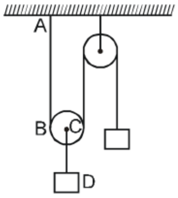
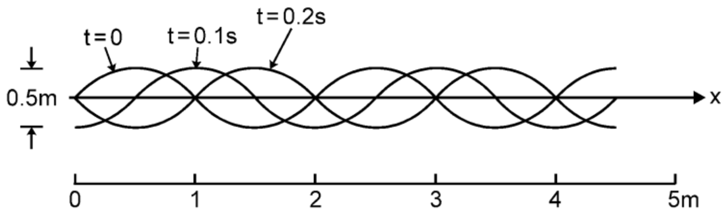
Self Practice Paper (SPP)

1. Both the strings, show in figure, are made of same material and have same cross-section. The pulleys are light. The wave speed of a transverse wave in the string AB is v_1 and in CD it is v_2. The v_1/v_2 is



(1) 1 (2) 2 (3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

2. Three consecutive flash photographs of a travelling wave on a string are reproduced in the figure here. The following observations are made. Mark the one which is correct. (Mass per unit length of the string = 3 g/cm.)



(1) displacement amplitude of the wave is 0.25 m, wavelength is 1 m, wave speed is 2.5 m/s and the frequency of the driving force is 0.2/s.
(2) displacement amplitude of the wave is 2.0 m, wavelength is 2 m, wave speed is 0.4 m/s and the frequency of the driving force is 0.7/s.
(3) displacement amplitude of the wave is 0.25 m, wavelength is 2 m, wave speed is 5 m/s and the frequency of the driving force is 2.5 /s.
(4) displacement amplitude of the wave is 0.5 m, wavelength is 2 m, wave speed is 2.5 m/s and the frequency of the driving force is 0.2/s.

3. A heavy ball is suspended from the ceiling of a motor car through a light string. A transverse pulse travels at a speed of 60 cm/s on the string when the car is at rest and 62 cm/s when the car accelerates on a horizontal road. Then acceleration of the car is : (Take g = 10 m/s^2.)
(1) 2.7 m/s^2 (2) 3.7 m/s^2 (3) 2.4 m/s^2 (4) 1.4 m/s^2

4. A copper wire is held at the two ends by rigid supports. At 30ºC the wire is just taut, with negligible tension. The speed of transverse waves in this wire at 10ºC is :
($\alpha = 1.7 \times 10^{-5}$ / ºC, $Y = 1.3 \times 10^{11}$ N/m^2, $d = 9 \times 10^3$ kg/m^3). **[JEE - 79]**
(1) 80 m/sec (2) 90 m/sec (3) 100 m/sec (4) 70 m/sec

5. Two small boats are 10m apart on a lake. Each pops up and down with a period of 4.0 seconds due to wave motion on the surface of water. When one boat is at its highest point, the other boat is at its lowest point. Both boats are always within a single cycle of the waves. The speed of the waves is
(1) 2.5 m/s (2) 5.0 m/s (3) 14 m/s (4) 40 m/s

6. A sinusoidal wave with amplitude y_m is travelling with speed V on a string with linear density ρ. The angular frequency of the wave is ω. The following conclusions are drawn. Mark the one which is correct.
(1) doubling the frequency doubles the rate at which energy is carried along the string
(2) if the amplitude were doubled, the rate at which energy is carried would be halved
(3) if the amplitude were doubled, the rate at which energy is carried would be doubled
(4) the rate at which energy is carried is directly proportional to the velocity of the wave.

7. Sinusoidal waves 5.00 cm in amplitude are to be transmitted along a string having a linear mass density equal to 4.00×10^{-2} kg/m. If the source can deliver a maximum power of 90 W and the string is under a tension of 100 N, then the highest frequency at which the source can operate is (take $\pi^2 = 10$):

(1) 45 Hz (2) 50 Hz (3) 30 Hz (4) 62 Hz

8. A wave pulse, travelling on a two piece string, gets partially reflected and partially transmitted at the junction. The reflected wave is inverted in shape as compared to the incident one. If the incident wave has wavelength λ and the transmitted wave λ',
(1) $\lambda' > \lambda$ (2) $\lambda' = \lambda$ (3) $\lambda' < \lambda$
(4) nothing can be said about the relation of λ and λ'.

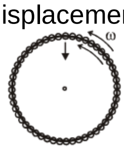
9. Equations of a stationary wave and a travelling wave are $y_1 = a \sin kx \cos \omega t$ and $y_2 = a \sin(\omega t - kx)$. The phase difference between two points $x_1 = \frac{\pi}{3k}$ and $x_2 = \frac{3\pi}{2k}$ are ϕ_1 and ϕ_2 respectively for the two waves. The ratio $\frac{\phi_1}{\phi_2}$ is :
(1) 1 (2) 5/6 (3) 3/4 (4) 6/7

10. An observer standing at the sea-coast observes 54 waves reaching the coast per minute. If the wavelength of wave is 10m, The velocity. of wave is **[REE - 79]**
(1) 19 m/sec (2) 29 m/sec (3) 9 m/sec (4) 39 m/sec

11. One end of two wires of the same metal and of same length (with radius, r and 2r) are joined together. The wire is used as sonometer wire and the junction is placed in between two bridges. The tension T is applied to the wire. If at a junction a node is formed then the ratio of number of loops formed in the wires will be: **[JEE - 85]**
(1) 1 : 2 (2) 2 : 3 (3) 3 : 4 (4) 4 : 5

12. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of its volume is submerged. The new fundamental frequency in Hz is **[JEE - 95]**
(1) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ (2) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$ (3) $300\left(\frac{2\rho}{2\rho-1}\right)$ (4) $300\left(\frac{2\rho-1}{2\rho}\right)$

13. (a) A circular loop of rope of length L rotates with uniform angular velocity ω about an axis through its centre on a horizontal smooth platform. Velocity of pulse (with respect to rope) produced due to slight radial displacement is given by



(1) ωL (2) $\frac{\omega L}{2\pi}$ (3) $\frac{\omega L}{\pi}$ (4) $\frac{\omega L}{4\pi^2}$

(b) In the above question if the motion of the pulse and rotation of the loop, both are in same direction then the velocity of the pulse w.r.t. to ground will be :
(1) ωL (2) $\frac{\omega L}{2\pi}$ (3) $\frac{\omega L}{\pi}$ (4) $\frac{\omega L}{4\pi^2}$

(c) In the above question if both are in opposite direction then the velocity of the pulse w.r.t. to ground will be:
(1) ωL (2) $\frac{\omega L}{2\pi}$ (3) $\frac{\omega L}{\pi}$ (4) 0

14. Three waves of equal frequency having amplitudes 10 μm, 4 μm and 7 μm arrive at a given point with a successive phase difference of $\pi/2$. The amplitude of the resulting wave is μm in given by
(1) 7 (2) 6 (3) 5 (4) 4

15. A metalled wire with tension T and at temperature 30°C vibrates with its fundamental frequency of 1 kHz. The same wire with the same tension but at 10°C temperature vibrates with a fundamental frequency of 1.001 kHz. The coefficient of linear expansion of the wire is

(1) 2×10^{-4} °C^{-1} (2) 1.5×10^{-4} °C^{-1} (3) 1×10^{-4} °C^{-1} (4) 0.5×10^{-4} °C^{-1}

16. A steel wire of length 1 m and mass 0.1 kg and having a uniform cross-sectional area of 10^{-6} m^2 is rigidly fixed at both ends. The temperature of the wire is lowered by 20°C. If the transverse waves are set up by plucking the string in the middle, the frequency of the fundamental note of vibration is

($Y_{steel} = 2 \times 10^{11}$ N/m^2, $\alpha_{steel} = 1.21 \times 10^{-5}$/°C)

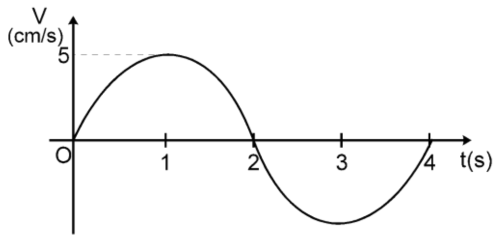
(1) 44 Hz (2) 88 Hz (3) 22 Hz (4) 11 Hz

17. A heavy but uniform rope of length L is suspended from a ceiling. A particle is dropped from the ceiling at the instant when the bottom end is given the jerk. Where will the particle meet the pulse :

(1) at a distance lf $\frac{2L}{3}$ from the bottom (2) at a distance $\frac{L}{3}$ from the bottom

(3) at a distance $\frac{L}{3}$ from the bottom (4) None of these

18. A certain transverse sinusoidal wave of wavelength 20 cm is moving in the positive x direction. The transverse velocity of the particle at x = 0 as a function of time is shown. The amplitude of the motion is



(1) $\frac{5}{\pi}$ cm (2) $\frac{\pi}{2}$ cm (3) $\frac{10}{\pi}$ cm (4) 2π cm

19. A 20 cm long rubber string obeys Hook's law. Initially when it is stretched to make its total length of 24 cm, the lowest frequency of resonance is υ_0. It is further stretched to make its total length of 26 cm. The lowest frequency of resonance will now be :

(1) the same as υ_0 (2) greater than υ_0 (3) lower than υ_0 (4) None of these

20. The wave–function for a certain standing wave on a string fixed at both ends is $y(x,t) = 0.5 \sin (0.025\pi x) \cos 500 t$ where x and y are in centimeters and t is in seconds. The shortest possible length of the string is :

(1) 126 cm (2) 160 cm (3) 40 cm (4) 80 cm

21. A wave travels on a light string. The equation of the wave is $Y = A \sin (kx - \omega t + 30°)$.It is reflected from a heavy string tied to an end of the light string at x = 0.If 64% of the incident energy is reflected the equation of the reflected wave is

(1) $Y = 0.8 A \sin (kx - \omega t + 30° + 180°)$ (2) $Y = 0.8 A \sin (kx + \omega t + 30° + 180°)$

(3) $Y = 0.8 A \sin (kx + \omega t - 30°)$ (4) $Y = 0.8 A \sin (kx + \omega t + 30°)$

22. A string of length 'ℓ' is fixed at both ends. It is vibrating in its 3_{rd} overtone with maximum amplitude 'a'. The amplitude at a distance $\frac{\ell}{3}$ from one end is :

(1) a (2) 0 (3) $\frac{\sqrt{3}a}{2}$ (4) $\frac{a}{2}$

23. A wire having a linear density of 0.05 gm/cm is stretched between two rigid supports with a tension of 4.5×10^7 dynes. It is observed that the wire resonates at a frequency of 420 cycles/sec. The next higher frequency at which the same wire resonates is 490 cycles/sec. The length of wire is approximately. **[JEE - 71]**

(1) 314 cm (2) 254 cm (3) 214 cm (4) 354 cm

24. A wire of density 9 gm/cm^3 is stretched between two clamps 1.00 m apart while subjected to an extension of 0.05 cm. The lowest frequency of transverse vibrations in the wire is
(Assume Young's modulus $Y = 9 \times 10^{10}$ N/m^2) **[JEE - 75]**

(1) 35 Hz (2) 45 Hz (3) 75 Hz (4) 90 Hz

25. Two wave pulses travel in opposite directions on a string and approach each other. The shape of the one pulse in inverted with respect to the other.

(1) the pulses will collide with each other and vanish after collision

(2) the pulses will reflect from each other i.e., the pulse going towards right will finally move towards left and vice versa.

(3) the pulses will pass through each other but their shapes will be modified

(4) the pulses will pass through each other without any change in their shape

SPP Answers

1.	(4)	**2.**	(3)	**3.**	(2)	**4.**	(4)	**5.**	(2)	**6.**	(4)	**7.**	(3)
8.	(3)	**9.**	(4)	**10.**	(3)	**11.**	(1)	**12.**	(1)	**13.**	(a)	(2)	
(b)	(3)	(c)	(4)	**14.**	(3)	**15.**	(3)	**16.**	(4)	**17.**	(2)	**18.**	(3)
19.	(2)	**20.**	(3)	**21.**	(3)	**22.**	(3)	**23.**	(3)	**24.**	(1)	**25.**	(4)

SPP Solutions

1. $V \propto \sqrt{T}$

$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{T/2}{T}} = \frac{1}{\sqrt{2}}$

2. Clear from the figure

3. $60 = \sqrt{\frac{mg}{\mu}}$ ....(1)

$62 = \sqrt{\frac{m(g^2 + a^2)^{1/2}}{\mu}}$ ....(2)

Solve (1) and (2)

$\therefore a = 3.7 \frac{m}{sec^2}$

4. $V = \sqrt{\frac{T}{\mu}} = \frac{YA\alpha\Delta T}{\mu}$ Put values

5. Distance between boat = $\frac{\lambda}{2}$ = 10 m $\Rightarrow$ λ = 20m

time penod T = 4 sec .

$\therefore$ V = λ / T = 20 m / 4sec.

= 5m/s.

6. By defination

7. As $\langle P \rangle = 2\pi_2 f_2 A_2 \mu v$ put values

8. Second string is denser so speed will decrease hence wavelength also $\lambda_1 < \lambda$

9. At $x_1 = \frac{\pi}{3K}$ and $x_2 = \frac{3\pi}{2K}$

Nodes are not formed because neither x_1 nor x_2 gives sin kx = 0

$\therefore \Delta x = x_2 - x_1 = \frac{7\pi}{6K}$

As this Δx is between λ and $\frac{\lambda}{2}$ $\therefore \phi_1 = \pi$

and $\phi_2 = K\Delta x = \frac{7\pi}{6}$ $\therefore \frac{\phi_1}{\phi_2} = \frac{6}{7}$

10. $v = f\lambda = \frac{54}{60} \times 10 = 9$ m/sec.

11. $\mu = \frac{m}{\ell} = \rho A$ $\therefore m_1 = \rho\pi r_2$

$m_2 = \rho 4 \pi r_2$ $\therefore \frac{v_1}{v_2} = \frac{\sqrt{T/\mu_1}}{\sqrt{T/\mu_2}}$

Let P loops and q loops are formed respectively 1st and 2nd ωire.

∴ $\frac{p}{2\ell} V_1 = \frac{q}{2\ell} V_2 \Rightarrow \frac{p}{q} = \frac{1}{2}$

12. If $T = mg = v\rho g$

∴ $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = 300$ .....(1)

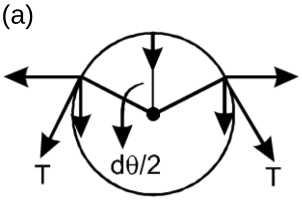
Now $T' = mg - f_b = v\rho g - \frac{V}{2} g$.

$T' = vg\left(\frac{2\rho - 1}{2}\right)$

∴ $f' = \frac{1}{2\ell}\sqrt{\frac{vg}{\mu}\frac{(2\rho-1)}{2}}$ ....(2)

∴ $\frac{f'}{f} = \left(\frac{2\rho-1}{2\rho}\right)^{½}$ $f' = 300\left(\frac{2\pi-1}{2P}\right)^{½}$

13. (a)



$2T \sin\frac{d\theta}{2} = dm.\omega_2 R$

$2T\frac{d\theta}{2} = \frac{m}{\ell} Rd\theta.\omega_2 R$ $T = \frac{m\omega^2 R^2}{\ell}$

∴ $V = \sqrt{\frac{T}{\mu}} = \sqrt{T/\frac{m}{\ell}} = \omega R_2$ ∴ $V = \frac{\omega L}{2\pi}$

(b) $V_{P/R} = \frac{\omega L}{2\pi}$

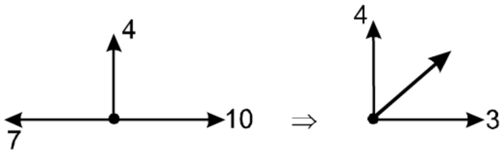
$V_{P./G} - V_{R/G} = \frac{\omega L}{2\pi} \Rightarrow V_{P/G} - \frac{\omega L}{2\pi} = \frac{\omega L}{2\pi}$

∴ $V_{P/G} = \left(\frac{wL}{2\pi}\right)2$ $V_{P/G} = \frac{\omega L}{\pi}$

(c) $V_{P/R} = \frac{\omega L}{\pi}$ $V_{P/G} - \left(-\frac{wL}{2\pi}\right) = \frac{wL}{2\pi}$

∴ $V_{P/G} = 0$

14.



Resultant Amplitude $= \sqrt{3^2 + 4^2} = 5\mu m$

15. $f_1 = \frac{1}{2\ell_1}\sqrt{\frac{T\ell_1}{m}}$ $f_2 = \frac{1}{2\ell_2}\sqrt{\frac{T\ell_2}{m}}$ ∴ $\frac{f_1}{f_2} = \sqrt{\frac{\ell_2}{\ell_1}} = \sqrt{\frac{\ell_1(1-20\alpha)}{\ell_1}}$

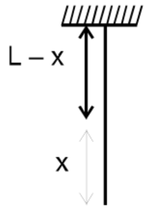
$\frac{f_1}{f_2} = (1 - 10\alpha)$ (By Binomial theorem)

$\alpha = \frac{f_2 - f_1}{10 f_2} = 10^{-4}\ ^{o}C^{-1}$

16. $n = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$ $\quad n = \frac{1}{2\ell}\sqrt{\frac{YA\alpha T}{\mu}}$

Put values $\quad \therefore n = 11$ Hz

17.



For the pulse $V = \sqrt{\frac{\mu x g}{\mu}} = \sqrt{xg} = \frac{dx}{dt}$

$\frac{dx}{dt} = \sqrt{xg} \Rightarrow \int_o^x \frac{dx}{\sqrt{x}} = \sqrt{g}\int_o^t dt$

$t = 2\sqrt{\frac{x}{g}}$ ———(1)

for the particle $\quad L - x = \frac{1}{2} gt^2$

$t = \sqrt{\frac{2(L-x)}{g}}$ ———(2)

1 = 2 $\Rightarrow$ $\quad x = \frac{L}{3}$ from the bottom

18. $V_{max} = A\omega = 5 \Rightarrow A\frac{2\pi}{4} = 5 \Rightarrow A = \frac{10}{\pi}$ cm.

19. As, $T = \frac{YA}{20}4 \Rightarrow v_0 = \frac{1}{2\times 24}\sqrt{\frac{YA}{20}.\frac{4}{\mu}}$ ....(1)

and $T' = \frac{YA}{20}6 \Rightarrow v' = \frac{1}{2\times 26}\sqrt{\frac{YA}{20}.\frac{6}{\mu}}$ ....(2)

$\therefore \frac{\nu'}{\nu_0} = \frac{24}{26}\sqrt{\frac{6}{4}}$ $\quad \therefore v' > v_0$

20. $K = 0.025\pi = \frac{2\pi}{\lambda}$ $\quad \lambda = \frac{2cm}{0.025}$

Required length $= \frac{\lambda}{2} = \frac{1}{0.025} = 40$ cm

21. As $y = A\sin(Kx - \omega t + 30^o)$ for incident wave

Now for reflected wave : Energy α Amp^2

$\therefore Y = 0.8A\sin(-Kx - \omega t + 30 + 180)$

$Y = 0.8A\sin(-Kx - \omega t + 210)$

$Y = -0.8\ A \sin (kx + \omega t - 210)$
$Y = -0.8\ A \sin [Kx + \omega t - 30 - 180]$
$Y = 0.8\ A \sin [180 - (Kx + \omega t - 30)]$
$Y = 0.8\ A \sin (Kx + \omega t - 30)$

22. $\Rightarrow\ 2\lambda = \ell$

$\Rightarrow \lambda = \frac{\ell}{2}$

As x = 0 is a node

$\therefore A_{s.\omega.} = A \sin \frac{2\pi}{\lambda} x$

$= a \sin \left(\frac{2\pi}{\lambda} \frac{\frac{\lambda}{2}}{3} \right)$

$= a \frac{\sqrt{3}}{2}$

23. $V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{4.5\times10^7}{0.05}}$

$\frac{n}{2\ell}\sqrt{\frac{4.5\times10^7}{0.05}} = 420$ ———(1)

$\frac{n+1}{2\ell}\sqrt{\frac{4.5\times10^7}{0.05}} = 490$ ———(2)

$1 \div 2 \Rightarrow \frac{n}{n+1} = \frac{6}{7}$

$\Rightarrow$ n = 6

Put n in (1)

$\therefore \frac{6}{2\ell} 3 \times 10^2 = 420$

$\therefore \frac{6}{2\ell} 3 \times 10^2 = 420$

$\ell = \frac{30000}{140}$

$\ell = \frac{1500}{7} = 214$ cm

24. $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

$= \frac{1}{2\times1}\sqrt{\frac{\frac{YA}{\ell}.\Delta\ell}{\rho A}}$

$= \frac{1}{2}\sqrt{\frac{9\times10^{10}\times5\times10^{-14}}{9\times10^3}}$

= 35 Hz

25. By difination