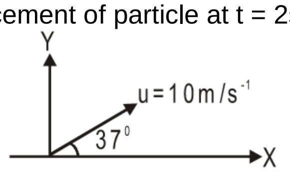
PROJECTILE MOTION

GALILEO STATEMENT :-

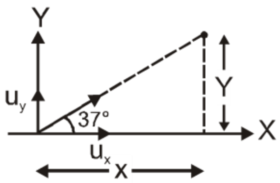
Motion along two mutually perpendicular direction are independent of each other.
In other words, any vector quality directed along a direction unaffected by a vector perpendicular to it.

Solved Examples

Example 1 A particle start from origin at t=0 with speed 10 m/s along the direction making 37^0 with positive X-axis. Find x and y displacement of particle at t = 2sec.

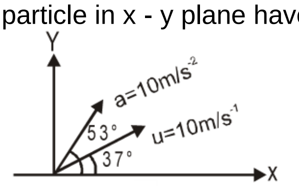


Solution Since X and Y are mutually perpendicular direction, their motion are independent.
X - displacement = u cos 37^0 × t = 8 × 2 = 16 m.
Y - displacement = u sin 37^0 × t = 6 × 2 = 12 m.



$$\vec{u} = 10 \quad \cos 37^0 \hat{i} + 10 \quad \sin 37^0 \hat{j} = 8\hat{i} + 6\hat{j}$$

Example 2 In example - 1, if the same particle in x - y plane have acceleration 10 m/s^2

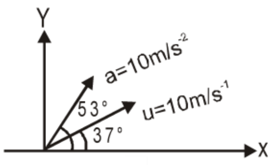


along the direction making 53° with positive x-axis. Find x and y displacement of particle at t = 2sec.

Solution

$$\vec{u} = 10 \quad \cos 37^0 \hat{i} + 10 \quad \sin 37^0 \hat{j} = 8\hat{i} + 6\hat{j}$$

$$\vec{a} = 10 \quad \cos 53^0 \hat{i} + 10 \quad \sin 53^0 \hat{j} = 6\hat{i} + 8\hat{j}$$



Since $\vec{a}$ = constant, therefore

X — displacement, $x = u_x t + \frac{1}{2} a_x t^2 = 8 \times 2 + \frac{1}{2} \times 6 \times (2)^2 = 28$ m.

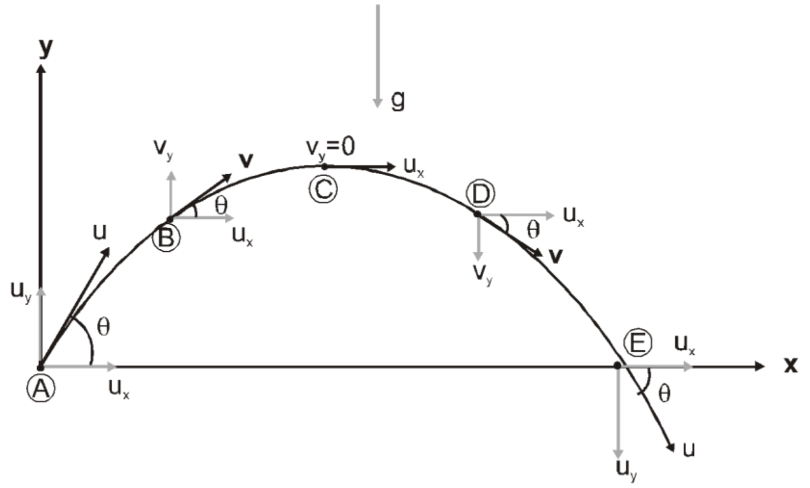
Y — displacement, $y = u_y t + \frac{1}{2} a_y t^2 = 6 \times 2 + \frac{1}{2} \times 8 \times (2)^2 = 28$ m.

PROJECTILE MOTION :-

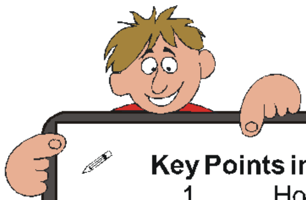
One of the common example of projectile motion is when a particle is projected with some initial velocity obliquely in vertical plane experiencing a constant free fall acceleration vertically downward. Such a particle is called Projectile (meaning that it is projected or launched) and its motion is called Projectile Motion.

In such a motion

- We shall consider only trajectories that are of sufficiently short range so that the gravitational force can be considered constant in both magnitude and direction.
- All effects of air resistance will be ignored.
- Earth is assumed to be flat.



The parabolic path of a projectile that leaves the origin with a velocity . The velocity vector changes with time in both magnitude and direction. This change is the result of acceleration in the negative y direction. The x component of velocity remains constant in time because there is no acceleration along the horizontal direction. The y component of velocity is zero at the peak of the path.



Key Points in the Diagram :

1. Horizontal component of velocity during the motion remains constant.
2. Vertical velocity first decreases to zero at maximum height and then increases during the flight.
3. Speed of particle is equal at same horizontal level
   (i.e. Speed at A = Speed at E.
   Also, Speed at B = Speed at D)
4. Angle between velocity vector and x-axis are equal at same horizontal level but having opposite sense
5. Motion is symmetrical about vertical line passing through point C
   i.e (i) Time taken from B to C = time taken from C to D
   (ii) Time taken from A to C = Time taken from C to E

- Consider a projectile thrown with a velocity u making an angle θ with the horizontal.
- Initial velocity u is resolved in components in a coordinate system in which horizontal direction is taken as x-axis, vertical direction as y-axis and point of projection as origin.

	Horizontal direction	Vertical direction
(a)	Initial velocity $u_x = u \cos\theta$	Initial velocity $u_y = u \sin\theta$
(b)	Acceleration $a_x = 0$	Acceleration $a_y = g$
(c)	Velocity after time t, $v_x = u \cos\theta$	Velocity after time t,$v_y = u \sin\theta - gt$
(d)	displacement $x = u \cos\theta\ t$	displacement $y = u \sin\theta\ t - gt^2$

Time of flight :- Time for which particle remains in air.
The displacement along vertical direction is zero for the complete flight.
In figure, vertical displacement from A to E = 0

$\Rightarrow \quad y = u_y t + \frac{1}{2} a_y t^2 = u \sin\theta . T - \frac{1}{2} g T^2 \quad \Rightarrow \quad$ **T =** $\frac{2u\sin\theta}{g}$

Horizontal range :- Horizontal distance travelled during its flight.
Horizontal displacement from A to E is the range of projectile (in figure)

$R = u_x . T \quad \Rightarrow \quad R = u \cos\theta . \frac{2u\sin\theta}{g}$

R = $\frac{u^2 \sin 2\theta}{g}$

Maximum height :- At the highest point of its trajectory, particle moves horizontally, and hence vertical component of velocity is zero.
Using 3rd equation of motion along y-axis.

$v_y^2 = u_y^2 + 2a_y \cdot y \quad \Rightarrow 0 = u^2 \sin^2\theta - 2gH$

$\Rightarrow$ **H =** $\frac{u^2 \sin^2\theta}{2g}$

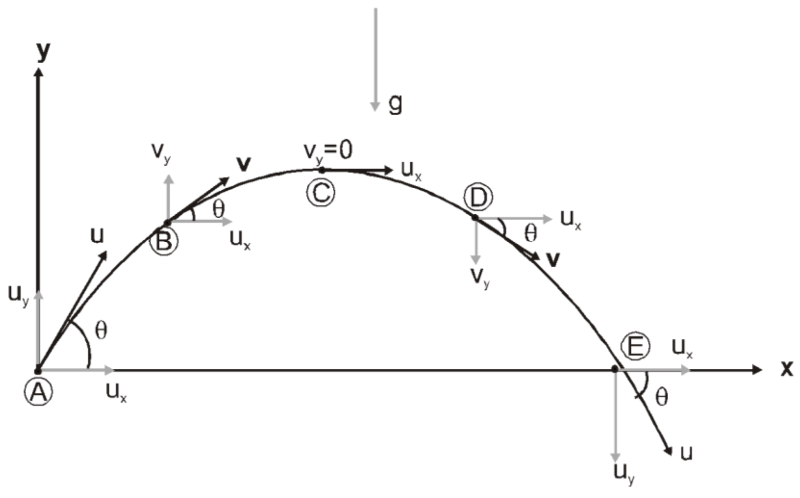
Resultant velocity at time t :- $\vec{v} = v_x \hat{i} + v_y \hat{j} = u \cos\theta \, \hat{i} + (u \sin\theta - gt) \, \hat{j}$

Resultant speed at time t :- $|v| = \sqrt{u^2 \cos^2\theta + (u\sin\theta - gt)^2}$

Angle Substended by particle at time t with Horizontal:-

From figure : $\tan\alpha = \frac{v_y}{v_x} = \frac{v_y}{u_x} = \frac{u\sin\theta - gt}{u\cos\theta}$

Also, $v\cos\alpha = u\cos\theta \quad \Rightarrow \quad v = \frac{u\cos\theta}{\cos\alpha}$



For ground to ground projectile

- $T = \dfrac{2u\sin\theta}{g}$ $\quad R = \dfrac{u^2\sin 2\theta}{g}$ $\quad H = \dfrac{u^2\sin^2\theta}{2g}$

Important Note :

- For maximum range $\theta = 45^o$

  $R_{max} = \dfrac{u^2}{g}$ $\Rightarrow$ $H_{max} = \dfrac{R_{max}}{2}$
- We get the same range for two angle of projections α and $(90 - \alpha)$ (Complementary angles) but in both cases, maximum heights attained by the particles are different.

  This is because, $R = \dfrac{u^2\sin 2\theta}{g}$, and $\sin 2(90 - \alpha) = \sin 180 - 2\alpha = \sin 2\alpha$
- If $R = H$ i.e. $= \dfrac{u^2\sin^2\theta}{2g}$ $\Rightarrow$ $\tan\theta = 4$
- Range can also be expressed as

  $R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{2u\sin\theta . u\cos\theta}{g} = \dfrac{2u_x u_y}{g}$
- Time of flight T and maximum hight H depends on vertical component of velocity.

Solved Examples

Example 3 A projectile is thrown with a speed of 100 m/s making an angle of 60^o with the horizontal. Find the time after which its inclination with the horizontal is 45^o ?

Solution

$u_x = 100 \times \cos 60^o = 50$

$u_y = 100 \times \sin 60^o = 50\sqrt{3}$

$v_y = u_y + a_y t = 50\sqrt{3} - gt$ and $v_x = u_x = 50$

When angle is 45^o ,

$\tan 45^o = \dfrac{v_y}{v_x}$ $\Rightarrow$ $v_y = v_x$

$50\sqrt{3} - gt = 50$ $\Rightarrow$ $50(\sqrt{3} - 1) = gt$ $\Rightarrow$ $t = 5(\sqrt{3} - 1)$ s

Example 4 A large number of bullets are fired in all directions with the same speed v. What is the maximum area on the ground on which these bullets will spread ?

Solution Maximum distance upto which a bullet can be fired is its maximum range, therefore

$R_{max} = \dfrac{v^2}{g}$ Maximum area $= \pi(R_{max})^2 = \dfrac{\pi v^4}{g^2}$.

Example 5 The velocity of projection of a projectile is given by : $\vec{u} = 5\hat{i} + 10\hat{j}$. Find

(a) Time of flight,

(b) Maximum height,

(c) Range

Solution We have $u_x = 5$ $u_y = 10$

(a) Time of flight = $\frac{2u\sin\theta}{g} = \frac{2u_y}{g} = \frac{2\times10}{10} = 2$ s

(b) Maximum height = $\frac{u^2\sin^2\theta}{2g} = \frac{{u_y}^2}{2g} = \frac{10\times10}{2\times10} = 5$ m

(c) Range = $\frac{2u\sin\theta.u\cos\theta}{g} = \frac{2u_xu_y}{g} = \frac{2\times10\times5}{10} = 10$ m

Example 6 A body is projected with a speed of 30 ms^{-1} at an angle of 30º with the vertical. Find the maximum height, time of flight and the horizontal range of the motion. [Take g = 10 m/s^2]

Solution Here u = 30 ms^{-1}, Angle of projection, θ = 90 – 30 = 60º

Maximum height,

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{30^2\sin^2 60^o}{2\times10} = \frac{900}{20}\times\frac{3}{4} = \text{m}$$

Time of flight,

$$T = \frac{2u\sin\theta}{g} = \frac{2\times30\times\sin60^0}{10} = 3\sqrt{3}\ \text{sec.}$$

$$\text{Horizontal range} = R = \frac{u^2\sin2\theta}{g} = \frac{30\times30\times2\sin60^o\cos60^o}{10} = 45\sqrt{3}\ \text{m}$$

Example 7 Two projectiles are thrown with different speeds and at different angles so as to cover the same maximum height. Find the ratio of time taken by particle to reach maximum height.

Solution $H_1 = H_2$ (given)

$$\frac{u_1^2\ \sin^2\theta_1}{2g} = \frac{u_2^2\ \sin^2\theta_2}{2g}$$

$u_1^2 \sin^2\theta_1 = u_2^2 \sin^2\theta_2$ .... (1)

Time taken to reach the maximum height is half the time of flight.

$\therefore T_1 = \frac{u_1\sin\theta_1}{g}$ and $T_2 = \frac{u_2\sin\theta_2}{g}$ ⇒ $\therefore$ $T_1 = T_2$ **(Using 1)**

Answer : 1 : 1

Self Practice Problems

1. A body is projected with a speed 'u' at an angle to the horizontal to have maximum range at the highest point the velocity is :

(1) zero (2) u (3) $\frac{u}{\sqrt{2}}$ (4) u$\sqrt{2}$

2. If two stones projected from the same point with same initial speed but an angle π/3 and π/6 respectively have their ranges R_1 and R_2 then –

(1) $R_1 = 2R_2$ (2) $R_1 = R_2$ (3) $R_1 = 5R_2$ (4) $R_1 = 25R_2$

3. The time of flight of projectile is 10 second and its range is 500m. The maximum height reached by it will be (g = 10 m/s^2)

(1) 25 m (2) 50 m (3) 82 m (4) 125 m

4. If four balls A, B, C, D are projected with same speed at angles of 15º, 30º, 45º and 60º with the horizontal respectively, the two balls which will fall at the same place will be
(1) A and B (2) A and D (3) B and D (4) A and C

5. A man can throw a stone 80 m. The maximum height to which it will rise in meters is :
(1) 10 m (2) 20 m (3) 40 m (4) 50 m

6. A body is projected at an angle of 30º to the horizontal with a speed of 40 m/s. The angle with the horizontal after 2 seconds will be
(1) 10º (2) 30º (3) 45º (4) 0º

7. Two bodies are projected at angles θ and (90 – θ) to the horizontal with the same speed. The ratio of their times of flight is.
(1) sinθ (2) cos θ (3) tan θ (4) $\tan^2 \theta$

8. In the above question find the ratio of maximum vertical heights is.
(1) sinθ (2) cos θ (3) tan θ (4) $\tan^2 \theta$

9. A body is so projected in the air that the horizontal range covered by the body is equal to the maximum vertical height attained by the body during the motion. The angle of projection is ?
(1) $\theta = \sin^{-1}(4)$ (2) $\theta = \cos^{-1}(4)$ (3) $\theta = \tan^{-1}(4)$ (4) $\theta = \cot^{-1}(4)$

Answer Key : **1.** (3) **2.** (2) **3.** (4) **4.** (3) **5.** (2) **6.** (4) **7.** (3) **8.** (4) **9.** (3)

EQUATION OF TRAJECTORY

The path followed by a particle (here projectile) during its motion is called its **Trajectory**. Equation of trajectory is the relation between instantaneous coordinates (Here x & y coordinate) of the particle.

Along Horizontal direction : $x = u_x . t$

$x = u \cos \theta . t$ ...(1)

Along vertical direction : $y = u_y . t - 1/2\, g\, t^2$

$= u \sin \theta . t - 1/2\, g\, t^2$ ...(2)

Eliminating 't' from equation (1) & (2)

$$y = u \sin \theta . \frac{x}{u\cos\theta} - \frac{1}{2} g\left(\frac{x}{u\cos\theta}\right)^2 \Rightarrow \mathbf{y = x \tan \theta - \frac{gx^2}{2u^2\cos^2\theta}}$$

This is an equation of parabola called as trajectory equation of projectile motion.

Other forms of trajectory equation :

- $$\mathbf{y = x \tan \theta - \frac{gx^2(1+\tan^2\theta)}{2u^2}}$$
- $$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

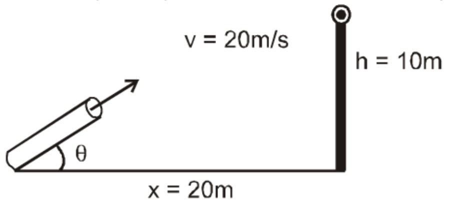
$$\Rightarrow y = x \tan \theta \left[1 - \frac{gx}{2u^2\cos^2\theta\tan\theta}\right]$$

$$\Rightarrow y = x \tan \theta \left[1 - \frac{gx}{2u^2 \sin\theta \cos\theta}\right]$$

$$\Rightarrow \mathbf{y = x \tan \theta} \left[1 - \frac{x}{R}\right]$$

Solved Examples

Example 8. Find the value of θ in the diagram given below so that the projectile can hit the target.



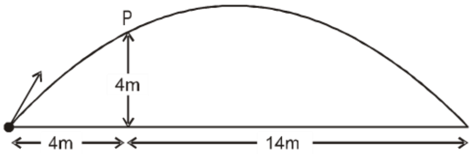
Solution $y = x\tan\theta - \dfrac{gx^2(1+\tan^2\theta)}{2u^2}$

$\Rightarrow 10 = 20\tan\theta - \dfrac{5\times(20)^2}{(20)^2}(1+\tan^2\theta)$

$\Rightarrow 2 = 4\tan\theta - (1+\tan^2\theta)$ $\quad\Rightarrow \tan^2\theta - 4\tan\theta + 3 = 0$

$\Rightarrow (\tan\theta - 3)(\tan\theta - 1) = 0$ $\quad\Rightarrow \tan\theta = 3, 1$ $\quad\Rightarrow \theta = 45^\circ, \tan^{-1}(3)$

Example 9. A ball is thrown from ground level so as to just clear a wall 4 m high at a distance of 4 m and falls at a distance of 14 m from the wall. Find the magnitude and direction of initial velocity of the ball. figure is given below.



Solution

The ball passes through the point P(4, 4). Also range = 4 + 14 = 18 m.
The trajectory of the the ball is,

$y = x\tan\theta\left(1 - \dfrac{x}{R}\right)$

Now $x = 4\text{m}, y = 4\text{m}$ and $R = 18$ m

$\therefore$ $4 = 4\tan\theta\left[1-\dfrac{4}{18}\right] = 4\tan\theta\,\dfrac{7}{9}$.

or $\tan\theta = \dfrac{9}{7}$ $\quad\Rightarrow\quad \theta = \tan^{-1}\dfrac{9}{7}$ $\quad$ And $\quad R = \dfrac{2u^2\sin\theta\cos\theta}{g}$

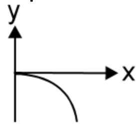
or $18 = \dfrac{2}{9.8}\times u^2\times\dfrac{9}{\sqrt{130}}\times\dfrac{7}{\sqrt{130}}$ $\quad\Rightarrow\quad u = \sqrt{182}$

Example 10 The radius vector of a point A relative to the origin varies with time t as $\vec{r} = at\,\hat{i} - bt^2\,\hat{j}$, where a and b are positive constants and $\hat{i}$ and $\hat{j}$ are the unit vectors of the x and y axes. Find:
(i) The equation of the point's trajectory y (x) ; plot this function
(ii) The time dependence of the velocity **v** and acceleration **a** vectors as well as of the moduli of these quantities.

Solution Comparing $\vec{r} = at\,\hat{i} - b\,t^2\,\hat{j}$ with $\vec{r} = x\,\hat{i} + y\,\hat{j}$
we get $x = at$
& $y = -bt^2$

$\Rightarrow \quad y = -b\left(\frac{x}{a}\right)^2$ equation of trajectory

(i) $y = -\frac{bx^2}{a^2}$



Ans

(ii) $\vec{v} = a\hat{i} - 2bt\hat{j}$, acceleration = $-2b\hat{j}$,

$|\vec{v}| = \sqrt{a^2 + 4b^2t^2}$, |acceleration | = 2b

$\vec{v} = a\hat{i} - 2bt\hat{j}$, $= -2b\hat{j}$, $|\vec{v}| = \sqrt{a^2 + 4b^2t^2}$, = 2b

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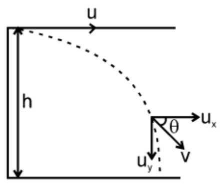
(PROJECTION FROM TOWER)

1. PROJECTILE THROWN PARALLEL TO THE HORIZONTAL FROM SOME HEIGHT

Consider a projectile thrown from point O at some height h from the ground with a velocity u. Now we shall study the characteristics of projectile motion by resolving the motion along horizontal and vertical directions.



	Horizontal direction	**Vertical direction**
(i)	Initial velocity $u_x = u$	Initial velocity $u_y = 0$
(ii)	Acceleration $a_x = 0$	Acceleration $a_y = g$ (downward)
(iii)	Velocity after time t: $v_x = u_x = u$	Velocity after time t: $v_y = u_y + a_yt = 0 - gt = -gt$
(iv)	displacement, $x = u_xt = ut$	displacement, $y = u_yt - (1/2)gt^2 = -(1/2)gt^2$

Fill in the Blanks

Time of flight : Time taken by the projectile to reach to ground.
Since vertical displacement to reach the ground, y = - h
Using, $y = u_yt + 1/2\, a_yt^2$

$\Rightarrow$ _ _ _ _ _ _ _ _ _ _ _ _

$\Rightarrow$ **t =** _ _ _ _ _ _ _ _

Horizontal range : Distance covered by the projectile along the horizontal direction between the point of projection to the point on the ground.
Along Horizontal, $R = u_x . t$
R = _ _ _ _ _ _ _ _ _

Velocity at time t : $\vec{v} = v_x\hat{i} + v_y\hat{j}$

Horizontal velocity of particle after time t : v_x = _ _ _ _
Vertical velocity of particle after time t : v_y = _ _ _ _ _ _ _ _ _

Speed a time t : $v = \sqrt{v_x^2 + v_y^2}$ = _ _ _ _ _ and $\tan\theta = v_y/v_x$= _ _ _ _ _

Speed with which the projectile hits the ground :

$v_x = u$

$v_y^2 = 0^2 - 2g(-h)$

$v_y = \sqrt{2gh}$ $\Rightarrow$ $v = \sqrt{V_x^2 + V_y^2}$ $\Rightarrow$ **V =** $\sqrt{\mathbf{u^2 + 2gh}}$

Trajectory equation : The path followed by a particle (here projectile) during its motion is called its **Trajectory**. Equation of trajectory is the relation between instantaneous coordinates (Here x & y coordinate) of the particle.

Along Horizontal : $x = ut$ ....(1)

Along vertical : $y = \frac{-1}{2} gt^2$ ....(2)

Put the value of t from eq. (1) in eq. (2) $\mathbf{y = \frac{-1}{2}\, g\, \frac{x^2}{u^2}}$ **.**

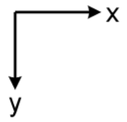
This is trajectory equation of the particle projected horizontally from some height.

Solved Examples

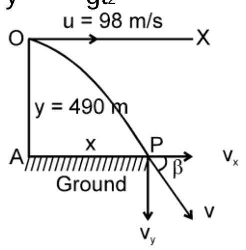
EXAMPLES BASED ON HORIZONTAL PROJECTION FROM SOME HEIGHT :

Example 11 A projectile is fired horizontally with a speed of 98 ms^{-1} from the top of a hill 490 m high. Find (i) the time taken to reach the ground (ii) the distance of the target from the hill and (iii) the velocity with which the projectile hits the ground. (take g = 9.8 m/s^2)

Solution (i) The projectile is fired from the top O of a hill with speed u = 98 ms^{-1} along the horizontal as shown as OX. It reaches the target P at vertical depth
OA, in the coordinate system as shown, OA = y = 490 m



As, $y = \frac{1}{2} gt^2$



∴ $490 = \frac{1}{2} \times 9.8\, t^2$

or $t = \sqrt{100} = 10$ s.

(ii) Distance of the target from the hill is given by,
AP = x = Horizontal velocity × time = 98 × 10 = 980 m.

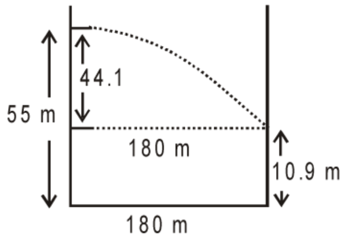
(iii) The horizontal and vertical components of velocity v of the projectile at point P are
$v_x = u = 98\ ms^{-1}$
$v_y = u_y + gt = 0 + 9.8 \times 10 = 98\ ms^{-1}$
$V = \sqrt{v_x^2 + v_y^2} = \sqrt{98^2 + 98^2} = 98\sqrt{2}\ ms^{-1}$
Now if the resultant velocity v makes an angle β with the horizontal, then
$\tan\beta = \frac{v_y}{v_x} = \frac{98}{98} = 1$
∴ β = 45º

Example 12 An object is thrown between two tall buildings. 180 m from each other. The object is thrown horizontally from a window 55 m above ground from one building through a window 10.9 m above ground in the other building. Find out the speed of projection. (use g = 9.8 m/s2)

Solution $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 44.1}{9.8}}$

t = 3 sec.

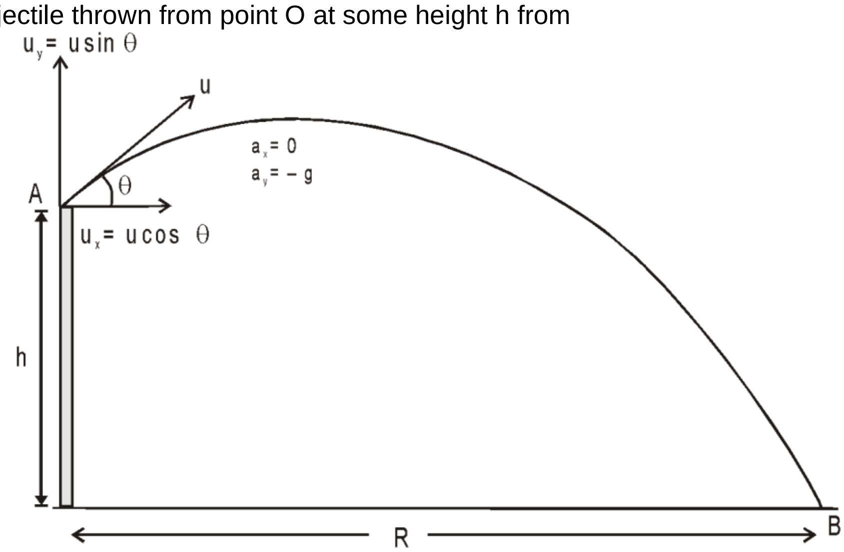
R = uT



$\frac{180}{3} = u$; u = 60 m/s

2. PROJECTION FROM A TOWER (With some angle of projection above horizontal)

Consider a projectile thrown from point O at some height h from



the ground with a velocity u as shown. Now we shall study the characteristics of projectile motion by resolving the motion along horizontal and vertical directions.

	Horizontal direction	Vertical direction
(i)	Initial velocity $u_x = u\cos\theta$	Initial velocity $u_y = u\sin\theta$
(ii)	Acceleration $a_x = 0$	Acceleration $a_y = g$ (downward)
(iii)	Velocity after time t: $v_x = u_x = u\cos\theta$	Velocity after time t: $v_y = u_y + a_y t = u\sin\theta - gt$
(iv)	displacement, $x = u_x t = u\cos\theta\, t$	displacement, $y = u_y t - (1/2)\, gt^2 = u\sin\theta\, t - (1/2)\, gt^2$

Fill in the Blanks

Time of flight : Time taken by the projectile to reach to ground.

Since vertical displacement to reach the ground, y = - h

Using, $y = u_y t + 1/2\, a_y t^2$

⇒ _ _ _ _ _ _ _ _ _ _ _ _

⇒ **t =** _ _ _ _ _ _ _ _

Horizontal range : Distance covered by the projectile along the horizontal direction between the point of projection to the point on the ground.

Along Horizontal, $R = u_x \cdot t$

R = _ _ _ _ _ _ _ _ _ _

Velocity at time t : $\vec{v} = v_x\hat{i} + v_y\hat{j}$

Horizontal velocity of particle after time t : v_x = _ _ _ _

Vertical velocity of particle after time t : v_y = _ _ _ _ _ _ _ _ _ _

Speed a time t : $v = \sqrt{v_x^2 + v_y^2}$ = _ _ _ _ _ and $\tan\theta = v_y/v_x$= _ _ _ _ _

Speed with which the projectile hits the ground :

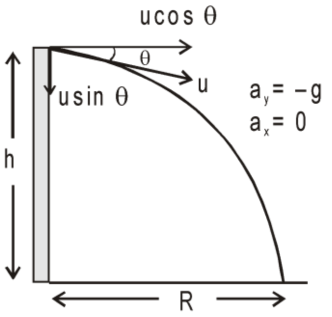
$v_x = u\cos\theta$

$v_{y2} = (u\sin\theta)_2 - 2g(-h)$

$v = \sqrt{V_x^2 + V_y^2}$ $\Rightarrow$ $\mathbf{v = \sqrt{u^2 + 2gh}}$

3. PROJECTION FROM A TOWER (With some angle of projection below horizontal)

Consider a projectile thrown from point O at some height h from the ground with a velocity u. Again we study the characteristics of projectile motion by resolving the motion along horizontal and vertical directions.



	Horizontal direction	Vertical direction
(i)	Initial velocity $u_x = u\cos\theta$	Initial velocity $u_y = -u\sin\theta$
(ii)	Acceleration $a_x = 0$	Acceleration $a_y = g$ (downward)
(iii)	Velocity after time t: $v_x = u_x = u\cos\theta$	Velocity after time t: $v_y = u_y + a_y t = -u\sin\theta - gt$
(iv)	displacement, $x = u_x t = u\cos\theta\, t$	displacement, $y = u_y t - (1/2)\, gt_2 = -u\sin\theta\, t - (1/2)\, gt_2$

Fill in the Blanks

Time of flight : Time taken by the projectile to reach to ground.

Since vertical displacement to reach the ground, $y = -h$

Using, $y = u_y t + 1/2\, a_y t_2$

$\Rightarrow$ _ _ _ _ _ _ _ _ _ _ _ _

$\Rightarrow$ **t =** _ _ _ _ _ _ _ _ _

Horizontal range : Distance covered by the projectile along the horizontal direction between the point of projection to the point on the ground.

Along Horizontal, $R = u_x \cdot t$

R = _ _ _ _ _ _ _ _ _ _

Velocity at time t : $\vec{v} = v_x\hat{i} + v_y\hat{j}$

Horizontal velocity of particle after time t : v_x = _ _ _ _

Vertical velocity of particle after time t : v_y = _ _ _ _ _ _ _ _ _ _

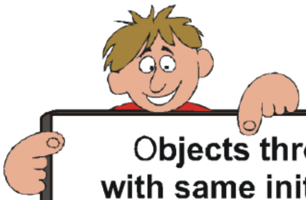
Speed a time t : $v = \sqrt{v_x^2 + v_y^2}$ = _ _ _ _ _ and $\tan\theta = v_y/v_x$= _ _ _ _ _

Speed with which the projectile hits the ground :

$v_x = u\cos\theta$

$v_y^2 = (-u\sin\theta)^2 - 2g(-h)$

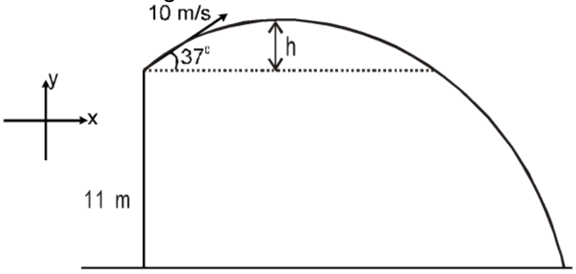
$v = \sqrt{V_x^2 + V_y^2}$ $\Rightarrow$ $\mathbf{v = \sqrt{u^2 + 2gh}}$



Objects thrown from same height in different directions with same initial speed will strike the ground with the same final speed. But the time of flight will be different.

Solved Examples

Example 13 From the top of a 11 m high tower a stone is projected with speed 10 m/s, at an angle of 37º as shown in figure. Find



(a) Speed after 2s

(b) Time of flight.

(c) Horizontal range.

(d) The maximum height attained by the particle.

(e) Speed just before striking the ground.

Solution

(a) Initial velocity in horizontal direction = 10 cos 37 = 8 m/s

Initial velocity in vertical direction = 10 sin 37º = 6 m/s

Speed after 2 seconds

$v = v_x\hat{i} + v_y\hat{j} = 8\hat{i} + (u_y + a_y t)\hat{j} = 8\hat{i} + (6 - 10 \times 2)\hat{j} = 8\hat{i} - 14\hat{j}$

(b) $S_y = u_y t + \frac{1}{2} a_y t^2 \quad \Rightarrow \quad -11 = 6 \times t + \frac{1}{2} \times (-10)\, t^2$

$5t^2 - 6t - 11 = 0 \quad \Rightarrow \quad (t+1)(5t - 11) = 0 \quad \Rightarrow \quad t = \frac{11}{5}$ sec.

(c) Range $= 8 \times \frac{11}{5} = \frac{88}{5}$ m

(d) Maximum height above the level of projection,

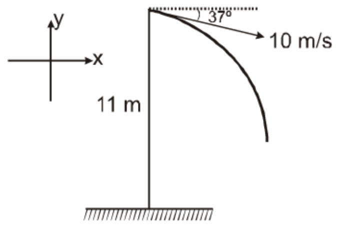
$h = \frac{u_y^2}{2g} = \frac{6^2}{2 \times 10} = 1.8$ m

$\therefore$ maximum height above ground = 11 + 1.8 = 12.8 m

(e) $v = \sqrt{u^2 + 2gh} = \sqrt{100 + 2 \times 10 \times 11}$

$\Rightarrow v = 8\sqrt{5}$ m/s

Example 14 From the top of a 11 m high tower a stone is projected with speed 10 m/s, at an angle of 37º as shown in figure. Find



(a) Time of flight.
(b) Horizontal range.
(c) Speed just before striking the ground.

Solution $u_x = 10 \cos 37^\circ = 8$ m/s , $u_y = -10 \sin 37^\circ = -6$ m/s

(a) $S_y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow -11 = -6 \times t + \frac{1}{2} \times (-10) t^2$

$5t^2 + 6t - 11 = 0$

$\Rightarrow$ $(t-1)(5t+11) = 0 \Rightarrow t = 1$ sec

(b) Range $= 8 \times 1 = 8$ m

(c) $v = \sqrt{u^2 + 2gh} = \sqrt{100 + 2 \times 10 \times 11}$

$v = \sqrt{320}$ m/s $= 8\sqrt{5}$ m/s

Self Practice Problems

10. An aeroplane is moving with a horizontal velocity u at a height h above the ground, if a packet is dropped from it the speed of the packet when it reaches the ground will be :

(1) $\sqrt{u^2 + 2gh}$ (2) $\sqrt{2gh}$ (3) $\sqrt{u^2 - 2gh}$ (4) 2gh

11. A ball is thrown horizontally and the same time another ball is dropped down from the top of a tower
(A) Both the balls will reach the ground at the same time
(B) Both will strike the ground with the same velocity
(1) A is true and B is false (2) A is true and B is true
(3) A is false and B is true (4) A is false and B is false

12. A body is thrown downward at an angle of 30º with the horizontal from the top of a tower 160m high. If its initial speed is 40 m/s the time taken to reach the ground will be :
(1) 2s (2) 3s (3) 4s (4) 5s

13. From the top of a tower of height h a body of mass m is projected in the horizontal direction with a velocity v, it falls on the ground at a distance x from the tower if a body of mass 2m is projected from the top of another tower of height 2h in the horizontal direction so that it falls on the ground at a distance 2x from the tower, the horizontal velocity of the second body is :

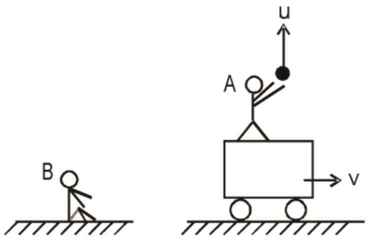
(1) 2v (2) $\sqrt{2}$ v (3) v/2 (4)

14. The equation of a projectile is $y = \sqrt{3}\,x - \frac{gx^2}{2}$, find the angle of projection. Where at t = 0, x = 0 and y = 0 also $\frac{d^2x}{dt^2} = 0$ & $\frac{d^2y}{dt^2} = -g$.
(1) $\theta = 30°$ (2) $\theta = 60°$ (3) $\theta = 53°$ (4) $\theta = 90°$

15. In the above question find the speed of projection
(1) 1 m/s (2) 2 m/s° (3) 3 m/s (4) 4 m/s

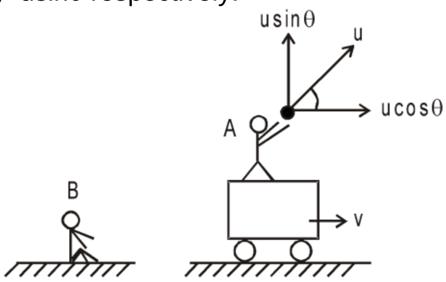
Answer Key : **10.** (1) **11.** (1) **12.** (3) **13.** (2) **14.** (2) **15.** (2)

1. PROJECTION FROM A MOVING PLATFORM

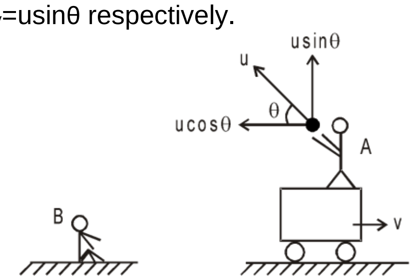


CASE (1) : When a ball is thrown upward from a truck moving with uniform speed, then observer A standing in the truck, will see the ball moving in straight vertical line (upward & downward).
The observer B sitting on road, will see the ball moving in a parabolic path. The horizontal speed of the ball is equal to the speed of the truck.

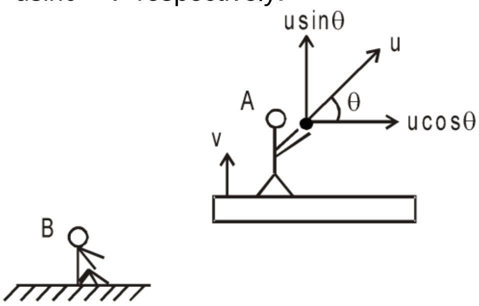
CASE (2) : When a ball is thrown at some angle 'θ' in the direction of motion of the truck, horizontal & vertical component of ball's velocity w.r.t. observer A standing on the truck, is ucosθ, and usinθ respectively.
Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground, is
u_x = ucosθ + v and u_y=usinθ respectively.



CASE (3) : When a ball is thrown at some angle 'θ' in the opposite direction of motion of the truck, horizontal & vertical component of ball's velocity w.r.t. observer A standing on the truck, is ucosθ, and usinθ respectively.
Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground, is
u_x = ucosθ – v and u_y=usinθ respectively.



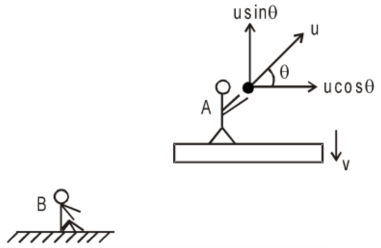
CASE (4) : When a ball is thrown at some angle 'θ' from a platform moving with speed v upwards, horizontal & vertical component of ball's velocity w.r.t. observer A standing on the moving platform, is ucosθ and usin θ respectively.
Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground, is
u_x = ucosθ and u_y = usinθ + v respectively.



CASE (5) : When a ball is thrown at some angle 'θ' from a platform moving with speed v downwards, horizontal & vertical component of ball's velocity w.r.t. observer A standing on the moving platform, is $u\cos\theta$ and $u\sin\theta$ respectively.

Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground, is

$u_x = u\cos\theta$ and $u_y = u\sin\theta - v$ respectively.



Solved Examples

Example 15 A boy standing on a long railroad car throws a ball straight upwards. The car is moving on the horizontal road with an acceleration of 1 m/s^2 and the projection speed in the vertical direction is 9.8 m/s. How far behind the boy will the ball fall on the car ?

Solution Let the initial velocity of car be 'u'.

time of flight, $t = \frac{2u_y}{g} = 2$

where u_y = component of velocity in vertical direction

Distance travelled by car $x_c = u \times 2 + \frac{1}{2} \times 1 \times 2^2 = 2u + 2$

distance travelled by ball $x_b = u \times 2$

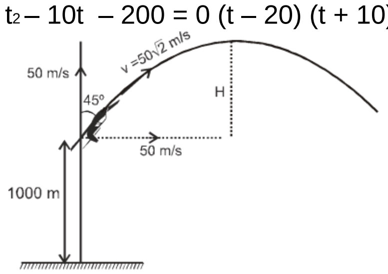
$x_c - x_b = 2u + 2 - 2u = 2m$ **Ans.**

Example 16 A fighter plane moving with a speed of $50\sqrt{2}$ m/s upward at an angle of 45º with the vertical, releases a bomb. Find

(a) Time of flight (b) Maximum height of the bomb above ground

Solution (a) $y = u_y t + \frac{1}{2}a_y t^2$ $-1000 = 50t - \frac{1}{2}\times 10 \times t^2$

$t^2 - 10t - 200 = 0$ $(t - 20)(t + 10) = 0$ ⇒ $t = 20$ sec



(b) $H = \frac{u_y^2}{2g} = \frac{50^2}{2g} = \frac{50 \times 50}{20} = 125$ m

Hence maximum height above ground

H = 1000 + 125 = 1125 m

DO IT YOURSELF - 1

A Bomber flying upward at an angle of 53º with the vertical releases a bomb at an altitude of 800 m. The bomb strikes the ground 20 sec after its release. Find :

(i) The velocity of the bomber at the time of release of the bomb .

(ii) The maximum height attained by the bomb .

(iii) The horizontal distance travelled by the bomb before it strikes the ground

(iv) The velocity (magnitude and direction) of the bomb just when it strikes the ground .

[Given sin 53º = 0.8 ; $g = 10\ ms^{-2}$]

Ans. (i) 100 m/s (ii) 980 m (iii) 1600 m (iv) $\theta = \tan^{-1}(7/4)$, $v = 20\sqrt{65}$ m/s