OBJECTIVE QUESTIONS

Section (A) : Definition of limit, LHL & RHL, Indeterminate forms

A-1. **Sol.** L.H.L. = $\lim\limits_{x\to 0^-} f(x)$

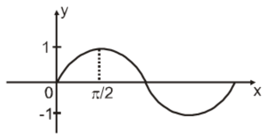
$= \lim\limits_{h\to 0^-} 4(0 - h) = 0$

R.H.L. = $\lim\limits_{h\to 0^-} f(0 + h)$

$= \lim\limits_{h\to 0^-} 3(0 + h)2 = 0$

$\Rightarrow$ RHL = RHL = 0

A- 2. ***Sol.*** $\ell\text{im}_{x\to 2} \left\{\frac{x}{2}\right\}$



R.H.L. = $\ell\text{im}_{h\to 0} \left\{\frac{2+h}{2}\right\} = \ell\text{im}_{h\to 0} \left\{1+\frac{h}{2}\right\} = \ell\text{im}_{h\to 0} \left\{\frac{h}{2}\right\} = \ell\text{im}_{h\to 0} \frac{h}{2} = 0$

L.H.L. = $\ell\text{im}_{h\to 0} \left\{\frac{2-h}{2}\right\} = \ell\text{im}_{h\to 0} \left\{1-\frac{h}{2}\right\} = \ell\text{im}_{h\to 0} \left(1-\frac{h}{2}\right) = 1$

L.H.L. ≠ R.H.L.

so $\ell\text{im}_{x\to 0} \left\{\frac{x}{2}\right\}$ *does not exist.*

A-3. **Sol.** $\lim\limits_{x\to \frac{\pi}{2}} \sin x = 1$

A-4. **Sol.** (1) L.H.L. = R.H.L. = 0

(2) L.H.L. = R.H.L. = 0

(3) L.H.L. = R.H.L. = 0

so all are exist.

A-5. **Sol.** $\ell\text{im}_{x\to \pi}$ sgn [tan x]

L.H.L. = $\ell\text{im}_{x\to \pi^-}$ sgn [tan x]

= $\ell\text{im}_{h\to 0}$ sgn [tan (π – h)]

= $\ell\text{im}_{h\to 0}$ sgn (– ve) = –1

R.H.L. = $\ell\text{im}_{x\to \pi^+}$ sgn [tan x] = $\ell\text{im}_{h\to 0}$ sgn [tan(π + h)]

= $\ell\text{im}_{h\to 0}$ sgn (+ ve) = 0

L.H.L. ≠ R.H.L.

so limit does not exist

A-6. **Sol.** $\lim\limits_{x\to 0^+} \sec x > 1$

So Limit not exist

A-7. **Sol.** L.H.L. = $\lim\limits_{h\to 0^-} \frac{-h}{+h+h^2} = -1$

R.H.L. = $\lim_{h\to 0^-} \frac{h}{h+h^2}$ = 1

A-8. **Sol.** Nr is exactly 0 as x $\to$ 0
so not an indeterminate form

A-9. **Sol.** Indeterminate form of type $\infty - \infty$

A-10. **Sol.** $\ell im_{x\to 1} (1 - x + [x - 1] + [1 - x])$

L.H.L.= $\ell im_{x\to 1^-} (1 - x + [x - 1] + [1 - x])$

$= \ell im_{h\to 0} (1 - (1-h) + [1 - h - 1] + [1 - 1 + h])$

$= \ell im_{h\to 0} (h + [-h] + [h])$

$= 0 - 1 + 0 = -1$

R.H.L. $= \ell im_{x\to 1^+} (1 - x + [x - 1] + [1 - x])$

$= \ell im_{h\to 0} (1 - (1+ h) + [1 + h - 1] + [1 - (1 + h)]$

$= \ell im_{h\to 0} (- h + [h] + [-h])$

$= 0 + 0 - 1 = -1$

L.H.L. = R.H.L. = – 1

so $\ell im_{x\to 1} (1 - x + [x - 1] + [1 - x]) = -1$

A-11. **Sol.** L.H.L. = $(-1)_{[2-h]} = -1$

R.H.L. = $\lim_{h\to 0^+} (-1)^{[2+h]}$ = 1

A-12. ***Sol.*** *f(x)* = $\frac{|x + \pi|}{\sin x}$

$f(-\pi_+) = \ell im_{h\to 0} \frac{|-\pi+h+\pi|}{\sin(-\pi+h)}$

$= \ell im_{h\to 0} \frac{|h|}{-\sin h} = -1$

$f(-\pi_-) = \ell im_{h\to 0} \frac{|-\pi-h+\pi|}{\sin(-\pi-h)}$

$= \ell im_{h\to 0} \frac{|h|}{\sin h} = 1$

$f(-\pi_+) \neq f(-\pi_-)$

so f(x) does not exist

Section (B) : Factorisation, Rationalisation, Use of standard limits, use of substitution

B-1. **Sol.** $\lim_{x\to 1} \frac{(x-1)}{2x(x-1)-5(x-1)}$

$= \lim_{x\to 1} \frac{1}{(2x-5)} = -\frac{1}{3}$

B-2. **Sol.** $\lim_{x\to 0} (x^2 - 9)^{\left(\frac{2x}{x^2-9}\right)}$

$\lim_{x\to 0} 2x = 0$

B-3. **Sol.** $\lim_{x\to 1} \frac{\sqrt{7x+2}-3}{\sqrt{5x-1}-\sqrt{6x-2}}$

$= \lim_{x\to 1} \frac{(7x+2)-9}{\sqrt{7x+2}+3} \cdot \frac{\sqrt{5x-1}+\sqrt{6x-2}}{(5x-1)-(6x-2)}$

$= \lim_{x\to 1} \frac{7(x-1)}{\sqrt{7x+2}+3} \cdot \frac{\sqrt{5x-1}+\sqrt{6x-2}}{(-x+1)}$

$= -7. \left(\frac{2+2}{3+3}\right) = -\frac{14}{3}$

B-4. **Sol.** $\operatorname{Lim}_{x\to a} \frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} = \operatorname{Lim}_{x\to a} \frac{a+2x-3x}{3a+x-4x} . \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$

$\operatorname{Lim}_{x\to a} \frac{(a-x)}{3(a-x)} . \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$

$= \frac{1}{3} . \frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}} = \frac{2}{3\sqrt{3}}$

B-5. **Sol.** $\lim_{x\to 1} \frac{(3x-4)(\sqrt{x}-1)}{2x^2+x-3}$

$= \lim_{x\to 1} \frac{(3x-4)(x-1)}{(2x+3)(x-1)(\sqrt{x}+1)}$

$= \lim_{x\to 1} \frac{(3x-4)}{(2x+3)(x-1)(\sqrt{x}+1)} = \frac{-1}{10}$

B-6. **Sol.** $\lim_{x\to 4} \left(\frac{(\sqrt{x})^3-(2)^3}{(\sqrt{x}-2)(\sqrt{x}+2)}\right) = \lim_{x\to 4} \left(\frac{(\sqrt{x}-2)(x+2\sqrt{x}+4)}{(\sqrt{x}-2)(\sqrt{x}+2)}\right) = \lim_{x\to 4} \left(\frac{x+2\sqrt{x}+4}{\sqrt{x}+2}\right) = 3$

B-7. **Sol.** $\lim_{x\to 0} \frac{\sqrt{2}\left(1-\cos\frac{x}{2}\right)}{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}$

$= \lim_{x\to 0} \frac{2\sqrt{2}\sin^2\left(\frac{x}{4}\right)}{16\sin^2\frac{x}{4}.\cos^2\frac{x}{4}\cos^2\frac{x}{2}} = \frac{\sqrt{2}}{8}$

B-8. **Sol.** $\lim_{x\to a} \frac{(x-b)-(a-b)}{(x^2-a^2)\ (\sqrt{x-b}+\sqrt{a-b})}$

$= \lim_{x\to a} \frac{(x-a)}{(x+a)\ (x-a)\ [\sqrt{x-b}+\sqrt{a-b}]}$

$= \frac{1}{4a\sqrt{a-b}}$

B-9. Sol. Given = $\lim_{x\to 0}\frac{\sin 3x}{3x} \cdot \frac{5x}{\tan 5x}\cdot\frac{3}{5} = \frac{3}{5}$

B-10. Sol. $\lim_{x\to 0}\frac{\tan mx}{\ell n(1-5x)} = \lim_{x\to 0}\frac{\tan mx}{mx}\cdot\frac{5x}{\ell n(1-5x)}\cdot\frac{m}{5} = -\frac{m}{5}$

B-11. Sol. $\lim_{x\to a}\frac{x^{5/3}-a^{5/3}}{x-a}$

$= \frac{5}{3}a^{\frac{5}{3}-1} = \frac{5}{3}a^{\frac{2}{3}}$

B-12. Sol. $\ell im_{x\to 0^+} \frac{\sqrt{x}}{\left(\frac{e^x-1}{x}\right)^{3/2}} \cdot \frac{\tan x}{x\sqrt{x}} = 1$

B-13. Sol. $\lim_{x\to 0}\frac{e^{\alpha x}-e^{\beta x}}{x} = \lim_{x\to 0}\left(\frac{e^{\alpha x}-1}{x}\right) - \left(\frac{e^{\beta x}-1}{x}\right)$

$= \alpha - \beta$

B-14. ***Sol.*** $\ell im_{x\to 0}\frac{(4^x-1)^3}{\sin\left(\frac{x}{p}\right)\ \ell n\left(1+\frac{x^2}{3}\right)} = \ell im_{x\to 0}\frac{x^3\left(\frac{4^x-1}{x}\right)^3}{\left(\frac{x}{p}\right)\left[\frac{\sin\left(\frac{x}{p}\right)}{\frac{x}{p}}\right].\ell n\left(1+\frac{x^2}{3}\right)}$

$= 3p\ \ell im_{x\to 0}\frac{\left(\frac{4^x-1}{x}\right)^3}{\left[\frac{\sin\left(\frac{x}{p}\right)}{\frac{x}{p}}\right]} \cdot \frac{\left(\frac{x^2}{3}\right)}{\ell n\left(1+\frac{x^2}{3}\right)} = 3\,p\,(\ell n\,4)^3$

B-15. Sol. $\lim_{x\to 0}\frac{2^x-1}{(1+x)^{1/2}-1} = \lim_{x\to 0}\frac{2^x-1}{x}\cdot\frac{x}{\sqrt{(1+x)}-1}$

$= \lim_{x\to 0}\frac{2^x-1}{x}\cdot\frac{x}{(1+x)-1}\cdot(\sqrt{1+x}+1)$

$= 2\ell n\,2 = \ell n\,4$

B-16. Sol. $\lim_{x\to 0}\frac{\sin 4x(1+\sqrt{1-x})}{1-1+x}$

$\lim_{x\to 0}\frac{\sin 4x}{4x}.(1+\sqrt{1-x}).4$

= 8

B-17. **Sol.** $\ell im_{x\to 0} \dfrac{1}{(\sqrt{1+x^2})\ \ (1+\sqrt{1-x^2})\ \left(\dfrac{\sin^{-1}x}{x}\right)^3} = \dfrac{1}{1.2.1} = \dfrac{1}{2}$

B-18. **Sol.** $\lim_{x\to 0}\dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x} =$

$\lim_{x\to 0}\dfrac{2\sin x}{x}.\dfrac{1}{\sqrt{1+\sin x}+\sqrt{1-\sin x}} = 2.1.\dfrac{1}{1+1} = 1$

B-19. **Sol.** $\lim_{x\to 0}\dfrac{\alpha\ \left(\dfrac{e^{\alpha x}-1}{\alpha x}\right)-\left(\dfrac{e^{\beta x}-1}{\beta x}\right)\ \beta}{\alpha\ \left(\dfrac{\sin\alpha x}{\alpha x}\right)-\beta\ \left(\dfrac{\sin\beta x}{\beta x}\right)}$

$= \dfrac{\alpha-\beta}{\alpha-\beta} = 1$

B-20. **Sol.** $\ell im_{x\to 0}\dfrac{(\cos 2x-\cos 4x).\cos x.\cos 3x}{(\cos x-\cos 3x).\cos 2x.\cos 4x}$

$= \ell im_{x\to 0}\dfrac{2\sin 3x.\sin x.\cos x.\cos 3x}{2\sin 2x.\sin x.\cos 2x.\cos 4x}$

$= \ell im_{x\to 0}\dfrac{3}{2}.\dfrac{\sin\ \ (3x)}{3x}.\dfrac{2x}{\sin 2x}.\dfrac{\cos x\cos 3x}{\cos 2x.\cos 4x}$

$= \dfrac{3}{2}$

B-21. **Sol.** $\ell im_{x\to 0}\dfrac{\ell n(1+\tan x)}{x} = \ell im_{x\to 0}\dfrac{\ell n(1+\tan x)}{\tan x}.\dfrac{\tan x}{x} = 1\ .1 = 1$

B-22. **Sol.** $\ell im_{x\to 0}\dfrac{\ell n-(2+x)+\ell n-0.5}{x}$

$= \ell im_{x\to 0}\dfrac{\ell n[(2+x)\ \ (0.5)]}{x} = \ell im_{x\to 0}\dfrac{\ell n\left(1+\dfrac{x}{2}\right)}{x} = \ell im_{x\to 0}\dfrac{\ell n\left(1+\dfrac{x}{2}\right)}{\dfrac{x}{2}}.\dfrac{1}{2} = \dfrac{1}{2}$

B-23 **Sol.** $\ell im_{x\to 1}\dfrac{\ell n(1-\ell n x)}{\ell n x^2} = \ell im_{x\to 0}\dfrac{\ell n(1-\ell n x)}{2\ell n x} = -\dfrac{1}{2}$

B-24. **Sol.** $\ell im_{x\to 0}\dfrac{e^{\tan x}-e^x}{\tan x-x} = \ell im_{x\to 0}\dfrac{e^x\left(e^{\tan x-x}-1\right)}{(\tan x-x)} = e_0\ .\ 1 = 1$

B-25. **Sol.** $\ell im_{x\to 2}\dfrac{(x+6)^{1/3}-2}{2-x}$ put x = 2 + h j[kus ij

$$= \lim_{h \to 0} \frac{(h+8)^{1/3} - 2}{-h} = \lim_{h \to 0} \frac{2\left\{\left(1+\frac{h}{8}\right)^{\frac{1}{3}} - 1\right\}}{-h}$$

$$= \lim_{h \to 0} \frac{2\left\{1 + \frac{1}{3} \cdot \frac{h}{8} + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{h}{8}\right)^2 + \ldots - 1\right\}}{-h} = -\frac{1}{12}$$

Section (C) : Infinite limits

C-1. Sol. $\lim_{x \to \infty} \frac{x\left(\sqrt{3 - \frac{1}{x^2}} - \sqrt{2 - \frac{1}{x^2}}\right)}{x(4 + 3/x)} = \frac{\sqrt{3} - \sqrt{2}}{4}$

C-2. Sol. $\lim_{x \to \infty} \frac{x\sqrt{1 - \frac{1}{x^2}}}{x\left(2 + \frac{1}{x}\right)} = \frac{1}{2}$

C-3. Sol. $\lim_{n \to \infty} \frac{n^2(n+1)^2}{4(1-n^4)}$

$= \lim_{n \to \infty} = \frac{n^4\left(1 + \frac{1}{n}\right)^2}{n^4(4)\left(\frac{1}{n^4} - 1\right)} = -\frac{1}{4}$

C-4. Sol. $\lim_{n \to \infty} \frac{1 \quad (1 - 5^n)}{(1-5) \quad (1+5^n) \quad (1-5^n)} = 0$

C-5. Sol. $\lim_{n \to \infty} \frac{(n+2)\ ! \quad (n+4)}{(n+4) \quad (n+3) \quad (n+2)\ !} = 0$

C-6. Sol. $\lim_{n \to \infty} \frac{\sqrt{x}(x + c - x)}{\sqrt{x+c} + \sqrt{x}}$

$= \lim_{n \to \infty} \frac{c\sqrt{x}}{\sqrt{x}(\sqrt{1 + c/x} + 1)} = \frac{c}{2}$

C-7. Sol. $\lim_{n \to \infty} \left(\sqrt{(x+a)\,(x+b)} - x\right)$

$= \lim_{n \to \infty} \frac{(x+a)\,(x+b) - x^2}{\sqrt{(x+a)\,(x+b)} + x}$

$$= \lim_{n\to\infty} \frac{(a+b)x+ab}{x\left[\sqrt{1+\frac{(a+b)}{x}+\frac{ab}{x^2}}+1\right]}$$

$$= \frac{a+b}{1+1} = \frac{a+b}{2}$$

C-8. ***Sol.*** $\lim\limits_{n\to\infty} \frac{5^{n+1}+3^n-2^{2n}}{5^n+2^n+3^{2n+3}}$

$$= \lim_{n\to\infty} \frac{5.5^n+3^n-4^n}{5^n+2^n+27.9^n}$$

$$= \lim_{n\to\infty} \frac{5.\left(\frac{5}{9}\right)^n+\left(\frac{3}{9}\right)^n-\left(\frac{4}{9}\right)^n}{\left(\frac{5}{9}\right)^n+\left(\frac{2}{9}\right)^n+27}$$

$$= \frac{0+0-0}{0+0+27} = 0$$

C-9. **Sol.** $\lim\limits_{x\to-\infty} \frac{x^3\left(\frac{\sin\ (1/x)}{1/x}+\frac{1}{x}\right)}{x^3\left(\frac{1}{x^3}-1\right)}$ $(\because\ |x|^3 = -x^3)$

$= -1$

C-10. **Sol.** $\lim\limits_{x\to\infty} \sqrt{\frac{x-\sin x}{x+\cos^2 x}}$

$$= \lim_{y\to 0} \sqrt{\frac{\frac{1}{y}-\sin\frac{1}{y}}{\frac{1}{y}+\cos^2\frac{1}{y}}}$$

$$= \lim_{y\to 0} \sqrt{\frac{1-y\sin\frac{1}{y}}{1+y\cos^2\frac{1}{y}}} = \sqrt{\frac{1-0}{1+0}} = 1$$

C-11. **Sol.** $\lim\limits_{x\to\infty} \frac{x^{10}\left[\left(1+\frac{1}{x}\right)^{10}+\left(1+\frac{2}{x}\right)^{10}+......+\left(1+\frac{100}{x}\right)^{10}\right]}{x^{10}\left(1+\left(\frac{10}{x}\right)^{10}\right)}$

= 1 + 1 +.......+ 100 terms

= 100

Section (D) : Use of expansion, L-Hospital rule

D-1. **Sol.** $\lim\limits_{x\to 0} \dfrac{2\left(1+\dfrac{x^2}{2\,!}+\dfrac{x^4}{4\,!}+.....\right)-2\left(1-\dfrac{x^2}{2\,!}+\dfrac{x^4}{4\,!}-\dfrac{x^6}{6\,!}+....\right)}{x\left(x-\dfrac{x^3}{3\,!}+.....\right)} = \lim\limits_{x\to 0}$

$\dfrac{x^2\left(2+2\left(\dfrac{1}{4\,!}+\dfrac{1}{6\,!}\right)x^4+....\right)}{x^2\left(1-\dfrac{x^2}{3\,!}+....\right)} = 2$

D-2. **Sol.** $\lim\limits_{x\to 0} \dfrac{1+\left(x-\dfrac{x^3}{3\,!}+\dfrac{x^5}{5\,!}......\right)-\left(1-\dfrac{x^2}{2\,!}+\dfrac{x^4}{4\,!}-.....\right)-\left(x+\dfrac{x^2}{2}+\dfrac{x^3}{3}+....\right)}{x^3}$

$= \lim\limits_{x\to 0} \dfrac{\dfrac{-3x^3}{3\,!}-\left(\dfrac{1}{4\,!}+\dfrac{1}{4!}\right)x^4+.....}{x^3}$

$= -\dfrac{3}{6} = -\dfrac{1}{2}$

D-3. **Sol.** By L.H. Rule

$= \lim\limits_{x\to 1} \dfrac{\dfrac{1}{x}-1}{-2+2x}$

again by L.H. rule

$= \lim\limits_{x\to 1} \dfrac{1}{-2x^2}$

$= -1/2$

D-4. **Sol.** $\lim\limits_{h\to 0} \dfrac{1}{2h}\left[\left(1+\dfrac{h}{8}\right)^{-\frac{1}{3}}-1\right]$

$= \lim\limits_{h\to 0} \dfrac{1}{2h}\left[1-\dfrac{h}{24}+\dfrac{\dfrac{1}{3}\left(\dfrac{1}{3}+1\right)}{2\,!}\left(\dfrac{h}{8}\right)^2......\infty-1\right] = -\dfrac{1}{48}$

D-5. **Sol.** $\lim\limits_{x\to 0} \dfrac{ax\left(1+x+\dfrac{x^2}{2\,!}+.........\right)-b\left(x-\dfrac{x^2}{2}+\dfrac{x^3}{3}-........\right)+cx\left(1-x+\dfrac{x^2}{2!}-\dfrac{x^3}{3!}+.......\right)}{x^2\left(x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}..........\right)} = 2$

$\Rightarrow \lim\limits_{x\to 0}$

$\dfrac{x(a-b+c)+x^2\left(a+\dfrac{b}{2}-c\right)+x^3\left(\dfrac{a}{2}-\dfrac{b}{3}+\dfrac{c}{2}\right)+x^4 \text{ and term containing higher power}}{x^3-\dfrac{x^5}{3!}+\dfrac{x^7}{5!}-..........} = 2$

then $a - b + c = 0$ .....(1)

$2a + b - 2c = 0$ .....(2)

$3a - 2b + 3c = 12$ .....(3)

On solving (1), (2), (3)

we get $a = 3$, $b = 12$, $c = 9$

D-6. **Sol.** $\lim_{x\to\pi/2} \frac{\ell n \sin x}{\cot x}$

By L.H. rule

$= \lim_{x\to\pi/2} - \frac{\cot x}{\operatorname{cosec}^2 x}$

$= 0$

D-7. **Sol.** $\lim_{x\to 1} \frac{\ell n x}{\cos\left(\frac{\pi}{2x}\right)}$

By L.H. rule

$= \lim_{x\to 1} -1 \frac{1}{x.\sin\left(\frac{\pi}{2x}\right).\frac{\pi}{2x^2}}$

$= \lim_{x\to 1} \frac{2x}{\pi \sin\left(\frac{\pi}{2x}\right)} = \frac{2}{\pi}$

D-8. **Sol.** $\lim_{x\to 1} \frac{1+2x+3x^2+.....+nx^{n-1}-0}{1-0}$ By L.H. rule

$= 1 + 2 + ... + n = \frac{n\ (n+1)}{2}$

D-9. **Sol.** $\lim_{x\to a} \frac{a^x - x^a}{x^x - a^a} = -1$

$\Rightarrow \lim_{x\to a} \frac{a^x \ell na - ax^{a-1}}{x^x \ell nex} = -1$ By L.H. rule

$\Rightarrow \frac{a^a \ell na - a.a^{a-1}}{a^a \ell nea} = -1$

$\Rightarrow \ell n(a/e) = -\ell n\ ea$

$\Rightarrow \frac{a}{e} = \frac{1}{ea} \Rightarrow a = 1$

D-10. **Sol.** $\lim_{x\to 0} \frac{(a+x)^2 \sin(a+x) - a^2 \sin\ a}{x}$ $\left(\frac{0}{0}\text{form}\right)\left(\frac{0}{0}\ :\ i\ \right)$

using L' Hospital rule

L' Hospital fu;e ds iz;ksx ls

$= \lim_{x\to 0} \frac{2(a+x)\sin(a+x)\ \ +(a+x)^2\cos(a+x)}{1}$

$= 2a \sin a + a_2 \cos a$

Section (E) : Limits of form ∞ – ∞ , 0º, ∞º , 1∞ , $\ell im_{x\to\infty} \frac{x}{e^x}$, $\ell im_{x\to\infty} \frac{\ell nx}{x}$, Sandwitch theorem

E-1. Sol. $\lim_{x\to1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)$

$= \lim_{x\to1}\frac{1+x+x^2-3}{(1-x^3)} = \lim_{x\to1}\frac{x^2+x-2}{-(x-1)(x^2+x+1)}$

$= \lim_{x\to1}\frac{(x-1)(x+2)}{-(x-1)(x^2+x+1)} = -1$

E-2. Sol. $\lim_{x\to0}\left(\frac{a}{x}-\cot\frac{x}{a}\right) = \lim_{x\to0}\left(\frac{a\tan\frac{x}{a}-x}{x\tan\frac{x}{a}}\right)$

$= \lim_{x\to0}\left(\frac{\sec^2\frac{x}{a}-1}{\tan\frac{x}{a}+\frac{x}{a}\sec^2\frac{x}{a}}\right) = \lim_{x\to0}\frac{2\sec^2\frac{x}{a}\tan\frac{x}{a}}{\frac{2}{a}\sec^2\frac{x}{a}+\frac{2x}{a^2}\sec^2\frac{x}{a}\tan\frac{x}{a}}$

$= \lim_{x\to0}\frac{\tan\frac{x}{a}}{1+\frac{x}{a}\tan\frac{x}{a}} = \frac{0}{1}=0$

E-3. Sol. $\lim_{x\to\infty}\left(\sqrt{x^2-5x+6}-x\right) = \lim_{x\to\infty}\frac{(x^2-5x+6)-x^2}{\sqrt{x^2-5x+6}+x}$

$= \lim_{x\to\infty}\frac{-5x+6}{\sqrt{x^2-5x+6}+x}$

$= \lim_{\frac{1}{x}\to\infty}\frac{-5+\frac{6}{x}}{\sqrt{1-\frac{5}{x}+\frac{6}{x^2}}+1} = \frac{-5+0}{1+1} = -\frac{5}{2}$

E-4. Sol. $\ell im_{x\to\infty}\frac{ax}{\sqrt{a^2x^2+ax+1}+\sqrt{a^2x^2+1}}$

$= \ell im_{x\to\infty}\frac{ax}{x\left[\left(\sqrt{a^2+\frac{a}{x}+\frac{1}{x^2}}\right)+\sqrt{a^2+\frac{1}{x^2}}\right]} = \frac{1}{2}$

E-5. Sol. $\lim_{x\to0}(1+\tan x)^{1/x} = e^{\lim_{x\to0}\frac{\tan x}{x}} = e$

E-6. Sol. $\lim_{x\to0}\left(\frac{1+x}{1-x}\right)^{\frac{1}{x}} = \lim_{x\to0}\frac{(1+x)^{1/x}}{(1-x)^{1/x}} = \frac{e}{e^{-1}} = e^2$

E-7. Sol. $\lim_{x\to\infty}\left(1+\frac{2}{x^2}\right)^{x^2} = e^{\lim_{x\to\infty}\frac{2}{x^2}.x^2} = e^2$

E-8. Sol. $\lim\limits_{x \to 0^+} \left(1 + \tan^2\sqrt{x}\right)^{\frac{5}{x}}$

$= \lim\limits_{h\to 0} \left(1 + \tan^2\sqrt{0+h}\right)^{\frac{5}{0+h}} = \lim\limits_{h\to 0} \left(1 + \tan^2\sqrt{h}\right)^{\frac{5}{h}}$ (1^∞ form :i)

$= e^{\lim\limits_{h\to 0} \frac{5\tan^2\sqrt{h}}{h}} = e^{\lim\limits_{h\to 0} 5\left(\frac{\tan\sqrt{h}}{\sqrt{h}}\right)^2} = e^5$

E-9. Sol. $\lim\limits_{x\to\infty} \left(\frac{x+1}{2x+1}\right)^{x^2} = \lim\limits_{x\to\infty} \left(\frac{1+\frac{1}{x}}{2+\frac{1}{x}}\right)^{x^2} = \left(\frac{1}{2}\right)^{\infty} = 0$

E-10. Sol. $\lim\limits_{x\to 1} (1 + \log_e x)^{1/1-x} = e^{\lim\limits_{x\to 1} \frac{\log_e x}{(1-x)}} = e^{-1} = \frac{1}{e}$

E-11. Sol. α, β are the roots of the equation $ax^2 + bx + c = 0$

∵ $ax^2 + bx + c = a(x-\alpha)\ (x-\beta)$

$= \lim\limits_{x\to\alpha} \left(1 + ax^2 + bx + c\right)^{\frac{1}{x-\alpha}}$

$= e^{\lim\limits_{x\to\alpha} \frac{ax^2+bx+c}{x-\alpha}}$

$= e^{\lim\limits_{x\to\alpha} \frac{a(x-\alpha)\ (x-\beta)}{(x-\alpha)}}$

$= e^{a(\alpha-\beta)}$

E-12. Sol. $\lim\limits_{x\to\infty} \left(\frac{x^2 - 2x + 1}{x^2 - 4x + 2}\right)^{x}$

$= e^{\lim\limits_{x\to\infty} \left(\frac{x^2-2x+1-x^2+4x-2}{x^2-4x+2}\right)x}$

$= e^{\lim\limits_{x\to 0} \left(\frac{(2x-1)x}{x^2-4x+2}\right)}$

$= e^{\lim\limits_{x\to\infty} \left(\frac{2-\frac{1}{x}}{1-\frac{4}{x}+\frac{2}{x^2}}\right)} = e^2$

E-13. Sol. Let $A = \lim\limits_{x\to 0} (\sin x)^x$

$\ell nA = \lim\limits_{x\to 0} x\ell n\sin x$

$= \lim\limits_{x\to 0} \frac{\ell n\sin x}{1/x} = \lim\limits_{x\to 0} \left(\frac{\cot x}{-\frac{1}{x^2}}\right)$

$= \lim\limits_{x\to 0} \left(-\frac{x^2}{\tan x}\right) = 0$

⇒ A = 1

E-14. Sol. $y = \lim\limits_{x\to 0^+} (\text{cosec } x)^{1/\ell nx}$

$\ell ny = \lim_{x\to 0^+} \frac{1}{\ell nx} \cdot \ell n(\text{cosec}(x))$

$= -\lim_{x\to 0^+} \frac{\ell n\ (\sin x)}{\ell nx}$

$= -\lim_{x\to 0^+} \frac{1}{\sin x} \cdot \frac{\cos x}{1/x}$

$= -\lim_{x\to 0^+} \cos x \cdot \frac{1}{\frac{\sin x}{x}}$

$= -1$ $\qquad y = 1/e$

E-15. **Sol.** $\lim_{h\to 0^-} -\frac{\ell nh}{\coth}$

$= \lim_{h\to 0^-} \frac{\ell n\ (1/h)}{1/h} \cdot \left(\frac{\tanh}{h}\right)$

$= 0$

E-18 ***Sol.*** $\lim_{x\to 0^+} \frac{x}{5}\cdot\left[\frac{2}{x}\right] = \lim_{x\to 0^+} \frac{x}{5}\left(\frac{2}{x}-\left\{\frac{2}{x}\right\}\right) = \frac{2}{5} - \lim_{x\to 0^+} \frac{x}{5}\left\{\frac{2}{x}\right\} = \frac{2}{5}$

E-19. **Sol.** $\lim_{n\to\infty} \frac{[1.2x]+[2.3x]+.....+[n.(n+1)x]}{n^3}$

$(1.2)x - 1 < [1.2x] \le (1.\ 2)x$

$(2.3)x - 1 < [2.3x] \le (2.3)x$

$n(n + 1)\ x - 1 < [n\ (n + 1)x] \le n(n + 1)x$

so vr% $(1.2)x + (2.3)x + \ldots n\ (n + 1)x - n$

$< [1.2\ x] + [2.\ 3\ x] + \ldots.\ [n(n + 1)x]$

$\le (1.\ 2)\ x + (2.\ 3)\ x + \ldots.\ n(n + 1)x$

$x\ .\ (\Sigma n_2 + \Sigma n) - n \le [1.\ 2x] + [2.\ 3x] + \ldots\ldots[n(n+1)x] \le x\ (\Sigma n_2 + \Sigma n)$

$\Rightarrow \lim_{n\to\infty} \frac{x.\left[\frac{n(n+1)\ (2n+1)}{6}+\frac{n\ (n\ +1)}{2}\right]-n}{n^3} < \lim_{n\to\infty}\frac{[1.2x]\ +\ [2.3x]+.....+[n(n+1)x]}{n^3} \le \lim_{n\to\infty}\frac{x\left[\frac{n(n+1)(2n+1)}{6}+\frac{n(n\ +1)}{2}\right]}{n^3}$

$\lim_{n\to\infty} x\left[\frac{1.\left(1+\frac{1}{n}\right)\ \left(2+\frac{1}{n}\right)}{6}+\frac{\left(\frac{1}{n}+\frac{1}{n^2}\right)}{2}\right]-\frac{1}{n^2} < \lim_{n\to\infty}\frac{[1.2x]\ +\ [2.3x]+.....+[n(n+1)x]}{n^3} \le \lim_{n\to\infty} x\left[\frac{1.\left(1+\frac{1}{n}\right)\ \left(2+\frac{1}{n}\right)}{6}+\frac{\left(\frac{1}{n}+\frac{1}{n^2}\right)}{2}\right]$

$\frac{x}{3} < \lim_{n\to\infty}\frac{[1.2\ \ x]+[2.3\ \ x]+.....+[n(n+1)x]}{n^3} \le \frac{x}{3}$

so $\ell im_{n \to \infty} \frac{[1.2\ \ x] + [2.3\ \ x] + ... + [n(n+1)x]}{n^3} = \frac{x}{3}$

Section (F) : Continuity at a point

F-1. **Sol.** $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x \cos \frac{1}{x} = 0$

k = 0

F-2. **Sol.** $f(x) = \begin{cases} \frac{x^2 - (a+2)x + a}{x-2} & , \quad x \neq 2 \\ 2 & , \quad x = 2 \end{cases}$

$\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{x(x-2) - a(x-1)}{x-2} = 2$

if function is continuous then a = 0

F-3. **Sol.** $\lim_{x \to 0} f(x) = e^{\frac{1}{x} \log \cos x}$

$= \lim_{x \to 0} e^{\frac{\log(\cos x - 1 + 1)}{(\cos x - 1)} \left(\frac{\cos x - 1}{x} \right)}$

$= e^0 = 1$ so a = 1

F-4. **Sol.** $f(x) = \frac{1 - \cos 7(x-\pi)}{x - \pi}$

$\lim_{x \to \pi} f(x) = \lim_{x \to \pi} \frac{2 \sin^2 \frac{7}{2}(x-\pi)}{(x-\pi)}$

$= \lim_{x \to \pi} 2 \left(\frac{\sin \frac{7}{2}(x-\pi)}{\frac{7}{2}(x-\pi)} \right) . \frac{7}{2} \sin \frac{7}{2} (x - \pi)$

= 0

F-5. For function $f(x) = \begin{cases} \left(1 + \frac{4x}{5}\right)^{1/x} & x \neq 0 \\ e^{4/5}, & x = 0 \end{cases}$, the correct statement is -

(1) $f(0_+)$ and $f(0_-)$ do not exist (2) $f(0_+) \neq f(0_-)$

(3*) f(x) is continuous at x = 0 (4) $\lim_{x \to 0} f(x) \neq f(0)$

Qyu $f(x) \begin{cases} \left(1 + \frac{4x}{5}\right)^{1/x} & x \neq 0 \\ e^{4/5}, & x = 0 \end{cases}$ ds fy, dkSulk dFku lR; gS &

(1) $f(0_+)$ rFkk $f(0_-)$ fo|eku ugha gS (2) $f(0_+) \neq f(0_-)$

(3*) f(x), x = 0 ij larr gS (4) $\lim_{x \to 0} f(x) \neq f(0)$

Sol. $f(x) = \begin{cases} \left(1 + \frac{4x}{5}\right)^{1/x} & , \quad x \neq 0 \\ e^{4/5} & , \quad x = 0 \end{cases}$

$\lim\limits_{x \to 0} f(x) = e^{1/x \ell n\left(1+\frac{4x}{5}\right)}$

$= e^{\frac{\ell n\left(1+\frac{4x}{5}\right)}{\left(x.\frac{4}{5}\right)} \cdot \left(\frac{4}{5}\right)}$

$= e^{4/5}$

hence function is continuous at x = 0

F-6. **Sol.** $f(x) = (\tan x \ \ \cot \alpha)^{\frac{1}{x-\alpha}}$

$\lim\limits_{x \to \alpha} f(x) = e^{\frac{1}{(x-\alpha)} \log(\tan x \ \cot \alpha)}$

$= e^{\frac{\log\,[(\tan x \ \cot \alpha - 1) + 1]}{(\tan x \ \cot \alpha - 1)} \frac{(\tan x \ \cot \alpha - 1)}{(x-\alpha)}}$

$= e^{1 \times \frac{(\sin x \cos \alpha - \cos x \sin \alpha)}{(x-\alpha)\cos x \sin \alpha}}$

$= e^{2 \text{ cosec } 2\alpha}$

F-7. **Sol.** $f(x) = e^{\frac{1}{(x-\alpha)} \log\left(\frac{\sin x}{\sin \alpha}\right)}$

$\lim\limits_{x \to \alpha} f(x) = e^{\frac{1}{(x-\alpha)} \frac{\log\left[\left(\frac{\sin x}{\sin \alpha}-1\right)+1\right]}{\left(\frac{\sin x}{\sin \alpha}-1\right)} \left(\frac{\sin x}{\sin \alpha}-1\right)}$

$= e^{1.\frac{\sin x - \sin \alpha}{(x-\alpha)\sin \alpha}}$

$= e^{\frac{2 \sin\left(\frac{x-\alpha}{2}\right) . \cos\left(\frac{x+\alpha}{2}\right)}{(x-\alpha)\sin \alpha}}$

$= e^{\cot \alpha}$

F-8. **Sol.** $f(x) = \begin{cases} \frac{x^2 - 4x + 3}{x^2 - 1} & , \ x \neq 1 \\ 2 & , \ x = 1 \end{cases}$

$\lim\limits_{x \to 1} \frac{x^2 - 4x + 3}{x^2 - 1} = -1$

f(1) = 2 hence discontinuous

f(–1) is not defined hence discontinuous.

F-9. **Sol.** $f(x) = \begin{cases} ax^2 - b & , \ 0 \le x < 1 \\ 2 & , \ x = 1 \\ x + 1 & , \ 1 < x \le 2 \end{cases}$

at x = 1, L.H.L. = a – b

f(1) = 2

a – b = 2

F-10. **Sol.** f (x) is continous at x = 0 therefore

$\lim\limits_{x \to 0^-} f(x) = f(0) = \lim\limits_{x \to 0^+} f(x)$

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} \left\{\frac{(a+1 \ ) \ \sin \ (a+1)x}{(a+1)x} + \frac{\sin x}{x}\right\} = a + 2$

and $\lim_{x\to0^+} f(x) = \lim_{x\to0^+} \frac{(1+bx)^{\frac{1}{2}}-1}{bx} = \frac{1}{2}$ here b should not be equal to zero.

so a + 2 = c = 1/2 and b ≠ 0

F-11. Sol. $f(0_+) = 0$, $f(0_-) = 1$, f(0) is not defined so discontinuous.

F-12. Sol. f(x) = [x] + [–x]

L.H.L. = $\lim_{x\to m-h} f(x) = m - 1 - m = -1$

R.H.L. = $\lim_{x\to m+h} f(x) = m - m - 1 = -1$

Section (G) : Continuity in an interval, Theorems on continuity, Continuity of composite functions, intermediate mean value theorem

G-1. Sol. f(x) = 3 – |sin x| is continuous everywhere

G-2. Sol. $f(x) = \begin{cases} \frac{\tan x}{\sin x} & , \ x \neq 0 \\ 1 & , \ x = 0 \end{cases}$

$\lim_{x\to0} \frac{\tan x}{\sin x} = 1$, continuous lrr~ gSA

G-3. Sol. f(x) not defined at x = 1

hence discontinuous at x = 1

G-4. Sol. $f[g(x)] = \begin{cases} [g(x)]^2 + 2 & , \ x \geq 2 \\ 1 - g(x) & , \ x < 2 \end{cases}$

$= \begin{cases} (2x)^2 + 2 & ; \ 2x \geq 2 \text{ and } x > 1 \\ (3-x)^2 + 2 & ; \ 3 - x \geq 2 \text{ and } x \leq 1 \\ 1 - 2x & ; \ 2x < 2 \text{ and } x > 1 \\ 1 - (3 - x) & ; \ 3 - x < 2 \text{ and } x \leq 1 \end{cases}$

$= \begin{cases} 4x^2 + 2 & ; \ x \geq 1 \text{ and } x > 1 \\ x^2 - 6x + 11 & ; \ x \leq 1 \text{ and } x \leq 1 \\ 1 - 2x & ; \ x < 1 \text{ and } x > 1 \\ x - 2 & ; \ x > 1 \text{ and } x \leq 1 \end{cases}$

$f[g(x)] = \begin{cases} 4x^2 + 2 & , \ x > 1 \\ x^2 - 6x + 11 & , \ x \leq 1 \end{cases}$

so $\lim_{x\to1} f[g(x)]$

R.H.L. $= \lim_{x\to1^+} (4x^2 + 2) = 6$

L.H.L. $= \lim_{x\to1^-} (x^2 - 6x + 11) = 6$

L.H.L. = R.H.L.

so $\lim_{x\to1} f[g(x)] = 6$

G-5. **Sol.** f(x) = $\begin{cases} \sin\frac{1}{x} & , \; x \neq 0 \\ 0 & , \; x = 0 \end{cases}$

L.H.L. & R.H.L. are not defined

G-6. **Sol.** L.H.L. at x = 2 is 10

continuous at x = 2

discontinuous at x = 0

G-7. **Sol.** $\lim_{x \to 1^-} f(x) = \frac{1}{a}$

$\frac{1}{a} = a$

a = ±1

a = $\frac{2b^2 - 4b}{2}$

$b^2 - 2b = a$

at a = 1, b = $\frac{2 \pm \sqrt{12}}{2}$

at a = –1, b = 1

G-8. **Sol.** Obviously

G-9. **Sol.** $f(x) = 4x^3 + 3x^2 + e^{\cos x} + \log(a^x - 1) + x^{1/3}$

at x = 0, $\log(a^x - 1)$ is not defined

hence f(x) is discontinuous at x = 0

G-10. **Sol.** f(x) = | x – 2 | + $\frac{x^2 - 5x + 6}{x - 1}$ + tanx

continuous in domain of f(x).

G-11. **Sol.** f(x) = $\left| \left(x + \frac{1}{2}\right) [x] \right|$ = $\begin{cases} -(2x+1) & , \; -2 \le x < -1 \\ |x + 1/2| & , \; -1 \le x < 0 \\ 0 & , \; 0 \le x < 1 \\ x + 1/2 & , \; 1 \le x < 2 \\ 5 & , \; x = 2 \end{cases}$

f(x) is discontinous at x = –1, 0, 1, 2

G-12. **Sol.** f(x) = $\frac{1}{1 - x}$ discontinuous at x = 1

f(f(x)) = $\frac{1 - x}{-x}$ discontinuous at x = 0

g(x) = f(f(x)) = x

g(x) is discontinuous at x = 0 & x = 1

G-13. **Sol.** t = $\frac{1}{x - 1} \Rightarrow x \neq 1$

$$y = \frac{1}{\left(\frac{1}{x-1}\right)^2 + \left(\frac{1}{x-1}\right) - 2} = \frac{-(x-1)^2}{2(x-1)^2 - (x-1) - 1} = \frac{-(x-1)^2}{(2(x-1)+1)((x-1)-1)}$$

$\Rightarrow x - 1 \neq \frac{-1}{2}, 1$

$\Rightarrow x \neq \frac{1}{2}, 2$

So discontinuous at $x = \frac{1}{2}, 1, 2$

G-14. Sol. $fog(x) = \frac{\tan^2 x + 1}{\tan^2 x - 1}$

discontinuous when $\tan x = \pm 1$

$\Rightarrow$ $x = n\pi \pm \frac{\pi}{4}$, $n \in I$ and $x = n\pi + \frac{\pi}{2}$, $n \in I$

G-15. Sol. Let $f(x) = 2 \tan x + 5x - 2$

$f(0) = -2$ $\quad f(\pi/4) = 2 \tan \frac{\pi}{4} + \frac{5\pi}{4} - 2 = \frac{5\pi}{4}$

Now $x \in \left[-2, \frac{5\pi}{4}\right]$ and $f(x)$ is continuous on $[0, \pi/4]$

$\therefore$ By intermediate value theorem $c \in [0, \pi/4]$ for which $f(c) = 0$

$\therefore$ (b) is correct.

Section (H) : Derivability at a point, Derivability in intervals, Relation between continuity and differentiability

H-1_. Sol. $f(x) = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

RHD = $\lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{h-0}{h} = 1$

H-2_. Sol. LHD = $\lim_{h\to 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h\to 0} \frac{0-0}{-h} = 0$

H-3_. Sol. $\lim_{x\to 0} \frac{\tan x}{x} = 1$ = f(0) hence f(x) is continuous at x =0

$\lim_{x\to 0} \frac{\tan x}{x} = 1$ = f(0) vr % f(x), x =0 ij lrr~ gS

Now vc LHD = $\lim_{h\to 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h\to 0} \frac{\frac{\tan h}{h} - 1}{-h}$

$= \lim_{h\to 0} \frac{\tan h - h}{-h^2} = 0$

RHD = $\lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{\frac{\tan h}{h} - 1}{h}$

$= \lim\limits_{h\to 0} \dfrac{\tan h - h}{h^2} = 0$

H-4. **Sol.** L.H.D. = $\lim\limits_{x\to 0^-} \dfrac{-h + |-h| - 0}{-h}$ = 0

R.H.D. = $\lim\limits_{x\to 0^+} \dfrac{h + (h) - 0}{h}$ = 2

Hence not differentiable at x = 0

H-5_. **Sol.** $\lim\limits_{x\to 1^+} f(x) = 0$ = $\lim\limits_{x\to 1^-} f(x) = f(1)$ Hence f(x) is continuous at x = 1

LHD = $\lim\limits_{h\to 0} \dfrac{f(1-h) - f(1)}{-h} = \lim\limits_{h\to 0} \dfrac{(1-h-1) - 0}{-h} = 1$

RHD = $\lim\limits_{h\to 0} \dfrac{f(1+h) - f(1)}{h} = \lim\limits_{h\to 0} \dfrac{(1+h)^3 - 1}{h} = \lim\limits_{h\to 0} \dfrac{h(3 + 3h + h^2)}{h}$ =3

LHD ≠ RHD

H-6 **Sol.** f(x) is continuous at x = 0 vr % f(x), x = 0 ij lrr~ gS

$\Rightarrow \lim\limits_{x\to 0^-} f(x) = f(0) = \lim\limits_{x\to 0^+} f(x)$

⇒ e° + 0 = b (0–1)$_2$

⇒ b = 1

f(x) is differentiable at x = 0 vr % f(x), x = 0 ij vodyuh; gS

⇒ LHD = RHD

⇒ $\lim\limits_{x\to 0}$ (e$_x$+a) = $\lim\limits_{x\to 0}$ 2b (x–1)

⇒ 1+a = – 2b

⇒ a = – 3

H-7. **Sol.** f(0$_+$) = $\lim\limits_{h\to 0^-} \sqrt{1 - e^{-h^2}}$ = 0

f(0$_-$) = 0

Hence continuous at x = 0 vr% lrr~ gS x = 0 ij

f'(0$_+$) $\lim\limits_{h\to 0^-}$ = $\dfrac{\sqrt{1-e^{-h^2}}}{h}$ = $\lim\limits_{h\to 0^-} \sqrt{\dfrac{e^{h^2} - 1}{h^2}}$ = $\dfrac{1}{\sqrt{e^{h^2}}}$ 1

f'(0$_-$) $\lim\limits_{h\to 0^-} \dfrac{\sqrt{1-e^{-h^2}}}{-h}$ = $\lim\limits_{h\to 0^-} -\sqrt{\dfrac{e^{h^2} - 1}{h^2}} \dfrac{1}{\sqrt{e^{h^2}}}$ = = –1

Hence not differentiable at x = 0

H-8. **Sol.** $\lim\limits_{x\to 0^-}$ f(x) = 0

$\lim\limits_{x\to 0^+}$ f(x) = 1

Discontinuous at x = 0

at x = 1, L.H.D. = $\lim\limits_{x\to 1^-} \dfrac{1-1}{-h}$ = 0

R.H.D. = $\lim\limits_{x\to 1^+} \left(\dfrac{\frac{1}{1+h} - \frac{1}{1}}{h} \right)$ = –1

H-9. Sol. $f(x) = \begin{cases} \dfrac{x(3e^{1/x}+4)}{2-e^{1/x}}, & x \neq 0 \\ 0 & x = 0 \end{cases}$

LHD (x = 0) = $\lim\limits_{n\to 0}\dfrac{\dfrac{-n(3e^{-1/n}+4)}{2-e^{-1/n}}-0}{-n} = 2$

RHD (x = 0) = $\lim\limits_{n\to 0}\dfrac{n\left(\dfrac{3e^{-1/n}+4}{2-e^{-1/n}}\right)-0}{n} = -3$ not differentiable

$\lim\limits_{x\to 0} f(x) = 0 = \lim\limits_{n\to 0^+} f(x) = f(0)$ continuous

H-10. Sol. f'(0+) = $\ell\text{im}_{h\to 0}\dfrac{h(\sqrt{h}-\sqrt{h+1})-0}{h}$ = – 1

& for x < 0, f(x) is not defined

Hence f(x) is differentiable at x = 0

H-11. Sol. $\lim\limits_{x\to 0}\left(\dfrac{f(1+x)}{f(1)}\right)^{1/x}$ (1^∞ form) (1^∞ :i)

= $e^{\lim\limits_{x\to 0}\frac{1}{x}\left(\frac{f(1+x)-f(1)}{f(1)}\right)}$ = $e^{\frac{f'(1)}{f(1)}}$ = $e^{6/3}$ = e^2 .

H-12. Sol. All are differentiable for x ∈ R

H-13. Sol. Not differentiable at x = 0

H-14. Sol. f(x) = |x – 1| + |x – 2|

Not differentiable at x = 1 & x = 2

H-15. Sol. $f(x) = \begin{cases} x\left(\dfrac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}\right) & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$

L.H.L. = R.H.L. = 0

L.H.D. = $\lim\limits_{x\to 0^-}\dfrac{(-h)\left(\dfrac{e^{-1/h}-e^{1/h}}{e^{-1/h}+e^{-1/h}}\right)-0}{-h} = -1$

R.H.D. = $\lim\limits_{x\to 0^+}\dfrac{h\dfrac{(e^{1/h}-e^{-1/h})}{e^{1/h}+e^{-1/h}}-0}{h} = 1$

not differentiable at x = 0

H-16. Sol. $f(x) = \begin{cases} x^n \sin\dfrac{1}{x} & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$

at x = 0, L.H.L. = $(-h)^n \sin\left(\frac{-1}{h}\right)$

R.H.L. = $(h)^n \sin\left(\frac{1}{h}\right)$

L.H.L. = R.H.L. if n > 0

L.H.D. = $\frac{(-h)^n \sin\left(\frac{-1}{h}\right)-0}{-h} = (-h)^{n-1} \sin\left(\frac{-1}{h}\right)$

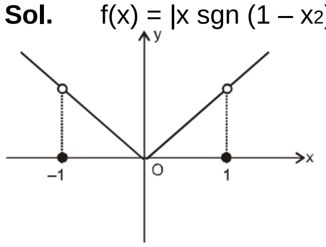
R.H.D. = $h^{n-1} \sin\left(\frac{1}{h}\right)$

L.H.D. = R.H.D. if n – 1 > 0

so f(x) is continuous but not differentiable

for n ∈ (0, 1]

H-17. **Sol.** $f(x) = |x \text{ sgn } (1 - x^2)| = \begin{cases} -x & x \in (-\infty,-1)\cup(-1, \infty) \\ 0 & x = -1, 0, 1 \\ x & x \in (0, 1)\cup(1, \infty) \end{cases}$



function is discontinous at x = –1, 1

and non differentiable at x = –1, 0,1

H-18. **Sol.** Except x = 0

$f'(x) = \begin{bmatrix} 1, & x \ge 0 \\ -1, & x < 0 \end{bmatrix}$

H-19. **Sol.** $f(x) = \cos^{-1}(\cos x)$

$f'(x) = -\frac{1}{\sqrt{1-\cos^2 x}} \quad (-\sin x)$

$= +\frac{1}{|\sin x|}\sin x$

= sgn (sinx)

Section (I) : Theorems in derivability, functional equations

I-1. **Sol.** |f(x)| may not be differentiable at f(x) = 0

$|f|^2$ is differentiable everywhere

I-2. **Sol.** |x – 0.5| is not differentiable at x = 0.5

|x – 1| is not differentiable at x = 1

tanx is not differentiable at $x = \frac{\pi}{2}$

⇒ in (0, 2) f(x) is not differentiable at x = 0.5, 1,

I-3. **Sol.** f(x) = $\sum_{k=0}^{n} a_k |x|^k$

= $a_0 + a_1 |x| + a_2|x|^2 + a_3|x|^3 + ...+ a_n|x|^n$

f(0) = a_0 we know that $\lim_{x\to 0} |x| = 0$

$\lim_{n\to 0} f(0) = a_0$

f(x) is continous for x = 0

$|x|^n$ is differentiable if n ≠ 1, n ∈ N

f(x) is not differentiable at x = 0, due to presence of |x|

If all a_{2k+1} = 0, f(x) does not contains |x|

f(x) is differentiable at x = 0

I-4. **Sol.**

RHD = $\lim_{h\to 0} \frac{\sinh - h - 0}{h}$

= $\lim_{h\to 0} \frac{\sinh}{h} - 1$

= 0

⇒ LHD = RHD

LHD = $\lim_{h\to 0} \frac{\sin(|-h|) - |h| - 0}{-h}$

= $\lim_{h\to 0} -\left(\frac{\sinh}{h} - \frac{h}{h}\right)$

= 0

I-5_. **Sol.** f(x+y) = f(x) +f(y) ⇒ f(x) =kx

as f(1) = 2 ⇒ k =2

⇒ f(x) = 2x

⇒ f'(x) = 2

I-6_. **Sol.** f(x+y) = f(x).f(y) ⇒ f(x) = a^x

Now f(1) =2 ⇒ a =2 ⇒ f(x) = 2^x

$\sum_{r=0}^{9} f(r) =$ $1+2^1+2^2+.....+2^9 = \frac{1(2^{10}-1)}{2-1} = 2^{10}-1 =$ 1023

I-7_. **Sol.** f(x.y) = f(x) +f(y) $\forall x, y \in R^+$ and differentiable vk\$j vodyuh;

⇒ f(x) = $\log_a x$

also f(e) =1 ⇒ a =e

⇒ f(x) = $\log_e x$

⇒ $\lim_{x\to 0} \frac{f(x+1)}{2x} = \lim_{x\to 0} \frac{\ell n(1+x)}{2x} = \frac{1}{2}$

I-8. **Sol.** f(x + y) = f(x).f(y)

f'(0) = $\frac{f(h) - f(0)}{h}$ = 11

f'(3) = $\lim_{x\to 3^+} \frac{f(3+h) - f(3)}{h}$

= $\lim_{h\to 0} \frac{f(3).f(h) - f(3)\ f(0)}{h}$

= $\lim_{h\to 0} f(3) \left(\frac{f(h) - f(0)}{h}\right)$

= f(3).f'(0)

= 3 × 11

= 33

New Typed

I-9_ **Sol.** ∴ $f(x) = x^n + 1$ or
$f(x) = -x^n + 1$
and $f(3) = -80$
∴ $n = 4$ in second equation
$f(x) = -x^4 + 1$