac{T}{(216)^{1/2}2^{1/4}}$ = $\frac{6000}{14.7\times 1.189}$ = 343 K

∴ T′ = 70ºC

Problem 7. Behaving like a black body sun emits maximum radiation at wavelength 0.48µm. The mean radius of the sun is 6.96 × 10^8m. stefan's constant is 5.67 × 10^{-8}wm^{-2}k^{-4} and wien's constant is 0.293 cm-k. The loss of mass per second by the emission of radiation from sun is -

Solution : Using wien's law

T = T = $\frac{b}{\lambda_m}$ = $\frac{0.293\times 10^{-2}}{0.48\times 10^{-6}}$ = 6104 K

Energy given out by sun per second

= $A\sigma T^4 = 4\pi (6.96\times 10^8)^2 \times 5.67\times 10^{-8} (6104)^4$ ⇒ 49.285 × 10^{25}J

Loss of mass per second

m = $\frac{E}{c^2}$ = $\frac{49.285\times 10^{25}}{9\times 10^{16}}$ ⇒ m = 5.4 × 10^9 kg/s

Problem 8. 50g of water and an equal volume of alcohol (relative density 0.8) are placed one after the other in the same calorimeter. They are found to cool from 60ºC to 50ºC in 2 minutes and 1 minute respectively if the water equivalent of the calorimeter is 2g then what is the specific heat of the alcohol ?

Solution : Given t_w = 2min, t_{alco} = 1 min

m_w = 50g, m_{alco} = 50 × 0.8 = 40g

S_w = 1 in cgs units, w = 2g

Therefore,

S_{alco} = $\frac{1}{m_{alco}}\left\{\frac{t_{alco}}{t_w}[m_w+W]-W\right\}$

⇒ S_{alco} = $\frac{1}{40}\left\{\frac{1}{2}[50+2]-2\right\}$ ⇒ S_{alco} = $\frac{24}{50}$ S_{alco} = 0.6 cgs units = 0.6 cal/gºC