Self Practice Paper (SPP)

1. Two spherical bodies of mass M and 5M and radii R and 2R respectively are released in free space with initial separation between their centres equal to 12R. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is : **[RPMT-2008]**

(1) 2.5R (2) 4.5 R (3) 7.5R (4) 1.5 R

2. A ball collides with an inclined plane of inclination θ after falling through a distance h. If it moves horizontally just after the impact, the coefficient of restitution is :

(1) $\tan^2\theta$ (2) $\cot^2\theta$ (3) $\tan\theta$ (4) $\cot\theta$

3. A plate in the form of a semicircle of radius a has a mass per unit area of *kr* where *k* is a constant and *r* is the distance from the centre of the straight edge. By dividing the plate into semicircular rings, find the distance of the centre of mass of the plate from the centre of its straight edge.

(1) $3a/2\pi$ (2) $a/2\pi$ (3) $3a/\pi$ (4) a/π

4. A particle of mass m moving with a speed v hits elastically another stationary particle of mass 2m in a fixed smooth horizontal circular tube of radius r. Find the time when the next collision will take place?

(1) $t = \frac{2\pi r}{v}$ (2) $t = \frac{\pi r}{v}$ (3) $t = \frac{\pi r}{2v}$ (4) $t = \frac{3\pi r}{2v}$

5. A point mass of 1kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses ? **[JEE-2010, 3/163]**

(1) Total momentum of the system is 5 kg ms^{-1}

(2) Momentum of 5 kg mass after collision is 4 kg ms^{-1}

(3) Kinetic energy of the centre of mass is 0.75 J

(4) Total kinetic energy of the system is 4 J

SPP Answers

1. (3) **2.** (1) **3.** (1) **4.** (1) **5.** (3)

SPP Solutions

1. Let at O there will be a collision. If smaller sphere moves x distance to reach at O, then bigger sphere will zmove a distance of

$F = \dfrac{GM \times 5M}{(12R - x)^2}$

$a_{small} = \dfrac{F}{M} = \dfrac{G \times 5M}{(12R - x)^2}$

$a_{big} = \dfrac{F}{5M} = \dfrac{GM}{(12R - x)^2}$

$x = \dfrac{1}{2} a_{small} t^2 = \dfrac{1}{2} \dfrac{G \times 5M}{(12R - x)^2} t^2$ .....(i)

$(9R - x) = \dfrac{1}{2} a_{big} t^2 = \dfrac{1}{2} \dfrac{GM}{(12R - x)^2} t^2$ .....(ii)

Thus, dividing Eq. (i) by Eq. (ii), we get

$\therefore \dfrac{x}{9R - x} = 5 \Rightarrow x = 45R - 5x \Rightarrow 6x = 45R \Rightarrow x = 7.5R$

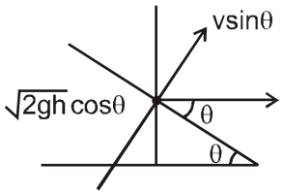
2. $e = \dfrac{v \sin\theta}{\sqrt{2gh}\cos\theta}$

apply conservation of momentum

$m\sqrt{2gh} = m\, v\cos\theta$ .....(i)

$e\sqrt{2gh}\cos\theta \times m = mv\cos\theta$ .....(ii)

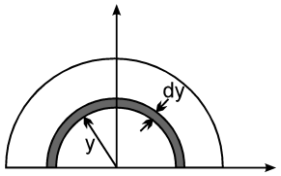
$\dfrac{\tan\theta}{e} = \cot\theta.$



$\therefore e = \tan^2\theta$ on solving

3. Take an elementary ring of radius y and width dy

Then y-coordinate of centre of mass of half disc is



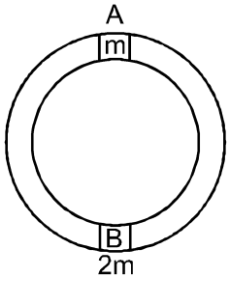
$$\bar{y} = \frac{\int_0^a y_{cm}\,dm}{\int_0^a dm} = \frac{\int_0^a \left(\frac{2y}{\pi}\right)\sigma\pi y\,dy}{\int_0^a \sigma\pi y\,dy} = \frac{\int_0^a \frac{2y}{\pi}(ky)\pi y\,dy}{\int_0^a (ky)\pi y\,dy} = \frac{3}{2}\frac{a}{\pi}$$

4. Particle B is a rest

$mv + 0 = mv_1 + 2mv_2$

$\therefore v = v_1 + 2v_2$ .....(i)

$\frac{v_2 - v_1}{v + 0} = 1$



$\Rightarrow v_2 - v_1 = v$ .....(ii)

Adding (i) + (ii)

$3v_2 = 2v$

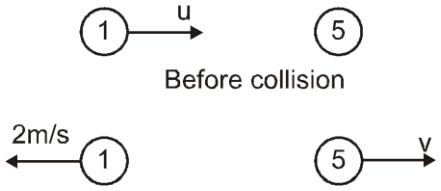
$\therefore v_2 = \frac{2}{3}v$ $v_1 = v_2 - v = \frac{-v}{3}$

Now, (iii) + (iv)

$t = \frac{2\pi r}{v_2 - v_1} = \frac{2\pi r}{\frac{2v}{3} + \frac{v}{3}}$

$t = \frac{2\pi r}{v}$ **Ans.**

5.



Since collision is elastic, so e = 1

Velocity of approach = velocity of separation

So, $u = v + 2$ .....(i)

By momentum conservation :

$1 \times u = 5v - 1 \times 2$

$u = 5v - 2$

$v + 2 = 5v - 2$

So, $v = 1$ m/s

and $u = 3$ m/s

Momentum of system = $1 \times 3 = 3$ kgm/s

Momentum of 5kg after collision = $5 \times 1 = 5$ kgm/s

So, kinetic energy of centre of mass = $\frac{1}{2}(m_1 + m_2)\left(\frac{m_1 u}{m_1 + m_2}\right)^2$

$= \frac{1}{2}(1 + 5)\left(\frac{1 \times 3}{6}\right)^2$

$= 0.75$ J

Total kinetic energy = $\frac{1}{2} \times 1 \times 3^2 = 4.5$ J.