Exercise-1

PART - I : OBJECTIVE QUESTIONS

Section (A):Addition and Subtraction laws of vectors, Position vector, Distance Formula, Section Formula, Direction Ratios & Direction cosines

A-1. **Sol.** Obviously

A-2. **Sol.**(1) $|\hat{i}+\hat{j}| = \sqrt{2}$, (2) $\left|\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{2}}\right| = \sqrt{\frac{3}{2}}$ (3) $|\hat{i}+\hat{j}+\hat{k}| = \sqrt{3}$ (4) $\left|\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}\right| = 1$

A-3. **sol.** $\bar{a}+\bar{b} = (2\hat{i} + 5\hat{j}) + (2\hat{i} - \hat{j})$

$= 4\hat{i} + 4\hat{j} = 4(\hat{i}+\hat{j})$

unit vector in the direction of $(\bar{a}+\bar{b}) = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$

A-4. **Sol.** $\overline{BC} = \hat{i}+\hat{j}$ $\Rightarrow$ $\overline{AB} = \hat{i}-\hat{j}$

$\overline{AB}+\overline{BC} = 2\hat{i}$ $\Rightarrow$ $\overline{AC} = 2\hat{i}$

A-5. **Sol.** $\overline{OA} = 2\hat{i} - \hat{j} + \hat{k}$, $\overline{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$, $\overline{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

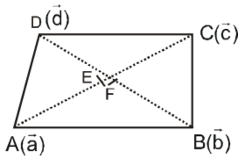
$\overline{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}$, $\overline{BC} = 2\hat{i} - \hat{j} + \hat{k}$ $\overline{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$

$\overline{AB}^2 = \overline{BC}^2 + \overline{CA}^2$

$\therefore$ Right angle Δ

A-6. **Sol.** $\overrightarrow{OE} = \frac{\vec{a}+\vec{c}}{2}$

$\overrightarrow{OF} = \frac{\vec{b}+\vec{d}}{2}$



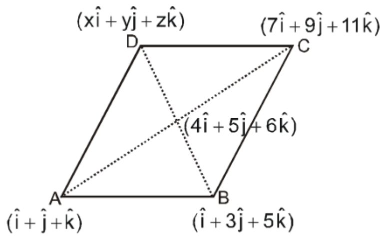
$\overrightarrow{AB}+\overrightarrow{AD}+\overrightarrow{CB}+\overrightarrow{CD}$

$= \vec{b}-\vec{a}+\vec{d}-\vec{a}+\vec{b}-\vec{c}+\vec{d}-\vec{c}$

$= 2[\vec{b}+\vec{d}-\vec{a}-\vec{c}]$

$= 4\left[\frac{\vec{b}+\vec{d}}{2} - \frac{(\vec{a}+\vec{c})}{2}\right] = 4\overrightarrow{EF}$

A-7. **Sol.**



x + 1 = 8

y + 3 = 10

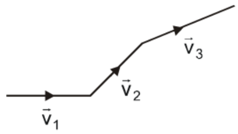
z + 5 = 12

D($7\hat{i}+7\hat{j}+7\hat{k}$)

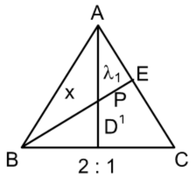
A-8. **Sol.** Clearly Δ is not possible.

Since $\vec{v}_3 = \vec{v}_1 + \vec{v}_2$

Hence $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are coplaner



A-9. Sol.



E(i – j + 4k)

D = $\dfrac{-2\hat{i}-2\hat{j}+12\hat{k}+5\hat{i}+2\hat{j}+4\hat{k}}{3}$ = i + 0 + $\dfrac{16\hat{k}}{3}$ = i + $\dfrac{16\hat{k}}{3}$

$\vec{P} = \dfrac{\mu(\hat{i}-\hat{j}+4\hat{k})+5\hat{i}+2\hat{j}+4\hat{k}}{\mu+1}$

$\vec{P} = \dfrac{\lambda\left(\hat{i}+\dfrac{16\hat{k}}{3}\right)+1\left(\dfrac{3\hat{i}-\hat{j}+2\hat{k}}{1}\right)}{\lambda+1}$

$\dfrac{\mu+5}{4+1} = \dfrac{\lambda+3}{\lambda+1}$ ...... (1) $\dfrac{\mu+2}{\mu+1} = \dfrac{-1}{\lambda+1}$ ..... (2) $\dfrac{4+4\mu}{\mu+1} = \dfrac{\frac{16}{3}\lambda+2}{\lambda+1}$ .... (3)

5λ + 5 + μλ + μ - λμ + λ + 3μ + 3

4λ + 5 – 3 = 2μ

μ = 2l + 1

put value of μ in equation (2)

$\dfrac{2-(2\lambda+1)}{2\lambda+2} = \dfrac{-1}{\lambda+1}$

1 – 2λ = –2

λ = $\dfrac{3}{2}$

μ = 2λ + 1 = 3 + 1 = 4

BP : PE = μ : 1 = 4 : 1

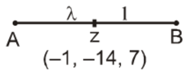
A-10. **Sol.** at x-axis

Q(1, 0, 0)

PQ = $\sqrt{4+9} = \sqrt{13}$

A-11. **Sol.** λ = $\sqrt{4^2+5^2} = \sqrt{41}$ ⇒ ∴5λ_2 = 5 × 41 = 205

A-12. **Sol.** $\dfrac{\lambda+3}{\lambda+1}$ = 7



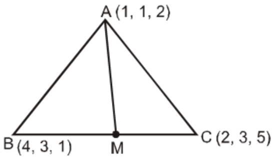
$\Rightarrow \quad \lambda = -\frac{2}{3}$

A-13. Sol. $x^2 + y^2 + y^2 + z^2 + z^2 + x^2 = 36$

$2(x^2 + y^2 + z^2) = 36$

$\sqrt{x^2+y^2+z^2} = 3\sqrt{2}$

A-14. Sol.



AB = $\sqrt{9+4+1} = \sqrt{14}$

AC = $\sqrt{1+4+9} = \sqrt{14}$

M ≡ (3, 3, 3)

$\overrightarrow{AM} = 2\hat{i} + 2\hat{j} + \hat{k}$

A-15. Sol. Let point P is (p, q, r)

$PA^2 - PB^2 = 2k^2$

⇒ $[(p-3)^2 + (q-4)^2 + (r-5)^2] - [(p+1)^2 + (q-3)^2 + (r+7)^2] = 2k^2$

⇒ $-6p - 2p - 8q + 6q - 10r - 14r + 9 + 25 + 25 - 1 - 9 - 49 = 2k^2$

⇒ $8p + 2q + 24r + 9 + 2k^2 = 0$

hence locus is [put p = x, q = y, r = z]

$8x + 2y + 24z + 9 + 2k^2 = 0$

A-16. Sol. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

A-17. Sol. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ $\alpha + \beta = 90^\circ$

$\sin^2\beta + \cos^2\beta + \cos^2\gamma = 1$ $\alpha = 90^\circ - \beta$

$\cos^2\gamma = 0$ $\cos\alpha = \sin\beta$

$\gamma = 90^\circ$

A-18. Sol. $\alpha = \beta = \gamma$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\alpha = \frac{1}{3}$ ⇒ $\cos\alpha = \pm\frac{1}{\sqrt{3}}$

A-19. Sol. Dr's of AB = 1, – 3 – α, 0

Dr's of CD = 3 – β, 2, – 2

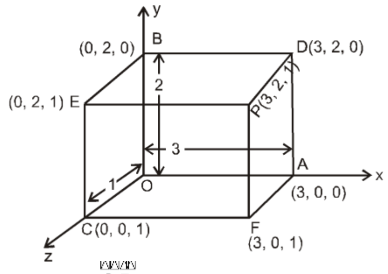
AB ⊥ CD

∴ $1(3-\beta) + (-3-\alpha) \cdot 2 + 0 = 0$

$3 - \beta - 6 - 2\alpha = 0$

$2\alpha + \beta + 3 = 0$ ∴ $\alpha = -1, \beta = -1$

A-20. Sol.



dr's of $\vec{OP}$ are 3, 2, 1

dr's of $\vec{FB}$ are – 3, 2, – 1

dr's of $\vec{AE}$ are – 3, 2, 1

dr's of $\vec{CD}$ are 3, 2, – 1

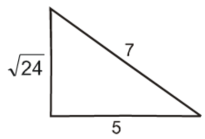
$$\cos\theta_1 = \left|\frac{-9+4-1}{14}\right| = \frac{3}{7}, \cos\theta_2 = \left|\frac{-9+4+1}{14}\right| = \frac{2}{7}$$

$$\cos\theta_3 = \left|\frac{9+4-1}{14}\right| = \frac{6}{7}$$

So angles are $\cos^{-1}\frac{2}{7}$, $\cos^{-1}\frac{3}{7}$, $\cos^{-1}\frac{6}{7}$

Section (B) : Dot Product, Projection of a line segement on other line, Cross Product

B-1. **Sol.**



$$\cos\theta = \frac{(\hat{i}+2\hat{j}+3\hat{k}).(3\hat{i}+2\hat{j}+\hat{k})}{\sqrt{9+5}\sqrt{9+5}} = \frac{3+4+3}{14} = \frac{10}{14} = \frac{5}{7}$$

$\sin\theta = \frac{2\sqrt{6}}{7}$ **Ans.**

B-2. **Sol.** $|\vec{x}| = |\vec{y}| = 1$

$|\vec{x}-\vec{y}|^2 = |\vec{x}|^2 + |\vec{y}|^2 - 2|\vec{x}||\vec{y}|\cos\theta$

$= 2\left(2\sin^2\frac{\theta}{2}\right) \Rightarrow \frac{1}{2}|\vec{x}-\vec{y}| = \left|\sin\frac{\theta}{2}\right|$ **Ans.**

B-3. **Sol.** diagonals are $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$

$$\cos\theta = \frac{(\vec{a}+\vec{b}).(\vec{a}-\vec{b})}{|\vec{a}+\vec{b}|\,|\vec{a}-\vec{b}|} = \frac{1}{3}$$

$\therefore \theta = \cos^{-1}\left(\frac{1}{3}\right)$

B-4. **Sol.** $\bar{a}+\bar{b}+\bar{c} = 0$

$\bar{a}+\bar{b} = -\bar{c}$

$|\bar{a}|^2 + |\bar{b}|^2 + 2\bar{a}\cdot\bar{b} = |\bar{c}|^2$

$9 + 25 + 2 \times 3 \times 5 \cos\theta = 49$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

B-5. **Sol.** $\bar{a} = \hat{i}$ Let $\bar{b} = x\hat{i} + y\hat{j}$

$\tan 120^{o} = \frac{y}{x}$

$\therefore \frac{y}{x} = -\sqrt{3}$ $\therefore y = -\sqrt{3}x$

$\therefore \bar{b} = x(\hat{i} - \sqrt{3}\hat{j})$

Unit vector $k\bar{b} = \pm\frac{\hat{i} - \sqrt{3}\hat{j}}{2}$

$\therefore \bar{b} = \frac{-\hat{i} + \sqrt{3}\hat{j}}{2}$

$\bar{a} + \bar{b} = \frac{\hat{i} + \sqrt{3}\hat{j}}{2}$

B-6. **Sol.** $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$

$\left.\begin{array}{l}\vec{a}.\vec{b} + \vec{a}.\vec{c} = 0\\ \vec{b}.\vec{c} + \vec{a}.\vec{b} = 0\\ \vec{c}.\vec{a} + \vec{b}.\vec{c} = 0\end{array}\right] \Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{a}.\vec{c} = 0$

$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{9 + 16 + 25} = 5\sqrt{2}$ **Ans.**

B-7. **Sol.** $|\vec{a} + \vec{b}|^2 = 100 = \vec{a}^2 + \vec{b}^2 + 2\vec{a}\cdot\vec{b}$

$|\vec{a} - \vec{b}|^2 = 64 = \vec{a}^2 + \vec{b}^2 - 2\vec{a}\cdot\vec{b}$

$\Rightarrow$ $164 = 2(\vec{a}^2 + \vec{b}^2)$

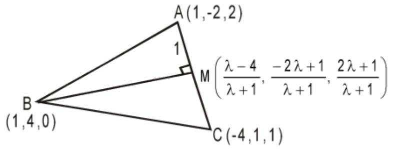
$\Rightarrow$ $\vec{b}^2 = 82 - 25 = 57$

B-8. **Sol.** $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$|\vec{a} + \vec{b} + \vec{c}|^2 = 0 \Rightarrow 3 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -\frac{3}{2}$

B-9. **Sol.**



$M\left(\frac{\lambda - 4}{\lambda + 1}, \frac{-2\lambda + 1}{\lambda + 1}, \frac{2\lambda + 1}{\lambda + 1}\right)$

$\overrightarrow{BM} \cdot \overrightarrow{CA} = 0$

$\Rightarrow \quad \left(\frac{\lambda-4}{\lambda+1}-1\right)(1 + 4) + \left(\frac{-2\lambda+1}{\lambda+1}-4\right)(-2-1) + \left(\frac{2\lambda+1}{\lambda+1}-0\right)(2-1) = 0$

$\left(\frac{(-5)(5)}{\lambda+1}\right) + \left(\frac{-6\lambda-3}{\lambda+1}\right)(3) + \left(\frac{2\lambda+1}{\lambda+1}\right) = 0$

$-25 + 18\lambda + 9 + 2\lambda + 1 = 0$

$\therefore \quad \overrightarrow{BM} = \frac{-5}{\lambda+1}\hat{i} + \frac{-6\lambda-3}{\lambda+1}\hat{j} + \frac{2\lambda+1}{\lambda+1}\hat{k}$

$\overrightarrow{BM} = \frac{10}{7}(-2\hat{i} - 3\hat{j} + \hat{k}) \Rightarrow \lambda = \frac{3}{4}$

B-10. Sol. $\vec{a}+\vec{b} = 5\hat{i}-3\hat{j}+3\hat{k}$

$(\vec{a}+\vec{b}).= (5\hat{i}-3\hat{j}+3\hat{k}).\frac{(\hat{i}-2\hat{j}+2\hat{k})}{3} = \frac{5+6+6}{3} = \frac{17}{3}$

B-11. Sol. $\vec{c} = \pm\ \lambda(3\hat{i}-4\hat{j})$

$\hat{c} = \pm\left(\frac{3\hat{i}-4\hat{j}}{5}\right)$

$\vec{V} = \hat{b}+2\hat{c} = \frac{1}{5}\left[(4\hat{i}+3\hat{j})\pm 2(3\hat{i}-4\hat{j})\right] = \frac{1}{5}[10\hat{i}-5\hat{j}]$ or $\frac{1}{5}[-2\hat{i}+11\hat{j}]$

B-12. Sol. $\overrightarrow{AB} = (-\hat{i}-2\hat{j}+2\hat{k})$

$\overrightarrow{CD} = (-3\hat{i}+6\hat{j}-2\hat{k})$

Projection $\mathbf{i}\ \frac{\overrightarrow{CD}\ .\ \overrightarrow{AB}}{|\vec{A}\cdot\vec{B}|} = \frac{3-12-4}{3} = -\frac{13}{3}$

B-13. Sol. $|x_2-x_1| = 12$

$|y_2-y_1| = 4$

$|z_2-z_1| = 3$

PQ = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$= \sqrt{144+16+9} = 13$

B-14. Sol. $\vec{a} = \hat{i}+\hat{j}$

$\vec{b} = 3\hat{i}+4\hat{k}$

$\vec{b}_2 = \vec{b}-\vec{b}_1 = \frac{-3\ \hat{i}}{2} + \frac{3\ \hat{j}}{2} + 4\hat{k}$

B-15. Sol. (1) $\left|\frac{2\hat{i}-2\hat{j}+\hat{k}}{3}\right| = 1$

(2) $\frac{1}{3}(2\hat{i}-2\hat{j}+\hat{k}) = -\frac{2}{3}\left(-\hat{i}+\hat{j}-\frac{\hat{k}}{2}\right)$

(3) $\frac{1}{3}(2\hat{i}-2\hat{j}+\hat{k}) \;.\; (3\hat{i}+2\hat{j}-2\hat{k}) = 0$

B-16. **Sol.** Component of $\vec{r}$ in direction of $\vec{a}$ = $\frac{\vec{r}\,.\,\vec{a}}{|\vec{a}|}\hat{a}$

$= \left\{\frac{(\hat{i}-8\hat{j}-7\hat{k})\;.(2\hat{i}+2\hat{j}+\hat{k})}{3}\right\}\frac{(2\hat{i}+2\hat{j}+\hat{k})}{3}$

$= -\frac{21}{9}(2\hat{i}+2\hat{j}+\hat{k}) = -\frac{7}{3}(2\hat{i}+2\hat{j}+\hat{k})$

B-17. $\bar{u}\times\bar{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -1 \\ 6 & -3 & 2 \end{vmatrix} = \hat{i} - 10\hat{j} - 18\hat{k}$

unit vector along $(\bar{u}\times\bar{v}) = \pm\frac{\hat{i}-10\hat{j}-18\hat{k}}{\sqrt{425}}$

B-18. **Sol.** $\vec{b} = \lambda(2\sqrt{2}\;\hat{i}-\hat{j}+4\hat{k})$; $|\vec{b}| = 10$

$\Rightarrow |\lambda|\;\sqrt{8+1+16} = 10 \quad \Rightarrow \quad \lambda = \pm 2 \Rightarrow \vec{b} = \pm\; 2\vec{a}$

B-19. **Sol.** $\hat{i} + 2\hat{j} + 3\hat{k} = \mu(\hat{i} + (\lambda+3)\hat{j} + 3\hat{k})$

$\mu = 1,\ \frac{2}{\mu} = \lambda + 3 = 2,\ \lambda = -1$ **Ans.**

B-20. **Sol.** Area = $\frac{1}{2}|\bar{d}_1\times\bar{d}_2|$

$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = \frac{1}{2}|-2\hat{i} - 14\hat{j} - 10\hat{k}|$

$= \frac{\sqrt{300}}{2} = 5\sqrt{3}$

B-21. **Sol.** Area $|\vec{a}\times\vec{b}| = |(\vec{p}+2\vec{q})\times(2\vec{p}+\vec{q})|$

$= |\vec{p}\times\vec{q}\; + 4\vec{q}\times\vec{p}| = |3\vec{p}\times\vec{q}| = 3\times\frac{1}{2} = \frac{3}{2}$

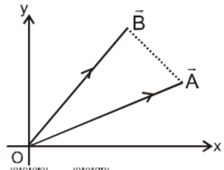
B-22. **Sol.** $(\vec{a}\times\vec{b})^2 + (\vec{a}\;.\;\vec{b})^2 = 144$

$= |\vec{a}|^2|\vec{b}|^2(\sin^2\theta + \cos^2\theta) = 144$

$|\vec{a}||\vec{b}| = 12$

$|\vec{b}| = \frac{12}{4} = 3$

B-23. Sol.



$\overrightarrow{OA} \times \overrightarrow{OB}$ = a fixed vector

⇒ $|\overrightarrow{OA} \times \overrightarrow{OB}|$ = const. number

⇒ ΔOAB = const.

⇒ B is on the line || to base OA

B-24. Sol. $\vec{A}\ .\ \vec{B} = \vec{A}.\vec{C}$ ⇒ $\vec{A}.\vec{B} - \vec{A}.\vec{C} = 0$

or () $\vec{A}.(\vec{B} - \vec{C}) = 0$ ... (1)

$\vec{A} \times \vec{B} = \vec{A} \times \vec{C}$ ⇒ $\vec{A} \times \vec{B} - \vec{A} \times \vec{C} = \vec{0}$

or () $\vec{A} \times (\vec{B} - \vec{C}) = \vec{0}$ ... (2)

(1) & (2) both possible if $\vec{B} - \vec{C} = \vec{0}$ ((1) & (2)

i.e. $\vec{B} = \vec{C}$

B-25. Sol. $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ & $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$

⇒ $\vec{a} \times \vec{b} - \vec{a} \times \vec{c} = \vec{c} \times \vec{d} - \vec{b} \times \vec{d}$

⇒ $\vec{a} \times (\vec{b} - \vec{c}) = (\vec{c} - \vec{b}) \times \vec{d}$

⇒ $(\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = 0$

Section (C) : Straight Line in three dimensional geometry

C-1. Sol. A(2, 1, 3) B(–1, 3, 1)

$\vec{r}$ = (–i + 3i + k) + k(3i – 2j+ 2k)

or $\frac{x-2}{3} = \frac{y-1}{-2} = \frac{z-3}{2}$

$\vec{r}$ = (8 + 3μ)i + j(–3 –2μ) + k(7 + 2μ)

8 + 3μ = 2

μ = – 2 (2, 1, 3)

8 + 3μ2 – 1

μ2 – 3 (1, 3, 1)

C-2. Sol. Point (2 + λ, 2λ – 1, –2λ + 3)

(2, – 1, 3) d = 6

$36 = \lambda^2 + (2\lambda)^2 + (-2\lambda)^2$

$9\lambda^2 = 36$

λ = 2, –2

λ = 2 P(4, 3, –1)

λ = –2 P(0, –5, 7)

C-3. Sol. 1 + λ = 2 +2μ 2 + 2λ = 4 + 2μ

λ = 2 + 2μ λ – μ = 1 3 + 3λ = 1 – 2μ

$\lambda - 2\mu \;\; 2$

$\mu_2 = -1$ $\quad\quad \lambda = 0$ $\quad\quad$ point (1, 2, 3)

$$\frac{x}{\frac{1}{2}} = \frac{y}{\frac{1}{3}} = \frac{z}{-1}$$

C-4. **Sol.**
DR's are 3, 2, –6

$$\frac{x}{\frac{1}{6}} = \frac{y}{-1} = \frac{z}{-\frac{1}{4}}$$

DR's are 2, –12, –3

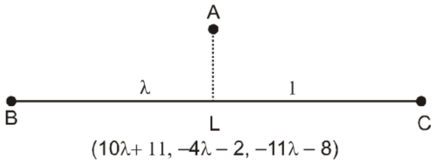
$\cos\theta = 0$

$\therefore \quad \theta = 90^\circ$

C-5. **Sol.** Angle between the given lines is = angle between the vectors $(-2\hat{i} + \hat{j} + 2\hat{k})$ and $(3\hat{i} - 2\hat{j} + 6\hat{k})$

$$\Rightarrow \quad \cos\theta = \frac{(-2\hat{i} + \hat{j} + 2\hat{k}).(3\hat{i} - 2\hat{j} + 6\hat{k})}{|-2\hat{i} + \hat{j} + 2\hat{k}|\;|3\hat{i} - 2\hat{j} + 6\hat{k}|}$$

C-6. **Sol.**



DR's of line BC ≡ (10, –4, – 11)

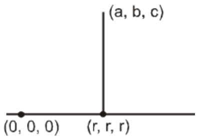
DR's of line AL = $(10\lambda + 9, -4\lambda - 1, -11\lambda - 13)$

$10(10\lambda + 9) + (4\lambda + 1)4 + (11\lambda + 13)11 = 0$

$\lambda = -1$

$\therefore \quad$ L(1, 2, 3) AL = $\sqrt{14}$

C-7. **Sol.**



$x = y = z = r$

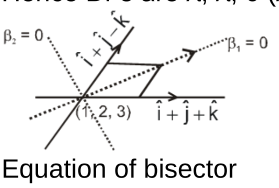
$(r - a) + (r - b) + (r - c) = 0$

$a + b + c = 3r$

C-8. **Sol.** Dr's of bisector

$$\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}} + \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}} = \lambda(\hat{i} + \hat{j})$$

Hence Dr's are λ, λ, 0 (λ ∈ R)

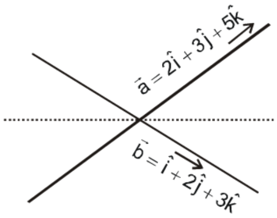


Equation of bisector

$$\frac{x - 1}{\lambda} = \frac{y - 2}{\lambda} = \frac{z - 3}{0}$$

$$\frac{x-1}{2}=\frac{y-2}{2};\quad z-3=0$$

C-9. Sol.



$$\frac{x}{2}=\frac{y}{3}=\frac{z}{5} \qquad \textbf{... (i)}$$

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{3} \qquad \textbf{... (ii)}$$

$$\hat{a}+\hat{b}=\frac{2\hat{i}+3\hat{j}+5\hat{k}}{\sqrt{38}}+\frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{14}}$$

⇒ (A) and (B) will be incorrect

Let the dr's of line ⊥ to (1) and (2) be a, b, c

⇒ 2a + 3b + 5c = 0 ... (iii)

and a + 2b + 3c = 0 ... (iv)

∴ $\frac{a}{9-10}=\frac{b}{5-6}=\frac{c}{4-3}$

⇒ $\frac{a}{-1}=\frac{b}{-1}=\frac{c}{1}$

⇒ $\frac{a}{1}=\frac{b}{1}=\frac{c}{-1}$

∴ equation of line passing through (0, 0, 0) and is ⊥r to the lines (1) and (2) is

$\frac{x}{1}=\frac{y}{1}=\frac{z}{-1}$ Ans.

C-10. Sol. $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}=r$ ... (1)

$\frac{x-1}{3}=\frac{y-2}{-1}=\frac{z-3}{4}$ ... (2)

$\frac{x+k}{3}=\frac{y-1}{2}=\frac{z-2}{h}$ ... (3)

∵ P(r, 2r, 3r) lies on (2)

∴ $\frac{r-1}{3}=\frac{2r-2}{-1}=\frac{3r-3}{4}$

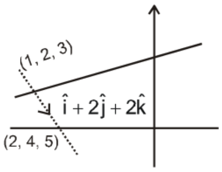
⇒ r = 1

∴ point of intersection of (1) and (2) be (1, 2, 3).

(1, 2, 3) will also satisfy (3) as the lines are concurrent

∴ $\frac{1+k}{3}=\frac{1}{2}=\frac{1}{h}$ ⇒ h = 2 ; k = $\frac{1}{2}$ Ans.

C-11. **Sol.** Common normal



$$\begin{vmatrix} \hat{i} & \hat{i} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \hat{i}(15-16)-\hat{i}(10-12)+\hat{k}(8-9)$$

$$= -\hat{i}+2\hat{j}-\hat{k}$$

SD = projection of $\hat{i}+2\hat{j}+2\hat{k}$ on $-\hat{i}+2\hat{j}-\hat{k}$

$$= \left|\frac{-1+4-2}{\sqrt{1+4+1}}\right| = \frac{1}{\sqrt{6}}$$

C-12. **Sol.** $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\therefore$ lines are perpendicular

Section (D) : Scalar Triple Product, Tetrahedron, Vector Triple Product, Vector Equations, Linear Independent and Linear dependent vectors

D-1. **Sol.** $|(\vec{a}\times\vec{b}).\vec{c}| = |[\vec{a}\ \vec{b}\ \vec{c}]|$

$$[\vec{a}\ \vec{b}\ \vec{c}]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} = |\vec{a}|^2\,|\vec{b}|^2\,|\vec{c}|^2$$

$[\vec{a}\ \vec{b}\ \vec{c}] = |\vec{a}|\,|\vec{b}|\,|\vec{c}|$

where

$\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

D-2.Sol. $[\hat{n}\ \hat{p}\ \hat{m}] = [\ \hat{p}\,\hat{m}\,\hat{n}\ ] = \hat{p}.(\hat{m}\times\hat{n})$

$= |\hat{p}|.|\hat{m}\times\hat{n}|\ \cos\alpha = |\hat{m}\times\hat{n}|\ \cos\alpha$

$= \sin\alpha \cos\alpha$

D-3. **Sol.** $[\hat{i}\ \hat{j}\ \hat{k}] = 1$, $[\hat{j}\ \hat{i}\ \hat{k}] = -1$, $[\hat{k}\ \hat{i}\ \hat{j}] = 1$

D-4. **Sol.** $\left[\ (\vec{a}+\ 2\vec{b}\ -\vec{c})\ \ (\vec{a}\ -\ \vec{b})\ \ (\vec{a}-\vec{b}\ -\vec{c})\ \right]$

$$= \begin{vmatrix} 1 & 2 & -1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{vmatrix} \left[\ \vec{a}\ \ \vec{b}\ \ \vec{c}\ \right] = 3\left[\vec{a}\,\vec{b}\,\vec{c}\right]$$

D-5. **Sol.** $\vec{r}.(\vec{a}+\vec{b}+\vec{c}) = 0$

$\Rightarrow \ell[\vec{a}\ \ \vec{b}\ \ \vec{c}] + m[\vec{a}\ \ \vec{b}\ \ \vec{c}] + n[\vec{a}\ \ \vec{b}\ \ \vec{c}] = 0$

or () $(\ell + m + n)\ [\vec{a}\ \ \vec{b}\ \ \vec{c}] = 0$

or () $\ell + m + n = 0$

D-6. **Sol.** $\vec{a}.\vec{b} = 0 \Rightarrow x - y + 2 = 0$ ... (1)

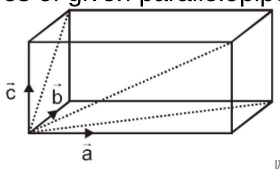
$\vec{a}.\vec{c} = 4 \Rightarrow x + 2y = 4$ ... (2)

$\Rightarrow x = 0,\ \ y = 2$

Hence $\vec{a} = 2\hat{j} + 2\hat{k}$

$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 0 & 2 & 2 \\ 1 & -1 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 8 = |\vec{a}|^2$

D-7. **Sol.** Diagonals of faces of given parallelopiped are $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$



Volume of parallelopipe using these vectors $= [\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}] = 2[\vec{a}\vec{b}\vec{c}]$

D-8. **Sol.** $\left[\vec{b}\ \vec{c}\ \vec{a}\right] < 0$

$\begin{vmatrix} 2 & 2x & 1 \\ 1 & 0 & 1 \\ x & 12 & -1 \end{vmatrix} < 0$

$2(0 - 12) - 2x(-1 - x) + 1(12) < 0$

or $-24 + 2x + 2x^2 + 12 < 0$

$\Rightarrow x^2 + x - 6 < 0$

$\Rightarrow x \in (-3, 2)$

D-9. **Sol.** Let $\vec{v} = \lambda\left[(\hat{i}+\hat{j}+\hat{k}) \times \left\{(\hat{i}+\hat{j}+\hat{k}) \times (2\hat{i} - 3\hat{j})\right\}\right]$

required vector is $3\hat{v}$

D-10. **Sol.** $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c}) - (\vec{a}.\vec{b})\vec{c} = -8\hat{i} - 3\hat{j} - \hat{k}$

D-11. **Sol.** $(\vec{d}+\vec{a})\ .\ \left[\vec{a} \times (\vec{b} \times (\vec{c} \times \vec{d}))\right]$

$= (\vec{d}+\vec{a})\ .\ \left[\vec{a} \times \{(\vec{b}.\vec{d})\vec{c} - (\vec{b}.\vec{c})\vec{d}\}\right]$

$= (\vec{d}+\vec{a})\ .\ \left[(\vec{b}.\vec{d})\vec{a} \times \vec{c} - (\vec{b}.\vec{c})\vec{a} \times \vec{d}\right]$

$= (\vec{b}.\vec{d})\ [\vec{a}\ \ \vec{c}\ \ \vec{d}]$

D-12. **Sol.** $\vec{a} \times [\vec{a} \times (\vec{a} \times \vec{b})] = \{\vec{a}.(\vec{a} \times \vec{b})\}\vec{a} - (\vec{a}.\vec{a})\ \ (\vec{a} \times \vec{b}) = 0\ \_\ \ |\vec{a}|^2\ (\vec{a} \times \vec{b})$

D-13. **Sol.** $\vec{b} \times \vec{d} = \vec{0} \Rightarrow \vec{b} = \lambda\vec{d}$

$\vec{a} = \vec{b} + \vec{c}$ & $\vec{c}.\vec{d} = 0$

$\Rightarrow \vec{a}.\vec{d} = \vec{b}.\vec{d} + \vec{c}.\vec{d}$

or $\vec{a}.\vec{d} = \vec{b}.\vec{d}$

Now $\dfrac{\vec{d}\times(\vec{a}\times\vec{d})}{d^2} = \dfrac{(\vec{d}.\vec{d})\vec{a} - (\vec{d}.\vec{a})\vec{d}}{d^2}$

$= \vec{a} - \dfrac{(\vec{b}.\vec{d})\vec{d}}{d^2} = \vec{a} - \dfrac{(\lambda\ \vec{d}.\vec{d})\vec{d}}{d^2}$

$= \vec{a} - \lambda\vec{d} = \vec{a} - \vec{b} = \vec{c}$

D-14. Sol. $\vec{A}\times\vec{x} = \vec{B}$ take cross product by $\vec{A}$

to get $\vec{A}\times(\vec{A}\times\vec{x}) = \vec{A}\times\vec{B}$ $\Rightarrow (\vec{A}\cdot\vec{x})\vec{A} - (\vec{A}\cdot\vec{A})\vec{x} = \vec{A}\times\vec{B}$

$\Rightarrow$ $\vec{x} = \dfrac{c\vec{A} - (\vec{A}\times\vec{B})}{|\vec{A}|^2}$

D-15. Sol. $\vec{a}\cdot\vec{d} = 0 = [\vec{b}\vec{c}\vec{d}]$ $\Rightarrow$ $\vec{d}$ is $\perp$ to $\vec{a}$ & coplanar to vectors $\vec{b}\,\&\,\vec{c}$

$\Rightarrow$ $\vec{d} = \lambda\left[\vec{a}\times(\vec{b}\times\vec{c})\right] = -\lambda(\hat{i}+\hat{j}-2\hat{k})$

since $\vec{d}$ is unit vector $|\vec{d}| = 1$ $\Rightarrow$ $\lambda = \pm\dfrac{1}{\sqrt{6}}$

D-16. Sol. $\vec{c} = \lambda\vec{a} + \mu\vec{b} + r(\vec{a}\times\vec{b})$ $\Rightarrow$ Since $\vec{a}\,.\,\vec{b} = 1$ $\Rightarrow$ $\vec{a}\,.\,\vec{c} = \vec{b}\,.\,\vec{c} = 1$

So $\vec{a}.\vec{c} = \lambda a.a + \mu\ \ a.b + 0$ $\Rightarrow$ $2\lambda + \mu = 1$ ... (1)

$\vec{b}.\vec{c} = \lambda\vec{a}\ .\ \vec{b} + \mu\ \ b.\ \ b + 0$

$\Rightarrow$ $\lambda + 2\mu = 1$ ... (2)

by (1) & (2) $\lambda = \mu = \dfrac{1}{3}$

Now $|c| = \sqrt{2}$

$\Rightarrow$ $\sqrt{2} = \sqrt{\dfrac{2}{9} + \dfrac{2}{9} + r^2\times 3 + \dfrac{2}{9} + 0 + 0}$

$\Rightarrow$ $r = \pm\dfrac{2}{3}$ $\Rightarrow \vec{c} = \hat{i} + \hat{k}$ or $\dfrac{-\hat{i} + 4\hat{j} - \hat{k}}{3}$

Section (E) : Plane

E-1. Sol. $y = 0$

$\dfrac{3\lambda - 1}{\lambda + 1} = 0$ $\Rightarrow$ $\lambda = \dfrac{1}{3}$

E-2. Sol. $x + 3y - 4z = -6$

$\dfrac{x}{-6} + \dfrac{y}{-2} + \dfrac{z}{3/2} = 0$

Algebraic sum of inteept $-6 - 2 + \dfrac{3}{2} = -\dfrac{13}{2}$

E-3. **Sol.** $x - 3y + 5z = 15$

$\frac{x}{15} + \frac{y}{-5} + \frac{z}{3} = 1$

E-4. **Sol.** $a(x - 2) + b(y + 3) + c(z - 1) = 0$

Dr's of the line joining (3, 4, – 1) & (2, – 1, 5) are – 1, – 5, 6

normal of the plane and above line are parallel

∴ equation of the plane

$-1(x - 2) - 5(y + 3) + 6(z - 1) = 0$

$x + 5y - 6z + 19 = 0$

E-5. **Sol.** DR's are 2, –1, 1

and 1, 1, 2

$\cos\theta = \frac{2-1+2}{6} = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

E-6. **Sol.** $\cos\theta = \left|\frac{6+4-10}{5\sqrt{2}\ .\ 3}\right| = 0$

E-7. **Sol.** $2x + 3y - 4z + \lambda = 0$

$2 + 6 - 12 + \lambda = 0$

$\lambda = 4$

E-8. **Sol.** $b(y - 3) + c(z + 4) = 0$ .... (i)

passes (1, –1, 3) $4b = 7c$

put in equation (i)

$7y + 4z - 5 = 0$

E-9. **Sol.** $a(x - 1) + b(y + 3) + c(z + 2) = 0$

$a + 2b + 2c = 0$ ... (1)

$3a + 3b + 2c = 0$ ... (2)

$\frac{a}{4-6} = \frac{b}{6-2} = \frac{c}{3-6}$

$\frac{a}{2} = \frac{b}{-4} = \frac{c}{3} = k$ (Let)

⇒ $a = 2k$

$b = -4k$

$c = 3k$

$2k(x - 1) - 4k(y + 3) + 3k(z + 2) = 0$

$2x - 4y + 3z - 8 = 0$

E-10. **Sol.** $y(x + z) = 0$

$y = 0$ $x + z = 0$

Dr's of normal of this plane are 0,1,0

Dr's of normal of this plane are 1,0,1

$a_1a_2 + b_1b_2 + c_1c_2 = 0 + 0 + 0 = 0$

Normals of these plane are perpendicular

Hence planes are perpendicular.

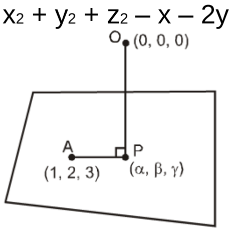
E-11. **Sol.** OP ⊥ AP

$\alpha(\alpha - 1) + \beta(\beta - 2) + \gamma(\gamma - 3) = 0$

∴ Locus of P(α, β, γ) is

$\therefore$ P (α, β, γ)

$x_2 + y_2 + z_2 - x - 2y - 3z = 0$



E-12. Sol. $2x - 3y + 6z - 5 = 0$ ....(i)

$2x - 3y + 6z + \frac{20}{3} = 0$ ....(ii)

distance $\left|\frac{\frac{20}{3}+5}{\sqrt{4+9+36}}\right| = \frac{5}{3}$

E-13. Sol. $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = -\ 2\left(\frac{ax_1+by_1+cz_1+\alpha}{a^2+b^2+c^2}\right)$

$\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{-1} = -\ 2\left(\frac{6+2-3-9}{9+4+1}\right)$

$\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{-1} = \frac{8}{14}$

x = $\frac{26}{7},\quad y = -\frac{15}{7},\quad z = \frac{17}{7}$

E-14. Sol. Vertices of the tetrahedron are (0, 0, 0), (6, 0, 0), (0, – 4, 0), (0, 0, 3)

$\therefore$ Volume = $\frac{1}{6}\begin{vmatrix} 0 & 0 & 0 & 1 \\ 6 & 0 & 0 & 1 \\ 0 & -4 & 0 & 1 \\ 0 & 0 & 3 & 1 \end{vmatrix} = 12$

Alternative : V = $\left|\frac{1}{6}\left[\vec{AB}\ \vec{AC}\ \vec{AD}\right]\right| = \left|\frac{1}{6}\begin{vmatrix} 6 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 3 \end{vmatrix}\right|$ = 12.

E-15. Sol. Required plane (x – y + 2z – 3) + λ(4x + 3y – z – 1) = 0

x(4λ + 1) + y(3λ – 1) + z(2 – λ) – (3 + λ) = 0 ... (1)

Now we can observe that from the given

options equation (1) can represent only first options

E-16. Sol. $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2} = r$ ... (1)

$\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ .... (2)

$\therefore$ coordinates of any point P on line (1)

$\therefore$ P(3r + 1, r + 2, 2r + 3)

for point of intersection of (1) and (2)

$$\frac{3r+1-3}{1}=\frac{r+2-1}{2}=\frac{2r+3-2}{3}$$

$$\frac{3r-2}{1}=\frac{r+1}{2}=\frac{2r+1}{3}$$

∴ r = 1

∴ point of intersection is (4, 3, 5)

∴ the equation of required plane

4(x – 4) + 3(y – 3) + 5(z – 5) = 0

4x + 3y + 52 = 50

E-17. Sol. Equation of line = $\frac{x-2}{1}=\frac{y+3}{-1}=\frac{z-1}{-6}=\lambda$

any point of the line (λ + 2, – λ – 3, – 6λ + 1)

it satisfy the plane 2λ + 4 – λ – 3 – 6λ + 1 = 7

⇒ – 5λ = 5 ⇒ λ = – 1

point (1,–2, 7)

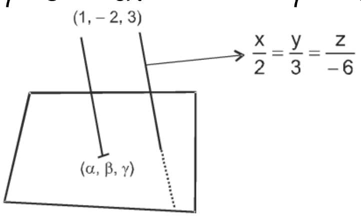
E-18. Sol. (3r + 2, – 1 + 4r, 2 + 12r) ≡ (2, –1, 2)

distance = $\sqrt{9+16+144}$ = 13

E-19. Sol. α – 1 = 2λ ⇒ α = 2λ + 1

β + 2 = 3λ ⇒ β = 3λ – 2

γ – 3 = – 6λ ⇒ γ = – 6λ + 3



2λ + 1 – 3λ + 2 – 6λ + 3 = 5

7λ = 1 ⇒ λ = 1/7

∴ Point on the plane is $\left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$

Distance = $\sqrt{(\alpha-1)^2+(\beta+2)^2+(\gamma-3)^2}$

= λ $\sqrt{4+9+36}=\frac{1}{7}.7$ = 1

E-20. Sol. Let A(2λ + 1, 4λ + 3, 3λ + 2)

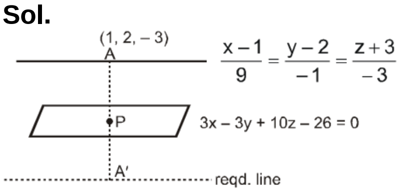
(2λ + 1 – 3) . 3 + (4λ + 3 – 8). 2 + (3λ + 2 – 2) (– 2) = 0

6λ – 6 + 8λ – 10 – 6λ = 0

λ = 2 ∴ A(5, 11, 8)

∴ AP = $\sqrt{(2)^2+(3)^2+(6)^2}=7$

E-21. Sol.



∵ A' is mirror image of A w.r.t the given plane

∴ $\frac{x-1}{3} = \frac{y-2}{-3} = \frac{z+3}{10} = -2\left(\frac{3-6-30-26}{9+9+100}\right)$

$\frac{x-1}{3} = \frac{y-2}{-3} = \frac{z+3}{10} = -2\left(-\frac{59}{118}\right)$ ∴ A'(4, – 1, 7)

∴ Equation of required line $\frac{x-4}{9} = \frac{y+1}{-1} = \frac{z-7}{-3}$

E-22. Sol. Dr's of the line

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 4 & 1 & -2 \end{vmatrix} = \hat{i}(-2-1) - \hat{j}(-2-4) + \hat{k}(1-4) = -3\hat{i} + 6\hat{j} - 3\hat{k}$

∴ Dr's are – 1, 2, – 1 or 1, – 2, 1

x + y + z – 1 = 0

4x + y – 2z + 2 = 0

Put z = 0

x + y = 1

4x + y = – 2

– 3x = 3

x = – 1, y = 2, z = 0

$\frac{x+1}{1} = \frac{y-2}{-2} = \frac{z-0}{1}$

put z = 1

$\left.\begin{array}{l} x+y=0 \\ 4x+y=0 \end{array}\right\}$ x = y = 0 & z = 1

∴ $\frac{x}{1} = \frac{y}{-2} = \frac{z-1}{1}$

put y = 1

x + z = 0

4x – 2z = – 3

2x + 2z = 0

⇒ $x = -\frac{1}{2}$, $z = \frac{1}{2}$, y = 1

∴ $\frac{x+\frac{1}{2}}{1} = \frac{y-1}{-2} = \frac{z-\frac{1}{2}}{1}$

Exercise-2

1. Sol. Before rotation $\vec{a} = 2p\hat{i} + \hat{j}$

after rotation $\vec{a} = (p+1)\hat{i}' + \hat{j}'$

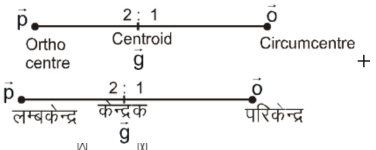
Since length of vector remains unaltered

$\sqrt{4p^2+1} = \sqrt{(p+1)^2+1}$

$4p^2 = (p+1)^2$ ⇒ p + 1 = ± 2P

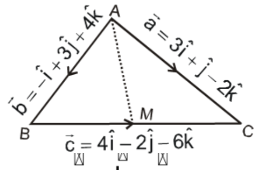
p = 1 or $-\frac{1}{3}$

2. Sol.



$\vec{g} = \dfrac{2.\vec{0} + 1.\vec{p}}{2+1}$ $\Rightarrow$ $\vec{p} = 3\vec{g}$ $\therefore$ k = 3

3. Sol.

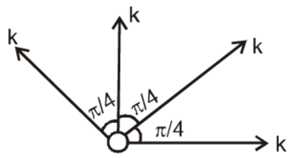


here $\vec{a} = \vec{b} + \vec{c}$

$\vec{AM} = \dfrac{1}{2}(\vec{a} + \vec{b}) = \dfrac{1}{2}[2\hat{i} + 4\hat{j} + 2\hat{k}] = \hat{i} + 2\hat{j} + \hat{k}$ $\Rightarrow$ $\lambda = \sqrt{6}$

4. Sol. These foes can be written in terms of vector as

$k\hat{i},\ \dfrac{k}{\sqrt{2}}\hat{i} + \dfrac{k}{\sqrt{2}}\hat{j},\ k\hat{j}$ and $-\dfrac{k}{\sqrt{2}}\hat{i} + \dfrac{k}{\sqrt{2}}\hat{j}$



Resultant = $k\hat{i} + (k + \sqrt{2}k)\hat{j}$

magnitude = $\sqrt{k^2 + (k + \sqrt{2}k)^2} = k\sqrt{4 + 2\sqrt{2}}$

5. Sol. Let the point be p(α, β, γ)

$\because$ $(\alpha - 1)^2 + (\alpha + 1)^2 + (\beta - 1)^2 + (\beta + 1)^2 + (\gamma - 1)^2 + (\gamma + 1)^2 = 10$

$\Rightarrow$ $2(\alpha^2 + \beta^2 + \gamma^2) = 4$

$\therefore$ $\alpha^2 + \beta^2 + \gamma^2 = 2$

$\therefore$ required locus is $x^2 + y^2 + z^2 = 2$

6. Sol. $\ell + m + n = 0$ ....(1)

$\ell^2 + m^2 = n^2$ ....(2)

Put $n = -(\ell + m)$ in (2)

$\ell^2 + m^2 = \ell^2 + m^2 + 2\ell m$

$\Rightarrow$ $\ell m = 0$

(i) if $\ell = 0$; $m \neq 0$ then from (1) $m = -n$

$\therefore \dfrac{\ell}{0} = \dfrac{m}{1} = \dfrac{n}{-1}$

direction cosine are : $0, \dfrac{1}{\sqrt{2}}, \dfrac{-1}{\sqrt{2}}$

(ii) if $\ell \neq 0$; $m = 0$, then from (1), $\ell = -n$

$\therefore \quad \frac{\ell}{1} = \frac{m}{0} = \frac{n}{-1}$

direction cosine are : $\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}$

Let θ be the angle between the lines

$\cos\theta = 0 + 0 + \frac{1}{2}$

$\cos\theta = \frac{1}{2}, \quad \theta = \frac{\pi}{3}$

7. Sol. Dr's of diagonal BD = a, – a, a or 1,–1,1

Dr's of diagonal AF = – a, a, a or –1, 1, 1

Angle between above diagonals

$\cos\theta = \left|\frac{-1-1+1}{\sqrt{3}\sqrt{3}}\right| = \frac{1}{3}$

8. Sol. $|\vec{e}_1 - \vec{e}_2|^2 < 1 \quad \Rightarrow \quad \vec{e}_1^{\,2} + \vec{e}_2^{\,2} - 2\vec{e}_1.\vec{e}_2 < 1$

$\Rightarrow$ 1 + 1 – 2cos(2θ) < 1

$\Rightarrow$ 2cos2θ > 1 $\quad \Rightarrow \quad \cos 2\theta > \frac{1}{2}$

$2\theta \in \left[0, \frac{\pi}{3}\right) \quad \Rightarrow \quad \theta \in \left[0, \frac{\pi}{6}\right)$

9. Sol. $(\vec{a} + P\vec{b}) . \vec{c} = 0 \quad \Rightarrow \quad P = -\frac{\vec{a}.\vec{c}}{\vec{b}.\vec{c}} = 5$

10. Sol. $\vec{u} + \vec{v} + \vec{w} = 0$

$\Rightarrow \quad |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 + 2(\vec{u}.\vec{v}) + 2(\vec{v}.\vec{w}) + 2(\vec{w}.\vec{u}) = 0$

$\Rightarrow \quad 9 + 16 + 25 + 2\,[\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}] = 0$

$\Rightarrow \quad \sqrt{|\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}|} = 5$

11. Sol. $\lambda = \frac{\frac{\vec{a}.\vec{b}}{|\vec{b}|}}{\frac{\vec{a}.\vec{b}}{|\vec{a}|}} = \frac{|\vec{a}|}{|\vec{b}|} = \frac{7}{3}$

12. Sol. Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

given expression is $x[y\hat{k} - z\hat{j}] + y[z\hat{i} - x\hat{k}] + z[x\hat{j} - y\hat{i}] = \vec{0}$

13. Sol. $\lambda \; \lambda(\vec{b} \times \vec{a}) + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0} \quad \{\vec{a} + \vec{b} + \vec{c} = \vec{0} \; \therefore \; \vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}\}$

$\Rightarrow \quad \lambda\vec{b} \times \vec{a} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b} = \vec{0}$

$\Rightarrow \quad \lambda = 2$

14. Sol. For option (ii)

$3\hat{i}+3\hat{j}=2\hat{j}+6\hat{j}+\lambda(\hat{i}-3\hat{j})$

λ = 1

option (iii)

$\hat{i}+9\hat{j} = 2\hat{i}+6\hat{j}+\lambda(\hat{i}-3\hat{j})$

λ = – 1

15. **Sol.** Equation of line $\vec{r}=3\hat{i}+\hat{j}-\hat{k}+\lambda(2\hat{i}-\hat{j}+2\hat{k})$ ......(i)

15 = |λ| × 3

λ = ± 5 put equation (i)

16. **Sol.** $(\vec{r}-\vec{a})\times\vec{b}=0$

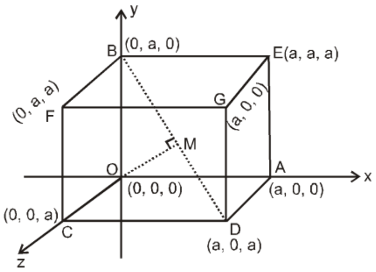
$\vec{r}=\vec{a}+\mu\vec{b} = \vec{a}+\vec{b}=3\hat{i}+\hat{j}-\hat{k}$

λ = μ = 1

$(\vec{r}-\vec{b})\times\vec{a}=0$

$\therefore\ \vec{r}=\vec{a}+\lambda\vec{b}$ λ = μ = 1

17. **Sol.**



Dr's of BD ≡ a, – a, a

Equation of line BD is

$$\frac{x-a}{a}=\frac{y-0}{-a}=\frac{z-a}{a}=\lambda$$

Let M = $\begin{cases} x=a\lambda+a \\ y=-a\lambda \\ z=a\lambda+a \end{cases}$

OM is $\perp_{ar}$ to BD

⇒ a(aλ + a) + (– a) (– aλ) + a(aλ + a) = 0

⇒ λ + 1 + λ + λ + 1 = 0 ⇒ λ = – 2/3

M $\left(\frac{a}{3},\frac{2a}{3},\frac{a}{3}\right)$

OM = $\sqrt{\frac{2}{3}}$ a

18. **Sol.** cos θ = 1 – 1 – 1 = – 1 < 0

acute angle bisector

$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})$ + t $(\hat{j}-\hat{k})$

19. **Sol.** $\vec{a}.\vec{b}=\vec{a}.\vec{c}=0$ ⇒ $\vec{a}=\lambda(\vec{b}\times\vec{c})$

also $|\vec{a}|=|\lambda(\vec{b}\times\vec{c})|$ = $\left|\lambda\frac{\sqrt{3}}{2}\right|$ = 1

$|[\vec a\ \vec b\ \vec c]| = |\vec a.(\vec b\times\vec c)| = |\lambda||(\vec b\times\vec c)|^2 = \frac{2}{\sqrt3}|\vec b|^2|\vec c|^2\sin^2\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt3}\times\left(\frac{\sqrt3}{2}\right)^2 = \frac{\sqrt3}{2}$

20.

Sol. $[\vec a\ \vec b\ \vec c]^2 = \begin{vmatrix}\vec a.\vec a & \vec a.\vec b & \vec a.\vec c\\ \vec b.\vec a & \vec b.\vec b & \vec b.\vec c\\ \vec c.\vec a & \vec c.\vec b & \vec c.\vec c\end{vmatrix} = \begin{vmatrix}1 & \cos\theta & \cos\theta\\ \cos\theta & 1 & \cos\theta\\ \cos\theta & \cos\theta & 1\end{vmatrix}$

$= 1(1 - \cos^2\theta) - \cos\theta(\cos\theta - \cos^2\theta) + \cos\theta(\cos^2\theta - \cos\theta)$

$= 1 - \cos^2\theta - \cos^2\theta + \cos^3\theta + \cos^3\theta - \cos^2\theta$

$= 2\cos^3\theta - 3\cos^2\theta + 1 = (\cos\theta - 1)^2(2\cos\theta + 1)$

$|[\vec a\ \vec b\ \vec c]| = (1 - \cos\theta)\sqrt{1+2\cos\theta}$.

21. **Sol.** $a(x - 2) + b(y - 3) + c(z - 1) = 0$ ... (1)

$2a - 2b - 3c = 0$

$4a + 0.b + 6c = 0$

$\frac{a}{-12-0} = \frac{b}{-12-12} = \frac{c}{0+8}$

$\frac{a}{3} = \frac{b}{6} = \frac{c}{-2} = \lambda$ (let) (½

Put these values of a, b, c in (1)

$3(x - 2) + 6(y - 3) - 2(z - 1) = 0$

$3x + 6y - 2z - 22 = 0$

$d = \left|\frac{-15-24-16-22}{\sqrt{9+36+4}}\right| = \left|\frac{77}{7}\right| = 11$

22. **Sol.** $\vec r = \lambda\vec b + \mu\vec c$

$\frac{\vec r.\vec a}{|\vec a|} = \sqrt{\frac{2}{3}} \Rightarrow \frac{\lambda(\vec b.\vec a)+\mu(\vec c.\vec a)}{|\vec a|} = \sqrt{\frac{2}{3}} \Rightarrow \frac{\lambda(-1)+\mu(-1)}{\sqrt6} = \sqrt{\frac{2}{3}}$

$\Rightarrow -\lambda - \mu = 2 \quad \Rightarrow \mu = -2 - \lambda$

So $\vec r = \lambda\vec b - (2 + \lambda)\vec c$

$\vec r = -2\hat i + (\lambda - 2)\hat j + (\lambda + 4)\hat k$

by option put $\lambda = 1$

$\vec r = -2\hat i - \hat j + 5\hat k$

23. **Sol.** Let $\vec v = \lambda_1\vec a + \lambda_2\vec b + \lambda_3\vec c$ [$\vec a$, $\vec b$, $\vec c$ are non coplanar]

$\vec v.\vec a = \vec v.\vec b = \vec v.\vec c = 0$

$\lambda_1\vec a.\vec a + \lambda_2\vec a.\vec b + \lambda_3\vec a.\vec c = 0$

$\lambda_1\vec b.\vec a + \lambda_2\vec b.\vec b + \lambda_3\vec b.\vec c = 0$

$\lambda_1\vec c.\vec a + \lambda_2\vec c.\vec b + \lambda_3\vec c.\vec c = 0$

Only possible values of $\lambda_1, \lambda_2, \lambda_3 = 0$ as

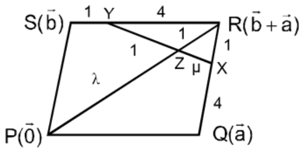
$$\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} \neq 0$$

24. **Sol.** PVs of vertex P,Q,R,S are (Let) $\vec{0},\vec{a},\vec{b}+\vec{a},\vec{b}$
using section rule PVs of

$X \equiv \dfrac{4(\vec{b}+\vec{a})+\vec{a}}{5}$ and $Y \equiv \dfrac{(\vec{b}+\vec{a})+4\vec{b}}{5}$ b

again Let $\dfrac{PZ}{ZR} = \lambda$ and $\dfrac{XZ}{YZ} = \mu$

PVs of point Z may be given as



$\dfrac{\lambda(\vec{b}+\vec{a})+\vec{0}}{\lambda+1}$ & also as $\dfrac{\mu\left(\vec{b}+\dfrac{\vec{a}}{5}\right)+1\left(\vec{a}+\dfrac{4\vec{b}}{5}\right)}{\mu+1}$

Equating both answers and coefficient of $\vec{a}$ & $\vec{b}$
(they are representing non collinear vectors $\overrightarrow{PQ}$ & $\overrightarrow{PS}$)

$\dfrac{\lambda}{\lambda+1} = \dfrac{\mu+\left(\dfrac{1}{5}\right)}{\mu+1}$ and $\dfrac{\lambda}{\lambda+1} = \dfrac{\left(\dfrac{4\mu}{5}\right)+1}{\mu+1}$

Solving these equations gives $\lambda = \dfrac{21}{4}$

5. **Sol.** $\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} = [\vec{a}\vec{b}\vec{c}]^2$ = 4_2 = 16

26. **Sol.** $\vec{a} \parallel (\vec{b}\times\vec{c}) \Rightarrow \vec{a} = \lambda\ (\vec{b}\times\vec{c})$

also $(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{c}) = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{c} \end{vmatrix} = \begin{vmatrix} \vec{a}.\vec{a} & 0 \\ 0 & \vec{b}.\vec{c} \end{vmatrix} \Rightarrow |\vec{a}|^2\ (\vec{b}\ .\ \vec{c})$

27. **Sol.** $\vec{a}.\vec{n}$ = d = 16 + 4 + 25 = 45
$\vec{r}.\vec{n} = \vec{a}.\vec{n}$
$\vec{r}.(4\hat{i}-2\hat{j}-5\hat{k})$ = 45

28. **Sol.** Let equation of plane is a(x – 1) + b(y – 1) + c(z –1) = 0
0 + b(–8) + 0 = 0 b = 0
– 8a – 4b – 6c = 0 ⇒ 4a + 2b + 3c = 0
⇒ 4a + 3c = 0
⇒ $\dfrac{a}{3} = \dfrac{b}{0} = \dfrac{c}{-4}$
Dr's of the normal are 3, 0, – 4
xy plane : z = 0 Dr's of the normal are 0, 0, 1
3(0) + 0(0) – 4(1)≠ 0 so planes are not perpendicular

29. Sol. Equation of plane passing through (1,0,0) is :

⇒ a(x – 1) + b(y – 0) + c(z – 0) = 0

a(x – 1) + b(y) + c(z) = 0.............(1)

(1) also passes through (0,1,0)

–a + b + c.0 = 0 ....(2)

angle between plane (1) and x + y – 3 = 0 is $\frac{\pi}{4}$

$\cos\frac{\pi}{4} = \left|\frac{a.1+b.1+c.0}{\sqrt{\Sigma a^2}\sqrt{1+1+0}}\right| = \frac{1}{\sqrt{2}}$

$\left|\frac{a+b}{\sqrt{a^2+b^2+c^2}\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$

⇒ $(a + b)^2 = a^2 + b^2 + c^2$

⇒ $2ab = c^2$ from equation (2) a = b

⇒ $2a^2 = c^2$ ⇒ $c = a\sqrt{2}$ or $c = -a\sqrt{2}$

∴ equation of planes are $x + y \pm \sqrt{2}\, z - 1 = 0$

so direction ratio of planes are $(1, 1, \pm\sqrt{2})$

30. Sol. 2x – y + z = 6 ... (1)

x + y + 2z = 7 ... (2)

x – y = 3 ... (3)

Let the equation of plane ⊥r to (2) and (3) be

ax + by + cz + d = 0 .... (4)

∴ a + b + 2c = 0

a – b + 0 . c = 0

∴ $\frac{a}{2} = \frac{b}{2} = \frac{c}{-1-1}$

dr's of normal to the plane ⊥r to (2) and (3) are 2, 2, – 2

now angle between both planes is $\cos\theta = \frac{(2)(2)+2(-1)+(-2)(1)}{3.2\sqrt{3}} = 0$

⇒ = 90°

31. Sol. $\vec{a}\times\vec{b}$ is vector perpendicular to plane containing $\vec{a}$ and $\vec{b}$

⇒ $\vec{a}\times\vec{b}$ lies in the plane which contains $\vec{c}$ and $\vec{d}$

⇒ $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = 0$

32. Sol. Equation of 2 planes $\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$ ....(i)

$\frac{x}{a_1}+\frac{y}{b_1}+\frac{z}{c_1} = 1$ ....(ii)

distance form (0, 0, 0) on 2 planes are equal

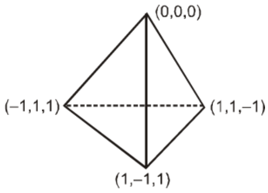
∴ $\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} = \frac{1}{\sqrt{\frac{1}{a_1^2+b_1^2+c_1^2}}}$

33. Sol. The planes are

y + z = 0 ....(1)

z + x = 0 ....(2)

$x + y = 0$ ....(3)
$x + y + z = 1$ ....(4)
Solving above equations we get vertices of the tetrahedron



as (0,0,0), (–1,1,1), (1,–1,1) and (1,1,–1)

Required volume = $\left|\frac{1}{6}\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}\right|$

$= \left|\frac{1}{6}\begin{vmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \\ 0 & 0 & -1 \end{vmatrix}\right| = \frac{4}{6} = \frac{2}{3} \Rightarrow \quad t = \frac{2}{3}$

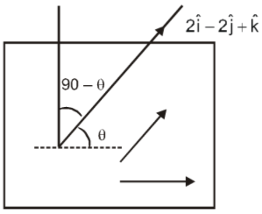
$729t = 729 \times \frac{2}{3} = 486$

34. **Sol.** $2x - y + 3z + 4 = 0 = ax + y - z + 2$ ... (1)
$\because$ equation of plane through (1) is
$(2x - y + 3z + 4) + \lambda(ax + y - z + 2) = 0$
$x(2 + a\lambda) + y(\lambda - 1) + z(3 - \lambda) + (4 + 2\lambda) = 0$ ....(2)
$x - 3y + z = 0 = x + 2y + z + 1$ ... (3)
$\because$ equation of plane passing through (3) is
$(x - 3y + z) + \mu(x + 2y + z + 1) = 0$
$x(1 + \mu) + y(2\mu - 3) + z(\mu + 1) + \mu = 0$ .... (4)
if lines (1) and (3) are coplanar, then
$$\frac{2+a\lambda}{\mu+1} = \frac{\lambda-1}{2\mu-3} = \frac{3-\lambda}{\mu+1} = \frac{4+2\lambda}{\mu}$$
Solving this we get $\lambda = -1$, $\mu = 1$
$\therefore$ $a = -2$

35. **Sol.** $a_1a_2 + b_1b_2 + c_1c_2 = 0$
$6 - 2 - 2m = 0$
$m = 2$

36. **Sol.** $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -1 \\ 1 & -1 & 2 \end{vmatrix} = \hat{i}(6-1) - \hat{j}(4+1) + \hat{k}(-2-3) = 5\hat{i} - 5\hat{j} - 5\hat{k}$

$\cos(90 - \theta) = \left|\frac{10+10-5}{5\sqrt{3}.\,3}\right|$



$\sin\theta = \frac{1}{\sqrt{3}}$ $\theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) = \cot^{-1}(\sqrt{2})$

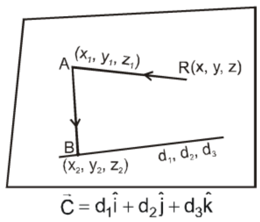
37. (3) $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = 0$ (4) $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = 0$

Sol. Vectors $\overrightarrow{AR}, \overrightarrow{AB}$ & $\overrightarrow{C}$ are coplanar

Equation of the required plane

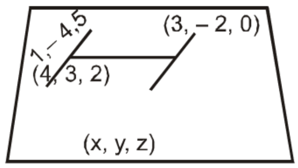
$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$$

or $\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$



38. **Sol.** $\frac{x-4}{1} = \frac{y-3}{-4} = \frac{z-2}{5}$ .... (1)

$\frac{x-3}{1} = \frac{y+2}{1/\lambda} = \frac{z-0}{1/\mu}$ .... (2)



Equation of the plane is

$$\begin{vmatrix} x-3 & y+2 & z \\ 1 & 5 & 2 \\ 1 & -4 & 5 \end{vmatrix} = 0$$

(x – 3) (25 + 8) – (y + 2) (5 – 2) + z(– 4 – 5) = 0
33x – 99 – 3y – 6 – 9z = 0
33x – 3y – 9z – 105 = 0
11x – y – 3z = 35

39. **Sol.** Equation of plane passing through (0, 0, 0) is

ax + by + cz = 0 .... (1)

$\because$ (1) is parallel to the line

$\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z+1}{-2}$

so 2a – b – 2c = 0

$a - c = \frac{b}{2}$ ... (2)

$\Rightarrow$ $\left|\frac{a-3b-c}{\sqrt{a^2+b^2+c^2}}\right| = \frac{5}{3}$

$\Rightarrow \quad \left|\dfrac{\frac{b}{2}-3b}{\sqrt{\frac{5b^2}{4}+2ac}}\right| = \dfrac{5}{3}$ as $a^2 + c^2 = \dfrac{b^2}{4}+2ac$

$\Rightarrow \quad ac = \dfrac{b^2}{2}$

$\Rightarrow \quad (a + c)^2 = (a - c)^2 + 4ac = \dfrac{b^2}{4}+2b^2 = \dfrac{9b^2}{4}$

$\Rightarrow \quad (a + c) = \dfrac{3b}{2}\ ;\ \dfrac{-3b}{2}$

$\therefore$ if a = b ; and $c = \dfrac{b}{2}$

$\therefore$ equation of plane $2x + 2y + z = 0$

or

$\therefore$ if $a = \dfrac{-b}{2}$; and $c = -b$

$\therefore$ equation of plane $x - 2y + 2z = 0$

40. **Sol.** $L_1 : \dfrac{x+4}{3} = \dfrac{y+6}{5} = \dfrac{z-1}{-2} = r$

$L_2 : 3x - 2y + z + 5 = 0 = 2x + 3y + 4z - k$

Any point on the first line is $(3r - 4, 5r - 6, - 2r + 1)$

As lines are coplanar therefore this point must lie on both the planes representing the second line

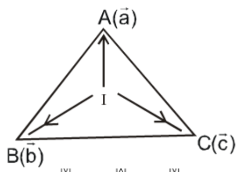
$3(3r - 4) - 2(5r - 6) + (- 2r + 1) + 5 = 0 \Rightarrow r = 2$

and $2(3r - 4) + 3(5r - 6) + 4(- 2r + 1) - k = 0 \Rightarrow k = 4$

PART - II : MISCELLANEOUS QUESTIONS

A-1. **Ans.** **(2)**

Sol.



$I \equiv \dfrac{a\vec{a}+b\vec{b}+c\vec{c}}{a+b+c}$

A-2. **Ans.** **(1)**

Sol. Statement-1

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2 \Rightarrow \cos^2\alpha + \cos^2\beta \neq \cos^2\gamma = 1$ (True)

Statement-2

$\ell^2 + m^2 + n^2 = 1$ (True)

$\ell = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma \Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Statement - 2 explains statement - 1.

A-3. **Ans.** **(2)**

Sol. Statement-1

$xy + yz = 0$

x = 0, y + z = 0
1, 0,0 0, 1, 1
$a_1a_2 + b_1b_2 + c_1c_2 = 0$
Planes are perpendicular. (True)
Statement-2 Obviously false

A-4. **Ans.** **(3)**

Sol. $\vec{a} + 5\vec{b} + 3\vec{c} = 0 \Rightarrow \vec{a} = -(5\vec{b} + 3\vec{c})$

$\vec{a}.(\vec{b} \times \vec{c}) = -(5\vec{b} + 3\vec{c}) . (\vec{b} \times \vec{c}) = 0$

while $\vec{b}.(\vec{a} + \vec{c}) \neq 0$ in general

Hence S_1 is false, S_2 is standard result.

A-5. **Ans.** **(1)**

Sol. Both the statements are independently true

$$\text{S.D.} = \left| \frac{\begin{vmatrix} -1 & 6 & -7 \\ -4 & 3 & 2 \\ -4 & 1 & 1 \end{vmatrix}}{\sqrt{\Sigma(mn' - m'n)^2}} \right| = \left| \frac{-1(1) - 6(4) - 7(8)}{\sqrt{1^2 + (4)^2 + (8)^2}} \right| = \frac{81}{9} = 9$$

Section (B) : MATCH THE COLUMN

Note : Only one answer type (1 × 1)

B-1. **Sol.** **Obvious**

B-2. **Ans.** **(A) → r ; (B) → s ; (C) → p ; (D) → q**

Sol. (A) $[\vec{a}\,\vec{b}\,\vec{c}] = 2$

$2(\vec{a} \times \vec{b}), 3(\vec{b} \times \vec{c}), (\vec{c} \times \vec{a})$

$6\,[\vec{a} \times \vec{b} \;\; \vec{b} \times \vec{c} \;\;] = 6\,[\vec{a}\;\vec{b}\;\vec{c}]^2$

$= 6 \times 4 = 24$

$P \rightarrow 3$

(B) $[\vec{a}\,\vec{b}\,\vec{c}] = 5$

$[3(\vec{a} + \vec{b})\;(\vec{b} + \vec{c})\;\;2(\vec{c} + \vec{a})]$

$= 6 \times 2\,[\vec{a}\,\vec{b}\,\vec{c}]$

$= 12 \times 5 = 60$

$Q \rightarrow 4$

(C) $\frac{1}{2}|\vec{a} \times \vec{b}| = 20$

$\Delta_1 = \frac{1}{2}|(2\vec{a} + 3\vec{b}) \times (\vec{a} - \vec{b})|$

$= \frac{1}{2}|-2\vec{a} \times \vec{b} - 3(\vec{a} \times \vec{b})|$

$= \frac{5}{2} |\vec{a} \times \vec{b}|$

= 5 × 20 = 100

R → 1

(D) $|\vec{a} \times \vec{b}| = 30$

$|(\vec{a} + \vec{b}) \times \vec{a}| = |\vec{b} \times \vec{a}| = 30$

S → 2

B-3. **Ans.** (A) → q, (B) → p, (C) → (s), (D) → (r)

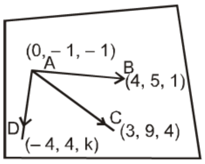
Sol. (A) $\vec{x} \times \vec{b} = \vec{a}$ and $\vec{a} . \vec{b} = 0$ $\Rightarrow \vec{x} = \alpha\vec{b} + \beta\vec{a} + \gamma\vec{a} \times \vec{b}$

$\Rightarrow \gamma = -\frac{1}{|\vec{b}|^2}$ & $\beta = 0 \Rightarrow$ $\vec{x} = \alpha\vec{b} - \frac{1}{|\vec{b}|^2} . \vec{a} \times \vec{b}$

(B) $\vec{b} \times (\vec{b} \times \vec{x}) = \vec{b} \times \vec{a}$ & $\beta\vec{b} - |\vec{b}|^2 \vec{x} = \vec{b} \times \vec{a}$

$\vec{x} = \frac{\beta\vec{b}}{|\vec{b}|^2} + \frac{\vec{a} \times \vec{b}}{|\vec{b}|^2}$

(C) $\begin{vmatrix} -4 & 5 & k+1 \\ 3 & 10 & 5 \\ 4 & 6 & 2 \end{vmatrix} = 0$



– 4(20 – 30) – 5(6 – 20) + (k + 1) (18 – 40) = 0 ⇒ 40 + 70 – 22 (k + 1) = 0 ⇒ k = 4

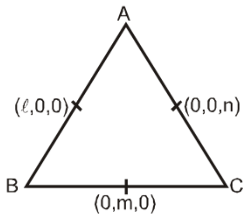
(D) AB = 2EF = $2\sqrt{m^2 + n^2}$

Similarly

BC = 2DF = $2\sqrt{\ell^2 + n^2}$

CA = $2\sqrt{\ell^2 + m^2}$

$AB^2 + BC^2 + CA^2 = 8(\ell^2 + m^2 + n^2)$



$\therefore \frac{AB^2 + BC^2 + CA^2}{\ell^2 + m^2 + n^2} = 8$

B-4. **Ans.** (A) → r ; (B) → q ; (C) → s ; (D) → p

Sol. $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$

Normal of plane P : $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix}$

$= \hat{i}\,(-16) - \hat{j}\,(-42 - 6) + \hat{k}\,(32)$

$= -16\hat{i} + 48\hat{j} + \hat{k}\,32$

$\Rightarrow \vec{n} = \hat{i} - 3\hat{j} - 2\hat{k}$

Point of intersection of L_1 and L_2

$2k_1 + 1 = k_2 + 4$

$-k_1 = k_2 - 3$

$1 = 3k_2 - 2$

$k_2 = 1$

Point of intersection (5, –2, –1)

Plane $(x-5)-3(y+7)-2(z+1)=0$

$x-3y-2z-5-6-2=0$

$x-3y-2z=13$

$\Rightarrow$ $a=1,\ b=3,\ c=-2,\ d=13$

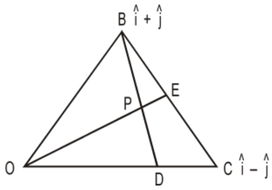
Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

CORRECT½

C-1. **Sol.** Here D$\left(\dfrac{2\hat{i}-2\hat{j}}{3}\right)$ and E$\left(\dfrac{\hat{i}-\hat{j}+2\hat{i}+2\hat{j}}{3}\right)$ or E$\left(\dfrac{3i+j}{3}\right)$

Now, let P divides $\overrightarrow{BD}$ in $\lambda : 1$ and P divides $\overrightarrow{OE}$ in $1 : \mu$.

So, p$\left(\dfrac{\lambda\dfrac{(2\hat{i}-2\hat{j})}{3}+\hat{i}+\hat{j}}{\lambda+1}\right) = \dfrac{3\hat{i}+\hat{j}}{3(\mu+1)}$



Solving we get $\lambda = 3 : 4$

$\mu = 1 : 6.$

C-2 **Sol.** **(3, 4)**

Let $\vec{r}$ be inclined at an angle α to each axis, then

$\ell = m = n = \cos\alpha$

Since $\ell^2 + m^2 + n^2 = 1 \Rightarrow 3\cos^2\alpha = 1$

$\Rightarrow$ $\cos\alpha = \pm\dfrac{1}{\sqrt{3}}$

So, $\ell = m = n = \dfrac{1}{\sqrt{3}}$ or $\ell = m = n = \dfrac{-1}{\sqrt{3}}$

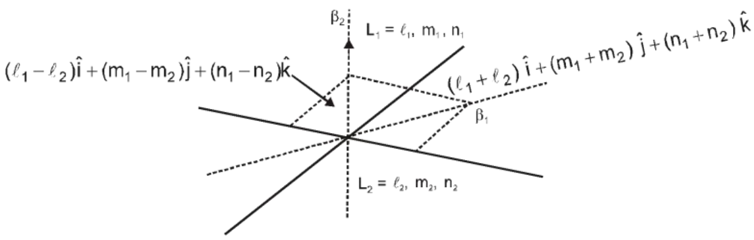
Hence, $\vec{r} = \pm 2\sqrt{3}\ (\hat{i}+\hat{j}+\hat{k})$.

C-3. **Sol.** $\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2$

Dc's of β_1(bisector)

$= \dfrac{\ell_1+\ell_2}{\sqrt{(\ell_1+\ell_2)^2+(m_1+m_2)^2+(n_1+n_2)^2}}$

$= \dfrac{\ell_1+\ell_2}{\sqrt{2+2(\ell_1\ell_2+m_1m_2+n_1n_2)}} = \dfrac{\ell_1+\ell_2}{\sqrt{2+2\cos\theta}} = \dfrac{\ell_1+\ell_2}{2\cos\theta/2}$



Similarly $\dfrac{m_1+m_2}{2\cos\theta/2}, \dfrac{n_1+n_2}{2\cos\theta/2}$

Similarly Dc's for bisector β_2

$\frac{\ell_1 - \ell_2}{2\sin\frac{\theta}{2}}, \frac{m_1 - m_2}{2\sin\frac{\theta}{2}}, \frac{n_1 - n_2}{2\sin\frac{\theta}{2}}$

C-4. **Sol.** Since $\vec{a}$ and $\vec{b}$ are non- collinear and $\vec{a}\lambda = \vec{b}\mu$ then λ = 0, μ = 0.

∴ $\vec{a}\cdot\vec{b} - \frac{\sqrt{3}}{2} = 0$ and $\vec{b}\cdot\vec{c} - \frac{1}{2} = 0$

⇒ $\alpha = \frac{\pi}{6}$ and $\beta = \frac{\pi}{3}$

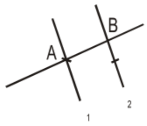
C-5. **Sol.** Let equation of line ℓ is

$\ell : \frac{x-0}{a} = \frac{y-0}{b} = \frac{z-0}{c}$ k

This line ℓ is perpendicular to given line ℓ_1 and ℓ_2.

Hence a + 2b + 2c = 0
2a + 2b + c = 0

$\frac{a}{-2} = \frac{b}{3} = \frac{c}{-2}$



Hence equation of ℓ is $\frac{x}{-2} = \frac{y}{3} = \frac{z}{-2} = k_1, \quad k_2$

↓ ↓
for ℓ_1 for ℓ_2

Now A($-2k_1, 3k_1, -2k_1$) B($-2k_2, 3k_2, -2k_2$)

Point A satisfied ℓ_1

$-2k_1\hat{i} + 3k_1\hat{j} - 2k_1\hat{k} = (3 + t)\hat{i} + (-1 + 2t)\hat{j} + (4 + 2t)\hat{k}$

$3 + t = -2k_1$ ....(1)
$-1 + 2t = 3k_1$ ....(2)
$4 + 2t = -2k_1$ ....(3)

(2) & (3) $-5 = 5k_1 \Rightarrow k_1 = -1 \Rightarrow$ A (2, –3, 2)

Let any point on ℓ_2 (3 + 2S, 3 + 2S, 2 + S)

Given $= \sqrt{(1+2S)^2 + (6+2S)^2 + (S)^2} = \sqrt{17}$

$9S_2 + 28S + 37 = 17$
$9S_2 + 28S + 20 = 0$
$9S_2 + 18S + 10S + 20 = 0$
$9S(S + 2) + 10 (S + 2) = 0$
$S = -2, -10/9$

Hence (–1, –1, 0) , (7/9, 7/9, 8/9)

C-6 **Sol.** Let $\vec{a} = (\vec{a}\cdot\hat{i})\hat{i} + (\vec{a}\cdot\hat{j})\hat{j} + (\vec{a}\cdot\hat{k})\hat{k} = (1)\hat{i} + (1)\hat{j} + (1)\hat{k} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = (\vec{b}\cdot\hat{i})\hat{i} + (\vec{b}\cdot\hat{j})\hat{j} + (\vec{b}\cdot\hat{k})\hat{k} = (1)\hat{i} + (-1)\hat{j} + 0\hat{k} = \hat{i} - \hat{j}$

and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$

Then, $\vec{a}, \vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$

∴ $\vec{a}\cdot\vec{b}\times\vec{c}$ = 6,

∴ $\vec{a}\cdot\vec{b}$ and $\vec{c}$ are mutually perpendicular.

Also, $\vec{a}\cdot\vec{b}\times\vec{c} = 6$

$\therefore$ $\vec{a}, \vec{b}, \vec{c}$, form the parallelopiped of volume 6 units.

C-7. **Sol.** Volume = $\frac{1}{6}[\overrightarrow{OA}\ \overrightarrow{OB}\ \overrightarrow{OC}]$

$\therefore$ $\frac{5}{6} = \frac{1}{6}\begin{vmatrix} 1 & -1 & 1 \\ \lambda & 0 & 1 \\ 0 & 1 & \lambda \end{vmatrix}$

$\Rightarrow$ $\lambda^2 + \lambda - 6 = 0 \Rightarrow (\lambda - 2)(\lambda + 3) = 0 \Rightarrow \lambda = 2, -3.$

C-8. **Ans.** **(1, 4)**

Sol. $\bar{a} = \hat{i} + \hat{j} + 2\hat{k}$

$\bar{b} = \hat{i} + 2\hat{j} + \hat{k}$

$\bar{c} = \hat{i} + \hat{j} + \hat{k}$

Required vector is $\lambda\vec{c} \times (\vec{a}\times\vec{b})$

$\lambda[(\vec{c}.\vec{b})\vec{a} - (\vec{c}.\vec{a})\vec{b}]$

$\lambda[(1+2+1)(\hat{i}+\hat{j}+2\hat{k}) - (1+1+2)(\hat{i}+2\hat{j}+\hat{k})]$

$\lambda[-4\hat{j}+4\hat{k}]$

so our vector is parallel to $-\hat{j} + \hat{k}$

C-9. **Sol.** We have $\vec{x}\times\vec{b} = \vec{c}\times\vec{b}$

$\Rightarrow$ $\vec{a}\times(\vec{x}\times\vec{b}) = \vec{a}\times(\vec{c}\times\vec{b})$ $\Rightarrow$ $(\vec{a}\cdot\vec{b})\vec{x} - (\vec{a}\cdot\vec{x})\vec{b} = \vec{a}\times(\vec{c}\times\vec{b})$

$\Rightarrow$ $\vec{x} = \frac{\vec{a}\times(\vec{c}\times\vec{b})}{\vec{a}\cdot\vec{b}}$ $(\because \vec{a}\perp\vec{x})$

Also, $\vec{a}\times(\vec{c}\times\vec{b}) = -(\vec{c}\times\vec{b})\times\vec{a} = (\vec{b}\times\vec{c})\times\vec{a}$

$\therefore$ $\vec{x} = \frac{(\vec{b}\times\vec{c})\times\vec{a}}{\vec{b}\cdot\vec{a}}$.

C-10. **Sol.** Vector $\overrightarrow{AB}$ is parallel to $\left[(2\hat{i}+3\hat{k})\times(4\hat{j}-3\hat{k})\right] \times \left[(\hat{j}-\hat{k})\times(3\hat{i}+3\hat{j})\right] = 54(\hat{j}-\hat{k})$

Let θ is the angle between the vectors, then

$\cos\theta = \pm\left(\frac{54+108}{3.54\sqrt{2}}\right) = \pm\frac{1}{\sqrt{2}}$

Hence $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$.

C-12. **Sol.** For co-planer lines $[\vec{a}-\vec{c}\ \vec{b}\ \vec{d}] = 0$

$\vec{a} \equiv (1, -1, 0)$, $= \vec{c}$ $(-1, -1, 0)$

$\vec{b} = 2\hat{i} + k\hat{j} + 2\hat{k}$ $\vec{d} = 5\hat{i} + 2\hat{j} + k\hat{k}$

Now $\begin{vmatrix} 2 & 0 & 0 \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix} = 0 \quad \Rightarrow \quad k = \pm 2$

$\vec{n}_1 = \vec{b}_1 \times \vec{d}_1 = 6\hat{j} - 6\hat{k}$, for $k = 2$

$\vec{n}_2 = = \vec{b}_2 \times \vec{d}_2 \; 14\hat{j} + 14\hat{k}$, for $k = -2$

so the equation of planes are $(\vec{r} - \vec{a}).\vec{n}_1 = 0 \Rightarrow y - z = -1$ ...... (1)

$(\vec{r} - \vec{a}).\vec{n}_2 = 0 \Rightarrow y + z = -1$ ...... (2)

so answer is (B,C)

C-13. Sol. $\frac{x-5}{0} = \frac{-y}{\alpha - 3} = \frac{z}{-2}$

$\frac{x-\alpha}{0} = \frac{y}{-1} = \frac{z}{2-\alpha}$

$\begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$

$(5 - \alpha)\,((3 - \alpha)\,(2 - \alpha) - 2) = 0$

$(\alpha^2 - 5\alpha + 6 - 2) = 0$

$(\alpha - 5)(\alpha^2 - 5\alpha + 4) = 0$

C-14. Sol. Let equation of plane is $ax + by + cz + d = 0$

Normal vector of plane $= \vec{n} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ -1 & 1 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 3i - 3j - 3k$

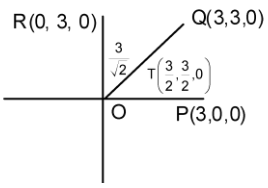
and plane passes through (1,1, 0)

so equation of plane is $x - y - z = 0$

$\therefore \quad p = 0, q = \frac{1}{\sqrt{3}}$

C-15. Ans. (2,3,4)

Sol.



$S \equiv \left(\frac{3}{2}, \frac{3}{2}, 3\right) \Rightarrow \overrightarrow{OQ} = 3\hat{i} + 3\hat{j} \quad \Rightarrow \quad \overrightarrow{OS} = \frac{3}{2}\hat{i} + \frac{3}{2}\hat{j} + 3\hat{k}$

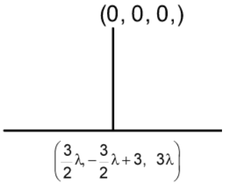
$\cos\theta = \frac{\frac{1}{2} + \frac{1}{2}}{\sqrt{2}\sqrt{\frac{1}{2} + \frac{1}{4} + 1}} = \frac{1}{\sqrt{2}\sqrt{\frac{3}{2}}} = \frac{1}{\sqrt{3}}$

$\vec{n} = \overrightarrow{OQ} \times \overrightarrow{OS} = (\hat{i} + \hat{j}) \times (\hat{i} + \hat{j} + 2\hat{k}) = \hat{k} - 2\hat{j} - \hat{k} + 2\hat{i} \Rightarrow 2\hat{i} - 2\hat{j}$

$x - y = \lambda \quad \Rightarrow \quad x = y \quad \Rightarrow \quad \perp \; (3, 0, 0) \Rightarrow \frac{3}{\sqrt{2}}$

RS → $\dfrac{x-0}{\frac{3}{2}} = \dfrac{y-3}{-\frac{3}{2}} = \dfrac{z-0}{3} = \lambda$ ⇒ $x = \frac{3}{2}\lambda,\ y = -\frac{3}{2}\lambda + 3,\ z = 3\lambda$

T distance ⇒ $\sqrt{\frac{3}{2} - 3 + 9}$ ⇒ $\sqrt{\frac{15}{2}}$



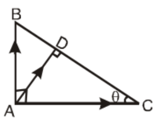
$D = \frac{9}{4}\lambda^2 + \left(3 - \frac{3}{2}\lambda\right)^2 + 9\lambda^2 = \frac{27}{2}\lambda^2 - 9\lambda + 9$ ⇒ $\lambda = \frac{9}{27} = \frac{1}{3}$

Exercise-3

1. **Sol.** $\left((\vec{c}.\vec{a})\,\vec{b} - (\vec{c}.\vec{b})\,\vec{a}\right) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$ ⇒ $(\vec{a}\cdot\vec{b})\vec{c} = (\vec{c}.\vec{b})\vec{a}$

2. **Sol.** Let $|\overrightarrow{BC}| = \ell$

∴ In ΔABC



$\ell = \sqrt{AB^2 + AC^2}$ and $\tan\theta = \frac{AB}{AC}$

⇒ $\sin\theta = \frac{AB}{\ell}$ and $\cos\theta = \frac{AC}{\ell}$

∴ Resultant vector

$= \frac{1}{AB}\hat{i} + \frac{1}{AC}\hat{j} = \left(\frac{1}{\ell\sin\theta}\hat{i} + \frac{1}{\ell\cos\theta}\hat{j}\right) = k\,\overrightarrow{AD}$

Now , AD = AC sinθ = ℓ cosθ sinθ = $\frac{AB.AC}{\ell}$ ... (i)

Magnitude of resultant vector

$\sqrt{\frac{1}{\ell^2}\left(\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}\right)} = \frac{\ell}{(AB)(AC)} = \frac{1}{AD}$ [from Eq (i)]

3. **Sol.** $\overrightarrow{AC}\cdot\overrightarrow{BC} = 0$ ⇒ $\{(a-2)\hat{i} - 2\hat{j}\}\cdot\{(a-1)\hat{i} + 6\hat{k}\} = 0$ ⇒ (a – 2) (a – 1) = 0

4. **Sol.** Given equations can be rewritten as $\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c}$

and $\frac{x-b'}{a'} = \frac{y-0}{1} = \frac{z-d'}{c'}$

These lines will be perpendicular, if aa' + cc' + 1 = 0

5. Sol. We know that, the image (x, y, z) of a points (x_1, y_1, z_1) in a plane ax + by + cz + d = 0 is given by

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = \frac{-2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}.$$

Thus, the image of point (–1, 3, 4) in a plane x – 2y = 0 is given by $\frac{x+1}{1} = \frac{y-3}{-2} = \frac{z-4}{0}$

$= \frac{-2[1\times(-1)+(-2)\times 3+0\times 4]}{1+4} \Rightarrow \frac{x+1}{1} = \frac{y-3}{-2} = \frac{z-4}{0} = \frac{-2(-7)}{5}$

$\Rightarrow$ $x = \frac{14}{5} - 1 = \frac{9}{5}$, $y = -\frac{28}{5} + 3 = -\frac{13}{5}$ and z = 4

Hence, the image of point (–1, 3, 4) is $\left(\frac{9}{5}, -\frac{13}{5}, 4\right)$.

6. Sol. $|2\hat{u}\times 3\hat{v}| = 6\sin\theta = 1$ $\Rightarrow$ $\sin\theta = \frac{1}{6}$

as $\theta \in \left(0, \frac{\pi}{2}\right)$

So only one value of θ is possible.

7. Sol. $[\vec{a}\ \vec{b}\ \vec{c}] = 0$ $\Rightarrow$ x = – 2

8. Sol. Let the direction cosines of line L be ℓ, m, n. Since, the line intersect the given planes, then the normal to the planes are perpendicular to the line L.

∴ $2\ell + 3m + n = 0$ .....(i)

and $\ell + 3m + 2n = 0$ .....(ii)

From equations (i) and (ii), we get $\frac{\ell}{3} = \frac{m}{-3} = \frac{n}{3} = k$ (say)

We, know, $\ell^2 + m^2 + n^2 = 1$

∴ $(3k)^2 + (-3k)^2 + (3k)^2 = 1$

$\Rightarrow$ $27k^2 = 1$ $\Rightarrow$ $k = \frac{1}{3\sqrt{3}}$

∴ $\ell = \frac{1}{\sqrt{3}}$ $\Rightarrow$ $\cos\alpha = \frac{1}{\sqrt{3}}$

9. Sol. Since, a line makes an angle of $\frac{\pi}{4}$ with positive directions of each of x and y-axis, therefore

$\alpha = \frac{\pi}{4}$, $\beta = \frac{\pi}{4}$

We know, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

∴ $\cos^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} + \cos^2\gamma = 1$

$\Rightarrow$ $\frac{1}{2} + \frac{1}{2} + \cos^2\gamma = 1$

⇒ $\cos^2\gamma = 0$ ⇒ $\gamma = \frac{\pi}{2}$

10. Sol. Given equation of sphere

$x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$

whose coordinates of centre are (3, 6, 1).

Since, one end of diameter are (2, 3, 5) and let the other end of diameter are (α, β, γ), then

$\frac{\alpha+2}{2} = 3, \quad \frac{\beta+3}{2} = 6, \quad \frac{\gamma+5}{2} = 1$

⇒ α = 4, β = 9 and γ = – 3.

Hence, the coordinates of other point are (4, 9, – 3).

11. Sol. $\vec{a} = \lambda(\hat{b}+\hat{c}) = \lambda\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}} + \frac{\hat{j}+\hat{k}}{\sqrt{2}}\right)$ ⇒ $\vec{a} = \lambda\left(\frac{\hat{i}+2\hat{j}+\hat{k}}{\sqrt{2}}\right)$

Now get α & β

12. Sol. $\cos\theta = \frac{\vec{a}.\vec{c}}{|\vec{a}||\vec{c}|} = \frac{-56|\vec{b}|^2}{56|\vec{b}|^2}$ ⇒ $\cos\theta = -1$ ⇒ $\theta = \pi$

13. Sol. Equation of line passing through (5, 1, a) and (3, b, 1) is

$\frac{x-3}{5-3} = \frac{y-b}{1-b} = \frac{z-1}{a-1}$ ....(i)

Point $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$ satisfies equation (i), we get $-\frac{3}{2} = \frac{\frac{17}{2}-b}{1-b} = \frac{-\frac{13}{2}-1}{a-1}$

⇒ $a - 1 = \frac{\left(-\frac{15}{2}\right)}{\left(-\frac{3}{2}\right)} = 5$

⇒ a = 6

Also, $-3(1-b) = 2\left(\frac{17}{2} - b\right)$

⇒ 3b – 3 = 17 – 2b

⇒ 5b = 20 ⇒ b = 4

14. Sol. Given, $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ ......(i)

and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ ....(i)

Since, lines intersect at a point. Then shortest distance between them is zero.

∴ $\begin{vmatrix} k & 2 & 3 \\ 3 & k & 2 \\ 1 & 1 & -2 \end{vmatrix} = 0$

⇒ $k(-2k-2) - 2(-6-2) + 3(3-k) = 0$

⇒ $-2k^2 - 5k + 25 = 0$

⇒ $2k^2 + 5k - 25 = 0$

⇒ $2k^2 + 10k - 5k - 25 = 0$

⇒ $2k(k + 5) - 5(k + 5) = 0$ ⇒ $k = \frac{5}{2}, -5$

Hence integer value of k is – 5.

15. **Sol.** $3p^2 - pq + 2q^2 = 0$

⇒ $p = 0, q = 0$

16. **Sol.** Parallel vector of line and normal vector of plane are ⊥

$3 + (-15) - 2\alpha = 0$

$\alpha = -6$

Now $2 + 3 - \alpha(-2) + \beta = 0$ [(2, 1, –2) lies on the plane]

⇒ $\beta = -5 + 12 = 7$

17. **Sol.** $\ell r = 6, mr = -3, nr = 2$

$\therefore r^2(\ell^2 + m^2 + n^2) = 36 + 9 + 4 = 49$

⇒ $r = 7$

$<\ell, m, n> \equiv < \frac{6}{7}, -\frac{3}{7}, \frac{2}{7} >$

18. **Ans.** **(1)**

Sol. Let image be (α, β, γ)

$$\frac{\alpha - 1}{1} = \frac{\beta - 3}{-1} = \frac{\gamma - 4}{1} = -2\left(\frac{1 - 3 + 4 - 5}{3}\right)$$

⇒ $\frac{\alpha - 1}{1} = \frac{\beta - 3}{-1} = \frac{\gamma - 4}{1} = 2$

⇒ $\alpha = 3, \beta = 1, \gamma = 6$

⇒ A(3, 1, 6) statement 1 is true

Now midpoint of A(3, 1, 6) and B(1, 3, 4) is (2, 2, 5)

equation of plane is $x - y + z = 5$

coordinates of midpoint lies on the plane so plane bisects the line segment AB. But it is not correct explanation of statement-1

Hence correct option is (1)

19. **Ans.** **(4)**

Sol. Since $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ ⇒ $\vec{a} \times (\vec{a} \times \vec{b}) + \vec{a} \times \vec{c} = \vec{0}$ ⇒ $(\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b} + \vec{a} \times \vec{c} = \vec{0}$

Since $\vec{a} \times \vec{c} = -2\hat{i} - \hat{j} - \hat{k}$

⇒ $3(\hat{j} - \hat{k}) - 2\vec{b} - 2\hat{i} - \hat{j} - \hat{k} = \vec{0}$ ⇒ $\vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$

Hence correct option is (4)

20. **Ans.** **(4)**

Sol. ∵ $\vec{a}, \vec{b}, \vec{c}$ are mutually orthogonal

∴ $\vec{a} \cdot \vec{c} = 0$ ⇒ $\lambda - 1 + 2\mu = 0$ ....(i)

and

$\vec{b} \cdot \vec{c} = 0$ ⇒ $2\lambda + 4 + \mu = 0$ ....(ii)

solving (i) and (ii), we get $\lambda = -3$ and $\mu = 2$

Hence correct option is (4)

21. **Ans. (2)**

Sol. $\ell = \frac{1}{\sqrt{2}}$, $m = -\frac{1}{2}$

$\ell^2 + m^2 + n^2 = 1 \Rightarrow n^2 = \frac{1}{4} \Rightarrow n = \pm\frac{1}{2}$

$\cos\theta = \frac{1}{2}$, $\theta = 60^\circ$ Hence correct option is (2)

22. **Sol. (1)**

$(2\vec{a} - \vec{b}) \cdot [(\vec{a}\times\vec{b}) \times (\vec{a} + 2\vec{b})]$

$= -(2\vec{a} - \vec{b}) \cdot [(\vec{a}+2\vec{b}) \times (\vec{a}\times\vec{b})]$

$= -(2\vec{a} - \vec{b}) \cdot [(\vec{a}+2\vec{b}) \cdot \vec{b}) - ((\vec{a}+2\vec{b}) \cdot \vec{a})\vec{b}]$

$= -(2\vec{a} - \vec{b}) \cdot [(\vec{a}\cdot\vec{b}) + 2\vec{b}\cdot\vec{b})\vec{a} - (\vec{a}\cdot\vec{a} + 2\vec{b}\cdot\vec{a})\vec{b})]$

$= -(2\vec{a} - \vec{b}) \cdot [0 + 2\vec{a} - (0 + \vec{b})]$

$= -(2\vec{a} - \vec{b}) \cdot (2\vec{a} - \vec{b})$

$= -(2\vec{a} - \vec{b})^2 = -4\vec{a}^2 + 4\vec{a}\cdot\vec{b} - \vec{b}^2$

$= -4 + 0 - 1 = -5$ **Ans.**

23. **Sol. (4)**

$\vec{a}\cdot\vec{b} \neq 0$, $\vec{b}\times\vec{c} = \vec{b}\times\vec{d}$, $\vec{a}\cdot\vec{d} = 0$

$(\vec{b}\times\vec{c}) \times \vec{a} = (\vec{b}\times\vec{d}) \times \vec{a}$

$(\vec{b}\cdot\vec{a})\vec{c} - (\vec{c}\cdot\vec{a})\vec{b} = (\vec{b}\cdot\vec{a})\vec{d} - (\vec{d}\cdot\vec{a})\vec{b}$

$\vec{d} = \vec{c} - \left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$

24. **Sol. (4)**

$\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} = 0$

$\Rightarrow$ p (qr – 1) + 1(1 – r) + 1(1 – q) = 0

$\Rightarrow$ pqr – p + 1– r + 1 – q = 0

$\Rightarrow$ pqr – (p + q + r) = – 2

25. **Sol. (3)**

$\vec{a} + 3\vec{b} = \lambda\vec{c}$ ....(1)

$\vec{b} + 2\vec{c} = \mu\vec{a}$ ....(2)

(1) – 3(2) gives $(1 + 3\mu)\vec{a} - (\lambda + 6)\vec{c} = 0$

As $\vec{a}$ and $\vec{c}$ are non collinear

$\therefore 1 + 3\mu = 0$ and $\lambda + 6 = 0$

From (1) $\vec{a} + 3\vec{b} + 6\vec{c} = \vec{0}$

26. **Sol. (1)**

$\frac{x-0}{1} = \frac{y-1}{2} = \frac{z-3}{\lambda}$ .... (1)

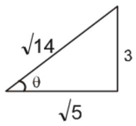
x + 2y + 3z = 4 .... (2)

Angle between the line and plane is

$\cos(90-\theta) = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} = \cos(90-\theta)$

⇒ $\sin\theta = \frac{1+4+3\lambda}{\sqrt{14}\times\sqrt{5+\lambda^2}} = \frac{5+3\lambda}{\sqrt{14}\times\sqrt{5+\lambda^2}}$ .... (3)

But given that angle between line and plane is



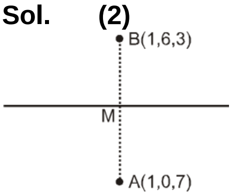
$\theta = \cos^{-1}\left(\sqrt{\frac{5}{14}}\right) = \sin^{-1}\left(\frac{3}{\sqrt{14}}\right)$ ⇒ $\sin\theta = \frac{3}{\sqrt{14}}$

∴ from (3) $\frac{3}{\sqrt{14}} = \frac{5+3\lambda}{\sqrt{14}\times\sqrt{5+\lambda^2}}$

⇒ $9(5+\lambda^2) = 25 + 9\lambda^2 + 30\lambda$ ⇒ $30\lambda = 20$

$\lambda = \frac{2}{3}$ **Ans.**

27. Sol. (2)



Mid- point of AB ≡ M(1,3,5)

M lies on line

Direction ratios of AB is < 0, 6, – 4 >

Direction ratios of given line is < 1, 2, 3 >

As AB is perpendicular to line

∴ 0.1 + 6.2 – 4.3 = 0

28. Sol. (1)

Line through P(1, – 5, 9) parallel to x = y = z is $\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda$ (say)

Q (x = 1 + λ, y = – 5 + λ, z = 9 + λ)

Given plane x – y + z = 5

∴ 1 + λ + 5 – λ + 9 + λ = 5

⇒ λ = –10

∴ Q(–9, – 15, – 1)

∴ $PQ = \sqrt{(1+9)^2 + (15-5)^2 + (9+1)^2} = \sqrt{300} = 10\sqrt{3}$

29. Sol. (3)

Let foot of perpendicular is (2α , 3α + 2, 4α + 3)

⇒ D' ratio of the perpendicular line < 2α – 3, 3α + 3, 4α – 8 >

and D' ratio of the line < 2, 3, 4 >

⇒ 2(2α – 3) + 3 (3α + 3) + 4 (4α – 8) = 0

⇒ 29 α – 29 = 0

$\Rightarrow$ $\alpha = 1$

$\Rightarrow$ feet of perpendicular is (2, 5, 7)

$\Rightarrow$ length of perpendicular is $\sqrt{1^2+6^2+4^2} = \sqrt{53}$

30. Sol. Equation of parallel plane $x - 2y + 2z + d = 0$

Now $\left|\frac{d}{\sqrt{1^2+2^2+2^2}}\right| = 1$

$d = \pm 3$

So equation required plane $x - 2y + 2z \pm 3 = 0$

$\left|\frac{d}{\sqrt{1^2+2^2+2^2}}\right| = 1$

$d = \pm 3$

31. Sol. $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$

= =

$\vec{a}$ (1, – 1, 1); $\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{b}$ (2, 3, 4)

$\vec{c}$ (3, k, 0); $\vec{r} = \vec{c} + \mu\vec{d}$

$\vec{d}$ (1, 2, 1)

These lines will intersect if lines are coplaner

$\vec{a} - \vec{c}$, $\hat{b}$ & $\vec{d}$ are coplaner

$\therefore$ $[\vec{a} - \vec{c}, \vec{b}, \vec{d}] = 0$

$$\begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $2(-5) - (k + 1)(-2) - 1(1) = 0$

$\Rightarrow$ $2(k + 1) = 11$

$\Rightarrow$ $k = \frac{9}{2}$

32. Sol. $\vec{c} = \hat{a} + 2\hat{b}$

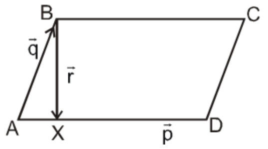
$\vec{d} = 5\hat{a} - 4\hat{b}$

$\vec{c}.\vec{d} = 0$

$\Rightarrow$ $(\hat{a} + 2\hat{b}).(5\hat{a} - 4\hat{b}) = 0$ $\Rightarrow$ $5 + 6\hat{a}.\hat{b} - 8 = 0$

$\Rightarrow$ $\hat{a}.\hat{b} = \frac{1}{2}$ $\Rightarrow$ $\theta = \frac{\pi}{3}$

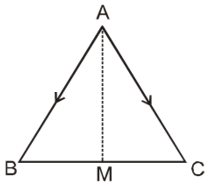
33. Sol. $\overrightarrow{AX} = \frac{\vec{p}.\vec{q}}{|\vec{p}|}\frac{\vec{p}}{|\vec{p}|} = \frac{\vec{p}.\vec{q}}{|\vec{p}|^2}\vec{p}$



$\vec{BX} = \vec{BA} + \vec{AX} = -\vec{q} + \dfrac{\vec{p}.\vec{q}}{|\vec{p}|^2}\vec{p}$

34. Sol. (3)

$\vec{AB} + \vec{BC} + \vec{CA} = 0 \Rightarrow \vec{BC} = \vec{AC} - \vec{AB}$



$\Rightarrow \vec{BM} = \dfrac{\vec{AC} - \vec{AB}}{2}$

$\Rightarrow \vec{AB} + \vec{BM} + \vec{MA} = 0 \Rightarrow \vec{AB} + \dfrac{\vec{AC} - \vec{AB}}{2} = \vec{AM} \Rightarrow \vec{AM} = \dfrac{\vec{AB} + \vec{AC}}{2} = 4\hat{i} - \hat{j} + 4\hat{k}$

$\Rightarrow |\vec{AM}| = \sqrt{33}$

35. Sol. (3)

$2x + y + 2z - 8 = 0$ ...(P_1)

$2x + y + 2z + \dfrac{5}{2} = 0$ ...(P_2)

Distance between P_1 and P_2 = $\left|\dfrac{-8 - \frac{5}{2}}{\sqrt{2^2 + 1^2 + 2^2}}\right| = \dfrac{7}{2}$

36. Sol. (3)

$\left[\vec{a} - \vec{c}\ \ \vec{b}\ \ \vec{d}\right] = 0$

$\begin{vmatrix} 2-1 & 3-4 & 4-5 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$

$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow 1(1 + 2k) + (1 + k_2) - (2 - k) = 0$

$\Rightarrow k_2 + 2k + k = 0$

$\Rightarrow k_2 + 3k = 0$

$\Rightarrow k = 0, -3$

Note : If 0 appears in the denominator, then the correct way of representing the equation of straight line is $\dfrac{x-2}{1} = \dfrac{y-3}{1}$; $z = 4$

37. Sol. Ans. (2)

LHS = $[\vec{a} \times \vec{b}\ \ \vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}]$

$= [\vec{p}\ \ \vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}]$ (where $\vec{p} = \vec{a} \times \vec{b}$)

$= \{\vec{p} \times (\vec{b} \times \vec{c})\}.(\vec{c} \times \vec{a})$

$= \{(\vec{p}.\vec{c})\vec{b} - (\vec{p}.\vec{b})\}.(\vec{c} \times \vec{a})$

$= \{[\vec{a}\ \vec{b}\ \vec{c}]\vec{b} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{c}\}.(\vec{c} \times \vec{a})$ (As $\vec{p} = \vec{a} \times \vec{b}$)

$= [\hat{a}\ \hat{b}\ \hat{c}][\hat{b}\ \hat{c}\ \hat{a}] - 0$ ($\because [\hat{a}\ \hat{b}\ \hat{b}] = 0$)

$= [\hat{a}\ \hat{b}\ \hat{c}]^2$ ($\because [\hat{b}\ \hat{c}\ \hat{a}] = [\hat{a}\ \hat{b}\ \hat{c}]$)

RHS $= \lambda [\hat{a}\ \hat{b}\ \hat{c}]^2$

$\therefore \lambda = 1$

38. Sol. Ans. (3)

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \frac{-2(2-3+4+3)}{4+1+1}$$

$$\Rightarrow \frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = -2$$

Image of point (1, 3, 4) on given line in the given plane is (–3, 5, 2)

Line is parallel to given plane 3, 1, –5

So, image $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

39. Sol. Ans. (3)

$\ell + m + n = 0$ ....(1)

$\ell^2 = m^2 + n^2$ ....(2)

$\Rightarrow \ell^2 - m^2 - (-\ell - m)^2 = 0$

$\Rightarrow 2m(m + \ell) = 0$

$m = 0$ or $\ell = -m$

so direction ratios are – 1, 0, 1 and – 1, 1, 0

$$\cos\theta = \left(\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right)$$

$$\Rightarrow \cos\theta = \left|\frac{1+0+0}{\sqrt{2}\ \ \sqrt{2}}\right| = \frac{1}{2}$$

$$\therefore \theta = \frac{\pi}{3}$$

40. Ans. (4)

Sol. Point of intersection

$(3\lambda + 2, 4\lambda - 1, 12\lambda + 2)$

$3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 = 16$

$11\lambda = 11 \Rightarrow \lambda = 1$

(5, 3, 14)

Distance $= \sqrt{16+9+144} = \sqrt{169} = 13$

41. Ans. (3)

Sol. Equation of real plane

$2x - 5y + z - 3 + \lambda(x + y + 4z - 5) = 0$

$x(2 + \lambda) + y(\lambda - 5) + z(4\lambda + 1) - 3 - 5\lambda = 0$

$$\Rightarrow \frac{\lambda+2}{1} = \frac{\lambda-5}{3} = \frac{4\lambda+1}{6} = -3 + \frac{55}{2}$$

$3\lambda + 6 = \lambda - 5$

$2\lambda = -11$

$\lambda = \frac{-11}{2}$

$\Rightarrow$ equation of plane $\frac{-7x}{2} - \frac{21y}{2} - 21z + \frac{49}{2} = 0$ $\Rightarrow$ $7x + 21y + 42z - 49 = 0$

$\Rightarrow$ $x + 3y + 6z - 7 = 0$

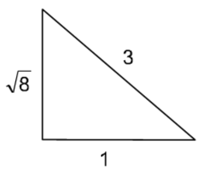
42. **Ans.** **(1)**

Sol. $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|b||c|\vec{a}$

$-\vec{c} \times (\vec{a} \times \vec{b})$

$-(\vec{c}.\vec{b})\vec{a} + (\vec{c}.\vec{a})\vec{b} = \frac{1}{3}|b||c|\vec{a}$

$\left(\frac{1}{3}|b||c| + (\vec{c}.\vec{b})\right)\vec{a} = (\vec{c}.\vec{a})\vec{b}$



Since $\vec{a}$ & $\vec{b}$ are not collinear

$\vec{c}.\vec{b} + \frac{1}{3}|b||c| = 0$ & $\vec{c}.\vec{a} = 0$

$\cos\theta + \frac{1}{3} = 0$

$\cos\theta = -\frac{1}{3}$ $\Rightarrow$ $\sin\theta = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}$

Aliter : $(\vec{a} \times \vec{b})\vec{c}$

$= (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$(\vec{a}.\vec{c})\vec{b} = \left(\frac{1}{3}|\vec{b}||\vec{c}| + \vec{b}.\vec{c}\right)\vec{a}$

$\vec{a}.\vec{c} = 0$

& $\frac{1}{3}|\vec{b}||\vec{c}| + \vec{b}.\vec{c} = 0$

$\frac{1}{3}|\vec{b}||\vec{c}| + |\vec{b}||\vec{c}| \cos\theta = 0$

$\cos\theta = -\frac{1}{3}$

$\sin\theta = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$

43. **Ans.** **(3)**

Sol. (i) (3, – 2, –4) lies on the plane $\therefore 3\ell - 2m + 4 = 9$ $\Rightarrow 3\ell - 2m = 5$ .... (i)

(ii) $2\ell - m - 3 = 0$ $\Rightarrow 2\ell - m = 3$ .... (ii)

from (i) and (ii) (i) (ii) $\ell = 1$ and $m = -1$

44. **Ans.** **(3)**

Sol. $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$

$(\vec{a}.\vec{c}) - (\vec{a}.\vec{b})\vec{c} = \frac{\sqrt{3}}{2}\vec{b} + \frac{\sqrt{3}}{2}\vec{c}$

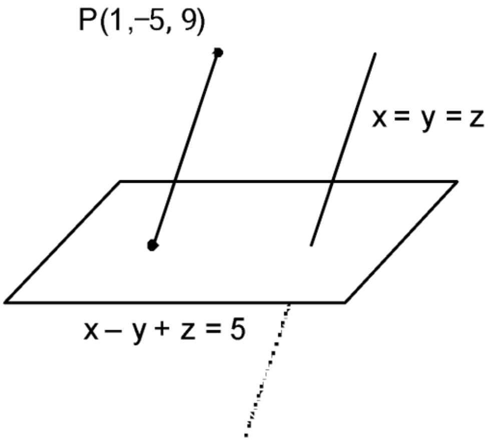
Hence $\vec{a}.\vec{c} = \frac{\sqrt{3}}{2}$ and $\vec{a}.\vec{b} = -\frac{\sqrt{3}}{2}$

$\vec{a}.\vec{b} = -\frac{\sqrt{3}}{2}$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$\theta = \frac{5\pi}{6}$

45. **Ans.** **(1)**



Sol.

Equation of line PQ : $\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda$

∴ Q can be taken as $(\lambda + 1, \lambda - 5, \lambda + 9)$

∴ Q $(\lambda + 1, \lambda - 5, \lambda + 9)$

As Q lies on plane $x - y + z = 5$

∴ $(\lambda + 1) - (\lambda - 5) + (\lambda + 9) = 5$

$\lambda = -10 \quad \Rightarrow \quad Q(-9, -15, -1)$

∴ Required distance PQ = $\sqrt{(1+9)^2 + (-5+15)^2 + (9+1)^2} = \sqrt{100+100+100} = 10\sqrt{3}$

46. **Ans.** **(2)**

Sol. Let R be the point of intersection of plane and line passing through P and parallel to given line.

So, R is $(1 + \lambda, -2 + 4\lambda, 3 + 5\lambda)$

substituting co-ordinates of R in plane

$2 + 2\lambda - 6 + 12\lambda - 12 - 20\lambda + 22 = 0 \quad \Rightarrow 6\lambda = 6 \Rightarrow \lambda = 1$

So , R is (2,2,8)

Hence PR = $\sqrt{1+16+25} = \sqrt{42}$

So, PQ = $2\sqrt{42}$

47. **Ans.** **(2)**

Sol. Let the plane be

$a(x - 1) + b(y + 1) + c(z + 1) = 0$

$a - 2b + 3c = 0$

$2a - b - c = 0$

$\frac{a}{5} = \frac{b}{7} = \frac{c}{3}$

$5(x - 1) + 7(y + 1) + 3(z + 1) = 0$

$5x + 7y + 3z + 5 = 0$

P(1, 3, –7)

$d = \left|\frac{5+21-21+5}{\sqrt{25+49+9}}\right| = \left|\frac{10}{\sqrt{83}}\right|$

48. **Ans. (2)**

Sol. $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k},\ \vec{b} = \hat{i} + \hat{j},\ |\vec{c} - \vec{a}| = 3$

$|(\vec{a}\times\vec{b})\times\vec{c}| = 3$, $\vec{c} \wedge \vec{a}\times\vec{b} = \frac{\pi}{6}$

Now $|\vec{a}\times\vec{b}||\vec{c}|\ \sin 30^0 = 3$, $|\vec{a}\times\vec{b}||\vec{c}| = 6$

$\Rightarrow |\vec{a}||\vec{b}||\vec{c}|\sin\theta = 6$, $\theta = \vec{a} \wedge \vec{b}$

$|\vec{a}| = 3$, $|\vec{b}| = \sqrt{2}$ $\theta = \cos^{-1}\left(\frac{2+1}{3\sqrt{2}}\right) = \frac{\pi}{4}$

$|\vec{c}| = \frac{6}{3\sqrt{2}}.\sqrt{2} = 2$

$|\vec{c} - \vec{a}| = 3$

Squaring, we get $|\vec{c}|^2 - 2\vec{a}.\vec{c} + |\vec{a}|^2 = 9 \Rightarrow \vec{a}.\vec{c} = \frac{|\vec{c}|^2}{2} = 2$

Exercise-3

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. **Sol.** Vector lying in the plane of $\vec{a}$ and $\vec{b}$ is $\vec{r} = \lambda_1\vec{a} + \lambda_2\vec{b}$ and its projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$

$\Rightarrow \left|\left[(\lambda_1+\lambda_2)\ \hat{i} + (2\lambda_1 - \lambda_2)\ \hat{j} + (\lambda_1+\lambda_2)\ \hat{k}\right] . \frac{[\hat{i}+\hat{j}-\hat{k}]}{\sqrt{3}}\right| = \frac{1}{\sqrt{3}}$

$\Rightarrow |2\lambda_1 - \lambda_2| = 1 \Rightarrow 2\lambda_1 - \lambda_2 = \pm 1$

taking negative sign $2\lambda_1 - \lambda_2 = -1$

$\Rightarrow \vec{r} = (3\lambda_1 + 1)\ \hat{i} - \hat{j} + (3\lambda_1 + 1)\ \hat{k}$

Hence the required vector is $4\hat{i} - \hat{j} + 4\hat{k}$

Alternate :

Vector lying in the plane of $\vec{a}$ and $\vec{b}$ is $\vec{a} + \lambda\vec{b}$, and its projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$

$\Rightarrow \left((1+\lambda)\ \hat{i} + (2-\lambda)\ \hat{j} + (1+\lambda)\ \hat{k}\ .\frac{(\hat{i}-\hat{j}-\hat{k})}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}}$

$\Rightarrow \lambda = 3$

Hence the required vector is $4\hat{i} - \hat{j} + 4\hat{k}$

2. Sol. For coplanarity,

$$\begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0 \quad \Rightarrow \quad \lambda^6 - 3\lambda^2 - 2 = 0$$

$(\lambda^2 - 2)(\lambda^4 + 2\lambda^2 + 1) = 0$

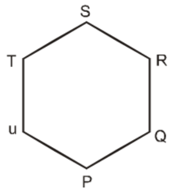
$\lambda^2 = 2, -1 \quad \Rightarrow \lambda = \pm\sqrt{2}$

3. Sol. Statement - 1

$\because \quad \overrightarrow{RS} + \overrightarrow{ST} = \overrightarrow{RT}$

and $\overrightarrow{RT}$ is not parallel to $\overrightarrow{PQ}$

so $\quad \overrightarrow{PQ} \times \left(\overrightarrow{RS} + \overrightarrow{ST}\right) \neq 0$



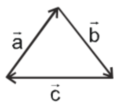
Statement - 2

while PQ & RS are also non parallel

So $\quad \overrightarrow{PQ} \times \overrightarrow{RS} \neq 0, \ \overrightarrow{PQ} \times \overrightarrow{ST} = 0$

4. Sol. $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\vec{a} + \vec{b} = -\vec{c}$



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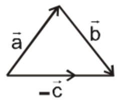


For a triangle

$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq 0$

5. Sol. Volume of parallelopiped = $|[\hat{a} \ \hat{b} \ \hat{c}]|$

Now, $$[\hat{a} \ \hat{b} \ \hat{c}]^2 = \begin{vmatrix} \hat{a}\cdot\hat{a} & \hat{a}\cdot\hat{b} & \hat{a}\cdot\hat{c} \\ \hat{b}\cdot\hat{a} & \hat{b}\cdot\hat{b} & \hat{b}\cdot\hat{c} \\ \hat{c}\cdot\hat{a} & \hat{c}\cdot\hat{b} & \hat{c}\cdot\hat{c} \end{vmatrix} = \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{vmatrix} = \frac{1}{2}$$

Hence volume = $\frac{1}{\sqrt{2}}$

6. Sol. **(C)**

$\ell = m = n = \frac{1}{\sqrt{3}}$

$2x + y + z = 9$

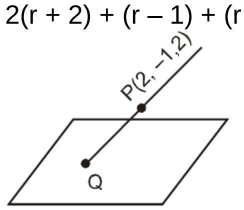
∴ equations of line are $\frac{x-2}{1/\sqrt{3}} = \frac{y+1}{1/\sqrt{3}} = \frac{z-2}{1/\sqrt{3}}$

$x - 2 = y + 1 = z - 2 = r$

$Q \equiv (r + 2, r - 1, r + 2)$

∵ Q Lies on the plane $2x + y + z = 9$

$2(r + 2) + (r - 1) + (r + 2) = 9$



⇒ $4r + 5 = 9$ ⇒ $r = 1$

∴ Q (3, 0, 3)

∴ $PQ = \sqrt{1+1+1} = \sqrt{3}$

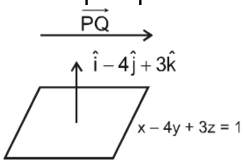
7. Sol. Given $\overrightarrow{OQ} = (1-3\mu)\hat{i} + (\mu-1)\hat{j} + (5\mu+2)\hat{k}$, $\overrightarrow{OP} = 3\hat{i} + 2\hat{j} + 6\hat{k}$ (where O is origin)

$\overrightarrow{PQ} = (1-3\mu-3)\hat{i} + (\mu-1-2)\hat{j} + (5\mu+2-6)\hat{k}$

$= (-2-3\mu)\hat{i} + (\mu-3)\hat{j} + (5\mu-4)\hat{k}$

∵ $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$

∴ $-2-3\mu-4\mu+12+15\mu-12 = 0$



⇒ $8\mu = 2$ ⇒ $\mu = \frac{1}{4}$

8. Sol. Direction ratio's of normal to plane containing the straight line

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = 8\hat{i} - \hat{j} - 10\hat{k}$$

Required plane $\begin{vmatrix} x-0 & y-0 & z-0 \\ 2 & 3 & 4 \\ 8 & -1 & -10 \end{vmatrix} = 0$ ⇒ $-26x + 52y - 26z = 0$ ⇒ $x - 2y + z = 0$

9. Sol. $a_1x + b_1y + c_1z = 1$

$a_2x + b_2y + c_2z = 0$

$a_3x + b_3y + c_3z = 0$

No three planes can meet at two distinct points. So number of matrices is 0

10. Sol. $D = \left|\frac{1-4-2-\alpha}{3}\right| = 5$

$\alpha + 5 = 15$ (∴ $\alpha > 0$)

⇒ $\alpha = 10$

⇒ plane is $x + 2y - 2z - 10 = 0$

Let foot of perpendicular is (α, β, γ)

$\frac{\alpha-1}{1} = \frac{\beta+2}{2} = \frac{\gamma-1}{-2} = -\left(\frac{1-4-2-10}{9}\right) = \frac{5}{3} \Rightarrow \alpha = \frac{8}{3}, \beta = \frac{4}{3}, \gamma = -\frac{7}{3}$

11. Ans. (C)

Sol. Let $\vec{v} = \lambda\vec{a} + \mu\vec{b}$

⇒ $\vec{v} = (\lambda+\mu)\hat{i} + (\lambda-\mu)\hat{j} + (\lambda+\mu)\hat{k}$

Now $\vec{v}.\hat{c} = \frac{1}{\sqrt{3}}$

⇒ $\frac{(\lambda+\mu)-(\lambda-\mu)-(\lambda+\mu)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

⇒ $\mu - \lambda = 1$

$\mu = \lambda + 1$

∴ $\vec{v} = (2\lambda + 1)\, i - \hat{j} + (2\lambda + 1)\hat{k}$

For $\lambda = 1$, $\vec{v} = 3\hat{i} - \hat{j} + 3\hat{k}$

12. **Ans.** **(C)**

Let $\vec{c} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{a} \times \vec{c} = \vec{c} \times \vec{b}$

$\Rightarrow$ $(\vec{a} + \vec{b}) \times \vec{c} = \vec{0}$

$\Rightarrow$ $(\vec{a} + \vec{b}) \,||\, \vec{c}$

Let $(\vec{a} + \vec{b}) = \lambda\vec{c}$

$\Rightarrow$ $|\vec{a} + \vec{b}| = |\lambda|\,|\vec{c}|$

$\Rightarrow$ $\sqrt{29} = |\lambda| . \sqrt{29}$

$\Rightarrow$ $\lambda = \pm 1$

$\therefore$ $\vec{a} + \vec{b} = \pm (2\hat{i} + 3\hat{j} + 4\hat{k})$

Now $(\vec{a} + \vec{b}).(-7\hat{i} + 2\hat{j} + 3\hat{k}) = \pm(-14 + 6 + 12)$
$= \pm 4$

13. **Sol.** **Ans.** **(A)**

Equation of required plane

$(x + 2y + 3z - 2) + \lambda(x - y + z - 3) = 0$

$\Rightarrow$ $(1 + \lambda)x + (2 - \lambda)y + (3 + \lambda)z - (2 + 3\lambda) = 0$

distance from point (3, 1, – 1)

$$= \left| \frac{3 + 3\lambda + 2 - \lambda - 3 - \lambda - 2 - 3\lambda}{\sqrt{(1+\lambda)^2 + (2-\lambda)^2 + (3+\lambda)^2}} \right| = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \left| \frac{-2\lambda}{\sqrt{3\lambda^2 + 4\lambda + 14}} \right| = \frac{2}{\sqrt{3}}$$

$\Rightarrow$ $3\lambda^2 = 3\lambda^2 + 4\lambda + 14$

$\Rightarrow$ $\lambda = -\frac{7}{2}$

equation of required plane

$5x - 11y + z - 17 = 0$

14. **XII**

Sol. **(C)**

$\vec{PR} = \vec{PQ} + \vec{PS}$ $\Rightarrow$ $\vec{SQ} = \vec{PQ} - \vec{PS}$ $\Rightarrow$ $\vec{PS} = \frac{\vec{PR} - \vec{SQ}}{2}$

$\vec{PQ} = \frac{\vec{PR} + \vec{SQ}}{2}$

$V = \left| \left[\vec{PQ}\ \vec{PS}\ \vec{PT}\right] \right|$ $\Rightarrow$ $V = \frac{1}{4}\left| \left[\vec{PR} + \vec{SQ}, \vec{PR} - \vec{SQ}, \vec{PT}\right] \right|$

$V = \frac{1}{2}\left| \left[\vec{PR}, \vec{SQ}, \vec{PT}\right] \right|$ $\Rightarrow$ $\frac{1}{2}\begin{vmatrix} 3 & 1 & -2 \\ 1 & -3 & -4 \\ 1 & 2 & 3 \end{vmatrix}$

$\frac{1}{2}(-3 - 7 - 10) = 10$

15. **Sol.** **(D)**

Any point on line $\frac{x+2}{2} = \frac{y+1}{-1} = \frac{z}{3} = \lambda$

Let any two points on this line are

A(– 2, – 1, 0), B (0, – 2, 3) Put (λ = 0, 1)

Let foot of perpendicular from A(– 2, – 1, 0) on plane is (α, β, γ)

$\Rightarrow \frac{\alpha+2}{1} = \frac{\beta+1}{1} = \frac{\beta-0}{1} = \mu$ (say)

Also, $\alpha + \beta + \gamma = 3$

$\Rightarrow \mu - 2 + \mu - 1 + \mu = 3 \Rightarrow \mu = 2$

$\Rightarrow$ M(0, 1, 2)

Similarly foot of perpendicular from B(0, – 2, 3) on plane is N $\left(\frac{2}{3}, \frac{-4}{3}, \frac{11}{3}\right)$

So, equation of MN is $\frac{x-0}{\frac{2}{3}} = \frac{y-1}{\frac{-7}{3}} = \frac{z-2}{\frac{5}{3}}$.

16. Ans. (C)

Sol. Line is

$\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-0}{1} = \alpha$ ......(1)

Q(α, α, 1)

Direction ratio of PQ are

$\lambda - \alpha, \lambda - \alpha, \lambda - 1$

Since PQ is perpendicular to (1)

$\therefore \lambda - \alpha + \lambda - \alpha + 0 = 0$

$\lambda = \alpha$

∴ Direction ratio of PQ are

$0, 0, \lambda - 1$

Another line is

$\frac{x-0}{-1} = \frac{y-0}{1} = \frac{z+1}{0} = \beta$ ......(2)

∴ R (– β, β, – 1)

∴ Direction ratio of PR are

$\lambda + \beta, \lambda - \beta, \lambda + 1$

Since PQ is perpendicular to (ii)

$\therefore -\lambda - \beta + \lambda - \beta = 0$

$\beta = 0$

∴ R(0, 0, – 1)

and Direction ratio of PQ are λ, λ, λ + 1

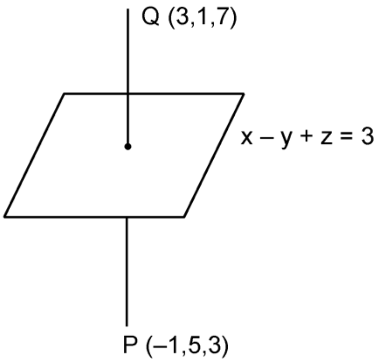
Since PQ ⊥ PR

$\therefore 0 + 0 + \lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1 \Rightarrow$ B, C

For λ = 1 the point is on the line so it will be rejected.

$\Rightarrow \lambda = -1$.

17. Ans. (C)



Sol.

$$\frac{x-3}{1}=\frac{y-1}{-1}=\frac{z-7}{1}=\frac{-2(6)}{3}=-4$$

x =– 1, y = 5, z = 3 P (–1, 5, 3)

a(x + 1) + b(y – 5) + c (z – 3) = 0

a + 2b + c = 0 ....(i)

a – 5b – 3c = 0

$$\frac{a}{-1}=\frac{b}{4}=\frac{c}{-7}$$

–(x + 1) + 4 (y – 5) – 7 (z – 3) = 0

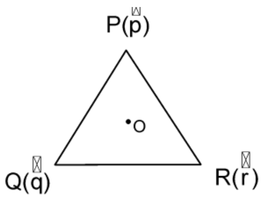
–x + 4y – 7z = 0

x – 4y + 7z = 0

18. (ciumcenter)

Ans. **(B)**

Sol.



$\vec{p}.\vec{q}+\vec{r}.\vec{s}=\vec{r}.\vec{p}+\vec{q}.\vec{s}=\vec{q}.\vec{r}+\vec{p}.\vec{s}$

$\Rightarrow \vec{p}.(\vec{q}-\vec{r})-\vec{s}.(\vec{q}-\vec{r})=0 \Rightarrow \overrightarrow{PS}.\overrightarrow{QR}=0$

Similarly $\overrightarrow{PQ}.\overrightarrow{SR}=0$

⇒ S is orthocentre of the triangle

19. **Ans.** **(D) Vector**

Sol. Let plane be

a(x – 1) + b(y – 1) + c(z – 1) = 0

Now, direction ratio of its normal = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = \hat{i}(-14) - \hat{j}(2) + \hat{k}(-15)$

So, $-14(x-1) - 2(y-1) - 15(z-1) = 0$

$14x + 2y + 15z = 31$