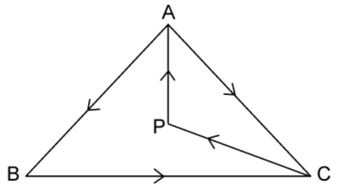
## Additional Problems For Self Practice (APSP)

## PART - I : PRACTICE TEST PAPER

**1.** **Sol.** $3\vec{p}+\vec{q}-2\vec{r} = \hat{i}+2\hat{j}+2\hat{k}$

unit vector = $\dfrac{\hat{i}+2\hat{j}+2\hat{k}}{3}$

**2.** **Sol.**




$\overrightarrow{PA}+\overrightarrow{AC}+\overrightarrow{CP}=0$ ........(1)

Since , $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$ (In ΔABC)

substituting it in (1)

$\overrightarrow{PA}+\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CP}=0$

$\overrightarrow{PA}+\overrightarrow{CP}=\overrightarrow{BA}+\overrightarrow{CB}$

**3.** **Sol.** Let $\overrightarrow{OA} = 2\hat{i}-\hat{j}-\hat{k}$; $\overrightarrow{OB} = \hat{i}-3\hat{j}-5\hat{k}$ and $\overrightarrow{OC}=3\hat{i}-4\hat{j}-4\hat{k}$

∴ a = $|\overrightarrow{BC}|=\sqrt{6}$, $b=|\overrightarrow{AC}|=\sqrt{35}$ and $c=|\overrightarrow{AB}|=\sqrt{41}$

cosA = $\dfrac{b^2+c^2-a^2}{2bc} = \sqrt{\dfrac{35}{41}}$

$\cos^2 A = \dfrac{35}{41}$

**4.** **Sol.** $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$

⇒ $\vec{a}.\vec{b}=0$ ⇒ 8α + 8β = 8 ⇒ α + β = 1

**5.** **Sol.** Parallel vector = $\dfrac{(2+\vec{b})\,\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{b^2+4b+44}}$

Acording to the condition = $\dfrac{(2+b)+6-2}{\sqrt{b^2+4b+44}}=1$ ⇒ b = 1

**6.** **Sol.** $\overrightarrow{AD}=\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}=\vec{a}+\vec{b}+\vec{c}$

$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC} = \vec{a}+\vec{b}$ or $\overrightarrow{CA} = -(\vec{a}+\vec{b})$

$\overrightarrow{BD}=\overrightarrow{BC}+\overrightarrow{CD}=\vec{b}+\vec{c}$

Therefore blfy,, $\overrightarrow{AB}.\overrightarrow{CD}+\overrightarrow{BC}.\overrightarrow{AD}+\overrightarrow{CA}.\overrightarrow{BD} = \vec{a}.\vec{c}+\vec{b}.(\vec{a}+\vec{b}+\vec{c})+(-\vec{a}-\vec{b}).(\vec{b}+\vec{c})$

$= \vec{a}.\vec{c}+\vec{b}.\vec{a}+\vec{b}.\vec{b}+\vec{b}.\vec{c}-\vec{a}.\vec{b}-\vec{a}.\vec{c}-\vec{b}.\vec{b}-\vec{b}.\vec{c} = 0$

**Alter.** $(\vec{b}-\vec{a}).(\vec{d}-\vec{a})+(\vec{c}-\vec{b}).(\vec{d}-\vec{a})+(\vec{a}-\vec{c}).(\vec{d}-\vec{b}) = 0$

**7.** **Sol.** Let $\vec{p}=\sqrt{3}(\vec{a}\times\vec{b})$ and $\vec{q}=\vec{b}-(\vec{a}\,.\,\vec{b})\vec{a}$

$\vec{p}$ is perpendicular to the plane of vectors $\vec{a}$ and $\vec{b}$

$\vec{q}$ is the vector lies in the plane of $\vec{a},\vec{b}$

$\therefore \vec{p}\,.\,\vec{q} = 0$

or the angle between them is $\frac{\pi}{2}$

**8.** **Sol.** $\vec{a}+\vec{b}=3\hat{i}-4\hat{j}-\hat{k}$

$\vec{b}-\vec{a}=2\hat{i}+3\hat{j}-6\hat{k}$

$\vec{b}=\frac{5}{2}\hat{i}-\frac{1}{2}\hat{j}-\frac{7}{2}\hat{k}$

length of the side $|\vec{b}|=\sqrt{\frac{1}{4}+\frac{49}{4}+\frac{25}{4}}=\frac{5\sqrt{3}}{2}$

**9.** **Sol.** $\vec{a}.\vec{b} = 0 \quad \Rightarrow \quad \vec{a}\perp\vec{b}$ or $\vec{a} = 0$ or $\vec{b} = 0$

$\vec{a}\times\vec{b} = 0 \quad \Rightarrow \quad \vec{a} \parallel \vec{b}$ or $\vec{a} = 0$ or $\vec{b} = 0$

Hence either $\vec{a}$ or $\vec{b}$ is null vector

**10.** **Sol.** $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -5\\ m & n & 12\end{vmatrix} = (36 + 5n)\hat{i} - (24 + 5m)\hat{j} + (2n - 3m)\hat{k} = 0$

m = $\frac{-24}{5}$, n = $\frac{-36}{5}$

**11.** **Sol.** $|\vec{a}\times\vec{b}|^2 + |\vec{a}.\vec{b}|^2 = |\vec{a}|^2.|\vec{b}|^2 = 4$

**12.** **Sol.** Let $\vec{a}=-2\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+\hat{j}+\hat{k}, \vec{c}=\hat{j}-\hat{k}, \vec{d}=\lambda\hat{j}+\hat{k}$

$\vec{b}-\vec{a}=3\hat{i}, \vec{c}-\vec{b}=-\hat{i}-2\hat{k}$

$\vec{d}-\vec{c}=(\lambda-1)\hat{j}+2\hat{k}$

$\begin{vmatrix}3 & 0 & 0\\ -1 & 0 & -2\\ 0 & \lambda-1 & 2\end{vmatrix} = 0 \Rightarrow \lambda = 1$

**13.** **Sol.** If the given vectors are coplaner, then their scalar tripple product is zero.

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0 \quad \Rightarrow \quad \lambda = -4$$

**14.** **Sol.** volume of cube $= [\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 12 & 4 & 3 \\ 8 & -12 & -9 \\ 33 & -4 & -24 \end{vmatrix} = 12\begin{vmatrix} 12 & 1 & 1 \\ 8 & -3 & -3 \\ 33 & -1 & -8 \end{vmatrix} = 3696$

**15.** **Sol.** $[\vec{a}\ \vec{b}\ \vec{c}] = 10 \quad \Rightarrow \quad \begin{vmatrix} 1 & 1 & 1 \\ 2 & \lambda & 1 \\ 1 & -1 & 4 \end{vmatrix} = 10$

$(4\lambda + 1) - (8 - 1) + (-2 - \lambda) = 10$

$4\lambda + 1 - 7 - 2 - \lambda = 10$

$3\lambda = 10 + 8 = 18 \quad \Rightarrow \quad \lambda = 6$

**16.** **Sol.** We have, projection of $\vec{a}$ on $\vec{b} = \frac{-9}{\sqrt{3}}$

$\frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{-9}{\sqrt{3}} \quad \Rightarrow \quad |\vec{a}|\cos\frac{5\pi}{6} = \frac{-9}{\sqrt{3}} \quad \Rightarrow \quad |a| = 6$

**17.** **Sol.** Since $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{a} - \lambda\vec{b} \quad \Rightarrow (\vec{a} + \lambda\vec{b}).(\vec{a} - \lambda\vec{b}) = 0$

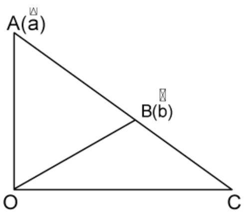
$\vec{a} + \lambda\vec{b}$, $\vec{a} - \lambda\vec{b} \Rightarrow (\vec{a} + \lambda\vec{b}).(\vec{a} - \lambda\vec{b}) = 0$

$\Rightarrow |\vec{a}|^2 - \lambda^2 |\vec{b}|^2 = 0 \quad \Rightarrow \lambda = \pm\frac{3}{4}$

**18.** **Sol.** $\overrightarrow{AB} \times \overrightarrow{AC} = (2\hat{i} - 4\hat{j} + 4\hat{k})$

Area of triangle ABC ¼ ABC $= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{4 + 16 + 16} = \frac{1}{2} \times 6$ = 3 sq.unit.

**19.** **Sol.** A($\vec{a}$)




$\overrightarrow{AC} = \vec{c} - \vec{a} = 3(\vec{b} - \vec{a}) \quad \Rightarrow \quad \vec{c} = 3\vec{b} - 2\vec{a}$

**20.** **Sol.** $\overrightarrow{AB} = 4\hat{i} - 5\hat{j} + 11\hat{k}$ direction cosine along y-axis $= \frac{-5}{\sqrt{16 + 25 + 121}} = \frac{-5}{\sqrt{162}}$

**21. Sol.** The plane is $2x - y + z = 4$ and the line $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{1}$

$\therefore \cos\theta = \frac{2+1+1}{\sqrt{6}\sqrt{3}} = \frac{2\sqrt{2}}{3}$ $\Rightarrow$ $\theta = \cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

**22. Sol.** Equation of plane

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 0 \\ 2 & 0 & 1 \end{vmatrix} = 4\hat{i} - 8\hat{k}$$

$4x - 8z = 0$ $\Rightarrow$ $x - 2z = 0$

equation of the line

$$\frac{x-1}{1} = \frac{y+2}{0} = \frac{z-1}{-2}$$

For intersection point, $(\lambda + 1) - 2(-2\lambda + 1) = 0$

$5\lambda - 1 = 0$ $\Rightarrow$ $\lambda = \frac{1}{5}$

Intersection point $\frac{6}{5}\hat{i} - 2\hat{j} + \frac{3}{5}\hat{k} = \frac{1}{5}(6\hat{i} - 10\hat{j} + 3\hat{k})$

**23. Sol.** $(\vec{r} - (\hat{i} - 2\hat{j} - 4\hat{k})).(2\hat{i} + 3\hat{j} + 6\hat{k}) = 0$

$2x + 3y + 6z = 2 - 6 - 2y$

$2x + 3y + 6z = -28$

$\vec{r}\,.\,(2\hat{i} + 3\hat{j} + 6\hat{k}) + 28 = 0$

**24. Sol.** Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$

$(\vec{r} - \vec{b}) \times \vec{a} = 0$ $\Rightarrow$ $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x-2 & y & z+1 \\ 1 & 1 & 0 \end{vmatrix} = 0$

$\therefore z = -1,\ x - y = 2$

Now $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ $\Rightarrow$ $(\vec{r} - \vec{a}) \times \vec{b} = 0$

$y = 1,\ x + 2z = 1$ $\Rightarrow$ $x = 3,\ y = 1,\ z = -1$

$r = 3\hat{i} + \hat{j} - \hat{k}$

**25. Sol.** For point P on the line $\vec{r} = \vec{a} + t\vec{b}$

$\therefore$ $\overrightarrow{PC} = (\vec{c} - \vec{a}) - t\vec{b}$ $\because$ $\overrightarrow{PC} \perp \vec{b}$

$((\vec{c} - \vec{a}) - t\vec{b}).\,\vec{b} = 0$ or $t = \frac{(\vec{c} - \vec{a}).\vec{b}}{\vec{b}^2}$

Distance of c from line $(|\overrightarrow{PC}| = |\vec{d}| = |\vec{c} - \vec{a} - t\vec{b}|$

$$|\vec{d}| = \left|\vec{c} - \vec{a} - \frac{(\vec{c} - \vec{a}).\vec{b}}{\vec{b}^2}\vec{b}\right| = \left|\frac{(\vec{c} - \vec{a})\vec{b}.\vec{b} - (\vec{c} - \vec{a}).\vec{b}\vec{b}}{\vec{b}^2}\right| = \left|\frac{\vec{b} \times (\vec{c} - \vec{a}) \times \vec{b}}{\vec{b}^2}\right| = \frac{|\vec{b}||(\vec{c} - \vec{a}) \times \vec{b}|}{\vec{b}^2} \sin 90°$$

$(\vec{b} \perp (\vec{c}-\vec{a}) \times \vec{b})$

$$|\vec{d}| = \frac{|(\vec{c}-\vec{a})\times\vec{b}|}{|\vec{b}|}$$

**26.** **Sol.** $\frac{\frac{2}{4}}{7} = \frac{\frac{-1}{-2}}{7} = \frac{-m}{2}$

$\frac{7}{2} = \frac{-m}{2}$ $\Rightarrow$ $m = -7$

**27.** **Sol.** Since no vector in given option are collinear with given vector. Hence then all form triangle.

**28.** **Sol.** $\frac{1}{6}\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = \frac{2}{3}$

**29.** **Sol.** $\frac{\frac{\vec{a}.\vec{b}}{|\vec{b}|}}{\frac{\vec{a}.\vec{b}}{|\vec{a}|}} = \frac{|\vec{a}|}{|\vec{b}|} = \frac{\sqrt{4+9+36}}{\sqrt{4+4+1}} = \frac{7}{3}$

**30.** **Sol.** $\vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{vmatrix} = \hat{i}(5) - \hat{j}(-5) + \hat{k}(-1-4) = 5\hat{i} + 5\hat{j} - 5\hat{k}$

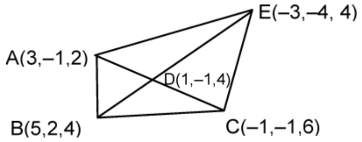
# Practice Test (JEE-Main Pattern)

## OBJECTIVE RESPONSE SHEET (ORS)

| Que. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | | | | | | | | | | |
| Que. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Ans. | | | | | | | | | | |
| Que. | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| Ans. | | | | | | | | | | |

## PART - II : PRACTICE QUESTIONS

**1.** **Sol.**



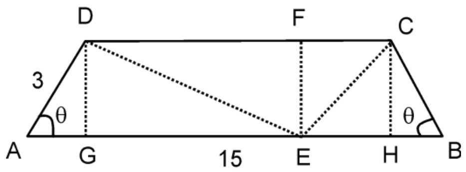

Area of ΔABD = $A_1 = \left|\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 2\\ -2 & 0 & 2\end{vmatrix}\right| = \left|\frac{1}{2}\{\hat{i}(6)-\hat{j}(4+4)+\hat{k}(6)\}\right| = \left|3\hat{i}-4\hat{j}+3\hat{k}\right| = \sqrt{9+16+9} = \sqrt{34}$

Area of quadrilateral ABCE = $A_2 = \left|\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -4 & 0 & 4\\ -8 & -6 & 0\end{vmatrix}\right| = \left|\frac{1}{2}\{\hat{i}(24)-\hat{j}(32)+\hat{k}(24)\}\right| = \left|12\hat{i}-16\hat{j}+12\hat{k}\right|$

$= 4\sqrt{9+16+9} = 4\sqrt{34}$

So $= \frac{A_1}{A_2} = \frac{1}{4}$

**2.** **Sol.**



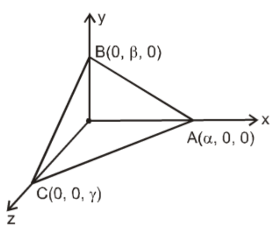

$$\frac{\text{Area of trapezium ABCD}}{\text{Area of } \Delta\text{CDE}} = \frac{\frac{1}{2}(AB+CD)(EF)}{\frac{1}{2}(CD)(EF)} = \frac{AB+CD}{CD} \quad \text{...... (i)}$$

$$AG = \frac{\vec{AB}\cdot\vec{AD}}{|\vec{AB}|} = \frac{-2+20+22}{\sqrt{225}} = \frac{40}{15} = \frac{8}{3}$$

Now CD = GH = AB – 2AG = $15 - \frac{16}{3} = \frac{29}{3}$

By equation (i) ratio of area = $\frac{15+\frac{29}{3}}{\left(\frac{29}{3}\right)} = \frac{74}{29}$

**3.** **Sol.**



Let the equation of plane be $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$ ... (1)

$\because$ it passes through (a, b, c) $\frac{a}{\alpha}+\frac{b}{\beta}+\frac{c}{\gamma}=1$ ... (2)

$\because$ the point common to the planes through A, B, C and parallel to the coordinate planes is (α, β, γ)

$\therefore$ from (2) we can say that locus of (α, β, γ) is

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1$$

**4.** **Sol.** We have

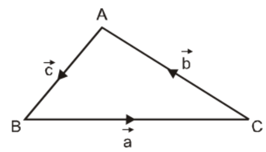
$\vec{a}+\vec{b}+\vec{c}=\vec{o}$

$\Rightarrow \vec{a}\times(\vec{a}+\vec{b}+\vec{c})=\vec{a}\times\vec{o}$

$\Rightarrow \vec{a}\times\vec{a}+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{o}$ $(\because \vec{a}\times\vec{a}=\vec{o})$

$\Rightarrow \vec{a}\times\vec{b}=\vec{c}\times\vec{a}$

Similarly $\vec{a}\times\vec{b}=\vec{c}\times\vec{a}=\vec{b}\times\vec{c}$




**5.**

**Sol.** Normal to plane $P_1$ is $\vec{N_1}=\vec{a}\times\vec{b}$

Normal to plane $P_2$ is $\vec{N_2}=\vec{c}\times\vec{d}$

We have $\vec{N_1}\times\vec{N_2}=0$

If θ is angle between $P_1$ and $P_2$ we get

$|\vec{N_1}|\times|\vec{N_2}|\sin\theta=0$ or $\sin\theta=0 \Rightarrow \theta=0$

**6.** **Sol.** As $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar vectors $2\vec{a}-b$, $2\vec{b}-\vec{c}$ and $2\vec{c}-\vec{a}$ are also coplanar vectors. Thus $[2\vec{a}-\vec{b} \quad 2\vec{b}-\vec{c} \quad 2\vec{c}-\vec{a}]=0$

**7.** **Sol.** $\because \left(\vec{a}+\vec{b}+\vec{c}\right)^2=\Sigma\vec{a}^2+2\Sigma\vec{a}.\vec{b}\geq 0 \Rightarrow 2\Sigma\vec{a}.\vec{b}\geq -3$

$\Rightarrow \Sigma|\vec{a}-\vec{b}|^2=2\Sigma\vec{a}^2-2\Sigma\vec{a}.\vec{b}$

$\Rightarrow \Sigma|\vec{a}-\vec{b}|^2\leq 2(3)+3$

$\Rightarrow \Sigma|\vec{a}-\vec{b}|^2\leq 9$

**8.** **Sol.** $[\vec{a}\vec{b}\vec{c}]=\begin{vmatrix}1 & 0 & -1\\ x & 1 & 1-x\\ y & x & 1+x-y\end{vmatrix}$

$C_3\to C_3+C_1=\begin{vmatrix}1 & 0 & 0\\ x & 1 & 1\\ y & x & 1+x\end{vmatrix}$

$C_3 \to C_3 - C_2 = \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 0 \\ y & x & 1 \end{vmatrix} \Rightarrow [\vec{a}\,\vec{b}\,\vec{c}] = 1$

So $[\vec{a}\,\vec{b}\,\vec{c}]$ does not depends on x and y .

**9. Sol.** $\left(\vec{a} + 2\vec{b}\right)\cdot\left(5\vec{a} - 4\vec{b}\right) = 0$

$5|\vec{a}|^2 + 6\vec{a}.\vec{b} - 8|\vec{b}|^2 = 0$

Since $\vec{a}$ and $\vec{b}$ are unit vector $|\vec{a}| = |\vec{b}| = 1$

Hence $5 + 6\,\vec{a}.\vec{b} - 8 = 0$

$\vec{a}.\vec{b} = \frac{1}{2} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$

**10. Sol.** $\vec{V} = 2\hat{i} + \hat{j} - \hat{k}$

$\vec{W} = \hat{i} + 3\hat{k}$

$[\vec{U}\,\vec{V}\,\vec{W}] = \vec{U}.[(2\hat{i} + \hat{j} - \hat{k}) \times (\hat{i} + 3\hat{k})]$

$= \vec{U}.(3\hat{i} - 7\hat{j} - \hat{k})$

Which is maximum if angle between $\vec{U}$ and $3\hat{i} - 7\hat{j} - \hat{k}$ is zero and maximum value

$= \sqrt{3^2 + 7^2 + 1^2} = \sqrt{59}$

**11. Sol.** Vector lying in the plane of $\vec{a}$ and $\vec{b}$ is $\vec{r} = \lambda_1\vec{a} + \lambda_2\vec{b}$ and its projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$

$\Rightarrow \left|\left[(\lambda_1 + \lambda_2)\hat{i} + (2\lambda_1 - \lambda_2)\hat{j} + (\lambda_1 + \lambda_2)\hat{k}\right].\frac{[\hat{i} + \hat{j} - \hat{k}]}{\sqrt{3}}\right| = \frac{1}{\sqrt{3}}$

$\Rightarrow |2\lambda_1 - \lambda_2| = 1 \Rightarrow 2\lambda_1 - \lambda_2 = \pm 1$

taking negative sign $2\lambda_1 - \lambda_2 = -1$

$\Rightarrow \vec{r} = (3\lambda_1 + 1)\hat{i} - \hat{j} + (3\lambda_1 + 1)\hat{k}$

Hence the required vector is $4\hat{i} - \hat{j} + 4\hat{k}$

**Alternate :**

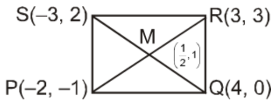
Vector lying in the plane of $\vec{a}$ and $\vec{b}$ is $\vec{a} + \lambda\vec{b}$, and its projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$

$\Rightarrow \left|\left[(\lambda_1 + \lambda_2)\hat{i} + (2\lambda_1 - \lambda_2)\hat{j} + (\lambda_1 + \lambda_2)\hat{k}\right].\frac{[\hat{i} + \hat{j} - \hat{k}]}{\sqrt{3}}\right| = \frac{1}{\sqrt{3}}$

$\Rightarrow \lambda = 3$

Hence the required vector is $4\hat{i} - \hat{j} + 4\hat{k}$

**12.** **Sol.** PQ = $\sqrt{36+1} = \sqrt{37}$ = RS,




PS = $\sqrt{1+9} = \sqrt{10}$ = QR,

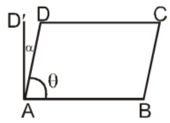
PQ ≠ PS

slope of PQ = $\frac{1}{6}$, slope of PS = – 3

PQ is not ⊥ to PS

So it is parallelogram, which is neither a rhombus nor a rectangle

**13.** **Sol.** $\cos\theta = \frac{-2+20+22}{15\times 3} = \frac{8}{9}$ [Using dot product]




θ + α = 90º

α = 90º – θ

$\cos\alpha = \sin\theta = \frac{\sqrt{17}}{9}$

**14.** **Sol.** $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = [\vec{a}\ \vec{b}\ \vec{c}]^2$

$= ((\vec{a}\times\vec{b})\cdot\vec{c})^2 = (ab\sin\theta\,\hat{c}\cdot\vec{c})^2 = \frac{a^2b^2}{4} = \frac{1}{4}(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)$

**15.** ***Sol.*** $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$C_1 \to C_1 - C_2,\ C_2 \to C_2 - C_3$

$\Rightarrow \begin{vmatrix} a-1 & 0 & 1 \\ 1-b & b-1 & 1 \\ 0 & 1-c & c \end{vmatrix} = 0$

$\Rightarrow$ (a – 1) {(b – 1) c – (1 – c)} + (1 – b) (1 – c) = 0

$\Rightarrow$ (1 – a) (1 – b) c + (1 – a) (1 – c) + (1 – b) (1 – c) = 0

$\Rightarrow \frac{c}{1-c} + \frac{1}{1-b} + \frac{1}{1-a} = 0 \quad \Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$

**16.** **Sol.** A vector along the angle bisector is $\hat{a}+\hat{b} = \frac{(-4\hat{i}+3\hat{k})}{5} + \frac{(14\hat{i}+2\hat{j}-5\hat{k})}{15}$

$= \frac{-12\hat{i}+9\hat{k}+14\hat{i}+2\hat{j}-5\hat{k}}{15} = \frac{2(\hat{i}+\hat{j}+2\hat{k})}{15}$

$\therefore \quad \vec{d} = \hat{i}+\hat{j}+2\hat{k}$

**17.** **Sol.** $|\overrightarrow{OP}| = |\hat{a}\cos t + \hat{b}\sin t| = \sqrt{\cos^2 t + \sin^2 t + 2\sin t\cos t\,(\hat{a}\cdot\hat{b})} = \sqrt{1+(\sin 2t)\,\hat{a}\cdot\hat{b}}$

$\left|\overrightarrow{OP}\right|_{max} = \sqrt{1+\hat{a}\cdot\hat{b}}$ when $t = \frac{\pi}{4}$

$\therefore$ $M = \sqrt{1+\hat{a}\cdot\hat{b}}$ and $\hat{u} = \frac{(\overrightarrow{OP})_{max}}{|\overrightarrow{OP}|_{max}} = \frac{\frac{(\hat{a}+\hat{b})}{\sqrt{2}}}{\left|\frac{\hat{a}+\hat{b}}{\sqrt{2}}\right|} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$

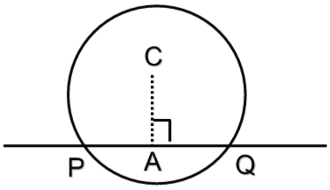
**18._** **Sol.** Centre of sphere C(2,–1,4)

radius $r = \sqrt{4+1+16-5} = 4$

let a point on line A $(3\lambda + 4, 2\lambda + 4, 3\lambda + 6)$ is foot of $\perp$ from C on line

$\therefore$ AC is $\perp$ to line

$\therefore 3(3\lambda + 2) + 2(2\lambda + 5) + 3(3\lambda + 2) = 0$




$22\lambda + 22 = 0 \Rightarrow \lambda = -1$

$\therefore$ A(1, 2, 3)

Now $AQ = \sqrt{CQ^2 - AC^2}$

$= \sqrt{16-11} = \sqrt{5}$

Length of chord $PQ = 2\sqrt{5}$

**19.** **Sol.** (1) Centroid is $\frac{(a\hat{i}+b\hat{j}+c\hat{k})+(b\hat{i}+c\hat{j}+a\hat{k})+(c\hat{i}+a\hat{j}+b\hat{k})}{3}$

**(2)** $(\hat{i}+\hat{j}+\hat{k}).(a\hat{i}+b\hat{j}+c\hat{k}) = (\hat{i}+\hat{j}+\hat{k}).(b\hat{i}+c\hat{j}+a\hat{k}) = (\hat{i}+\hat{j}+\hat{k}).(c\hat{i}+a\hat{j}+b\hat{k})$

**(3)** $\Delta ABC$ is equilateral $AB = BC = CA = \sqrt{(a-b)^2+(b-c)^2+(c-a)^2}$

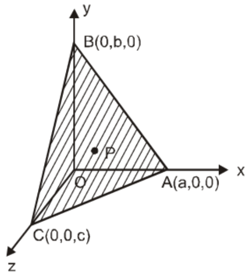
Also $OA = OB = OC = \sqrt{a^2+b^2+c^2}$ by symmetry $\perp$ from O on plane of $\Delta ABC$ meet at centroid.

**20.** **Sol.** As $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ cuts the coordinate axes at

A(a,0,0) B(0,b,0) C(0,0,c)

and its distance from origin = 1

$\Rightarrow$ $\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} = 1$




or $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$ = 1 ................(i)

where P is centroid of Δ.

P(x,y,z) = $\left(\frac{a+0+0}{3}, \frac{0+b+0}{3}, \frac{0+0+c}{3}\right)$

$x = \frac{a}{3}$ , $y = \frac{b}{3}$ , $z = \frac{c}{3}$ ............(ii)

Thus from (i) and (ii)

$$\frac{1}{9x^2}+\frac{1}{9y^2}+\frac{1}{9z^2}=1$$

or $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=9=k$

k = 9

**21.** **Sol.** Equation of plane is $\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$

x – 2y + z = 0 ..........(1)

Ax – 2y + z = d ..........(2)

Compare $\frac{A}{1} = \frac{-2}{-2} = \frac{1}{1}$ ⇒ A = 1

Distance between planes is $\left|\frac{d}{\sqrt{1+1+4}}\right| = \sqrt{6}$

⇒ |d| = 6

**22.** **Sol.** Equation of QR is

$$\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-5}{1}$$

Let P ≡ (2 + λ, 3 + 4λ, 5 + λ)

10 + 5λ – 12 – 16λ – 5 – λ = 1

– 7 – 12λ = 1

⇒ $\lambda = \frac{-2}{3}$

then P ≡ $\left(\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right)$

Let S = (2 + μ, 3 + 4μ, 5 + μ)

$\overrightarrow{TS} = (\mu)\hat{i} + (4\mu+2)\hat{j} + (\mu+1)\hat{k}$

$\overrightarrow{TS} \cdot (\hat{i}+4\hat{j}+\hat{k}) = 0$

μ + 16μ + 8 + μ + 1 = 0

$\mu = -\frac{1}{2}$

S = $\left(\frac{3}{2}, 1, \frac{9}{2}\right)$

PS = $\sqrt{\left(\frac{4}{3}-\frac{3}{2}\right)^2+\frac{4}{9}+\left(\frac{13}{3}-\frac{9}{2}\right)^2}$

= $\sqrt{\frac{1}{36}+\frac{4}{9}+\frac{1}{36}}$

= $\sqrt{\frac{1}{18}+\frac{4}{9}}$ = $\sqrt{\frac{9}{18}}$ = $\frac{1}{\sqrt{2}}$

**23.** **Sol.** Let the d.c's of reflected ray be $\langle \ell, m, n \rangle$, then

d.r's of the normal are $\langle \ell+1, m, n \rangle$

$\therefore$ $\frac{\ell+1}{1} = \frac{m}{-1} = \frac{n}{1}$ and so $\ell^2 + (\ell + 1)^2 + (\ell + 1)^2 = 1$

i.e. $\ell = -1$ or $-\frac{1}{3}$

if $\ell = -1$, then d.c's are $\langle -1, 0, 0 \rangle$ which is not possible

$\therefore$ $\ell = -\frac{1}{3}$

$\therefore$ d.c's of the reflected ray are $\left\langle -\frac{1}{3}, -\frac{2}{3}, \frac{2}{3} \right\rangle$

**24.** **Sol.** $\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$

$(1 - a^2) + c(-c - ab) - b(ac + b) = 0$

$1 - a^2 - c^2 - abc - abc - b^2 = 0$

$a^2 + b^2 + c^2 + 2abc = 1$

**25.** **Sol.** Any point of the line $x + y + 2z - 3 = 0$, $2x + 3y + 4z - 4 = 0$ is $(5, -2, 0)$ and drs of the line are $(-2, 0, 1)$

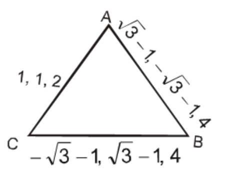
$\Rightarrow$ Equation of line in symmetrical form $\frac{x-5}{-2} = \frac{y+2}{0} = \frac{z}{1}$

Also Equation of z-axis is $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$,

d.c's of the line of shortest distance are $\langle 0, 1, 0 \rangle$

$\therefore$ shortest distance $= |(5 - 0) . 0 + (-2 - 0) . 1 + (0 - 0) . 0| = 2$

**26.** **Sol.** Angle between CA & CB




$\cos C = \left| \frac{1(-\sqrt{3}-1) + 1(\sqrt{3}-1) + 2.4}{\sqrt{1+1+4}\ \sqrt{(-\sqrt{3}-1)^2 + (\sqrt{3}-1)^2 + 4^2}} \right|$

$= \left| \frac{6}{\sqrt{6}\ \sqrt{24}} \right| = \frac{1}{2}$ $\therefore$ $\angle C = \frac{\pi}{3}$

Similarly $\angle A = \angle B = \frac{\pi}{3}$

it is given that lines are coplanar. So form a equilateral triangle

**27.*** **Sol.** Let $P_3$ be $P_2 + \lambda P_1 = 0$ $\Rightarrow$ $x + \lambda y + z - 1 = 0$

Distance from $(0, 1, 0)$ is 1

$\therefore$ $\frac{0 + \lambda + 0 - 1}{\sqrt{1 + \lambda^2 + 1}} = \pm 1$ $\Rightarrow$ $\lambda = -\frac{1}{2}$

∴ Equation of $P_3$ is $2x - y + 2z - 2 = 0$

Dist. from (α, β, γ) is 2

∴ $\left|\frac{2\alpha - \beta + 2\gamma - 2}{3}\right| = 2$ ⇒ $2\alpha - \beta + 2\gamma = 2 \pm 6$

∴ option (B, D) are correct.

**28.*** **Ans.** Let $\vec{v}$ be the vector along L

then $\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{vmatrix} = \hat{i} - 3\hat{j} - 5\hat{k}$

So any point on line L is A(λ, – 3λ, –5λ)

Foot of perpendicular from A to P, is

$$\frac{h-\lambda}{1} = \frac{k+3\lambda}{2} = \frac{\ell+5\lambda}{-1} = -\frac{(\lambda - 6\lambda + 5\lambda + 1)}{1+4+1} = -\frac{1}{6}$$

$h = \lambda - \frac{1}{6}$, $k = -3\lambda - \frac{1}{3}$, $\ell = -5\lambda + \frac{1}{6}$

so foot is $\left(\lambda - \frac{1}{6}, -3\lambda - \frac{1}{3}, -5\lambda + \frac{1}{6}\right)$

**29.*** **Ans.** **(1,3,4)**

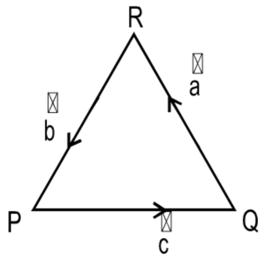
**Sol.** $\vec{a} + \vec{b} + \vec{c} = 0$

⇒ $\vec{b} + \vec{c} = -\vec{a}$

⇒ $48 + \vec{c}^2 + 48 = 144$

⇒ $\vec{c}^2 = 48$

⇒ $|\vec{c}^2| = 4\sqrt{3}$




∴ $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 24 - 12 = 12$ **Ans.** **(A)**

Further

$\vec{a} + \vec{b} = -\vec{c}$

⇒ $144 + 48 + 2\vec{a}.\vec{b} = 48$

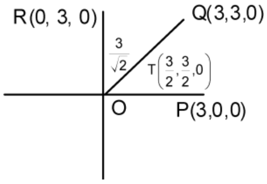
⇒ $\vec{a}.\vec{b} = -72$ **Ans.** **(D)**

∵ $\vec{a} + \vec{b} + \vec{c} = 0$

⇒ $\vec{a} \times \vec{b} + \vec{a} \times \vec{c} = 0$

∴ $|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 2\,|\vec{a} \times \vec{b}| = 2.\sqrt{144.48 - (72)^2} = 48\sqrt{3}$ **Ans.** **(C)**

**30.***




**Sol.**

$S \equiv \left(\frac{3}{2},\ \frac{3}{2},\ 3\right) \Rightarrow \overrightarrow{OQ} = 3\hat{i}+3\hat{j} \Rightarrow \overrightarrow{OS} = \frac{3}{2}\hat{i}+\frac{3}{2}\hat{j}+3\hat{k}$

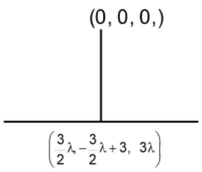
$\cos\theta = \dfrac{\frac{1}{2}+\frac{1}{2}}{\sqrt{2}\sqrt{\frac{1}{2}+\frac{1}{4}+1}} = \dfrac{1}{\sqrt{2}\sqrt{\frac{3}{2}}} = \dfrac{1}{\sqrt{3}}$

$\vec{n} = \overrightarrow{OQ}\times\overrightarrow{OS} = (\hat{i}+\hat{j})\times(\hat{i}+\hat{j}+2\hat{k}) = \hat{k}-2\hat{j}-\hat{k}+2\hat{i} \Rightarrow 2\hat{i}-2\hat{j}$

$x - y = \lambda \Rightarrow x = y \Rightarrow \perp\ (3, 0, 0) \Rightarrow \frac{3}{\sqrt{2}}$

$RS \rightarrow \dfrac{x-0}{\frac{3}{2}} = \dfrac{y-3}{-\frac{3}{2}} = \dfrac{z-0}{3} = \lambda \Rightarrow x = \frac{3}{2}\lambda,\ y = -\frac{3}{2}\lambda + 3,\ z = 3\lambda$

T distance $\Rightarrow \sqrt{\frac{3}{2}-3+9} \Rightarrow \sqrt{\frac{15}{2}}$




$D = \frac{9}{4}\lambda^2 + \left(3-\frac{3}{2}\lambda\right)^2 + 9\lambda^2 = \frac{27}{2}\lambda^2 - 9\lambda + 9 \Rightarrow \lambda = \frac{9}{27} = \frac{1}{3}$

**31._** **Sol.** Equation $= \begin{vmatrix} x & y+4 & z-10 \\ 1 & 2 & -4 \\ -2 & -1 & 5 \end{vmatrix} = \begin{matrix} 6x+3y+3z-18=0 \\ 2x+y+z-6=0 \end{matrix}$

## COMPREHENSION

**32.** **Sol.** Direction ratio of $L_1$ are 3, 1, 2 and of $L_2$ are 1, 2, 3

Vector perpendicular to $L_1$ and $L_2$ is $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$

$= -\hat{i} - 7\hat{j} + 5\hat{k}$ unit vector is $\dfrac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

Hence 'B' is correct

**33.** **Sol.** Shortest distance between $L_1$ and $L_2$ is

$\dfrac{|(\vec{b}-\vec{a}).\vec{p}\times\vec{q}|}{|\vec{p}\times\vec{q}|}$, Now $\vec{p}\times\vec{q} = -\hat{i} - 7\hat{j} + 5\hat{k}$

and $\vec{b}-\vec{a} = 3\hat{i} + 4\hat{k}$

Hence $\left|\dfrac{-3+20}{5\sqrt{3}}\right| = \dfrac{17}{5\sqrt{3}}$.

Hence (4) is correct

**34.** **Sol.** The equation of plane passing through (– 1, – 2, – 1)

and normal ⊥ to $L_1$ & to $L_2$ i.e. parallel to – i – 7j + 5k is

a (x + 1) + b (y + 2) + c (z + 1) = 0

or, – (x + 1) – 7 (y + 2) + 5 (z + 1) = 0

i.e. x + 7y – 5z + 10 = 0

Distance of (1, 1, 1) from this plane is

$\left|\dfrac{1+7-5+10}{5\sqrt{3}}\right| = \dfrac{13}{\sqrt{75}}$ Hence (3) is correct.

**35.** **Ans. (1)**

**Sol.** $\vec{a} = \dfrac{(\hat{u}+\hat{v})}{2\left|\cos\frac{\alpha}{2}\right|}$, $\vec{b} = \dfrac{(\hat{v}+\hat{w})}{2\left|\cos\frac{\beta}{2}\right|}$

$\vec{c} = \dfrac{(\hat{w}+\hat{u})}{2\left|\cos\frac{\gamma}{2}\right|}$

$\therefore\ [\vec{a}\ \vec{b}\ \vec{b}\times\vec{c}\ \vec{c}\times\vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$

$= \left[\dfrac{((\hat{u}+\hat{v})\times(\hat{v}+\hat{w})).(\hat{w}+\hat{u})}{8\left|\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}\right|}\right]^2 = \left[\dfrac{2[\hat{u}\ \hat{v}\ \hat{w}]}{8\left|\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}\right|}\right]^2 = \dfrac{[\hat{u}\,\hat{v}\,\hat{w}]^2}{16}$ $\sec^2(\alpha/2).\sec^2(\beta/2).\sec^2(\gamma/2)$

**36.** **Ans. (2)**

**Solution :** Here

$\hat{w} - \hat{v} = \lambda\hat{a}$, $\lambda \in R^+$

$|\hat{w} - \hat{v}|^2 = |\lambda\hat{a}|^2$

⇒ $1 + 1 - 2\cos2\theta = \lambda^2$

⇒ $\lambda = 2\sin\theta$

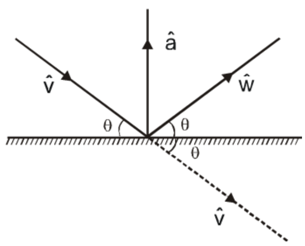
⇒ $\hat{w} - \hat{v} = 2\sin\theta\ \hat{a}$ ...........(1)

Let $\hat{a}.\hat{v} = 1.1.\cos(90^\circ + \theta)$

$\hat{a}.\hat{v} = -\sin\theta$  $\sin\theta = -\hat{a}.\hat{v}$ ...........(2)

from (1) & (2)

$\hat{w} = \hat{v} - 2(\hat{a}.\hat{v})\hat{a}$




**37.** **Ans. (3)**

**Sol.** 3 (3 + 14t) – 6(1 + 2t) – 2(15t) = 15

9 + 42t – 6 – 12t – 30 t = 15

3 = 15 which is not possible

statement 1 is false

$$\bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix} = \hat{i}\,(12 + 2) - \hat{j}\,(-6 + 4) + \hat{k}\,(3 + 12)$$

$= 14\hat{i} + 2\hat{j} + 15\hat{k}$

statement 2 is true

**38.** **Ans. (3)**

**Sol.** $\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 2\hat{j} + 2\hat{k}$ $\Rightarrow$ $\vec{n}_2 \times \vec{n}_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -3 & 3 \end{vmatrix} = -4\hat{j} - 4\hat{k}$

$\vec{n}_3 \times \vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 3 \\ 1 & -1 & 1 \end{vmatrix} = 2\hat{j} + 2\hat{k}$

The three lines are respectively parallel to $\vec{n}_1 \times \vec{n}_2$, $\vec{n}_2 \times \vec{n}_3$, $\vec{n}_3 \times \vec{n}_1$. Also $\vec{n}_1 \times \vec{n}_2$, $\vec{n}_2 \times \vec{n}_3$, $\vec{n}_3 \times \vec{n}_1$ are parallel

$\therefore$ Thus the lines are parallel.

$\Rightarrow$ Statement 1 is wrong $\Rightarrow$ Option D

**Aliter :** $[\vec{n}_1\ \vec{n}_2\ \vec{n}_3] = 0$ lines are parallel.