Self Practice Paper (SPP)

1. A spherical condenser has 10 cm and 12 cm as the radii of inner and outer spheres. The space between the two spheres is filled with a dielectric of dielectric constant 5. The capacity when;

(i) the outer sphere is earthed.

(1) $\frac{2}{3} \times 10^{-10}$ F (2) $\frac{8}{3} \times 10^{-10}$ F (3) $\frac{10}{3} \times 10^{-10}$ F (4) $\frac{16}{3} \times 10^{-10}$ F

(ii) the inner sphere is earthed.

(1) $\frac{104}{30} \times 10^{-10}$ F (2) $\frac{52}{30} \times 10^{-10}$ F (3) $\frac{26}{30} \times 10^{-10}$ F (4) 6×10^{-11} F

2. A parallel plate condenser of capacity C is connected to a battery and is charged to potential V. Another condenser of capacity 2C is connected to another battery and is charged to potential 2V. The charging batteries are removed and now the condensers are connected in such a way that the positive plate of one is connected to negative plate of another. The final energy of this system is–

(1) zero (2) $\frac{25CV^2}{6}$ (3) $\frac{3CV^2}{2}$ (4) $\frac{9CV^2}{2}$

3. An uncharged capacitor of capacitance C is connected to a battery of emf ε at t = 0 through a resistance R, then

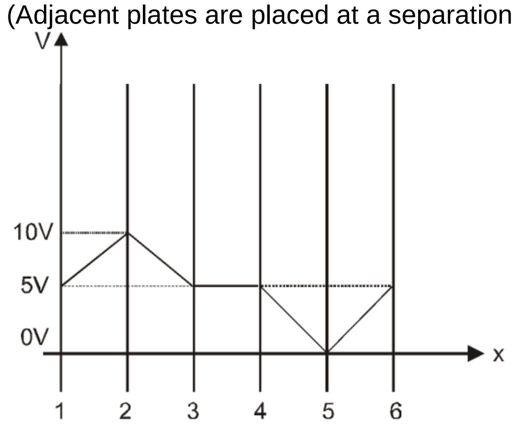
(i) the maximum rate at which energy is stored in the capacitor is :

(1) $\frac{\varepsilon^2}{4R}$ (2) $\frac{\varepsilon^2}{2R}$ (3) $\frac{\varepsilon^2}{R}$ (4) $\frac{2\varepsilon^2}{R}$

(ii) time at which the rate has this maximum value is

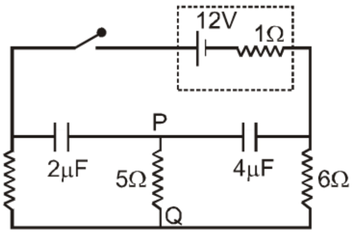
(1) 2CR ln2 (2) $\frac{1}{2}$ CR ln2 (3) CR ln2 (4) 3CR ln2

4. The V versus x plot for six identical metal plates of cross-sectional area A is as shown. The equivalent capacitance between 2 and 5 is (Adjacent plates are placed at a separation d) :



(1) $\frac{2\epsilon_0 A}{d}$ (2) $\frac{\epsilon_0 A}{d}$ (3) $\frac{3\epsilon_0 A}{d}$ (4) $\frac{\epsilon_0 A}{2d}$

5. In the circuit shown in figure the capacitors are initially uncharged. The current through resistor PQ just after closing the switch is :

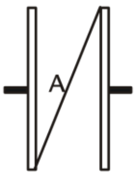


(1) 2A from P to Q (2) 2A from Q to P (3) 6A from P to Q (4) zero

6. The plates of a parallel plate condenser are being moved away with a constant speed v. If the plate separation at any instant of time is d then the rate of change of capacitance with time is proportional to–

(1) $\frac{1}{d}$ (2) $\frac{1}{d^2}$ (3) d_2 (4) d

7. A parallel plate capacitor of capacitance C is as shown. A thin metal plate A is placed between the plates of the given capacitor in such a way that its edges touch the two plates as shown. The capacity now be comes.



(1) 0 (2) 3C (3) 4C (4) ∞

8. A capacitor of capacitance C_0 is charged to a potential V_0 and then isolated. A capacitor C is then charged from C_0, discharged and charged again ; the process is repeated n times. Due to this, potential of the larger capacitor is decreased to V, then value of C is :

(1) $C_0\,[V_0/V]_{1/n}$ (2) $C_0[(V_0/V)_{1/n} - 1]$ (3) $C_0\,[(V_0/V) - 1]$ (4) $C_0\,[(V/V_0)_n + 1]$

9. A 3 mega ohm resistor and an uncharged 1 μF capacitor are connected in a single loop circuit with a constant source of 4 volt. At one second after the connection is made what are the rates at which;

(i) the charge on the capacitor is increasing.

(1) $4(1-e^{-1/3})$ μ C/s (2) $4e^{-1/3}$ μ C/s

(3) $\frac{4}{3}e^{-1/3}$ μ C/s (4) $\frac{4}{3}(1-e^{-1/3})$ μ C/s

(ii) energy is being stored in the capacitor.

(1) $\frac{16}{3}(1-e^{-1/3})e^{-1/3}$ μJ/s (2) $\frac{16}{3}(1-e^{-2/3})$ μJ/s

(3) $\frac{16}{3}e^{-2/3}$ μJ/s (4) None of these

(iii) joule heat is appearing in the resistor.

(1) $\frac{16}{3}e^{-1/3}\mu$ J/s (2) $\frac{1}{2}e^{-1/3}\mu$ J/s

(3) $\frac{16}{3}(e^{-2/3})$ μ J/s (4) $\frac{16}{3}(1-e^{-1/3})^2$ μ J/s

(iv) energy is being delivered by the source.

(1) $16(1-e^{-1/3})$ μ J/s (2) 16μ J/s

(3) $\frac{16}{3}e^{-1/3}$ μ J/s (4) $\frac{16}{3}(1-e^{-1/3})$ μ J/s

10. An uncharged capacitor of capacitance 100μF is connected to a battery of emf 20V at t = 0 through a resistance 10Ω, then
(i) the maximum rate at which energy is stored in the capacitor is :
(1) 10J/s (2) 20 J/s (3) 40J/s (4) 5J/s

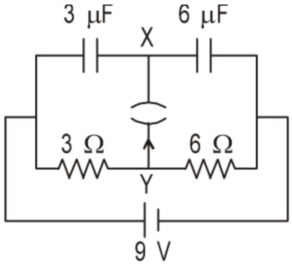
(ii) time at which the rate has this maximum value is
(1) (4 ln 2) ms (2) (2 ln 2) ms (3) (ln 2) ms (4) (3 ln 2) ms

11. (i) A 3μF capacitor is charged up to 300 volt and 2μF is charged up to 200 volt. The capacitor are connected so that the plates of same polarity are connected together. The final potential difference between the plates of the capacitor after they are connected is :
(1) 220 V (2) 160 V (3) 280 V (4) 260 V

(ii) If instead of this, the plates of opposite polarity were joined together, then amount of charge that flows is :
(1) 6×10^{-4} C (2) 1.5×10^{-4} C (3) 3×10^{-4} C (4) 7.5×10^{-4} C

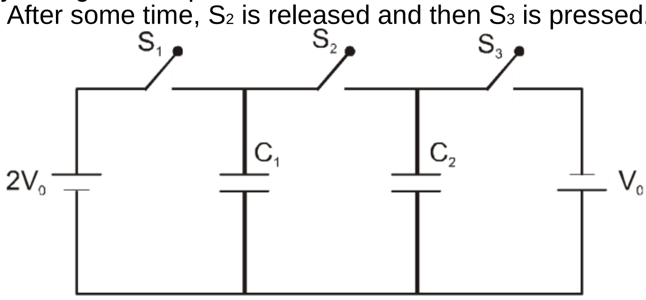
12. An uncharged capacitor of capacitance 4μF, a battery of emf 12 volt and a resistor of 2.5 MΩ are connected in series. The time after which $V_C = 3V_R$ is (take $\ell n2 = 0.693$)
(1) 6.93 seconds (2) 13.86 seconds (3) 7 seconds (4) 14 seconds

13. A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is



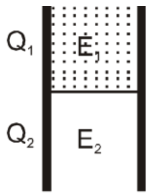
(1) 0 (2) 54 μC (3) 27 μC (4) 81 μC

14. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time.



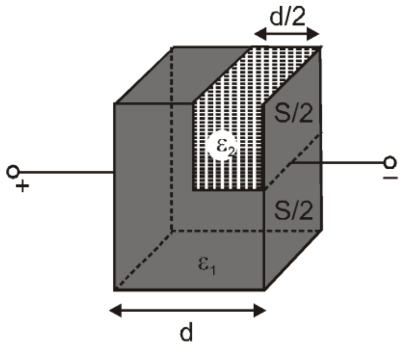
(1) the charge on the upper plate of C_1 is $2CV_0$ (2) the charge on the upper plate of C_1 is CV_0
(3) the charge on the upper plate of C_2 is 0 (4) the charge on the upper plate of C_2 is $-2CV_0$

15. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. Choose the correct option/options, igonoring edge effects.



(1) $\frac{E_1}{E_2} = 1$ /2 (2) $\frac{E_1}{E_2} = \frac{1}{K}$ (3) $\frac{Q_1}{Q_2} = \frac{3}{K}$ (4) $\frac{C}{C_1} = \frac{2+K}{K}$

16. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\frac{C_2}{C_1}$ is



(1) 6/5 (2) 5/3 (3) 7/5 (4) 7/3

17. A fully charged capacitor has a capacitance 'C'. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity 's' and mass 'm'. If the temperature of the block is raised by 'ΔT', the potential difference 'V' across the capacitance is :

(1) $\sqrt{\frac{2mC\Delta T}{s}}$ (2) $\frac{mC\Delta T}{s}$ (3) $\frac{ms\Delta T}{C}$ (4) $\sqrt{\frac{2ms\Delta T}{C}}$

SPP Answers

1.	(i)	(3)	(ii)	(1)	**2.**	(3)	**3.**	(i) (1)	(ii) (3)	**4.**	(2)	**5.**	(4)
6.	(2)	**7.**	(4)	**8.**	(2)	**9.** (i) (3)		(ii)(1)	(iii) (3)	(iv) (3)	**10.**	(i) (1) (ii) (3)	
11.	(i) (4)	(ii) (1)		**12.**	(2)	**13.**	(3)	**14.**	(2)	**15.**	(4)	**16.**	(4)
17.	(4)												

SPP Solutions

1. (i) outer sphere is earthed

$$C = \frac{4\pi \in_0 kab}{b-a} = \frac{4 \times 3.14 \times 8.85 \times 10^{-12} \times 5 \times 10 \times 10^{-2} \times 120 \times 10^{-2}}{(12-10) \times 10^{-2}}$$

$C = 3.34 \times 10^{-10} = \frac{10}{3} \times 10^{-10}$ F

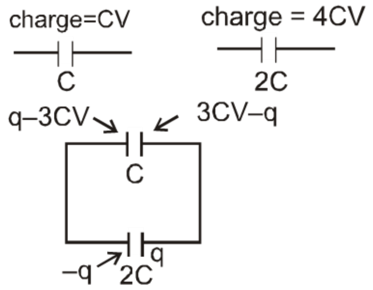
(ii) inner sphere is earthed

$C = \frac{4\pi \in_0 ab}{b-a} + 4\pi \in_0 b$

$= \frac{10}{3} \times 10^{-10}$ F $+ 4 \times 3.14 \times 8.85 \times 10^{-12} \times 12 \times 10^{-2}$

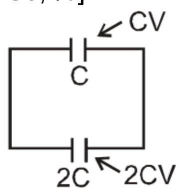
$= 3.34 \times 10^{-10} + 0.13338 \times 10^{-10}$

$= \left(\frac{10}{3} + \frac{1.4}{10}\right) \times 10^{-10} = \frac{104}{30} \times 10^{-10}$ F

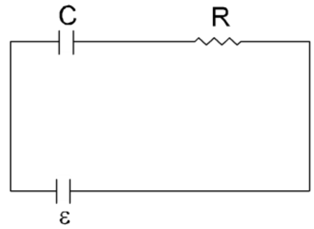
2. 

Total charge = 4 CV – CV = 3 CV
Now, let it is distributed as shown, potential across the capacitors is same

So, %] $\frac{q}{2C} = \frac{3CV - q}{C}$ $\Rightarrow$ q = 2 CV



Total potential energy = $\frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} = \frac{C^2V^2}{2C} + \frac{4C^2V^2}{2\times 2C} = \frac{3CV^2}{2}$

3. 

energy stored in capacitor = $\frac{Q^2}{2C}$

Rate at which energy is stored = $\frac{d}{dt}\frac{Q^2}{2C} = \frac{Q}{C}\cdot\frac{dQ}{dt} = \frac{Qi}{C}$

$Q = \varepsilon C\{1 - e^{-t/RC}\}$

$i = \frac{\varepsilon e^{-t/RC}}{R}$

Rate of energy storage = $\frac{\varepsilon^2}{R}\{1 - e^{-t/RC}\}\{e^{-t/RC}\} = \frac{\varepsilon^2}{R}\{e^{-t/RC} - e^{-2t/RC}\}$ .... (1)

It will be maximum when, $e^{-t/RC} - e^{-2tRC}$ will be maximum let $y(t) = e^{-t/RC} - e^{-2t/RC}$

for maximum , $y'(t) = 0$

$y'(t) = \frac{-e^{-t/RC}}{RC} + \frac{2e^{-2t/RC}}{RC}$

$e^{-t/RC} = \frac{1}{2}$

putting it back in eq. (1)

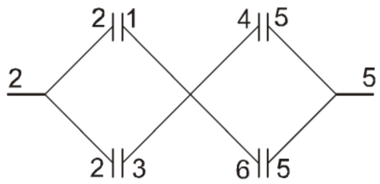
(i) maximum rate of energy storage = $\frac{\varepsilon^2}{R}\left\{\frac{1}{2} - \left(\frac{1}{2}\right)^2\right\} = \frac{\varepsilon^2}{4R}$

(ii) This will occur when , $e^{-t/RC} = \frac{1}{2}$

$$\frac{-t}{RC} = \ln \frac{1}{2}$$

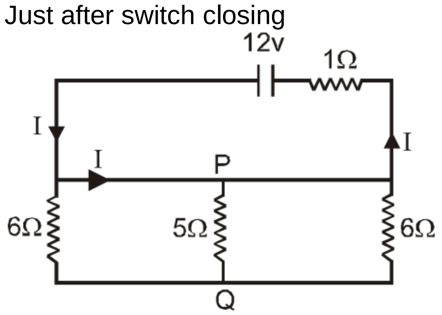
$$t = RC \ln 2$$

4.



$C_{eq.} = \frac{C}{2} + \frac{C}{2} = C = \frac{\epsilon_0 A}{d}$ **Ans.**

5. Just after switch closing



current through resistor PQ is zero just after closing the switch.

6. $x = Vt$, $\Rightarrow$ $d \propto t$ $C = \frac{\epsilon_0 A}{Vt}$ $\frac{dc}{dt} = -\frac{\epsilon_0 A}{V}\frac{1}{t^2}$ $\frac{dc}{dt} \propto \frac{1}{d^2}$ **Ans**

7. Theoritical capacitance = ∞, because d become zero

8. Charge on C_0, $Q_1 = C_0V_0$,

Initial charge on C_1, $Q_2 = 0$

Common potential $V_1 = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_0V_0}{C + C_0}$ $\Rightarrow Q_1 = C_0V_1 = \frac{C_0^2}{C + C_0} V_0$

Similarly $V_2 = \frac{C_0V_1}{C + C_0} = \left(\frac{C_0}{C + C_0}\right)^2 V_0 \Rightarrow Q_2 = C_0V_2 = \frac{C_0^3}{(C + C_0)^2} V_0$

for n times n $V_n = \left(\frac{C_0}{C + C_0}\right)^n V_0 = V$ $\Rightarrow$ $C = \left[\left(\frac{V_0}{V}\right)^{1/n} - 1\right] C_0$ **Ans**

9. (i) **(C)** $q_0 = 4\mu C$

$$i = \frac{dq}{dt} = \frac{q_0}{RC} e^{-t/RC} = \frac{4 \times 10^{-6}}{1 \times 10^{-6} \times 3 \times 10^{6}} e^{-1/3} = \frac{4}{3} e^{-1/3} \ \mu C/sec$$

(ii) **(A)** $U = \frac{q_0^2}{2C}(1 - e^{-t/RC})^2$

$$\frac{dU}{dt} = \frac{q_0^2}{RC^2}(1 - e^{-t/RC})\, e^{-t/RC}$$

$$= \frac{(4 \times 10^{-6})^2}{3 \times 10^6 \times (1 \times 10^{-6})^2}(1 - e^{-1/3})\, e^{-1/3}$$

$= \frac{16}{3} (1 - e^{-1/3})\, e^{-1/3}$ μJ/sec.

(iii) **(C)** $H = \int i^2 R dt \Rightarrow \frac{dH}{dt} = i^2 R$

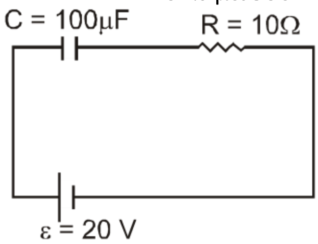
$\frac{dH}{dt} = i_0^2 R e^{-2t/RC} = \left(\frac{4}{3\times10^6}\right)^2 3\times10^6\, e^{-2/3} = \frac{16}{3} e^{-2/3}$ μJ /s

(iv) **(C)** $U = qV \Rightarrow \frac{dU}{dt} = V\frac{dq_0}{dt}(1 - e^{-t/RC})$

$\frac{dU}{dt} = \frac{q_0 V}{RC} e^{-t/RC}$

$= \frac{4\times10^{-6}\times4}{3\times10^6\times1\times10^6} e^{-1/3}$

$= \frac{16}{3} e^{-1/3}$ μJ/sec.

10. 

energy stored in capacitor $= \frac{Q^2}{2C}$

Rate at which energy is stored $= \frac{d}{dt}\left(\frac{Q^2}{2C}\right) = \frac{Q}{C}\cdot\frac{dQ}{dt} = \frac{Qi}{C}$

$Q = \varepsilon C\{1 - e^{-t/RC}\}$

$i = \frac{\varepsilon e^{-t/RC}}{R}$

Rate of energy storage $= \frac{\varepsilon^2}{R}\{1 - e^{-t/RC}\}\{e^{-t/RC}\} = \frac{\varepsilon^2}{R}\{e^{-t/RC} - e^{-2t/RC}\}$ .... (1)

It will be maximum when, $e^{-t/RC} - e^{-2tRC}$ will be maximum let $y(t) = e^{-t/RC} - e^{-2t/RC}$

for maximum , $y'(t) = 0$

$y'(t) = \frac{-e^{-t/RC}}{RC} + \frac{2e^{-2t/RC}}{RC}$

$e^{-t/RC} = \frac{1}{2}$

putting it back in eq. (1)

(i) maximum rate of energy storage

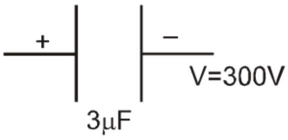
$= \frac{\varepsilon^2}{R}\left\{\frac{1}{2} - \left(\frac{1}{2}\right)^2\right\} = \frac{\varepsilon^2}{4R} = \frac{(20)^2}{4\times10} = 10$ J/s

(ii) This will occur when , $e^{-t/RC} = \frac{1}{2}$

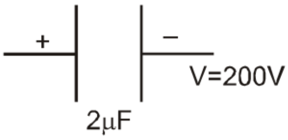
$$\frac{-t}{RC} = \ell n\,\frac{1}{2}$$

t = RC ℓn 2 = 10 × 100 × 10^{-6} × ℓn2 = (ℓn 2) ms

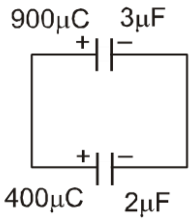
11.



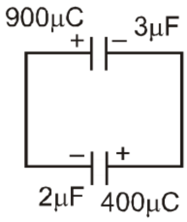
$Q_1 = C_1V_1 = 900\mu C$



$Q_2 = C_2V_2 = 400\mu C$



(i) (D) $V = \dfrac{Q_1+Q_2}{C_1+C_2} = \dfrac{900+400}{3+2}$ = 260V



(i) (A) $V = \dfrac{Q_1+Q_2}{C_1+C_2} = \dfrac{900-400}{3+2}$ = 100V

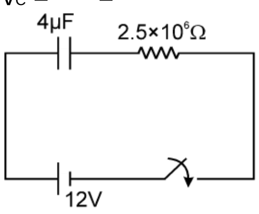
Charge on 3μF = C_1V= 300μC

amount of charge flow is = 900μC – 300μC = 600μC = 6 × 10^{-4} C

12. $v_C + v_R = 12$

$v_C + \dfrac{v_C}{3} = 12 \Rightarrow$ $v_C = 9$ volt

$v_C = \dfrac{q}{C} = \dfrac{CE(1-e^{-t/RC})}{C}$



$v_c = 2v_R$

= E (1 – $e^{-t/RC}$)

9 = 12 (1 – $e^{-t/4\times2.5)}$

9 = 12 – 12$e^{-t/10}$

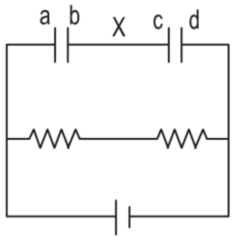
3 = 12$e^{-t/10}$; $e^{-t/10} = \dfrac{1}{4}$

$\dfrac{t}{10} = \ell n4 = 2\ell n2 = 2 \times .693$

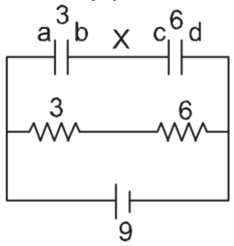
t = 2 × 6.93 = 13.86

13. When switch is opened, the circuit is as following

since the capacitors are in series plates b and c will have equal and opposite charges



$\therefore$ $q_b + q_c = 0$; also charge on each capacitor = 18μC
when switch is closed, the circuit is as following
In steady state, the current in the resistances is 1 amp. Potential difference across 3μF = potential difference across 3Ω = 3 volt.
Similarly p.d. across 6 μF is 6 volt



$\therefore$ charge on plate b = – 9 μC and charge on plate c = + 36 μC
$\therefore$ charge on plates b and c = +36 – 9 = +27 μC.
The change in charges on plates b and c goes through wire from Y to X. and Δq = 27μC

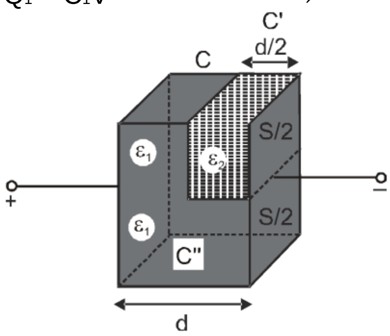
14. When switch S_1 is released charge on C_1 is $2CV_0$ (on upper plate)
When switch S_2 is released charge on C_1 is CV_0 (on upper plate) and charge on C_2 is CV_0 (on upper plate)
When switch S_3 is released charge on C_1 is CV_0 (on upper plate) and charge on C_2 is $-CV_0$ (on upper plate)

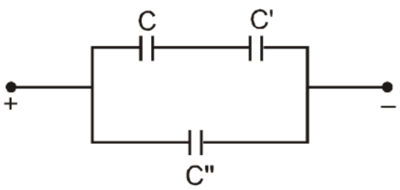
15. $C = \frac{K\varepsilon_0 A}{3d} + \frac{2\varepsilon_0 A}{3d}$ $\Rightarrow$ $C_1 = \frac{K\varepsilon_0 A}{3d}$

$\frac{C}{C_1} = \frac{2+K}{K}$ $\Rightarrow$ $E_1 = E_2 = \frac{V}{d}$ $\Rightarrow$ $\frac{E_1}{E_2} = 1$ Ans. (A)

$Q_1 = C_1V = \frac{K\varepsilon_0 A}{3d}V$ $\Rightarrow$ $Q_2 = C_2V = \frac{2\varepsilon_0 A}{3d}V$ $\Rightarrow$ $\frac{Q_1}{Q_2} = \frac{K}{2}$

16.





$C_1 = \frac{\epsilon_0 A}{d}$; $C = \frac{2\varepsilon_0 \frac{s}{2}}{\frac{d}{2}} = \frac{2\varepsilon_0 s}{d}$

$$C' = \frac{\frac{4\varepsilon_0 \frac{s}{2}}{d}}{\frac{}{2}} = \frac{4\varepsilon_0 s}{d} \quad ; \quad C'' = \frac{2\varepsilon_0 \frac{s}{2}}{d} = \frac{\epsilon_0 s}{d}$$

$$C_2 = \frac{C\,C'}{C + C'} + C'' = \frac{4}{3}\frac{\epsilon_0 s}{d} + \frac{\epsilon_0 s}{d} = \frac{7}{3}\frac{\epsilon_0 s}{d}$$

$$\frac{C_2}{C_1} = \frac{7}{3}$$

17. $E = (1/2)\, CV^2$ .... (i)

The energy stored in capacitor is lost in form of heat energy.

$H = ms\,\Delta T$ .... (ii)

From Eq. (i) and (ii), we have

$ms\Delta T = (1/2)\, CV^2$

$$V = \sqrt{\frac{2ms\Delta T}{C}}$$