

HINTS & SOLUTIONS

TOPIC : KINETIC THEORY OF GASES & THERMODYNAMICS EXERCISE # 1

SECTION (A)

1. For isothermal, $PV = \text{constant}$, $P \propto \frac{1}{V}$. As area decreases \therefore number of collision per unit area increases
2. $\vec{P} = M\vec{V}_{av}$, As $\vec{V}_{av} = 0$ (in equilibrium) $\therefore \vec{P}_{av} = 0$
3. As $\vec{P}_{av} = M\vec{V}_{av}$, it depend mass and \vec{V}_{av} of gas
5. Equation for ideal gas is
 $PV = RT$
6. According to gas equation
 $PV = nRT$
$$V = \frac{nRT}{P} = \frac{2/32 \times 8.31 \times 300}{1 \times 10^5}$$
9. Work does not characterize the thermodynamic state of matter, it is a path function gives only relationship between two quantities.
10. Clear from the definition.
11. At low pressure and high temperature inter molecular forces become ineffective. So a real gas behaves like an ideal gas.
12. $P_1 = \frac{\rho_1 RT}{M_1}$... (i)
 $P_2 = \frac{\rho_2 RT}{M_2}$... (ii)
by (i) and (ii)
$$\frac{\rho_1}{\rho_2} = \frac{8}{9}$$
13. A stationary rigid body rotating about its axis has three degree of freedom.
14. From ideal gas equation
 $PV = nRT \Rightarrow PV \propto T$ $\therefore \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow p_2 = \frac{P_1 V_1 T_2}{T_2 T_1}$
Here : $p_1 = 1 \times 10^5 \text{ N/m}^2$, $v_1 = 1 \text{ m}^3$
 $T_1 = 300 \text{ K}$, $v_2 = v_1 = 2 \text{ m}^3$,
 $T_2 = 2T_1 = 600 \text{ K}$ putting given values in eq. (1)
$$p_2 = \frac{1 \times 10^5 \times 1 \times 600}{2 \times 300} = 10^5 \text{ N/m}^2$$
15. $\Delta Q = \mu C_v \Delta T$

Here : $\mu = \frac{2}{4} = \frac{1}{2}$
 $C_V = ME_v = 4 \times 3 = 12$

At constant volume

$$\frac{p_2}{p_1} = \frac{T_2}{T_1}$$

But $p_2 = 2p_1$ (given) $\therefore T_2 = 2T_1 = 2 \times 273 \text{ K}$

or $\Delta T = T_2 - T_1 = 273 \text{ K}$ So $\Delta Q = \frac{1}{2} \times 12 \times 273 = 1638 \text{ J}$

16. We know that for ideal gas $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ (at constant pressure)

$$T_2 = \frac{V_2}{V_1} \times T_1 = \frac{3V}{V} \times 273 = 3 \times 273 \text{ K}$$

$$T_2 = 819 \text{ K} = 819 - 273 = 546^\circ\text{C}$$

17. Ideal gas equation can be written as

$$pv = nRT \quad \text{or} \quad \frac{n}{v} = \frac{p}{RT} = \text{constant}$$

So, at constant pressure and temperature, all gases, will contain equal number of molecules per unit volume.

18. $f = 3 + 2$
 $f = 5$

19. No. of moles $n = \frac{m}{\text{molecular weight}} = \frac{5}{32}$

So, from ideal gas equation $PV = nRT \Rightarrow PV = \frac{5}{32}RT$

20. There will be no change in number of moles if the vessels are joined by valve. Therefore, from gas equation

$$PV = nRT$$

$$\Rightarrow \frac{P_1 V_1}{RT_1} + \frac{P_2 V_2}{RT_2} = \frac{P(V_1 + V_2)}{RT} \Rightarrow \frac{P_1 V_1 T_2 + P_2 V_2 T_1}{T_1 T_2} = \frac{P(V_1 + V_2)}{T}$$

$$\Rightarrow T = \frac{P(V_1 + V_2) T_1 T_2}{(P_1 V_1 T_2 + P_2 V_2 T_1)}$$

Internal energy of the system remains same before and after opening of valve, so

$$\frac{f n_1 R T_1}{2} + \frac{f n_2 R T_2}{2} = \frac{f(n_1 + n_2) R T}{2} \Rightarrow n_1 T_1 + n_2 T_2 = (n_1 + n_2) T$$

$$P_1 V_1 + P_2 V_2 = P(V_1 + V_2) \quad \text{Hence } T = \frac{(P_1 V_1 + P_2 V_2) T_1 T_2}{(P_1 V_1 T_2 + P_2 V_2 T_1)}$$

SECTION (B)

1. self explanatory

$$\frac{Pm}{\rho}$$

2. $\rho = nRT$ slope of $T_1 >$ slope of T_2
 $\therefore T_1 > T_2$

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3. one molecule has some single value of speed which is equal average speed and rms speed of the gas
 $\therefore V_a = V_{rms}.$

4.
$$V_{RMS} = \sqrt{\frac{3RT}{M_0}} = v \quad \text{for oxygen}$$

$$V_{RMS} = \sqrt{\frac{3R \times 2T}{M_0 / 2}} = 2v.$$

5.
$$V_{av} \propto \frac{1}{\sqrt{M_0}}$$
 \therefore oxygen molecule hits the wall with smaller average speed

6.
$$PV = \frac{M}{M_0} RT.$$

$$PV = \frac{M}{M_0} K N_0 T \quad \frac{MKT}{PV} = m_1 \quad m_1 = \text{mass of each molecule it depends nature of gas.}$$

8. The gases carbon-monoxide (CO) and nitrogen (N₂) are diatomic, so both have equal kinetic energy $\frac{5}{2}KT$, i.e. $E_1 = E_2$.

9. Velocity of molecules depends on its temperature as $V \propto \sqrt{T}$.

10.
$$\frac{v_{Rms_{He}}}{v_{Rms_{Ar}}} = \frac{\sqrt{\frac{3RT}{m_{He}}}}{\sqrt{\frac{3RT}{m_{Ar}}}} = \sqrt{\frac{m_{Ar}}{m_{He}}} = \sqrt{\frac{40}{4}} = \sqrt{10} \approx 3.16$$

11. Average kinetic energy of gas depends upon its temperature.
 So, H₂ and O₂ both are at equal temperature ratio of their average kinetic energies is 1 : 1

12. RMS velocity $u_{rms} \propto \sqrt{T}$
 If temperature is increased to 3 times u_{rms} increases by $\sqrt{3}$ times.

13. According to kinetic theory of gases, energy of molecule does not change when they collided with walls of container. So, assumption is incorrect.

14. The rms velocity of the molecule of a gas of molecular weight M at kelvin temperature T is given by,

$$C_{rms} = \sqrt{\left(\frac{3RT}{M}\right)}$$

Let M_O and M_H are molecular weights of oxygen and hydrogen and T_O and T_H the corresponding Kelvin temperature at which C_{rms} is same for both gases.

$$C_{rms(O)} = C_{rms(H)}$$

$$\sqrt{\left(\frac{3RT_O}{M_O}\right)} = \sqrt{\left(\frac{3RT_H}{M_H}\right)}$$

$$T_O = 273 + 47 = 320 \text{ K}$$

Hence,
$$\frac{T_O}{M_O} = \frac{T_H}{M_H}$$

$$M_O = 32, M_H = 2 \quad \therefore T_H = \frac{2}{32} \times 320 = 20 \text{ K}$$

15. The kinetic energy of gas

$$E = \frac{3}{2} nkT$$

$$\therefore E \propto T \quad \therefore \frac{E'}{E} = \frac{T'}{T} = \frac{400}{300} \Rightarrow \frac{E'}{E} = \frac{4}{3} = 1.33$$

16. $U_{rms} = \sqrt{\left(\frac{3RT}{M}\right)}$

$$\Rightarrow U_{rms} \propto \sqrt{T} \quad \therefore \frac{U'_{rms}}{U_{rms}} = \sqrt{\left(\frac{T'}{T}\right)}$$

$$\text{Here : } U_{rms} = U, T' = 3T \quad \therefore U'_{rms} = \sqrt{\left(\frac{3T}{T}\right)} U = \sqrt{3} U$$

17. Law of pressure is

$$\frac{p_1}{p_2} = \frac{T_1}{T_2}$$

$$\Rightarrow \frac{p}{p \times 0.4} = \frac{x + 273}{x + (273 + 1)} \Rightarrow \frac{250}{251} = \frac{x + 273}{x + 243} \quad \therefore x = -23^\circ \text{C} = 250 \text{ K}$$

18. When temperature of a gas is increased then its internal energy increases.

19. The rms velocity of sound in gas is

$$V_{rms(\text{molecules})} = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.4 \times RT}{M}} \quad \dots(1)$$

when the oxygen dissociates, its molecular mass becomes atomic mass,

so $M = \frac{M}{2}$ and $T = 2T$ given and $= 1.66$

$$V_{rms(\text{atomic})} = \sqrt{\frac{1.66 \times R \times 2T}{M/2}} = \sqrt{\frac{1.66 \times R \times 2T \times 2}{M}} \quad \dots(2)$$

From equations (1) and (2)

$$\frac{V_{rms(\text{atomic})}}{V_{rms(\text{molecular})}} = \sqrt{\frac{1.66 \times R \times 2T \times 2}{M}} \times \sqrt{\frac{M}{1.4 \times RT}} = \sqrt{\frac{1.66 \times 4}{1.4}} = 2.18$$

Hence, $V_{rms}(\text{atomic}) = 2.18 V_{rms}(\text{molecular})$

20. We know that rms speed is directly proportional to square root of temperature

$$V_{rms} \propto \sqrt{T}$$

$$\text{Hence, } \frac{V_{rms(1)}}{V_{rms(2)}} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{400}{V_{rms(2)}} = \frac{\sqrt{27 + 273}}{\sqrt{227 + 273}} = \sqrt{\frac{300}{500}}$$

$$V_{rms(2)} = V = \sqrt{\frac{500}{300}} \times 400 = 1.29 \times 400$$

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$$= 516.39 \text{ m/s}$$

$$\approx 516 \text{ m/s}$$

21. Root mean square speed of molecules in a gas is defined as the square root of mean of squares of the speed of different molecules i.e.,

$$v_{rms} = \sqrt{(v_1^2 + v_2^2 + \dots) / N}$$

$$\text{or } v_{rms} = \sqrt{\frac{3RT}{M}}$$

(According to kinetic theory of gases)

While most probable speed is the speed which maximum number of molecules in a gas have at constant temperature and is given by

$$v_{mp} = \sqrt{\frac{2RT}{M}}$$

It is obvious that $v_{rms} > v_{mp}$.

22. Kinetic energy of gas molecules

$$E = \frac{3}{2} pV$$

$$p = \frac{2E}{3V} = \frac{2}{3} \times \frac{1.52 \times 10^5}{20 \times 10^{-3}} = \frac{10^5}{20 \times 10^{-3}} = 5 \times 10^6 \text{ N.m}^{-2}$$

23. Root mean square speed, $v_{rms} = \sqrt{\frac{3p}{\rho}}$ $p = \frac{v_{rms}^2 \rho}{3} = \frac{(3180)^2 \times 8.99 \times 10^{-2}}{3}$
 $= 3 \times 10^5 \text{ N/m}^2$
 3 atm.

24. KE average is dependent on temp. Only not on nature of gas. So ans.(a)

25. Speed of sound

$$u = \sqrt{\frac{\gamma p}{d}} \propto \frac{1}{\sqrt{d}}$$

(for given same pressure, γ and temperature)

$$\therefore \frac{v_1}{v_2} = \sqrt{\left(\frac{d_2}{d_1}\right)}$$

SECTION (C)

$$1. \quad v_{avr} = \bar{V} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8RT}{\pi M}} \Rightarrow \frac{\bar{V}_{H_2}}{\bar{V}_{N_2}} = \sqrt{\frac{1/2}{1/28}} = \sqrt{\frac{14}{1}}$$

$$2. \quad v_{av} = \sqrt{\frac{8RT}{\pi M_0}}, v_{AV} \propto \sqrt{T}$$

For same temp in vessel A, B and C, Average speed of O_2 molecule is same in vessel A and C and is equal to V_1 .

SECTION (D)

$$1. \quad \text{As translation K.E is } E = \frac{3}{2} nRT \quad E = \frac{3}{2} PV \quad P = \frac{2E}{3V}$$

where E = Translation K.E.

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2. For an ideal gas, the no of molecules in 1 mole of all gas at is same .
3. As ΔU is a state function i.e., it depends initial and final position in process A and B initial and final temp are same.
 $\therefore \Delta U_1 = \Delta U_2$
4. Internal energy

$$U = \frac{f}{2} KT \text{ per molecule} = \frac{f}{2} RT \text{ per mole}$$

$$U = \frac{3}{2} RT \quad f = 3 \text{ for mono atomic}$$
6. $U = U_{O_2} + U_{Ar} \quad \left(U = \frac{f}{2} nRT \right) = \frac{5}{2}(2)RT + \frac{3}{2}(4)RT = 11 RT$
7. $\Delta U = Q = 12 \times 100 \times 5 \times 60 J = 30 KJ$
8. $\Delta U = \Delta Q = 300 \text{ cal.}$

$$\Delta U = \frac{300}{50} = 6 \text{ cal.}$$
9. Temperature of a gas is determined by the total translational K.E. measured with respect to the centre of mass of the gas. Therefore, the motion of centre of mass of the gas does not affect the temperature. Hence, the temperature of gas will remain same.
11. In thermodynamic system, entropy and internal energy are state functions, entropy (ΔS) can be zero for adiabatic process. Work done in adiabatic process may be non-zero.
12. The Change in internal energy does not depend upon path followed by the process. It only depends on initial and final states.
Hence, $\Delta U_1 = \Delta U_2$
13. Equating internal energy

$$1 \times \frac{5}{2} RT_0 + 1 \times \frac{3}{2} R \left(\frac{7}{3} T_0 \right) = 1 \times \frac{3}{2} RT_f + \frac{5}{2} RT_f \Rightarrow T_f = \frac{3}{2} T_0 \quad \therefore (3) \text{ is correct.}$$
14. As no work is done and system is thermally insulated from surrounding, it means sum of internal energy of gas in two partitions is constant ie, $U = U_1 + U_2$
Assuming both gases have same degree of freedom, then
Assuming both gases have same degree of freedom, then

$$U = \frac{f(n_1 + n_2)RT}{2} \text{ and } U_1 = \frac{f n_1 R T_1}{2}, \quad U_2 = \frac{f n_2 R T_2}{2}$$

$$U = \frac{(p_1 V_1 + p_2 V_2) T_1 T_2}{p_1 V_1 T_2 + p_2 V_2 T_1}$$
Solving we get $T = \frac{(p_1 V_1 + p_2 V_2) T_1 T_2}{p_1 V_1 T_2 + p_2 V_2 T_1}$

SECTION (E)

1. As $\Delta U = nR\Delta T$
For closed path
 $\Delta T = 0 \quad \therefore \Delta U = 0.$
2. $T \propto P$ or $\frac{P}{T} = \text{constant}$

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As $\frac{P}{T} = \frac{nR}{V} = \text{constant}$ or $V = \text{constant} \therefore W = 0.$

3. $\Delta U = 0 \therefore T = \text{constant}$
clearly, option B is correct.

4. $\frac{V}{T} = \frac{nR}{P} \quad \frac{1}{P} \propto \text{slope} \quad \text{or} \quad P \propto \frac{1}{\text{slope}} \therefore P_2 < P_1$

5. In isothermal expansion
 $T = \text{constant} \quad \Delta U = 0 \quad W = \Delta Q$
 \therefore option (4) is correct.

6. W.D. = $\pi \times \text{Pressure Radius} \times \text{volume Radius (area of ellipse)}$

$$W = \pi \left(\frac{P_2 - P_1}{2} \right) \left(\frac{V_2 - V_1}{2} \right) = \frac{\pi}{4} (P_2 - P_1) (V_2 - V_1)$$

7. $L \rightarrow M \quad P = \text{constant} \quad V \propto T. \quad MN \quad T = \text{constant}$
Here, option B is constant

8. work done on the gas = negative work
 $W = PdV$
when $V \rightarrow$ decreases
then $W = -ve$
hence option D is correct.

9. As $W = P\Delta V$
 $\Delta V = \text{same in both process}$
As $P_B > P_A$
 $\therefore \Delta W_2 > W_1$

10. As initial and final state are same
 $\therefore T_i = T_f$ As V_{rms}, \vec{P}_{av} and \vec{K}_{av} depends on temperature
 \therefore all are equal.

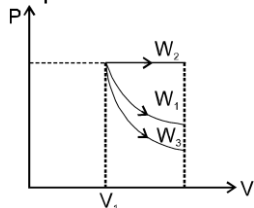
11. Area of cycle = $|W|$
$$= \frac{\pi(4-3)(4-2)}{2} + \frac{\pi(2-1)(3-2.5)}{2} = \frac{2.5\pi}{2} = \frac{5\pi}{4} \text{ atm L}$$

$$W = - \left(\frac{5\pi}{4} \right) \text{ atm L}$$

12. $\Delta Q = \Delta W + \Delta U$ for adiabatic process
 $\Delta Q = 0 \Rightarrow \Delta W = -\Delta U = (-100) = 100 \text{ J}$

13. $W = \text{Area under } P-V \text{ curve} = (P_2 - P_1) \times (V_2 - V_1) = P \Delta V$ for constant pressure = $10^5 (2 - 1) \text{ m}^3 = 10^5 \text{ J}$

19. All processes can be shown as



$$W = \int P dV = \text{Area under } P - V \text{ curve}$$

Therefore, $W_2 > W_1 > W_3$

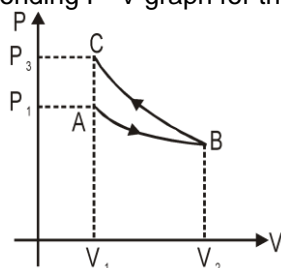
20. $\Delta Q = \Delta U + \Delta W$
 Process ABCA is cyclic, $\Delta U = 0$
 $\Delta Q = \Delta W$
 $\Rightarrow \Delta Q = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA} \Rightarrow 5 = 10(2 - 1) + 0 + \Delta W_{CA} \Rightarrow \Delta W_{CA} = -5J$

21. $W = \frac{1}{2} \times 3V_0 \times 4P_0 = 6P_0V_0$

22. Heat given $Q = 400 \times 4.2 J = 1680J$
 $\Delta Q = \Delta W + \Delta U$
 $\Delta U = 1680 - 1000 = 680 J$

23. As AB has more pressure at same volume than DC then
 $T_A > T_D$

24. Slope of adiabatic process at a given state (P,V,T) is more than the slope of isothermal process. The corresponding P-V graph for the two process



is as shown in figure.

In the graph, AB is isothermal and BC is adiabatic.

W_{AB} = positive (as volume is increasing)

and W_{BC} = negative (as volume is decreasing) plus,

$|W_{BC}| < |W_{AB}|$, as area under P-V graph gives the work done.

Hence, $W_{AB} + W_{BC} = W < 0$

From the graph itself, it is clear that $P_3 > P_1$.

Hence, the correct option is (C).

Note : At point B, slope of adiabatic (process BC) is greater than the slope of isothermal (process AB).

SECTION (F)

1. In process AB $T = \text{constant}$ $P = \text{increases}$ $P \propto \frac{1}{V}$
 or $V = \text{decreases}$ $\Delta Q = \Delta W$ $\Delta W = -ve$.
 or $\Delta Q = -ve \therefore$ heat is rejected out of the system.
2. $Q = \Delta W$ ($T = \text{constant}$)
 if heat is supplied then $\Delta W = +ve$
3. $\Delta U = \Delta Q - \Delta W$ is same in both methods as it is a state function
5. The efficiency of heat engine is $\eta = 1 - \frac{T_2}{T_1}$ or $\frac{W}{Q_1} = 1 - \frac{T_2}{T_1}$
 Here, Q_1 = heat absorbed from the source of heat = 6 kcal
 T_1 = temperature of source = $227 + 273 = 500 K$
 and T_2 = temperature of sink = $127 + 273 = 400 K$

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$$\text{Hence } \frac{W}{6} = 1 - \frac{400}{500} \quad \text{or} \quad \frac{W}{6} = 1 - \frac{100}{500} \quad \text{or} \quad W = 1.2 \text{ kcal}$$

6. At STP one mole of ideal gas occupy 22.4 lit. volume

$$\text{So no. of moles } \mu = \frac{44.8}{22.4} = 2$$

$$\text{Since } (\Delta Q)_v = \mu C_v \Delta T = 2 \times \frac{3}{2} R \times 10 = 30R \left(\because (C_v)_{\text{mono}} = \frac{3}{2} R \right)$$

7. (a) In a cyclic process, according to the diagram $Q_{\text{abcda}} = Q_{\text{ab}} + Q_{\text{bc}} + Q_{\text{cd}} + Q_{\text{da}}$ Here, ab, bc, cd and da represents isothermal expansion, isochoric compression, isothermal compression and isochoric expansion respectively.

$$Q_{\text{bc}} = Q_{\text{da}}$$

$$\therefore Q_{\text{abcda}} = Q_{\text{ab}} + Q_{\text{cd}} = \mu R T_1 \ln \left[\frac{2V_0}{V_0} \right] + \mu R T_2 \ln \left[\frac{V_0}{2V_0} \right] = 2 \times 8.3 \times 0.69 (500 - 300) = 2290.3 \text{ J}$$

8. In a cyclic process, system is not isolated from the surroundings.
In a cyclic process, a system starts from one point and ends at the same point. In this case, the change in the internal energy must again be zero and therefore the thermal energy added to the system must equal the work done during the cycle. That is, in a cyclic process.

$$\Delta U = 0 \quad \text{or} \quad E = 0$$

- 9*. Statement (a) and (d) are wrong. Concept of entropy is associated with second law of thermodynamics.

10. From first law of thermodynamics,

$$Q = \Delta U + W$$

For path iaf,

$$50 = \Delta U + 20$$

$$\therefore \Delta U = U_f - U_i = 30 \text{ cal}$$

For path ibf,

$$\text{or } Q = \Delta U + W$$

$$W = Q - \Delta U = 36 - 30 = 6 \text{ cal.}$$

SECTION (G)

1. $dQ = dW + dU$
 $dQ = PdV + dU$
 $dQ = nRdT + dU$

$$dQ = \frac{2dU}{f} + dU$$

$$\frac{dU}{dQ} = \frac{1}{\left(\frac{2}{f} + 1\right)}$$

$$\frac{dU}{dQ} = \frac{5}{7}$$

2. $s = \frac{Q}{m\Delta T}$
For changing state
 $T = \text{const or } \Delta T = 0$
 $\therefore s = \infty$ (infinite)

3. As $f = 5$
 $dU = nC_v dT = \frac{nfRdT}{2}$

$$C_v = \frac{fR}{2} \quad \therefore \quad C_v = \frac{5R}{2}$$

4. According to law of equipartition of energy, the internal energy associated per degree of freedom is $\frac{1}{2} kT$, where k is the Boltzmann's constant.

Thus, internal energy associated per molecule = $f \frac{1}{2} kT$

If N_A is Avagadro's number, then internal energy of one mole of an ideal gas is

$$U = N_A f \frac{1}{2} kT = \frac{1}{2} f (N_A k)T = \frac{1}{2} f RT$$

where $R = N_A k$ = gas constant

Molar heat capacity at constant volume

$$C_v = \frac{dU}{dT} = \frac{d}{dT} \left(\frac{1}{2} f RT \right) = \frac{1}{2} f R$$

Molar heat capacity at constant pressure

$$C_p = C_v + R \quad (\text{Mayor's relation}) = \frac{1}{2} f R + R = \left(\frac{1}{2} f + 1 \right) R$$

$$\frac{C_p}{C_v} = \frac{\left(\frac{1}{2} f + 1 \right) R}{\frac{1}{2} f R} = \frac{\frac{1}{2} f + 1}{\frac{1}{2} f} = 1 + \frac{2}{f}$$

Hence, $\gamma =$

5. Given : $P \propto T^3$ (i)

In adiabatic process

$$T V^{1-\gamma} = \text{constant}$$

$$T \propto \frac{1}{V^{(1-\gamma)/\gamma}}$$

$$T^{(\gamma/(\gamma-1))} \propto P \quad \text{.....(ii)}$$

Comparing equations (i) and (ii), we get

$$\therefore \frac{\gamma}{\gamma-1} = 3$$

$$3\gamma - 3 = \gamma$$

$$2\gamma = 3$$

$$\frac{C_p}{C_v} = \gamma = \frac{3}{2}$$

6. $C_p - C_v = R = 2 \frac{\text{cal.}}{\text{g.mol.K}}$ which is true for option (a) and (b) also value of $\frac{C_p}{C_v} = \gamma$ should be equivalent to that for monoatomic, diatomic or polyatomic gas.

$$\left(\frac{C_p}{C_v} \right)_{\text{monoatomic}} = \frac{5}{3}$$

With this point of view option (a) is correct because

7. $\Delta Q = ms \Delta T$.

8. Using the relation

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} \quad \Rightarrow \quad \frac{1+1}{\gamma-1} = \frac{1}{\left(\frac{5}{3} - 1 \right)} + \frac{1}{\left(\frac{7}{5} - 1 \right)}$$

$$\Rightarrow \frac{2}{\gamma-1} = \frac{3}{2} + \frac{5}{2} \quad \Rightarrow \quad \frac{2}{\gamma-1} = 4$$

$$\Rightarrow \gamma = \frac{3}{2} \quad \therefore \gamma = \frac{24}{16}$$

9. $C_V = 2.5 R$

As temperature increases, vibrational degree of freedom becomes 2 at high temperature.

$$C_V = \frac{7}{2} R = 3.5 R$$

10. $Q = ms \Delta T = 100 \times 1.0 \times 20 = 2000 \text{ cal.}$

11. $\Delta Q = n C_P \Delta T$

$$= 2 \left(\frac{f}{2} R + R \right) \Delta T = 2 \left[\frac{3}{2} R + R \right] \times 5 = 2 \times \frac{5}{2} \times 8.31 \times 5 = 208 \text{ J}$$

13. (a) The ratio of molar specific heat at constant pressure to molar specific heat at constant volume is denoted by γ .

$$\gamma = \frac{C_P}{C_V} = \frac{\left(\frac{f}{2} + 1 \right) R}{\frac{f}{2} R} = 1 + \frac{2}{f}$$

where, f is degrees of freedom. The term degrees of freedom of a molecule or gas are the number of independent motions that a molecule or gas can have. The independent motion of a system can be translational, rotational or vibrational or any combination of these. A monoatomic gas molecule consists of a single atom. It can have only 3 translational degrees of freedom, i.e., $f = 3$.

Hence, $\gamma = 1 + \frac{2}{3} = \frac{5}{3} = 1.67$

(b) Yes, On increasing pressure on the surface of water (as in pressure cooker), its boiling point is elevated. Similarly on decreasing pressure in a vessel having water, its boiling point will be lowered and so the water will begin to boil without supplying heat.

14. Mayer's formula is $C_P - C_V = R$ and $\gamma = \frac{C_P}{C_V}$
Therefore, using above two relations, we find

$$C_V = \frac{R}{\gamma-1}$$

For a mole of monoatomic gas ; $\gamma = \frac{5}{3} \therefore C_V = \frac{R}{3(5/3)-1} = \frac{3}{2} R$

When these two moles are mixed, then the heat required to raise the temperature to 1°C is

$$\therefore C_V = \frac{3}{2} R + \frac{5}{2} R = 4R$$

Hence, for one mole, heat required is $= \frac{4R}{2} = 2R$

$$\therefore C_V = 2R \Rightarrow \frac{R}{\gamma-1} = 2R \Rightarrow \gamma = \frac{3}{2}$$

Alternative :

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} \quad \text{Here, } n_1 = 1, n_2 = 1, \gamma_1 = \frac{5}{3}, \gamma_2 = \frac{7}{5}$$

$$\therefore \frac{1+1}{\gamma-1} = \frac{1}{(5/3)-1} + \frac{1}{(7/5)-1} \Rightarrow \frac{2}{\gamma-1} = \frac{3}{2} + \frac{5}{2} \Rightarrow \frac{2}{\gamma-1} = \frac{8}{2}$$

$$\Rightarrow \frac{2}{\gamma-1} = 4 \Rightarrow \gamma = \frac{2}{4} + 1$$

Hence, $\gamma = \frac{3}{2}$

15. $C_v = \frac{n_1 C_{v1} + n_2 C_{v2}}{n_1 + n_2}$

For helium, $n_1 = \frac{16}{4} = 4$ and $\gamma_1 = \frac{5}{3}$

For oxygen, $n_2 = \frac{16}{32} = \frac{1}{2}$ and $\gamma_2 = \frac{7}{5}$

$$C_{v1} = \frac{R}{\gamma_1 - 1} = \frac{R}{\frac{5}{3} - 1} = \frac{3}{2} R$$

$$C_{v2} = \frac{R}{\gamma_2 - 1} = \frac{R}{\frac{7}{5} - 1} = \frac{5}{2} R$$

$$C_v = \frac{4 \times \frac{3}{2} R + \frac{1}{2} \times \frac{5}{2} R}{4 + \frac{1}{2}} = \frac{6R + \frac{5}{4} R}{\frac{9}{2}} = \frac{29R \times 2}{9 \times 4} = \frac{29R}{18}$$

Now $C_v = \frac{R}{\gamma - 1}$

$$\Rightarrow \gamma - 1 = \frac{\frac{29R}{18}}{R} + 1 \Rightarrow \frac{C_p}{C_v} = \frac{18}{19} + 1 = \frac{18 + 19}{19} = 1.62$$

16. According to Mayer's relation,

$$C_p - C_v = \frac{R}{m} = \frac{R}{28}$$

SECTION (H)

1. As Volume decreases
 \therefore pressure of the gas in the cylinder increases

2. For adiabatic
 $P_A V_A^\gamma = C$ or $P_A^{1/\gamma} = C \dots(ii)$
 For isothermal
 $P_B V_B = C \dots(i)$
 from (i) and (ii)
 $P_A < P_B$

3. For adiabatic
 $T^{\frac{1}{\gamma-1}} V = C \dots(i)$
 For isothermal $T = \text{const} \dots(ii)$
 From (i) and (ii) $T_a < T_b$

Kinetic Theory of Gases & Thermodynamics

4. For W.D. by gas in isothermal is more as compare to adiabatic process

$$\therefore \Delta W_a < \Delta W_b$$

5. Isothermal $P \propto \frac{1}{V}$

Adiabatic $P \propto \frac{1}{V^\gamma}$

Also, slope of adiabatic is more as compare to isothermal
 \therefore option (3) is correct.

6. Self explanatory

7. Gas has different specific heat for different processes
 \therefore gas has infinite number of specific heats.

8. As compare to gas solid expand very less.
 $\therefore C_p$ is slightly greater then C_v .

9. Adiabatic process
 $\Delta Q = 0$
 For any process
 $\Delta U = nC_v\Delta T$
 Hence, option (3) is correct.

10. $B = \gamma P$ (for adiabatic process)
 $B = 1.4 \times 1 \times 10^5 = 1.4 \times 10^5 \text{ N/m}^2$

11. $B = \frac{VdP}{dV} = - \frac{(-PdV)}{dV}$ (for isothermal process)
 $B = P$

12. Slope $= -\gamma \frac{dP}{dV}$
 As slope of A > slope of B
 $\therefore \gamma$ of A > γ of B
 or A \rightarrow Helium
 B \rightarrow Hydrogen

13. B \rightarrow A
 $\Delta Q = 0$
 $0 = -30 + \Delta U_{BA}$
 $\Delta U_{BA} = 30 \text{ J}$
 $\therefore \Delta U_{AB} = -\Delta U_{BA} = -30 \text{ J}$

14. For free expansion
 $\Delta U = 0$ or $\Delta T = 0$
 $\therefore U$ or $T = \text{const}$

15. For free expansion
 $Q = 0, W = 0, \Delta U = 0$

16. $\frac{dP}{P} = -\gamma \frac{dV}{V}$ (For adiabatic)
 $0.5 = -1.4 \frac{dV}{V}$
 \therefore Volume decrease by 0.36%

17. XY Adiabatic compression
YZ Isothermal Expansion
ZX Compression at constant pressure

18. As W.D. is isobaric > W.D. in Isothermal > W.D in adiabatic
or $W_2 > W_1 > W_3$
Hence option (1) is correct.

19. Process ... (i) is isobaric
 $\Delta U_1 = \Delta Q - \Delta W = \text{positive}$
process (ii) is isothermal
 $\Delta U_2 = 0$
Process (iii) is adiabatic
 $\Delta Q = 0$
 $\Delta U = -\Delta W = \text{negative} \quad \therefore \quad \Delta U_1 > \Delta U_2 > \Delta U_3$

20. For polytropic process

$$C = \frac{R}{\gamma-1} + \frac{R}{1-x} \Rightarrow \text{As } PV_\gamma = K \Rightarrow \text{Put } x = \gamma \therefore C = 0$$

21. $V = kT^{2/3}$

$$dV = \frac{2}{3} k T^{-\frac{1}{3}} dT$$

$$W = \int P dV = \int \frac{nRT}{V} dV = R \int \frac{T}{V} dV = R \int \frac{T}{KT^{\frac{2}{3}}} dT = \frac{2}{3} R (T_2 - T_1)$$

$$= \frac{2}{3} R (30) = 20 (8.31) = 166.2 \text{ J}$$

22. $VP_n = \text{constant}$.
 $dV P_n + Vn P_{n-1} dP = 0$

$$-\frac{VdP}{dV} = \frac{P}{n} = \text{bulk modulus}$$

23. $V = k \left(\frac{nRT}{VT} \right)^{0.33}$
 $V_{1.33} = \text{const}$
 $V = \text{const}$

24. For adiabatic process $PV_\gamma = \text{constant}$

$$V^\gamma \frac{dp}{dV} + p\gamma V^{\gamma-1} = 0$$

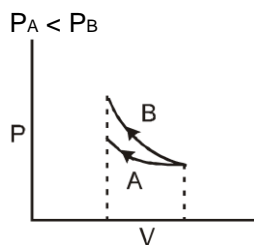
$$\frac{dp}{dV} = -\gamma \frac{p}{V}$$

$$\frac{dp}{p} = -\gamma \frac{dV}{V}$$

25. Modules of elasticity

$$B = - \frac{dp}{dV} \cdot V \text{ for isobaric process } dp = 0 \text{ then } B = 0$$

26.



27.

$$B = - \frac{dp}{dV} \cdot V$$

isothermal

$$B = \frac{+p}{V} = 1.5 \times 10^5 \text{ N/m}^2$$

adiabatic

$$B = \frac{\gamma p}{V} = 1.4 \times 1.5 \times 10^5 = 2.1 \times 10^5 \text{ N/m}^2$$

34.

Number of moles of He = $\frac{1}{4}$

Now $T_1 (5.6)_{\gamma-1} = T_2 (0.7)_{\gamma-1}$

$$T_1 = T_2 \left(\frac{1}{8} \right)^{2/3}$$

$$4T_1 = T_2$$

$$\text{Work done} = - \frac{nR[T_2 - T_1]}{\gamma - 1} = - \frac{\frac{1}{4}R[3T_1]}{\frac{2}{3}} = - \frac{9}{8}RT_1$$

35.

$$PT_2 = C$$

using $PV = nRT$ in $PT_2 = C$

$$\left(\frac{nRT}{V} \right) T^2 = C \Rightarrow T_3 \propto V$$

Differentiating we get

$$3 \frac{dT}{T} = \frac{dV}{V}$$

$$\text{Coefficient of volume expansion } (\gamma) = \frac{1}{V} \frac{dV}{dT} = \frac{3}{T}$$

36.

In an adiabatic process, there is no heat transfer into or out of a system i.e., $Q = 0$.

In an adiabatic process $Q = 0$

So, from 1st law of thermodynamics.

$$W = -\Delta U = -nC_v \Delta T$$

$$= -n \left(\frac{R}{\gamma - 1} \right) (T_f - T_i)$$

$$= \frac{nR}{\gamma - 1} (T_i - T_f) \quad (i)$$

Here : $W = 6R \text{ J}$, $n = 1 \text{ mol}$,

$$R = 8.31 \text{ J/mol-K}, \gamma = \frac{5}{3}, T_i = T_K$$

Substituting given values in Eq. (i), we get

$$\therefore 6R = \frac{R}{(5/3 - 1)}(T - T_f)$$

$$\Rightarrow 6R = \frac{3R}{2}(T - T_f) \Rightarrow T - T_f = 4$$

$$\therefore T_f = (T - 4) \text{ K}$$

Note : Adiabatic expansion of mono, dia and polyatomic gases are shown below

1 \rightarrow monoatomic, 2 \rightarrow diatomic, 3 \rightarrow polyatomic

37. In an isobaric process or at constant pressure

$$V \propto T$$

$$\text{or } \frac{V_2}{V_1} = \frac{T_2}{T_1} \quad \text{or } \frac{T_2}{T_1} = \frac{2V}{V} = 2 \quad \text{or } T_2 = 2T_1 = 2 \times 300 = 600 \text{ K}$$

Work done in isobaric process

$$\begin{aligned} W_1 &= P(V_2 - V_1) = \mu R(T_2 - T_1) \\ &= 2 \times 8.3(600 - 300) \\ &= 2 \times 8.3 \times 300 = 4980 \text{ J.} \end{aligned}$$

Now, in adiabatic process

$$TV^{\gamma-1} = \text{constant} \quad \text{or} \quad \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{T_1}{T_2}$$

Here, in adiabatic process

$$T_1 = 600 \text{ K}, T_2 = 300 \text{ K}$$

$$\therefore \left(\frac{V_2}{40}\right)^{\frac{5}{3}-1} = \frac{600}{300}$$

$$\text{or } V_2 = (2)^{3/2} \times 40 \quad \text{or } V_2 = 2\sqrt{2} \times 40$$

$$\therefore \text{Final volume} = 112.8 \text{ litre}$$

Again $PV^\gamma = \text{constant}$

$$\Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma = \left(\frac{1}{2}\right)^{5/2} \Rightarrow P_2 = \left(\frac{1}{2}\right)^{5/3} \times \frac{2 \times 8.3 \times 300}{20 \times 10^{-3}}$$

$$= \left(\frac{1}{2}\right)^{5/3} \times 249 \times 10^3 \text{ N-m}^{-2}$$

Thus, final pressure = $44.15 \times 10^3 \text{ N-m}^{-2}$

work done in adiabatic process

$$\begin{aligned} &= \frac{\mu R(T_1 - T_2)}{\gamma - 1} = \frac{2 \times 8.3(600 - 300)}{(5/3) - 1} = 2 \times 8.3 \times 300 \times 3/2 = 7470 \text{ J} \end{aligned}$$

SECTION (I)

$$1. \quad \text{Efficiency } (\eta) = \left(1 - \frac{T_2}{T_1}\right) \times 100 = \left(1 - \frac{300}{600}\right) \times 100 = 50\%$$

$$2. \quad \frac{\text{Work done}}{\text{Total Heat given}} \times 100 = \eta$$

$$W = \frac{50}{100} \times 1000 = 500 \text{ J}$$

$$3. \quad \text{Heat rejected} = \text{total heat given} - W = 1000 - 500 = 500 \text{ J}$$

$$4. \quad \eta = \left(1 - \frac{T_2}{T_1}\right) \times 100 \quad T_2 = 0^\circ\text{C} = 273 \text{ K} = \left(1 - \frac{273}{373}\right) \times 100 \quad T_1 = 100^\circ\text{C} = 373 \text{ K} = 26.81\%$$

$$5. \quad \eta = 26.81 + 20$$

$$= 46.81 = \left(1 - \frac{T_2}{T_1}\right) \times 100 \Rightarrow \frac{46.81}{100} = 1 - \frac{T_2}{T_1}$$

$$\frac{T_2}{T_1} = 1 - \frac{46.81}{100} \Rightarrow T_2 = 373 \left(1 - \frac{46.81}{100}\right)$$

$$= \frac{373 \times 53.19}{100} = 198.39 \text{ K}$$

$$6. \quad \text{Now if } T_2 = \text{constant}$$

$$T_1 = \text{Changed}$$

$$\eta = 26.81 + 20 = \left(1 - \frac{T_2}{T_1}\right) \times 100 \Rightarrow \frac{T_2}{T_1} = 1 - \frac{46.81}{100}$$

$$T_1 = \frac{273 \times 100}{53.19} = 513.2 \text{ K}$$

$$\Delta T = T_f - T_i = 513.2 - 373 = 140.2 \text{ K} \quad \text{or} \quad 140.2^\circ\text{C}$$

$$7. \quad \eta = \left(1 - \frac{T_2}{T_1}\right) \times 100 = \left(1 - \frac{273}{473}\right) \times 100 = \frac{200}{473} \times 100 \text{ in } \% \quad \text{or } \eta = \frac{200}{473} \text{ in fraction}$$

$$8. \quad \eta = \left(1 - \frac{T_2}{T_1}\right) \times 100 = \left(1 - \frac{73}{273}\right) \times 100 = \frac{200}{273}$$

$$9. \quad \frac{\eta_1}{\eta_2} = \frac{200}{473} / \frac{200}{273} = \frac{273}{473} = 0.58$$

$$10. \quad \text{Work done in first engine}$$

$$W_1 = \text{Heat given} \times \eta_1 = 4 \times 10^3 \text{ cal} \times \frac{200}{473} \text{ cal} = \frac{4 \times 10^3 \times 200}{473} \times 4.2 \text{ J} = 7.1 \times 10^3 \text{ J}$$

$$11. \quad \text{Work = output of second engine}$$

$$W_2 = \text{Heat given} \times \eta_2 = \frac{4 \times 10^3 \times 200}{273} \text{ cal} = 2.93 \times 10^3 \text{ cal.}$$

$$12. \quad \frac{W_1}{W_2} = \frac{1.69 \times 10^3 \text{ cal}}{2.93 \times 10^3 \text{ cal}} = 0.577$$

$$13. \quad \text{Efficiency of the Carnot engine is given by}$$

$$\eta = 1 - \frac{T_2}{T_1}$$

where T_1 = temperature of source
 T_2 = temperature of sink
 Given $\eta = 50\% = 0.5$, $T_2 = 500 \text{ K}$
 Substituting in relation (i), we have

$$0.5 = 1 - \frac{500}{T_1}$$

or

$$\frac{500}{T_1} = 0.5$$

$$\therefore T_1 = \frac{500}{0.5} = 1000 \text{ K}$$

Now, the temperature of sink is changed to T_2 and the efficiency becomes 60% i.e., 0.6.

Using relation (i), we get

$$0.6 = 1 - \frac{T_2}{1000} \quad \text{or} \quad \frac{T_2}{1000} = 1 - 0.6 = 0.4 \quad \text{or} \quad T_2 = 0.4 \times 1000 = 400 \text{ K}$$

14. The efficiency of Carnot engine is,

$$\eta = 1 - \frac{T_2}{T_1}$$

Where T_1 is the temperature of the source and T_2 that of sink.

$$\frac{T_2}{T_1} = \frac{Q_2}{Q_1}$$

Since, $\frac{T_2}{T_1} = \frac{Q_2}{Q_1}$ So, $\eta = 1 - \frac{Q_2}{Q_1}$

To obtain 100% efficiency (i.e., $\eta = 1$), Q_2 must be zero that is, if a sink at absolute zero would be available, all the heat taken from the source would have been converted into work. The temperature of sink means a negative temperature on the absolute scale at which the efficiency of engine is greater than unity. This would be a violation of the 2nd law of thermodynamics. Hence, a negative temperature on the absolute scale is impossible. Hence, we cannot reach absolute zero temperature.

15. Heat cannot flow itself from a lower temperature to a body of higher temperature. This corresponds to second law of thermodynamics.

16. $T_1 = 627 + 273 = 900\text{K}$
 $Q_1 = 3 \times 10^6 \text{ cal}$

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$T_2 = 27 + 273 = 300\text{K} \quad \therefore$$

$$\frac{T_2}{T_1} = \frac{300}{900}$$

$$Q_2 = \frac{T_2}{T_1} \times Q_1 = \frac{300}{900} \times 3 \times 10^6 = 1 \times 10^6 \text{ cal}$$

$$\text{Work done} = Q_1 - Q_2 = 3 \times 10^6 - 1 \times 10^6 = 2 \times 10^6 \text{ cal} = 2 \times 4.2 \times 10^6 \text{ J} = 8.4 \times 10^6 \text{ J}$$

17. Initially] $\eta = \frac{T_1 - T_2}{T_1} \Rightarrow 0.5 = \frac{T_1 - (273 + 7)}{T_1}$

$$\Rightarrow \frac{1}{2} = \frac{T_1 - 280}{T_1} \Rightarrow T_1 = 560\text{K}$$

$$\text{Finally] } \eta_1' = \frac{T_1' - T_2}{T_1'} \Rightarrow 0.7 = \frac{T_1' - (273 + 7)}{T_1'} \Rightarrow T_1' = 933\text{K}$$

$$\text{Increase in temperature} = 933 - 560 = 273\text{K}$$

18. Efficiency of all reversible cycles depends upon temperature of source and sink which will be different.

19. The heat converted to work is the amount of heat that remains after going through sink.
 From the relation

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$\text{Given } Q_1 = 6 \times 10^4 \text{ cal,}$$

$$T_1 = 227 + 273 = 500\text{K}$$

$$T_2 = 127 + 273 = 400 \text{ K}$$

$$\therefore \frac{Q_2}{6 \times 10^4} = \frac{400}{500} \Rightarrow Q_2 = \frac{4}{5} \times 6 \times 10^4 = 4.8 \times 10^4 \text{ cal}$$

$$\text{Now, heat converted to work} = Q_1 - Q_2$$

$$= 6.0 \times 10^4 - 4.8 \times 10^4$$

$$= 1.2 \times 10^4 \text{ cal}$$

Note : Carnot cycle consists of following four stages :

- (i) Isothermal expansion
- (ii) Adiabatic expansion
- (iii) isothermal compression
- (iv) Adiabatic compression

After doing the calculations for different processes, we achieve the relation

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

20. The efficiency of Carnot engine is defined as the ratio of work done to the heat supplied i.e.,

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

Here, T_1 is the temperature of source and T_2 is the temperature of sink

As given, $\eta = 40\% = \frac{40}{100} = 0.4$

and $T_2 = 300\text{K}$

So, $0.4 = 1 - \frac{300}{T_1} \Rightarrow T_1 = \frac{300}{1-0.4} = \frac{300}{0.6} = 500\text{K}$

Let temperature of the source be increased by $x\text{K}$, then efficiency becomes

$$\eta' = 40\% + 50\% \text{ of } \eta = \frac{40}{100} + \frac{50}{100} \times 0.4 = 0.4 + 0.5 \times 0.4 = 0.6$$

Hence, $0.6 = 1 - \frac{300}{500+x} \Rightarrow \frac{300}{500+x} = 0.4 \Rightarrow 500+x = \frac{300}{0.4} = 750$
 $\therefore x = 750 - 500 = 250\text{ K}$

21. Efficiency of engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\therefore \frac{T_2}{T_1} = 1 - \eta = 1 - \frac{1}{6} = \frac{5}{6} \quad \dots\dots\dots(i)$$

In other case,

$$\frac{T_2 - 62}{T_1} = 1 - \eta = 1 - \frac{2}{6} = \frac{2}{3} \quad \dots\dots\dots(ii)$$

Using Equation (i),

$$T_2 - 62 = \frac{2}{3} T_1 = \frac{2}{3} \times \frac{6}{5} T_2$$

or $\frac{1}{5} T_2 = 62$

$\therefore T_2 = 310\text{ K} = 310 - 273^\circ\text{C} = 37^\circ\text{C}$

Hence, $T_1 = \frac{5}{6} T_2 = \frac{6}{5} \times 310 = 372\text{ K} = 372 - 273 = 99^\circ\text{C}$

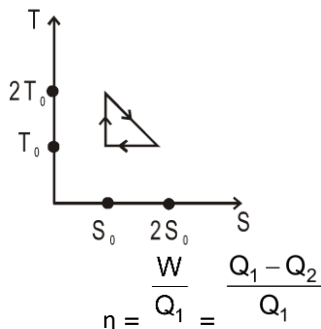
Hence, temperature of source is 99°C .

22. According to the figure

$$Q_1 = T_0 S_0 + \frac{1}{2} T_0 S_0 = \frac{3}{2} T_0 S_0$$

$$Q_2 = T_0 (2S_0 - S_0) = T_0 S_0$$

$$Q_3 = 0$$



$$= 1 - \frac{Q_2}{Q_1} = 1 - \frac{2}{3} = \frac{1}{3}$$

23. For Carnot engine using as refrigerator

$$W = Q_2 \left(\frac{T_1}{T_2} - 1 \right)$$

It is given $\eta = \frac{1}{10} \Rightarrow \eta = 1 - \frac{T_2}{T_1} \Rightarrow \frac{T_2}{T_1} = \frac{9}{10}$
 So, $Q_2 = 90 \text{ J}$ (as $W = 10 \text{ J}$)

24. For adiabatic, $W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{nR(T_1 - T_2)}{\gamma - 1}$
 Putting values, we get $\gamma = 1.4$, hence diatomic.

SECTION (J)

1. $\eta = \frac{T_1 - T_2}{T_1} - \frac{W}{Q} \Rightarrow Q = \left(\frac{T_1}{T_1 - T_2} \right) W = \frac{600}{(600 - 300)} \times 800 = 1600 \text{ J}$

2. $K = \frac{T_2}{T_1 - T_2} = \frac{273}{303 - 273} = \frac{273}{30} = 9$

3. In a refrigerator, the heat dissipated in the atmosphere is more than that taken from the cooling chamber, therefore the room is heated if the door of a refrigerator is kept open.

4. $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{400}{500} = \frac{1}{5} \therefore \eta = \frac{W}{Q} \Rightarrow \frac{1}{5} = \frac{W}{Q} \Rightarrow W = \frac{Q}{5} = \frac{6}{5} \times 10^4 = 1.2 \times 10^4 \text{ J}$

5. $\eta = 1 - \frac{T_2}{T_1} \Rightarrow -\frac{30}{100} = 1 - \frac{350}{T_1}$
 $\Rightarrow \frac{350}{T_1} = 1 - \frac{50}{100} = \frac{70}{100} = \frac{7}{10} \Rightarrow T_1 = 500 \text{ K} = 227^\circ \text{C}$

6. $\eta = 1 - \frac{T_2}{T_1}$ for 100% efficiency $\eta = 1$ which gives $T_2 = 0 \text{ K}$.

7. $\eta = -\frac{T_2}{T_1} = 1 - \frac{(273 + 123)}{(273 + 27)} = 1 - \frac{150}{300} = \frac{1}{2} = 50\%$

8. $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{500} = \frac{2}{5}$

9. In first case, $(\eta_1) = 1 - \frac{500}{800} = \frac{3}{8}$

and in Second case, $(\eta_2) = 1 - \frac{600}{x}$

Since $n_1 = n_2$ therefore $\frac{3}{8} = 1 - \frac{600}{x}$ or $\frac{600}{x} = 1 - \frac{3}{8} = \frac{5}{8}$ or $x = \frac{600 \times 8}{5} = 960 \text{ K}$

10. $\eta_{\max} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{400} = \frac{1}{4} = 25\%$

So 26 % efficiency is impossible

11. $\eta_1 = 1 - \frac{T_2}{T_1} = 1 - \frac{(273+0)}{(273+200)} = \frac{200}{473}$

In second case $\eta_2 = 1 - \frac{(273-0)}{(273+0)} = \frac{200}{473} \Rightarrow \frac{\eta_1}{\eta_2} = \frac{1}{\left(\frac{473}{273}\right)} = 1:1.73$

12. $\eta = 1 - \frac{T_2}{T_1} \Rightarrow \frac{1}{2} = 1 - \frac{500}{T} \Rightarrow \frac{500}{T_1} = \frac{1}{2}$ (i)

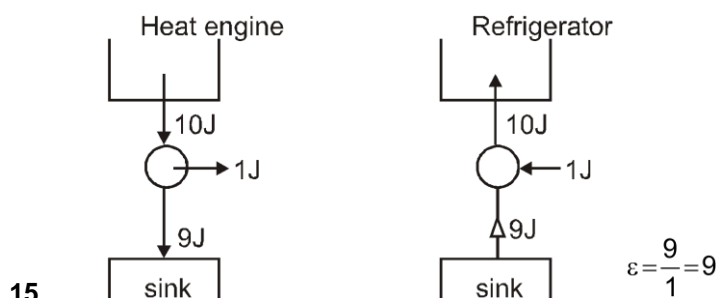
$\frac{60}{100} = 1 - \frac{T_2}{T_1} \Rightarrow \frac{T_2}{T_1} = \frac{2}{5}$ (ii)

Dividing equation (i) by (ii)

$\frac{500}{T_2} = \frac{5}{4}, \Rightarrow T_2' = 400 \text{ K}$

13. We know that there is no loss of heat in an ideal flask. Therefore mechanical energy wasted in shaking, is changed into heat energy and hence, temperature rises.

14. When the electric fan is switched on, the electric energy is converted into mechanical energy, which in turn is converted into heat energy. It is so because the rotating fan in a closed room will increase the kinetic energy of translation of molecules of air in the room and hence, temperature of the room increases.



EXERCISE # 2

1. From gas equation

$$PV = nRT$$

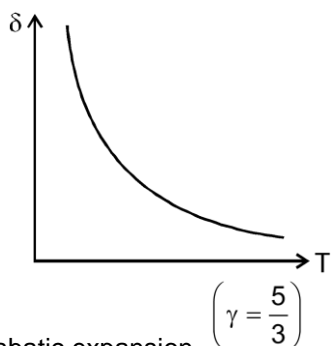
$$\frac{nRT}{V}$$

$$P = \frac{nRT}{V} = \text{Constant}$$

$$P \cdot \Delta V = nR \Delta T$$

$$\therefore \frac{\Delta V}{\Delta T} = \frac{nR}{P} = \left(\frac{nRT}{V} \right) = \frac{V}{T} \Rightarrow \frac{\Delta V}{V \cdot \Delta T} = \frac{1}{T} \Rightarrow \delta = \frac{1}{T}$$

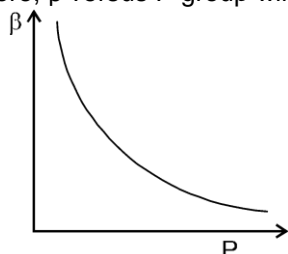
Hence, graph will be as shown



2. For adiabatic expansion, $\left(\gamma = \frac{5}{3}\right)$
 $TV_{\gamma-1} = \text{Constant} \Rightarrow T_1 V_{1\gamma-1} = T_2 V_{2\gamma-1}$
 $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{AL_2}{AL_1}\right)^{\frac{5}{3}-1} \Rightarrow \frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{2/3}$

3. $\beta = \left(\frac{-dV/dP}{V}\right)$
 At constant temperature;
 $PV = K$
 $PdV + VdP = 0$
 $\frac{-dV/dP}{V} = \frac{1}{P}$ Thus, $\beta = \frac{1}{P}$

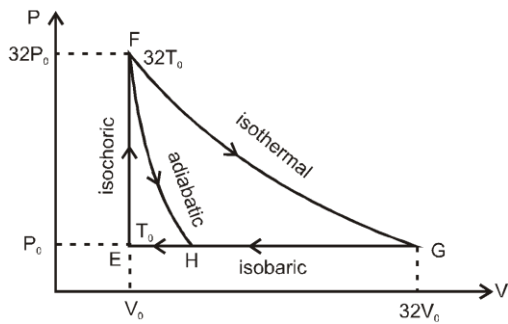
There fore, β versus P group will be a rectangular hyperbola.



4. Out of the alternatives provided, none appears completely correct.

AB is an isothermal process ($T = \text{constant}$, $P \propto \frac{1}{V}$). So $P - V$ graph should be rectangular hyperbola with P decreasing and V increasing.
 BC is an isobaric process. ($P = \text{constant}$ $V \propto T$). Temperature is increasing. Hence volume should also increase.
 CA is an adiabatic process ($PV_{\gamma} = \text{constant}$) Pressure is increasing. So volume should decrease. At point A, on isotherm AB and an adiabatic curve AC are meeting. We know that (slope of an adiabatic graph in $P-V$ diagram) $= \gamma$ (slope of an isothermal graph in the same diagram) with $\gamma > 1$ or (Slope)

5. In horizontal plane Kinetic Energy of the block is completely converted into heat due to Friction but in the case of inclined plane some part of this Kinetic Energy is also convert into gravitational Potential Energy. So decrease in the mechanical energy in second situation is smaller than that in the first situation. So statement-1 is correct.



6.

In $F \rightarrow G$ work done in isothermal process is $nRT \ln \left(\frac{V_f}{V_i} \right) = 32 P_0 V_0 \ln \left(\frac{32V_0}{V_0} \right)$
 $= 32 P_0 V_0 \ln 2 = 160 P_0 V_0 \ln 2$
 In $G \rightarrow E$, $\Delta W = P_0 \Delta V = P_0 (31 V_0) = 31 P_0 V_0$
 In $G \rightarrow H$ work done is less than $31 P_0 V_0$ i.e., $24 P_0 V_0$
 In $F \rightarrow H$ work done is $36 P_0 V_0$

7.

From ideal gas equation,
 $pV = \mu RT$

$$\therefore \mu = \frac{pV}{RT}$$

\therefore Number of moles of gas in first container,

$$\mu_1 = \frac{p_1 V}{RT_1}$$

Number of moles of gas in second container,

$$\mu_2 = \frac{p_2 V}{RT_2}$$

Number of moles in containers when joined with each other,

$$\mu = \frac{pV}{RT}$$

But, $\mu = \mu_1 + \mu_2$

$$\frac{p(2V)}{RT} = \frac{p_1 V}{RT_1} + \frac{p_2 V}{RT_2} \Rightarrow \frac{2p}{T} = \frac{p_1}{T_1} + \frac{p_2}{T_2} \Rightarrow \frac{p}{T} = \frac{p_1}{2T_1} + \frac{p_2}{2T_2}$$

8.

Heat Energy

$$dU = V_v dT = \left(\frac{5}{2} R \right) dT \Rightarrow dT = \frac{2(dU)}{5R}$$

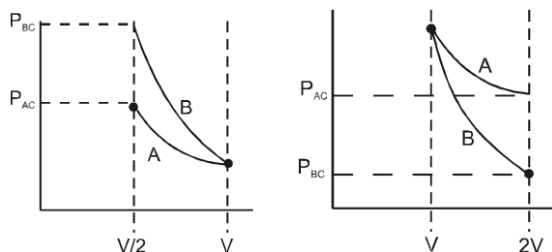
By First law of thermodynamics

$$dU = dQ - dW = Q - \frac{Q}{4}$$

$$dU = \frac{3Q}{4}$$

Now molar heat capacity is

$$C = \frac{dQ}{dT} = \frac{Q}{2 \left(\frac{dU}{5R} \right)} = \frac{5RQ}{2 \left(\frac{3Q}{4} \right)} = \frac{10}{3} R$$



9.

EXERCISE # 3

PART - I

1. Given $T_1 = 800\text{K}$

$$T_2 = 400\text{K and } W = Q_1 - Q_2 = 800\text{W}$$

The work done per unit amount of heat absorbed is given by

$$\frac{W}{Q_1} = \frac{T_1 - T_2}{T_1} = \frac{800 - 400}{800} = \frac{1}{2} \Rightarrow \frac{800}{Q_1} = \frac{1}{2} \Rightarrow Q_1 = 1600 \text{ W or Js}^{-1}$$

$$\text{or } Q_1 = \frac{1600}{\frac{1}{2}} = 3200 \text{ W or Js}^{-1} \text{ or } Q_1 = \frac{1600}{4.2} = 381 \text{ cal s}^{-1}$$

$$\text{Efficiency, } \eta = \frac{T_1 - T_2}{T_1} = \frac{800 - 400}{800} = \frac{1}{2} \text{ or } 50\%$$

2. (i) In adiabatic change ideal gases do not obey boyle's law but obey Poisson's law.

$$TV^{\gamma-1} = \text{constant}$$

$$\text{or } T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow T_2 = \left[\frac{V_1}{V_2} \right]^{\frac{\gamma}{\gamma-1}} \times T_1 = \left[\frac{V}{2\sqrt{2}V} \right]^{\frac{5}{3-1}} \times 300 = 150 \text{ K}$$

- (ii) Change in internal energy $\Delta U = \mu C_v dt$

$$\Delta U = 4 \times \frac{3}{2} R (150 - 300) = 2 \times 8.3 \times (-150) = -7500 \text{ J}$$

3. Heat given to a system (ΔQ) is equal to the sum of increase in the internal energy (Δu) and the work done (ΔW) by the system against the surrounding and $1 \text{ cal} = 4.2 \text{ J}$.

According to first law of thermodynamics

$$\Delta U = Q - W = 2 \times 4.2 \times 1000 - 500 = 8400 - 500 = 7900 \text{ J}$$

4. by first law of thermodynamics $\Delta Q = \Delta U + \Delta W$

for adiabatic expansion $\Delta Q = 0$

$$\Rightarrow \Delta U = -\Delta W$$

5. Let C_v and C_p be molar specific heats of the ideal gas at constant volume and constant pressure, respectively, then

$$C_p = MC_p \text{ and } C_v = MC_v \quad \therefore C_p - C_v = R \quad \therefore MC_p - MC_v = R \Rightarrow C_p - C_v = R/M$$

6. Ideal gas equation, for an adiabatic process is

7. $PV^\gamma = \text{constant} \quad T_1 = 273 + 27 = 300\text{K}$

$$P \left(\frac{T}{P} \right)^\gamma = \text{constant} \quad T_2 = 273 + 927 = 1200\text{K}$$

$$P^{1-\gamma} T^\gamma = \text{constant}$$

$$\Rightarrow P_1^{1-\gamma} T_1^\gamma = P_2^{1-\gamma} T_2^\gamma$$

$$\Rightarrow 2^{1-1.4} (300)^{1.4} = P_2^{1-1.4} (1200)^{1.4}$$

$$\Rightarrow \left(\frac{P_2}{P_1} \right)^{1-\gamma} = \left(\frac{T_1}{T_2} \right)^\gamma \Rightarrow \frac{P_2}{P_1} = \left(\frac{T_1}{T_2} \right)^{\frac{\gamma}{1-\gamma}}$$

$$\left(\frac{P_1}{P_2}\right)^{1-\gamma} = \left(\frac{T_2}{T_1}\right)^{\gamma} \Rightarrow \left(\frac{P_1}{P_2}\right)^{1-1.4} = \left(\frac{1200}{300}\right)^{1.4}$$

$$\left(\frac{P_1}{P_2}\right)^{-0.4} = (4)^{1.4} \Rightarrow \left(\frac{P_2}{P_1}\right)^{0.4} = 4^{1.4}$$

$$P_2 = P_1 4^{\left(\frac{1.4}{0.4}\right)} = P_1 4^{\left(\frac{7}{2}\right)} = P_1 (2^7) = 2 \times 128 = 256$$

8. In cyclic process $\Delta U = 0$

So heat absorbed

$$\Delta Q = W = \text{Area under the curve}$$

$$= -(2V)(P) = -2PV$$

$$\text{so heat rejected} = 2PV$$

9. 1st process is isothermal expansion which is only correct shown in option (4)
2nd process is isobaric compression which is correctly shown in option (4)

10. Initial and final condition is same for all process

$$\Delta U_1 = \Delta U_2 = \Delta U_3$$

$$\Delta Q = \Delta U + \Delta W$$

$$\text{Work done } \Delta W_1 > \Delta W_2 > \Delta W_3 \quad (\text{Area of P.V. graph})$$

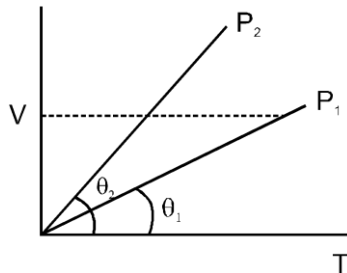
$$\text{So } \Delta Q_1 > \Delta Q_2 > \Delta Q_3$$

11. $W_{\text{net}} = \text{Area} = \frac{1}{2} AC \cdot BC$

$$= \frac{1}{2} \times 5 \times 10^{-3} \times 4 \times 10^5$$

$$= 10^3 \text{ J}$$

12. $P_1 > P_2$



$$\text{as } V = \text{constant} \Rightarrow P \propto T$$

13. $C_P - C_V = R$

$$\gamma = \frac{C_P}{C_V}$$

14. $\Delta Q = \Delta U + \Delta W = \frac{3}{2} \cdot \frac{1}{4} R (T_2 - T_1) + 0 = \frac{3}{8} Nk (T_2 - T_1)$

15. $P \propto T^{\frac{\gamma}{1-\gamma}}$
 $PT^{-\frac{\gamma}{1-\gamma}} = C$

$$\frac{C_P}{C_V} = \gamma = \frac{3}{2}$$

$$\frac{\gamma}{1-\gamma} = -3$$

$$r = -3 + 3r$$

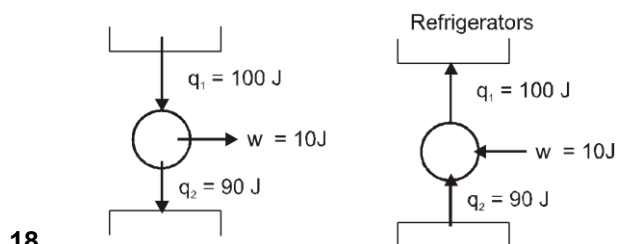
$$3 = 2r$$

$$r = \frac{3}{2}$$

16. Mean free path $\ell = \frac{1}{\sqrt{2}nd^2}$

So $\ell \propto \frac{1}{d^2} \Rightarrow \ell \propto \frac{1}{r^2}$

17. $\Delta U = \frac{f}{2} nR (T_f - T_i) = \frac{5}{2} \{P_f V_f - P_i V_i\} = \frac{5}{2} \{2 \times 10^3 \times 6 - 5 \times 10^3 \times 4\}$
 $= \frac{5}{2} \{12 - 20\} \times 10^3 \text{ J} = 5 \times (-4) \times 10^3 \text{ J} = -20 \text{ KJ}$



So, 90J heat is absorbed at lower temperature

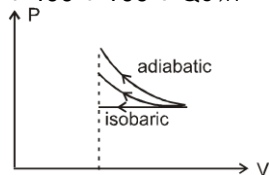
$$\gamma = \frac{C_P}{C_V} = \frac{\left(\frac{f}{2} + 1\right)R}{\left(\frac{f}{2}\right)R} = \left(1 + \frac{2}{f}\right)$$

19.

20. For a complete cycle

$$Q_{\text{cycle}} = W_{\text{cycle}}$$

$$+400 + 100 + Q_{C \rightarrow A} = \frac{1}{2} (2 \times 10^{-3}) (4 \times 10^4) \Rightarrow Q_{C \rightarrow A} = -460 \text{ J} \Rightarrow Q_{A \rightarrow C} = +460 \text{ J}$$



21.

Since area under the curve is max for adiabatic process so work done on the gas will be max for adiabatic process.

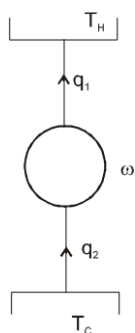
22. $\text{cop} = \frac{q_1}{w} = \frac{q_2}{q_1 - q_2} = \frac{T_C}{T_H - T_C} = 5$

$$T_C = 5T_H - 5T_C$$

$$6T_C = 5T_H$$

$$T_H = \frac{6}{5} \times 253 \text{ K}$$

$$= 303.6 \text{ K} = 30.6^\circ\text{C} = 31^\circ\text{C}$$

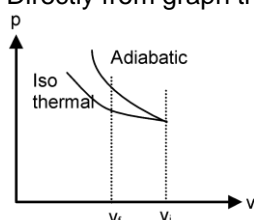


$$23. \quad P_A = \frac{\rho_A M_A}{RT}, \quad P_B = \frac{\rho_B M_B}{RT} = \frac{3}{2} \Rightarrow \frac{P_A}{P_B} = \frac{\rho_A M_A}{\rho_B M_B} = 2 \frac{M_A}{M_B} = \frac{3}{2}$$

$$\frac{M_A}{M_B} = \frac{3}{4}$$

so,

24. Directly from graph the magnitude of work done = Area under p-v plot is larger for adiabatic compression



$$25. \quad V_{RMS} = \sqrt{\frac{3RT}{M_0}}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{200}{V_2} = \sqrt{\frac{300}{400}}$$

$$\Rightarrow V_2 = \frac{400}{\sqrt{3}}$$

26. $PV^3 = \text{constant}$
for a polytropic process. $PV^\alpha = \text{constant}$

$$C = C_v + \frac{R}{1-\alpha} = \frac{3}{2}R + \frac{R}{1-3} = \frac{3R}{2} - \frac{R}{2} = R$$

$$27. \quad \frac{Q_{\text{more}}}{W} = \frac{Q_{\text{more}}}{Q_{\text{more}} - Q_{\text{less}}} = \frac{T_{\text{more}}}{T_{\text{more}} - T_{\text{less}}} = \frac{t_1 + 273}{(t_1 + 273) - (t_2 + 273)}$$

$$= \frac{t_1 + 273}{t_1 - t_2}$$

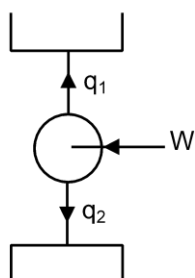
$$28. \quad n = \frac{PV}{RT} = \frac{\text{mass}}{\text{Molar mass}} = \frac{PV}{RT}$$

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{(\text{Molar mass})P}{RT} = \frac{(m.N_A)P}{RT} = \frac{mP}{KT}$$

29. Isochoric parallel to y axis
Isobaric \rightarrow parallel to x axis
Along the curve isothermal
So, correct option (2)

30.
$$U_{\text{total}} = \frac{f_1}{2} n_1 RT + \frac{f_2}{2} n_2 RT$$

$$= \frac{5}{2} \times 2RT + \frac{3}{2} \times 4RT = 11RT$$



31.
$$\frac{W}{q_1} = \frac{1}{10} = \frac{10}{q_1}$$

$$\Rightarrow q_1 = 100 \text{ J}$$

 so $q_2 = 100 - 10 = 90 \text{ J}$

32. $V \propto T \Rightarrow$ pressure = constant so the process is an isobaric process

$$\frac{W}{Q} = \frac{nR\Delta T}{nC_P\Delta T} = \frac{W}{Q} = \frac{nR\Delta T}{n(R + \frac{f}{2}R)\Delta T} = \frac{1}{1 + \frac{3}{2}} = \frac{2}{5}$$

33. $\eta = 1 - \frac{T_{\text{less}}}{T_{\text{more}}} = 1 - \frac{273}{373} = 26.8 \%$

34.
$$\sqrt{\frac{3kT}{m_0}} = V_e \Rightarrow \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times T}{2.76 \times 10^{-26}}} = 11.2 \times 10^3$$

$$\Rightarrow T = 8.36 \times 10^4 \text{ K}$$

35. In adiabatic process $\Delta Q = 0$

36. $KE \propto \text{Temp}$
 i.e. increasing temperature, increases KE of gas filled in container

37. $\gamma = 1 + \frac{2}{f}$

$$\gamma_{\text{H}_2} = 1 + \frac{2}{f} = \frac{7}{5} \Rightarrow \gamma_{\text{He}} = 1 + \frac{2}{3} = \frac{5}{3} \Rightarrow \gamma_3 = 1 + \frac{2}{7} = \frac{9}{7}$$

38. $Q = \Delta U + W$
 $\Delta U = Q - W$
 $Q = 2256 \text{ J}$
 $W = P\Delta V$
 $W = 10^5 (1671 - 1) \times 10^{-6}$
 $W = 167 \text{ J}$
 $\Delta U = 2256 - 167$
 $\Delta U = 2089 \text{ J}$

PART - II

1. At A, temperature = T_0 , volume = V_0 , pressure = P_0 for n moles of monoatomic gas.
 At A, $P_0 V_0 = nRT_0$ (cold)
 At B, $2P_0 V_0 = nRT'_{\text{high}} \Rightarrow T' = 2T_0$
 Maximum thermal efficiency of the reversible engine of this Carnot's cycle

$$= \eta = 1 - \frac{T_0}{T_1} = 1 - \frac{T_0}{2T_0} = \frac{1}{2} = 50\%$$

2. Using $\eta = 1 - \frac{T_2}{T_1}$ or $\frac{T_2}{T_1} = 1 - \eta$

According to first case

$$\frac{T_2}{T_1} = 1 - \frac{1}{6} = \frac{5}{6} \quad \dots\dots\dots(i)$$

According to second case

$$\frac{T_2 - 62}{T_1} = 1 - 2 \times \frac{1}{6} = \frac{2}{3} \quad \dots\dots\dots(ii)$$

$$\frac{T_2}{T_1} - \frac{62}{T_1} = \frac{2}{3}$$

From equations (i) and (ii), we get

$$\frac{5}{6} - \frac{62}{T_1} = \frac{2}{3} \quad \Rightarrow \quad \frac{5}{6} - \frac{2}{3} = \frac{62}{T_1}$$

$$\Rightarrow \quad \frac{1}{6} = \frac{62}{T_1}$$

$$T_1 = 372 \text{ K or } T_1 = 372 - 273$$

$$T_1 = 99^\circ\text{C}$$

4. According to gas equation

$$PV = nRT$$

$$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1} \quad \text{or} \quad \frac{P_2}{P_1} = \frac{V_1}{V_2} \cdot \frac{T_2}{T_1}$$

Here, $T_2 = 3000 \text{ K}$, $T_1 = 300 \text{ K}$

Since H_2 splits into hydrogen atoms, therefore volume becomes half

$$\therefore \frac{P_2}{P_1} = \frac{V_1}{\frac{1}{2}V_1} \times \frac{3000}{300}$$

$$\text{i.e., } V_2 = \frac{1}{2}V_1$$

$$\frac{P_2}{P_1} = 20$$

$$\text{or } P_2 = 20 P_1$$

PART - III

1. n (moles) = 2

A to B is isobaric process

$$W_{AB} = P\Delta V = nR\Delta T = (2)(R)(200) = 400 R$$

$$W_{AB} = 400 R$$

2. D to A is isothermal process

Work done by the gas in D to A is

$$W_{DA} = nRT \ln \frac{V_2}{V_1} = nRT \ln \frac{P_1}{P_2}$$

$$= (2)(R)(300) \ln \frac{10^5}{2 \times 10^5}$$

$$= (600 R) [-\ln 2]$$

$$= - (600 R) (0.693)$$

$$= - 414 R$$

$W_{DA} = - 414 R$, it is work done by the gas

Kinetic Theory of Gases & Thermodynamics

So work done on the gas is + 414 R

$$\begin{aligned}
 3. \quad W_{ABCD} &= W_{AB} + W_{BC} + W_{CD} + W_{DA} \\
 &= nR (\Delta T)_{AB} + nR (T_B) \ln \frac{P_B}{P_C} + nR (\Delta T)_{CD} + nR (T_D) \ln \frac{P_D}{P_A} \\
 &= nR (200) + 500 nR \ln 2 + nR (-200) + 300 nR \ln \frac{1}{2} \\
 &= 2 \ln 2 [500 R - 300 R] = (400 R) (\ln 2) = (400 R) (0.693) = 276 R \\
 W_{ABCD} &= 276 R
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \frac{m}{d} &= \frac{1}{4} = m_3 \\
 kE &= \text{for diatomic} \\
 kE &= \frac{5}{2} PV = \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} = 5 \times 10^4 \text{ J}
 \end{aligned}$$

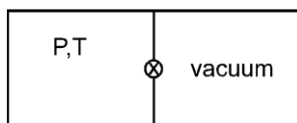
$$\begin{aligned}
 5. \quad TV_{\gamma-1} &= \text{constant} \\
 T_1 V_1^{\frac{7}{5}-1} &= T_2 (32V)^{\frac{7}{5}-1} \\
 \frac{T_2}{T_1} &= \frac{1}{(32)^{2/5}} = \frac{1}{4} \\
 \eta &= 1 - \frac{T_2}{T_1} = 1 - \frac{1}{4} = \frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \eta &= 1 - \frac{T_2}{T_1} = \frac{1}{6} \Rightarrow \frac{T_2}{T_1} = 1 - \frac{1}{6} = \frac{5}{6} \\
 \frac{1}{3} &= \frac{1 - \frac{(T_2 - 62)}{T_1}}{1} \Rightarrow \frac{T_2 - 62}{T_1} = \frac{2}{3} \\
 \frac{5(T_2 - 62)}{T_2} &= \frac{2}{3} \\
 5T_2 - 310 &= 4T_2
 \end{aligned}$$

$$\begin{aligned}
 T_2 &= 310 \\
 T_1 &= 372 \text{ K} \quad \text{and} \quad T_1 = \frac{6 \times 310}{5} \\
 \text{Ans.}
 \end{aligned}$$

$$7. \quad T = \frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$$

$$\begin{aligned}
 8. \quad \frac{1}{2} M v^2 &= C_V \Delta T \Rightarrow \frac{1}{2} M v^2 = \frac{R}{\gamma - 1} \Delta T \\
 \Delta T &= \frac{M v^2 (\gamma - 1)}{2R} = \frac{(\gamma - 1) M v^2}{2R}
 \end{aligned}$$



$$\begin{aligned}
 9. \quad &\text{It is the free expansion} \\
 &\text{So, T remain constant} \\
 &P_1 V_1 = P_2 V_2
 \end{aligned}$$

$$P \frac{V}{2} = P_2(V)$$

$$P_2 = \left(\frac{P}{2} \right)$$

$$10. \quad \eta = \frac{\frac{f}{2} p_0 v_0}{\frac{f}{2} p_0 v_0 + \frac{f}{2} (2p_0) v_0 + 2p_0 v_0} = \frac{1}{\frac{3}{2} + 3 + 2} = \frac{200}{13} = 15.4\%$$

11. For 1st case

$$\text{efficiency} = \eta = \left(1 - \frac{T_1}{T_2} \right) \times 100$$

$$\left(1 - \frac{T_1}{500} \right) \times 100 = 40$$

$$T_1 = 300 \text{ K}$$

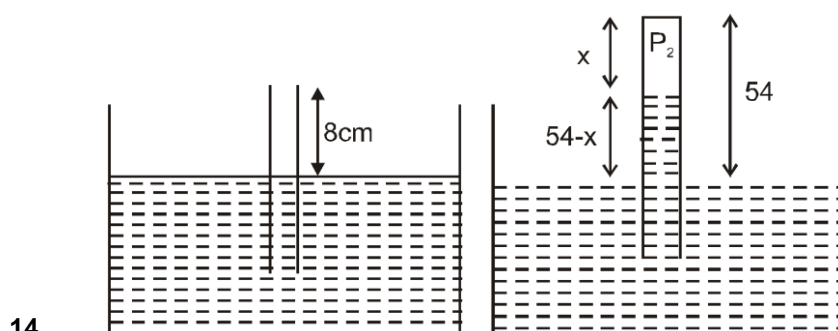
for 2nd case

$$\eta = \left(1 - \frac{300}{T_2} \right) \times 100 = 60$$

$$T_2 = 750 \text{ K}$$

$$12. \quad \frac{3}{2} P_0 V_0 + \frac{5}{2} 2P_0 V_0 = \frac{13}{2} P_0 V_0$$

13. $\Delta U = \frac{f}{2} nR\Delta T$
 For cyclic process $\Delta U = 0$
 For process CA
 $\Delta U = 1 \times \frac{5}{2} R (-200) = -500R$
 For process AB :-
 $\Delta U = 1 \times \frac{5}{2} R x(+400) = 1000R$
 For process BC :-
 $\Delta U = 1 \times \frac{5}{2} R x(-200) = -500R$



- For air trapped in tube $P_1 V_1 = P_2 V_2$
 $P_1 = P_{atm} = \rho g 76$
 $V_1 = A \cdot 8$ (A = area of cross section)
 $P_2 = P_{atm} - \rho g (54 - x) = \rho g (22 + x)$
 $V_2 = A \cdot x$
 $\rho g 76 \cdot A 8 = \rho g (22 + x) A x$
 $x^2 + 22x - 78 \times 8 = 0$
 $\Rightarrow x = 16 \text{ cm.}$

15. $p = \frac{1}{3} \frac{U}{V} \Rightarrow \frac{nRT}{V} \propto \frac{1}{3} T_4$
 $VT_3 = \text{const} \Rightarrow \frac{4}{3} \pi R^3 T_3 = \text{const}$
 $TR = \text{const} \Rightarrow T \propto \frac{1}{R}$

16. Since entropy is a state function, therefore change in entropy in both the processes should be same. Therefore correct option is (2)

17. since $\tau = \frac{1}{n\pi\sqrt{2}v_{rms}d^2}$
 $n \propto \frac{1}{V}$ and $v_{rms} \propto \sqrt{T} \Rightarrow \tau \propto \frac{V}{\sqrt{T}}$
 $n = C_1 V^{-1} \quad \langle v \rangle = C_2 T^{1/2} \quad \text{since } TV^{\gamma-1} = \text{constant} \Rightarrow \tau \propto V^{\frac{\gamma+1}{2}}$

$$18. \quad P - P_0 = -\frac{P_0}{V_0}(V - 2V_0)$$

$$P = 3P_0 - \frac{P_0}{V_0}V \quad \dots\dots(1)$$

$$\frac{nRT}{V} = 3P_0 - \frac{P_0}{V_0}V$$

$$nRT = 3P_0V - \frac{P_0}{V_0}V^2 \quad \text{differentiate w.r.t. Volume}$$

$$\frac{2P_0}{V_0}V = 0 \quad \Rightarrow \quad V = \frac{3V_0}{2}$$

Put in (1)

$$P = 3P_0 - \frac{P_0}{V_0}\left(\frac{3V_0}{2}\right) = \frac{3P_0}{2}$$

Now,

$$PV = nRT$$

$$\frac{9P_0V_0}{4} = nRT \quad \Rightarrow \quad T = \frac{9P_0V_0}{4nR}$$

$$19. \quad C = (M_0)s$$

For H_2 as well as N_2

$$C_P - C_V = R$$

$$(M_0) S_P - (M_0) S_V = R$$

$$S_P - S_V = \frac{R}{M_0}$$

For H_2 gas

$$S_P - S_V = \frac{R}{2} = a$$

For N_2 gas

$$S_P - S_V = \frac{R}{28} = b$$

$$\frac{a}{b} = \frac{\frac{R}{2}}{\frac{R}{28}} = 14 \Rightarrow a = 14b$$

So

$$20. \quad n_f - n_i = \left(\frac{PV}{RT}\right)_f - \left(\frac{PV}{RT}\right)_i$$

$$n_f - n_i = \frac{PV}{R} \left(\frac{1}{T_f} - \frac{1}{T_i}\right) = \frac{(10^5)(30)}{25} \left(\frac{1}{300} - \frac{1}{290}\right)$$

$$= -\frac{90 \times 10^5}{25} \left(\frac{10}{(300)(290)}\right) = -\frac{90 \times 10^4}{25 \times 3 \times 29} = \frac{3 \times 10^4}{25 \times 29} \text{ mole} = -\frac{3 \times 10^4}{25 \times 29} \times 6 \times 10^{23} = -2.48 \times 10^{25}$$

21. For adiabatic process

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \quad \gamma = \frac{5}{3} \Rightarrow 300(V)^{\frac{2}{3}} = T_2(2V)^{\frac{2}{3}} \Rightarrow T_2 = \frac{300}{2^{\frac{2}{3}}} \approx 189K$$

$$\Delta U = \frac{f}{2} n R \Delta T = \frac{3}{2} \cdot 2 \cdot \frac{25}{3} (189 - 300) = -2.7 \text{ kJ}$$

$$\frac{(V_{\text{RMS}})_{\text{He}}}{(V_{\text{RMS}})_{\text{Ar}}} = \sqrt{\frac{M_{\text{Ar}}}{M_{\text{He}}}} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16$$

22. For ABC $\Delta Q = \Delta U + \Delta W$
 $60 = \Delta U + 30$
 $\Delta U = 30 \text{ J}$
 For ADC
 $\Delta Q = \Delta U + \Delta W = 30 + 10 = 40 \text{ J}$

24. $V_{\text{rms}} = \sqrt{\frac{3RT}{M}} \Rightarrow V_{\text{rms}} \propto \sqrt{T}$
 To double the V_{rms} , Temp must be 4 times of the initial temperature
 $T_2 = 4 \times 300 = 1200 \text{ K}$

$$\text{heat added} = n C_v dT = \frac{15}{28} \times \frac{5}{2} \times 8.314 \times (1200 - 300) = 10021.3 \text{ J} \approx 10 \text{ kJ}$$

$$25. \eta_1 = 1 - \frac{T_2}{T_1} \Rightarrow \eta_2 = 1 - \frac{T_3}{T_2} \Rightarrow \eta_3 = 1 - \frac{T_4}{T_3}$$

Given $\eta_1 = \eta_2 = \eta_3$
 $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$
 $T_2^2 = T_1 T_3 \dots (1)$
 $T_2^2 = T_1 T_3 \dots (1)$
 (1) and (2)

$$T_3 = (T_1 T_4^2)^{1/3}, T_2 = (T_1^2 T_4)^{1/3}$$

26. $U = \frac{f}{2} n R T$
 $= \frac{f}{2} P V \Rightarrow \frac{f}{2} P \frac{M}{\rho}$
 $= \frac{3}{2} \times 4 \times 10^3 \times \frac{2}{8}$
 $= 1.5 \times 10^3 \text{ J}$

27. $W = n R \Delta T = 291 \text{ J}$

28. 1st condition:
 $\Delta Q_g = \Delta Q_\ell$
 $100 \times S_A \times (100 - 90) = 50 \times S_B \times (90 - 75) \dots (i)$
 2nd condition:
 $\Delta \theta_g = \Delta \theta_\ell$
 $100 \times S_A (100 - \theta) = 50 \times S_B (\theta - 50) \dots (ii)$
 dividing (ii) by (i)
 $\frac{100 - \theta}{100 - 90} = \frac{\theta - 50}{90 - 75}$
 $300 - 3\theta = 2\theta - 100$
 $\theta = 80^\circ \text{C}$

29. For adiabatic process

$$TV^{\gamma-1} = \text{Constant}$$

$$\gamma = 1 + \frac{2}{5} = \frac{7}{5} \quad (\text{For diatomic gas})$$

$$\text{So } x = \gamma - 1 = \frac{7}{5} - 1 = \frac{2}{5}$$

$$\tau = \frac{1}{\sqrt{2n\pi d^2 V_{\text{avg}}}}$$

30.

$$n = \frac{\text{Number of molecules}}{\text{Volume}}$$

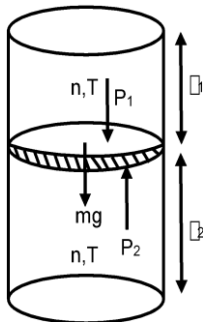
$$\tau \propto \frac{V}{\sqrt{T}}$$

$$\Rightarrow \tau \propto \frac{\sqrt{T}}{P}$$

$$\frac{\tau_1}{\tau_2} = \sqrt{\frac{T_2}{T_1}} \times \frac{P_1}{P_2}$$

$$\Rightarrow \tau_2 = 6 \times 10^{-8} \times \sqrt{\frac{5}{3}} \times \frac{1}{2}$$

$$\approx 4 \times 10^{-8} \text{ s}$$



31.

$$P_1 + \frac{mg}{A} = P_2$$

$$\Rightarrow \frac{nRT}{\rho_1 A} + \frac{mg}{A} = \frac{nRT}{\rho_2 A}$$

$$\Rightarrow m = \frac{nRT}{g} \left(\frac{1}{\rho_2} - \frac{1}{\rho_1} \right)$$

$$= \frac{nRT}{g} \left(\frac{\rho_1 - \rho_2}{\rho_1 \rho_2} \right)$$