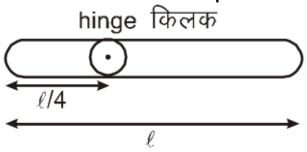
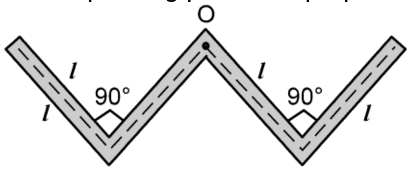
Self Practice Paper (SPP)

1. A uniform rod is hinged as shown in the figure and is released from a horizontal position. The angular velocity of the rod as it passes the vertical position is:



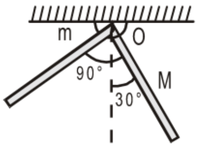
(1) $\sqrt{\frac{12g}{3\ell}}$ (2) $\sqrt{\frac{2g}{3\ell}}$ (3) $\sqrt{\frac{24g}{7\ell}}$ (4) $\sqrt{\frac{3g}{7\ell}}$

2. A thin uniform rod of length $4\,l$, mass 4m is bent at the points as shown in the fig. What is the moment of inertia of the rod about the axis passing point O & perpendicular to the plane of the paper.



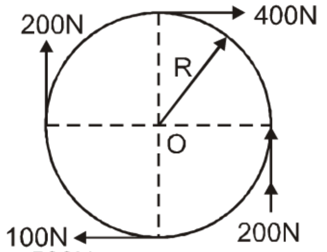
(1) $\frac{m\,\ell^2}{3}$ (2) $\frac{10\,m\ell^2}{3}$ (3) $\frac{m\,\ell^2}{12}$ (4) $\frac{m\ell^2}{24}$

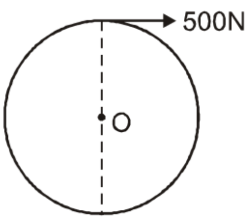
3. Two uniform rods of equal length but different masses are rigidly joined to form an L−shaped body, which is then pivoted about O as shown in the figure. If in equilibrium the body is in the shown configuration, ratio M/m will be:

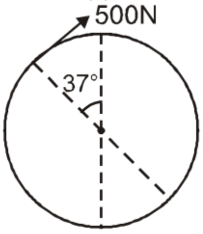


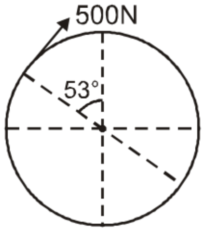
(1) 2 (2) 3 (3) $\sqrt{2}$ (4) $\sqrt{3}$

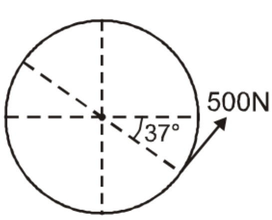
4. Four forces tangent to the circle of radius 'R' are acting on a wheel as shown in the figure. The resultant equivalent one force system will be :



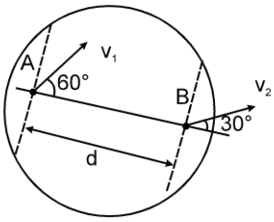
(1) 

(2) 

(3) 

(4) 

5. Two points A & B on a disc have velocities v_1 & v_2 at some moment. Their directions make angles 60° and 30° respectively with the line of separation as shown in figure. The angular velocity of disc is :

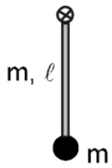


(1) $\frac{\sqrt{3}v_1}{d}$ (2) $\frac{v_2}{\sqrt{3}d}$ (3) $\frac{v_2 - v_1}{d}$ (4) $\frac{v_2}{d}$

6. The angular momentum of a particle about origin is varying as L = 4t + 8 (SI units) when it moves along a straight line y = x – 4 (x,y in meters). The magnitude of force acting on the particle would be :

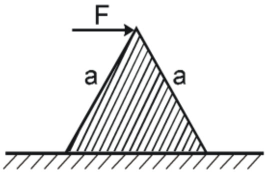
(1) 1 N (2) 2 N (3) $\sqrt{2}$ N (4) $\sqrt{3}$ N

7. A particle is attached to the lower end of a uniform rod which is hinged at its other end as shown in the figure. The minimum speed given to the particle so that the rod performs circular motion in a vertical plane will be :[length of the rod is ℓ, consider masses of both rod and particle to be same]



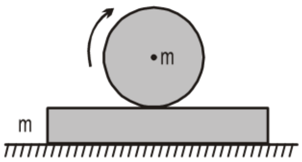
(1) $\sqrt{5g\ell}$ (2) $\sqrt{4g\ell}$ (3) $\sqrt{4.5g\ell}$ (4) none of these

8. An equilateral uniform prism of mass m rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the prism as shown in the figure. If the coefficient of friction is sufficiently high so that the prism does not slide before toppling, then the minimum force required to topple the prism is :



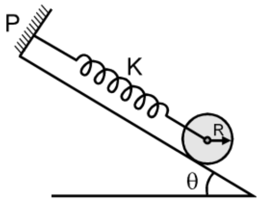
(1) $\frac{mg}{\sqrt{3}}$ (2) $\frac{mg}{4}$ (3) $\frac{\mu mg}{\sqrt{3}}$ (4) $\frac{\mu mg}{4}$

9. A uniform sphere of mass 'm' is given some angular velocity about a horizontal axis through its centre and gently placed on a plank of mass 'm'. The co-efficient of friction between the two is μ. The plank rests on a smooth horizontal surface. The initial acceleration of the centre of sphere relative to the plank will be :



(1) zero (2) μg (3) (7/5) μg (4) 2 μg

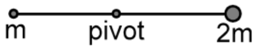
10. A uniform cylinder of mass M and radius R rolls without slipping down a slope of angle θ to the horizontal. The cylinder is connected to a spring constant K while the other end of the spring is connected to a rigid support at P. The cylinder is released when the spring is unstretched. The maximum displacement of cylinder is



(1) $\frac{3}{4}\frac{Mg\sin\theta}{K}$ (2) $\frac{Mg\sin\theta}{K}$ (3) $\frac{2Mg\sin\theta}{K}$ (4) $\frac{4}{3}\frac{Mg\sin\theta}{K}$

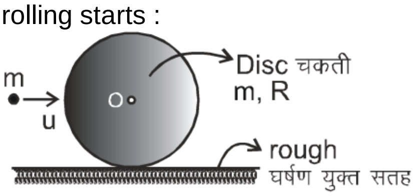
11. When a person throws a meter stick it is found that the centre of the stick is moving with a speed of 10 m/s vertically upwards & left end of stick with a speed of 20 m/s vertically upwards. Then the angular speed of the stick is:
(1) 20 rad/ sec (2) 10 rad/sec (3) 30 rad/sec (4) none of these

12. A rod of negligible mass and length ℓ is pivoted at its centre. A particle of mass m is fixed to its left end & another particle of mass 2 m is fixed to the right end. If the system is released from rest and after sometime becomes vertical, the speed v of the two masses and angular velocity at that instant.



(1) $\sqrt{g\ell/3}$, $\sqrt{4g/3\ell}$ (2) $\sqrt{4g\ell/3}$, $\sqrt{4g/3\ell}$ (3) $\sqrt{4g/3\ell}$, $\sqrt{4g\ell/3}$ (4) $\sqrt{g\ell/3}$, $\sqrt{g\ell/3}$

13. A point mass m collides with a disc of mass m and radius R resting on a rough horizontal surface as shown in figure. Its collision is perfectly inelastic and immediately system starts pure rolling. Find angular velocity of the system after pure rolling starts :



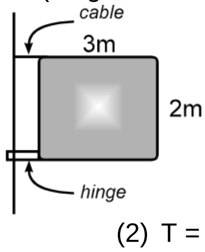
(1) $\left(\frac{2u}{7R}\right)$ (2) $\left(\frac{7u}{2R}\right)$ (3) $\left(\frac{5u}{2R}\right)$ (4) $\left(\frac{2u}{5R}\right)$

14. Moment of inertia of a uniform quarter disc of radius R and mass M about an axis through its centre of mass and perpendicular to its plane is :

(1) $\frac{MR^2}{2} - M\left(\frac{4R}{3\pi}\right)^2$ (2) $\frac{MR^2}{2} - M\left(\sqrt{2}\,\frac{4R}{3\pi}\right)^2$

(3) $\frac{MR^2}{2} + M\left(\frac{4R}{3\pi}\right)^2$ (4) $\frac{MR^2}{2} + M\left(\sqrt{2}\,\frac{4R}{3\pi}\right)^2$

15. In figure the uniform gate weighs 300 N and is 3 m wide & 2 m high. It is supported by a hinge at the bottom left corner and a horizontal cable at the top left corner, as shown , the tension in the cable and the force that the hinge exerts on the gate (magnitude & direction :



(1) T = 225 N , F_X = 225N, F_Y = 300 N (2) T = 300 N , F_X = 225 N, F_Y = 300 N
(3) T = 225 N , F_X = 300N, F_Y = 250 N (4) T = 300 N , F_X = 300 N, F_Y = 250 N

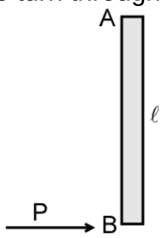
16. A bit of mud stuck to a bicycle's front wheel of radius R detaches and is flung horizontally forward when it is at the top of the wheel. The bicycle is moving forward at a speed v and it is rolling without slipping. The horizontal distance travelled by the mud after detaching from the wheel is:

(1) $\sqrt{2rv^2/g}$ (2) $\sqrt{8rv^2/g}$ (3) $\sqrt{4rv^2/g}$ (4) $\sqrt{16\ Rv^2/g}$

17. A uniform disk of mass 300kg is rotating freely about a vertical axis through its centre with constant angular velocity ω. A boy of mass 30kg starts from the centre and moves along a radius to the edge of the disk. The angular velocity of the disk now is

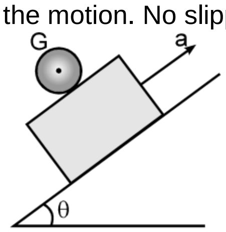
(1) $\frac{\omega_0}{6}$ (2) $\frac{\omega_0}{5}$ (3) $\frac{4\omega_0}{5}$ (4) $\frac{5\omega_0}{6}$

18. A uniform rod AB of mass m and length ℓ at rest on a smooth horizontal surface. An impulse P is applied to the end B. The time taken by the rod to turn through a right angle is:



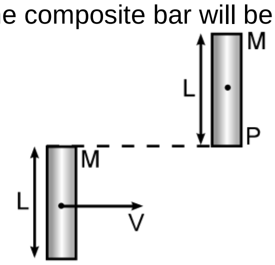
(1) $\frac{2\pi m\ell}{P}$ (2) $\frac{\pi m\ell}{3P}$ (3) $\frac{\pi m\ell}{12P}$ (4) $\frac{2\pi m\ell}{3P}$

19. Determine the acceleration *a* of the supporting surface required to keep the centre G of the circular pipe in a fixed position w.r.t. ground during the motion. No slipping takes place between pipe and its support.



(1) g sin θ (2) 2g sin θ (3) $\frac{g}{2}\sin\theta$ (4) $\sqrt{2}$ g sin θ

20. A bar of mass M & length L is in pure translatory motion with its centre of mass velocity V. It collides with and sticks to a second identical bar which is initially at rest. (Assume that it becomes one composite bar of length 2 L). The angular velocity of the composite bar will be:



(1) 3V/4L clockwise
(2) 4V/3L clockwise
(3) 3V/4L counter clockwise
(4) V/L counter clockwise

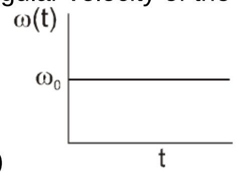
21. A body rolls down without slipping on an inclined plane. The fractional of its total energy associated with rotation will be (radius of gyration is k & radius of body is R)

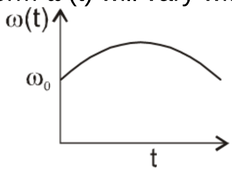
(1) $\frac{R^2}{R^2+k^2}$ (2) $\frac{k^2}{R^2+k^2}$ (3) $k_2 + R_2$ (4) $\frac{1}{k^2+R^2}$

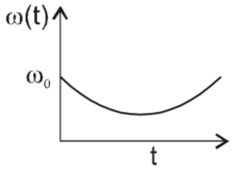
22. A body starts form rest, on applying a constant torque on a body
(1) linear velocity increases
(2) angular velocity increases
(3) it will rotate with constant angular velocity
(4) it will move with constant velocity

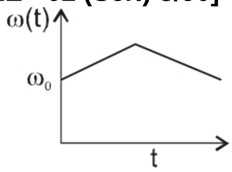
23. A body is rotating uniformly about a vertical axis fixed in an inertial frame. The resultant force on a particle of the body not on the axis is
(1) vertical
(2) horizontal and skew with the axis
(3) horizontal and intersecting the axis
(4) none of these.

24. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform . Now the platform is given an angular velocity ω_0 . When the tortoise moves along a chord of the platform with a constant velocity (with respect to the platform) the angular velocity of the platform ω (t) will vary with time t as : **[JEE - 02 (Scr.) 3/90]**

(1) 

(2) 

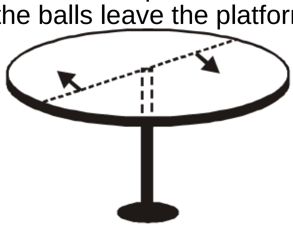
(3) 

(4) 

25. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0º. Then **[JEE 2009, 3/160, –1]**

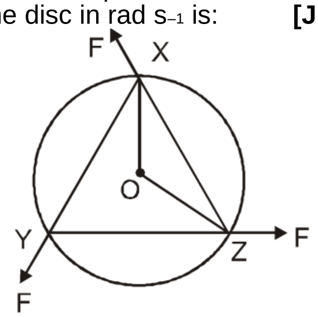
(1) at θ = 30º, the block will start sliding down the plane
(2) the block will remain at rest on the plane up to certain θ and then it will topple
(3) at θ = 60º, the block will start sliding down the plane and continue to do so at higher angles
(4) at θ = 60º, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

26. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platfrom at a distance 0.25 m from the centre on its either sides along its diameter (see figure).Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platfrom, the balls have horizontal speed of $9ms^{-1}$ with respect to the ground. The rotational speed of the platfrom in rad^{-1} after the balls leave the platform is **[JEE (Advanced)-2014,P-1, 3/60]**



(1) 1 (2) 2 (3) 3 (4) 4

27. A uniform circular disc of mass 1.5 kg and radius 0.5m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is: **[JEE (Advanced)-2014,P-1, 3/60]**



(1) 1 (2) 2 (3) 3 (4) 4

28. The dinsities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a contant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively, If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is : **[JEE(Advanced) 2015 ; 4/88]**

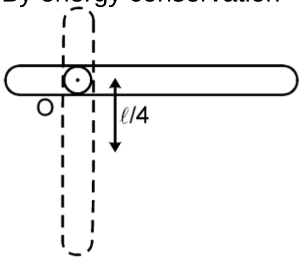
(1) 2 (2) 4 (3) 6 (4) 8

SPP Answers

1.	(3)	**2.**	(2)	**3.**	(4)	**4.**	(3)	**5.**	(4)	**6.**	(3)	**7.**	(3)
8.	(1)	**9.**	(4)	**10.**	(3)	**11.**	(1)	**12.**	(1)	**13.**	(1)	**14.**	(2)
15.	(1)	**16.**	(4)	**17.**	(4)	**18.**	(3)	**19.**	(1)	**20.**	(3)	**21.**	(2)
22.	(2)	**23.**	(3)	**24.**	(2)	**25.**	(2)	**26.**	(4)	**27.**	(2)	**28.**	(3)

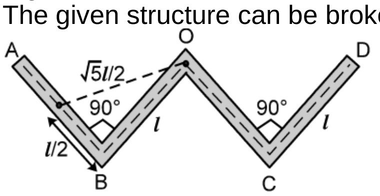
SPP Solutions

1. By energy conservation



$mg\frac{\ell}{4} = \frac{1}{2}\cdot\left(\frac{7}{48}m\ell^2\right)\omega^2$ [$I_{(about\ O)} = \frac{m\ell^2}{12} + m\left(\frac{\ell}{4}\right)^2$] $I_0 = \frac{7}{48}ml^2$ $\Rightarrow$ $\omega = \sqrt{\frac{24g}{7\ell}}$

2. The given structure can be broken into 4 parts



For AB $I = I_{CM} + m \times d^2 = \frac{m\ell^2}{12} + \frac{5m}{4}\ell^2$; $I_{AB} = \frac{4}{3}ml^2$

For BO $I = \frac{m\ell^2}{3}$ $\therefore$ For composite frame : (by symmetry)

$I = 2[I_{AB} + I_{OB}]$ $= 2\left[\frac{4m\ell^2}{3} + \frac{m\ell^2}{3}\right] = \frac{10}{3}ml^2$.]

3. In equilibrium, torques of forces mg and Mg about an axis passing through O balance each other.

$mg.\frac{L}{2}\cos30° = Mg\frac{L}{2}\cos60°$ $\Rightarrow$ $\frac{M}{m} = \sqrt{3}$

4. $\vec{F}_{net} = (400-100)\hat{i} + (200+200)\hat{j} = 300\hat{i} + 400\hat{j}$ $\Rightarrow |\vec{F}| = 500$ N

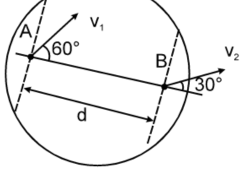
Angle made by $\vec{F}_{net}$ with the vertical is $\theta = \tan^{-1}\left(\frac{300}{400}\right) = 37°$

also τ = 500 R therefore point of application of the resultant force is at a distance R from the centre.

5. For rigid body separation between two point remains same.

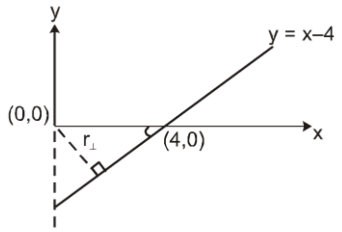
$v_1 \cos60° = v_2 \cos30°$

$\frac{v_1}{2} = \frac{\sqrt{3}\ v_2}{2}$ $\Rightarrow$ $v_1 = \sqrt{3}\ v_2$



$\omega_{disc} = \left|\frac{v_2\sin30° - v_1\sin60°}{d}\right| = \left|\frac{\frac{v_2}{2} - \frac{\sqrt{3}v_1}{2}}{d}\right| = \left|\frac{v_2 - \sqrt{3}\times\sqrt{3}v_2}{2d}\right| = \frac{2v_2}{2d} = \frac{v_2}{d} \Rightarrow \omega_{disc} = \frac{v_2}{d}$

6. $\tau = \frac{dL}{dt} = 4$



from figure $r_\perp = 2\sqrt{2}$ m

Hence $\tau = r_\perp F$

$4 = 2\sqrt{2}.F$ ∴ $F = \sqrt{2}$ N **Ans.**

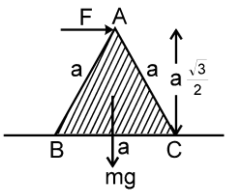
7. For (rod + particle) system :

$$\frac{1}{2}\left(\frac{m\ \ell^2}{3}\right)\left(\frac{v^2}{\ell^2}\right) + \frac{1}{2}mv^2 = 2\ mg\left(\frac{3\ \ell}{2}\right)$$

[Since, com will finally reach a height $2\left(\frac{3\ell}{4}\right)$] $\Rightarrow$ $v = \sqrt{4.5\ \ g\ell}$

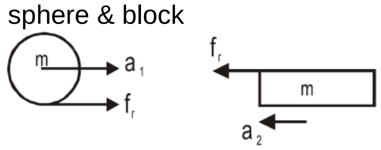
8. The tendency of rotating will be about the pont C. For minimum force, the torque of F about C has to be equal to the torque of mg about C.

∴ $F\left(a\frac{\sqrt{3}}{2}\right) = \left(\frac{a}{2}\right)mg$



$\Rightarrow$ $F = \frac{mg}{\sqrt{3}}$ **Ans.**

9. FBD for sphere & block



$a_1 = \frac{f_r}{m} = \frac{\mu mg}{m}$ $a_2 = \frac{f_r}{m} = \frac{\mu mg}{m}$

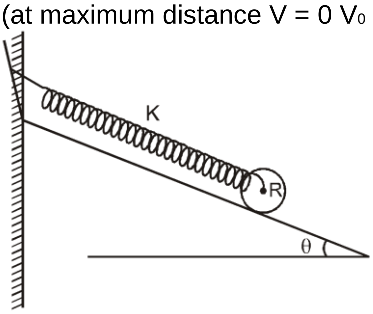
$\vec{a}_1 = \mu g\hat{i}$ $\vec{a}_2 = -\mu g\hat{i}$

$\vec{a}_{rel} = \vec{a}_1 - \vec{a}_2 = 2\mu g\hat{i}$

$a_{rel} = 2\mu g$.

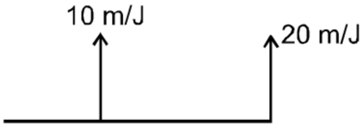
10. Using Energy conservation,

(at maximum distance V = 0 V_0 = 0)



$\frac{1}{2}Kx^2 = (mg\ x\ \sin\theta)$ $x = \left(\frac{2mg\ \ \ \sin\ \theta}{K}\right)$

11.

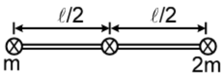


$20 = V_{cm} + wR$

$20 = 10 + \omega\left(\frac{\ell}{2}\right)$

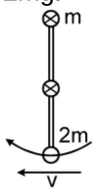
$10 = \frac{\omega}{2}$ $\quad$ (ω = 20 rad / sec)

12. Decrease in PE =



Increase in rotational K.E

$\Rightarrow \quad 2mg.\frac{\ell}{2} - mg.\frac{\ell}{2} = \frac{1}{2} I . \omega^2 \quad = \frac{1}{2}\left(2m\frac{\ell^2}{4} + m.\frac{\ell}{4}\right)\omega^2$

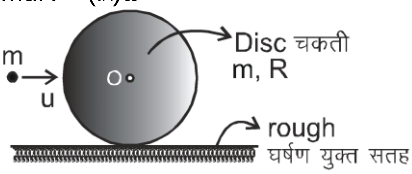


$\frac{mg\ell}{2} = \frac{1}{2} . \frac{3m\ell^2}{4} . \omega = \frac{3m\ell^2}{8}\omega^2 \quad \Rightarrow \omega = \sqrt{\frac{4g}{3\ell}}$ and $v = r\omega = \frac{\ell}{2}\sqrt{\frac{4g}{3\ell}} = \sqrt{\frac{g\ell}{3\ell}}$

[Ans.: (a) V = $\sqrt{g\ell/3}$, ω = $\sqrt{4g/3\ell}$]

13. Angluar momentum conservation about contact point A

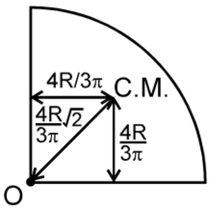
$muR = (I_A)\omega$



$I_A = \left(\frac{mR^2}{2} + mR^2\right) + m\left(\sqrt{2}\ R\right)^2 = \frac{7}{2}mR^2$

$\omega = \frac{muR}{\frac{7}{2}mR^2} = \frac{2u}{7R}$ **Ans.**

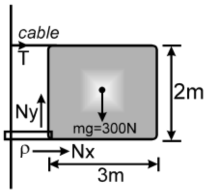
14. M.I. about 'O' is $\frac{MR^2}{2}$



By parallel-axis theorem

$\frac{MR^2}{2} = I_{cm} + M\left(\frac{4R}{3\pi} . \sqrt{2}\right)^2 \Rightarrow I_{cm} = \frac{MR^2}{2} - M\left(\sqrt{2} . \frac{4R}{3\pi}\right)^2$

15. From FBD



Equation in horizontal direction

$T = N_x$ ....(1)

For Rotational equation about P

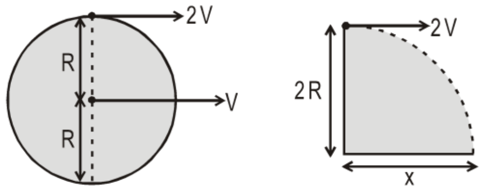
$T.\ 2 = 1.5 \times 300$

$T = 225$ N

$N_x = 225$ N

$N_y = 300$ N And Ng = mg = 300 N

16.



$x = 2v\sqrt{\frac{2R \cdot 2}{g}} = 2v\sqrt{\frac{4R}{g}} = \sqrt{\frac{16Rv^2}{g}}$

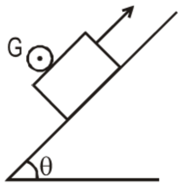
17. As $\Sigma\tau = 0$, angular momentum remains conseved :

$\therefore \quad L = \left(0 + \frac{300R^2}{2}\right)\omega_0 = \left(\frac{300R^2}{2} + 30R^2\right).\omega \Rightarrow \quad 150\,\omega_0 = 180\,\omega \quad \Rightarrow \quad \omega = 5/6\,\omega_0$

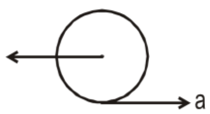
18. Impulse = change in momentum

$\therefore \quad P.\frac{\ell}{2} = \frac{m\ell^2}{12}.\omega$ (about centre of AB) $\quad \Rightarrow \quad \omega = \frac{6P}{m\ell}$

For $\theta = \frac{\pi}{2}$; $\quad \frac{\pi}{2} = \omega t \quad \Rightarrow \quad t = \frac{\pi}{2\omega} = \frac{\pi m\ell}{2 \times 6P} \quad \Rightarrow \quad t = \frac{\pi m\ell}{12P}$

19. 

g sinθ



Circular pipe is rest

so g sin θ = a

20. Cons. of ang. momentum about P gives

$MV\frac{L}{2} = \frac{(2M)\ (2\ L)^2}{12}\,w \quad \Rightarrow \quad \frac{V}{2} = \frac{2\ L\ w}{3} \qquad w = \frac{3\ V}{4\ L}$

21. Rotation energy = $\frac{1}{2}I\omega^2$ I = mK^2

linear energy = $\frac{1}{2}mv^2$ K = gyration radius

Total energy = $\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$ $v = \omega R$

Frictional of its total energy associated with rotation.= $\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2 + \frac{1}{2}mv^2} = \frac{mK^2\omega^2}{mK^2\omega^2 + m\omega^2R^2} = \left(\frac{K^2}{R^2 + K^2}\right)$

22. α = constant $\therefore \omega$ increases

23. Body is rotating uniformly so resultant force on particale is centripetal force which is horizontal and intersecting the axis of rotation.

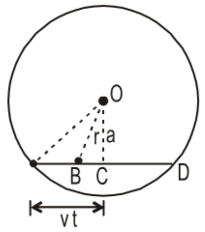
24. Since, there is no external torque, angular momentum will remain conserved. The moment of inertia will first decrease till the tortoise moves from A to C and then increase as it moves from C and D. Therefore ω will initially increase and then decrease.

Let R be the radius of platform m the mass of disc and M is the mass of platform.

Moment of inertia when the tortoise is at A

$$I_1 = mR^2 + \frac{MR^2}{2}$$

and moment of inertia when the tortoise is at B



$$I_2 = mr^2 + \frac{MR^2}{2}$$

here $r^2 = a^2 + [\sqrt{R^2 - a^2} - vt]^2$

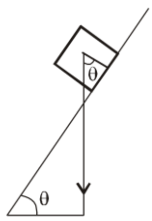
From conservation of angular momentum

$$\omega_0 I_1 = \omega(t) I_2$$

substituting the values we can see that variation of $\omega(t)$ is nonlinear.

25. Angle of repose $\theta_0 = \tan^{-1}\mu = \tan^{-1}\sqrt{3} = 60^\circ$

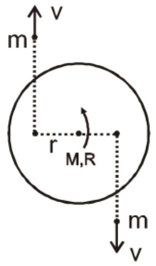
$\tan\theta = \frac{5}{15/2} = \frac{2}{3}$. $\theta < 45^\circ$.



Block will topple before it starts slide down.

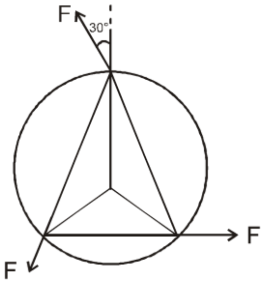
26. Applying conservation of angular momentum.

$$2mvr - \frac{MR^2}{2}\,\omega = 0$$



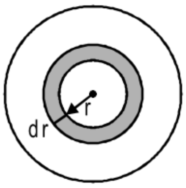
$$\omega = \frac{4mvr}{MR^2} \qquad \omega = \frac{(4)\ (5\times10^{-2})\ (9)\left(\frac{1}{4}\right)}{45\times10^{-2}\times\frac{1}{4}} \qquad \omega = 4 \text{ rad/s}$$

27.



$$\omega = \frac{\int \tau dt}{I} = \frac{\int_0^t 3F\sin 30^\circ R\ \ dt}{I} = \frac{3.\ \ (0.5)\ \ (0.5)\ \ (0.5)\ \ (1)}{\frac{1.5(0.5)^2}{2}} = 2 \text{ rad/s}$$

28. Consider a shell of radius r and thickness dr



$$dI = \frac{2}{3}(\rho.4\pi r^2 dr)r^2 \quad \Rightarrow \quad I = \int dI \quad \Rightarrow \quad \frac{I_B}{I_A} = \frac{\int_o^R \frac{2}{3}k\frac{r^5}{R^5}.4\pi r^2 dr\quad r^2}{\int_o^R \frac{2}{3}k\frac{r}{R}4\pi r^2 dr\quad r^2} = \frac{6}{10}$$