Exercise-1

OBJECTIVE QUESTIONS

Section (A) : Degree and order, Formation of diferential equation

A-1. **Sol.** Order = 2 Degree = 2

A-2. **Sol.** Obvious

A-3. **Sol.** $\left(\frac{d^2y}{dx^2}\right)^2 r_2 = \left[1+\left(\frac{dy}{dx}\right)\right]^3$
order : 2
degree : 2

A-4. **Sol.** $y_2\left(\frac{d^2y}{dx^2}\right)^2 + x_2y_2 - \sin x = -3x\left(\frac{dy}{dx}\right)^{1/3}$

$\left(y^2\left(\frac{d^2y}{dx^2}\right)^2 + x^2y^2 - \sin x\right)^3 = -27x_3\left(\frac{dy}{dx}\right)$

here order = 2 = p
Degree = 6 = q ∴ p < q

A-5. **Sol.** degree is not defined.

A-6. **Sol.** $y = k_1 \sin (x + C_3) - k_2 e_x$ $k_1 : C_1 + C_2 ; k_2 = c_4\, e^{C_5}$
order : 3

A-7. **Sol.** arbitary constant 1 so order is one.

A-8. **Sol.** $y_2 = 4(ae_b)\, e_x = 4ce_x$
arbitary constant 1 so order 1

A-9. **Sol.** $\ell na + \ell ny = be_x + c$

$\ell ny = be_x + c'$

order 2

A-10. **Sol.** $y = \frac{1+\tan ax}{1-\tan ax} \cdot \frac{1-\tan ax}{1+\tan ax} + (ce_d)e_{bx} = 1 + ke_{bx}$
two arbitary constant
∴ order 2

A-11. **Sol.** $y = Ax + A_3 \Rightarrow \frac{dy}{dx} = A$

∴ $y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3$ Degree = 3

A-12. **Sol.** Let equation of St. Line

$Y - y = m(X - x)$

Distance from origin $\Rightarrow \left|\dfrac{mx - y}{\sqrt{1+m^2}}\right| = 1$

$\therefore \quad (mx - y)^2 = 1 + m^2$

$\left(y - \dfrac{dy}{dx}x\right)^2 = 1 + \left(\dfrac{dy}{dx}\right)^2$

A-13. Sol. tangent to $x^2 = 4y$ $\qquad x^2 = 4y$

$x = my + \dfrac{1}{m}$

$m = \dfrac{dy}{dx} \Rightarrow \quad x = y\left(\dfrac{dy}{dx}\right) + \dfrac{1}{(dy/dx)}$

$\Rightarrow \quad x\left(\dfrac{dy}{dx}\right) = y\left(\dfrac{dy}{dx}\right)^2 + 1$

$\therefore$ order = 1 $\qquad$ degree = 2

$\therefore$ = 1 $\qquad$ = 2

A-14. Sol. $(x - 0)^2 + (y - \lambda)^2 = \lambda^2$

$x^2 + y^2 - 2\lambda y = 0$

$2x + 2y\dfrac{dy}{dx} - 2\lambda\dfrac{dy}{dx} = 0$

$\lambda = \dfrac{x + y\dfrac{dy}{dx}}{\dfrac{dy}{dx}}$

so differential equation is $\dfrac{dy}{dx}(x^2 + y^2) = 2y\left(x + y\dfrac{dy}{dx}\right)$

$\dfrac{dy}{dx}(x^2 - y^2) = 2xy$

so $g(x) = 2x$

A-15. Sol. $y\ell n|cx| = x$

$y(\ell n|c| + \ell n|x|) = x$

$yk + y\ell n|x| = x$ .....(i)

$k\dfrac{dy}{dx} + \dfrac{dy}{dx}\ell n|x| + \dfrac{y}{x} = 1$

$k = \dfrac{1 - \dfrac{y}{x} - \ell n|x|\dfrac{dy}{dx}}{\dfrac{dy}{dx}}$ .....(ii)

Now $y.\left(\dfrac{1 - \dfrac{y}{x} - \ell n|x|\dfrac{dy}{dx}}{\dfrac{dy}{dx}}\right) + y\ell n|x| = x$

$y - \dfrac{y^2}{x} - y\ell n|x|\dfrac{dy}{dx} + y\ell n|x|\dfrac{dy}{dx} = x\dfrac{dy}{dx}$

$x\frac{dy}{dx} = y - \frac{y^2}{x}$

$\frac{dy}{dx} = \frac{y}{x} - \frac{y^2}{x^2}$

so blfy, $\phi\left(\frac{x}{y}\right) = -\left(\frac{x}{y}\right)^{-2}$

$\varphi(2) = -(2)^{-2} = -\frac{1}{4}$

Section (B) : Variable separable, Homogeneous equation

B-1. **Sol.** $\frac{dy}{dx} = e^{-2y} \Rightarrow \frac{e^{2y}}{2} = x + c$

$y = 0, x = 5 \Rightarrow c = -\frac{9}{2}$

$y(x_0) = 3$

$\Rightarrow \frac{e^6}{2} = x_0 - \frac{9}{2} \Rightarrow x_0 = \frac{e^6+9}{2}$

B-2. **Sol.** $\varphi(x) = \varphi'(x)$ $\varphi(1) = 2$

$\frac{d\phi}{dx} = \varphi(x)$

$\ell n\, \varphi(x) = x + c$

$\ell n\, 2 = 1 + c \Rightarrow c = \ell n 2 - 1$

$\ell n \varphi(3) = 3 + c = 2 + \ell n 2$

$\Rightarrow \varphi(3) = 2e^2$

B-3. **Sol.** $\frac{dy}{dx} = 1 + x + y + xy = (1 + x)(1 + y)$

$\Rightarrow \int \frac{dy}{1+y} = \int (1+x)\, dx$

$\Rightarrow \ell n(1 + y) = x + \frac{x^2}{2} + c$

$y(-1) = 0 \Rightarrow c = \frac{1}{2}$

$\ell n(1 + y) = x + \frac{x^2}{2} + \frac{1}{2} = \frac{(1+x)^2}{2}$

$\Rightarrow y = e^{\frac{(1+x)^2}{2}} - 1$

B-4. **Ans.** **(2)**

Sol. $(x + 2)\frac{dy}{dx} = (x + 2)^2 - 13$ $\frac{dy}{dx} = (x + 2) - \frac{13}{x+2}$

$y = \frac{x^2}{2} + 2x - 13\, \ell n(x + 2) + C$ at $x = 0, y = 0 \Rightarrow c = 13\, \ell n 2$

$y = \frac{x^2}{2} + 2x - 13\ \ell n|x+2| + 13\ \ell n2$

Now $y(-4) = 8 - 8 - 13\ell n|-4+2| + 13\ell n2 = 0$

B-5. **Sol.** $\frac{dy}{dx} = \frac{x(2\ell n\ x+1)}{\sin y + y\cos y}$

$\int (\sin y + y\cos y)\ dy = \int (2x\ell n\ x + x)\ dx$

using by parts

$y \sin y = x^2\ \ell nx + C$

Sol. $\frac{dy}{dx} = \frac{x(2\ell n\ x+1)}{\sin y + y\cos y}$

$\int (\sin y + y\cos y)\ dy = \int (2x\ell n\ x + x)\ dx$

$y \sin y = x^2\ \ell nx + C$

B-6. **Sol.** $\frac{dy}{dx} - ky = 0$, $\frac{dy}{y} = kdx$

$\ell ny = kx + c$

at $x = 0, y = 1$ ∴ $c = 0$

$x = 0$ ĳ $y = 1$ ∴ $c = 0$

Now $\ell ny = kx$

$y = e^{kx}$

$\lim\limits_{x \to \infty} y = \lim\limits_{x \to \infty} e^{kx} = 0$

∴ $k < 0$

B-7. **Sol.** (1)

$\frac{dy}{dx} + 3y = 2 \Rightarrow \int \frac{dy}{2-3y} = \int dx$

$\Rightarrow \frac{-\ell n(2-3y)}{3} = x + c$

$\Rightarrow \ell n(2-3y) = -3x - c$

$\Rightarrow 2 - 3y = e^{-3x} . e^{-c} \Rightarrow y = \frac{2 - e^{-3x} .\ e^{-c}}{3}$

$\lim\limits_{x\to\infty} y = \lim\limits_{x\to\infty} \frac{2 - e^{-3x} .\ e^{-c}}{3} = \frac{2}{3}$

B-8. **Sol.** $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

$\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{-\ dx}{\sqrt{1-x^2}} \Rightarrow -\sin^{-1} y = \sin^{-1} x + C'$

$\sin^{-1} x + \sin^{-1} y = C$

B-9. **Ans** **(3)**

Sol. $ydy + \sqrt{1+y^2}dx = 0$

$$\frac{y}{\sqrt{1+y^2}}dy + dx = 0$$

$$\int \frac{y}{\sqrt{1+y^2}}dy + \int dx = 0$$

$\Rightarrow \quad \sqrt{1+y^2} + x = c \quad \Rightarrow \quad (c-x)^2 = (1+y^2) \quad \Rightarrow \quad (x-c)^2 - y^2 = 1$

hyperbola

B-10. Sol. $\frac{dy}{dx} = y - y^2 \Rightarrow \int \frac{dy}{y-y^2} = \int dx$

$\int \frac{1}{1-y} + \frac{1}{y}\, dy = x + c \Rightarrow \ell n \frac{y}{1-y} = x + c$

$\frac{y}{1-y} = ke^x \Rightarrow y = ke^x - kye^x \Rightarrow y = \frac{ke^x}{1+ke^x}$

$x = 0,\ y = 2;\ 2 = \frac{k}{1+k} \Rightarrow 2 + 2k = k$

$\Rightarrow k = -2,\ y = \frac{-2e^x}{1-2e^x} \Rightarrow y = \frac{-2}{e^{-x}-2}$

$\lim_{x\to\infty} (y(x)) = \lim_{x\to\infty} \frac{-2}{e^{-x}-2} = 1$

B-11. Sol. $\int \frac{dx}{x} = \int \frac{y}{1+y^2}dy$

$\ell n x = \frac{1}{2}\ell n(1+y^2) + c$

$\ell n \left(\frac{x^2}{1+y^2}\right) = k_1$

$\Rightarrow x^2 = k_2(1+y^2)$

at (1, 0) $\quad k_2 = 1 \quad$ (1, 0) ij $\quad k_2 = 1$

$\Rightarrow x^2 = 1 + y^2$

B-12. Sol. $x\,dy = y\,dx$

$\frac{dy}{y} = \frac{dx}{x} \Rightarrow \ell n\, y - \ell n\, x = c$

$y = kx$ ∴ straight line passing through origin

B-13. Sol. (4)

$\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right) \Rightarrow \frac{dy}{dx} = -2\cos\frac{x}{2}\sin\frac{y}{2}$

$\Rightarrow \int \text{cosec}\frac{y}{2}dy = \int -2\cos\frac{x}{2}dx \quad \Rightarrow 2\,\ell n\left(\tan\frac{y}{4}\right) = -4\sin\frac{x}{2} + c'$

$\Rightarrow \ell n\left(\tan\frac{y}{4}\right) + 2\sin\frac{x}{2} = c$

B-14. Sol. $\frac{dy}{dx} = \sin(x + y) + \cos(x + y)$

$x + y = u$

$1 + \frac{dy}{dx} = \frac{du}{dx}$

$\frac{du}{dx} - 1 = \sin u + \cos u$

$\int \frac{du}{1+\sin u+\cos u} = \int dx$

$\Rightarrow \ell n \left| \tan\left(\frac{x+y}{2}\right) + 1 \right| = x + c$

B-15. Sol. $\frac{dy}{dx} + e^{x-y} + e^{y-x} = 1$ put $y - x = t \Rightarrow \frac{dy}{dx} - 1 = \frac{dt}{dx}$

$\Rightarrow 1 + \frac{dt}{dx} + e^{-t} + e^{t} = 1$

$\Rightarrow \frac{e^t}{1+e^{2t}} dt + dx = 0$ $\Rightarrow \tan^{-1}(e^t) + x = c$ $\Rightarrow \tan^{-1}\left(\frac{e^y}{e^x}\right) + x = c$

B-16. Sol. $3y + 2x = v$ $\therefore 3\frac{dy}{dx} + 2 = \frac{dv}{dx}$

$\left(\frac{dv}{dx} - 2\right) = \frac{2v+5}{v+4}$

$\frac{dv}{dx} = \frac{6v+15}{v+4} + 2 = \frac{8v+23}{v+4}$

$\frac{8v+32}{8v+23} dv = 8dx$

$\int\left(1+\frac{9}{8v+23}\right) dv = \int\left(1+\frac{9}{8v+23}\right)$

$v + \frac{9}{8} \ell n(8v + 23) = 8x + c$

$y - 2x + \frac{3}{8} \ell n(24y + 16x + 23) = k$

B-17. Sol. $\frac{y}{x}$ is a term with zero degree

B-18. Sol. $\frac{dy}{dx} = \frac{y^2-2xy-x^2}{y^2+2xy-x^2}$

put $y = tx$

$\frac{dy}{dx} = t + x \frac{dt}{dx}$

$t + \frac{xdt}{dx} = \frac{t^2-2t-1}{t^2+2t-1}$

$$x\frac{dt}{dx} = \frac{t^2 - 2t - 1 - t^3 - 2t^2 + t}{t^2 + 2t - 1} \Rightarrow \int \frac{t^2 + 2t - 1}{-t^3 - t^2 - t - 1}\,dt = \int \frac{dx}{x}$$

$$\int \frac{t^2 + 2t - 1}{(t+1)\ (t^2+1)}\,dt = -\ell nx + c = \int\left(\frac{2t}{t^2+1} - \frac{1}{t+1}\right)dt = -\ell nx + c$$

$$\ell n\frac{t^2+1}{t+1} = c - \ell nx$$

$$\frac{y^2 + x^2}{y + x} = \frac{1}{k}$$

$x + y = k(x^2 + y^2)$

$k = 0$

$\therefore \quad x + y = 0$

B-19. **Sol.** $y' = \frac{x^2 + y^2}{x^2 - y^2}$

$y'_{(1,2)} = \frac{1+4}{1-4} = \frac{-5}{3}$

B-20. **Sol.** $\frac{dy}{dx} = \frac{\cos\frac{y}{x} + \frac{y}{x}\sin\frac{y}{x}}{\sin\frac{y}{x} - \frac{x}{y}\cos\frac{y}{x}}$ put $y = tx$ $\quad y = tx$

$$t + x\frac{dt}{dx} = \frac{\cos t + t\sin t}{\sin t - \frac{\cos t}{t}} \Rightarrow \int \frac{t\sin t - \cos t}{2t\cos t}\,dt = \int \frac{dx}{x}$$

$$-\frac{1}{2}\ \ell n(t\cos t) = \ell nx + c$$

$$-\frac{1}{2}\ \ell n\left(\frac{y}{x}\cos\frac{y}{x}\right) = \ell n\, x + c$$

Section (C) : Linear differential equation, Bernoulli's equation

C-1. **Sol.** $\frac{dv}{dt} + \frac{k}{m}v = -g$

Integrating factor (I.F.) = $e^{\int \frac{k}{m}dt} = e^{\frac{k}{m}t}$

$\therefore \quad v\, e^{\frac{k}{m}t} = -\int g\, e^{k \cdot t/m}\, dt$

$v\, e^{\frac{k}{m}t} = \frac{-gm}{k} e^{\frac{k}{m}t} + c$

$v = c \cdot e^{\frac{-k}{m}t} - \frac{mg}{k}$

C-2. **Sol.** $\frac{dy}{dx} = y\tan x - 2\sin x$

$\frac{dy}{dx} - y\tan x = -2\sin x$

I.F. = $e^{-\int \tan x\, dx} = |\cos x|$

$y\cos x = \frac{\cos 2x}{2} + k$

$\Rightarrow \quad y = \frac{\cos 2x}{2\cos x} + k\sec x \quad \Rightarrow \quad y = \cos x + c\sec x$

C-3. **Sol.** $(1 + x^2)\frac{dy}{dx} + 2xy = \cos x$

$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{\cos x}{1+x^2}$

I.F. = $1 + x^2$

$\frac{d}{dx}(y(1 + x^2)) = \cos x$

$(1 + x^2)y = \sin x + c$

C-4. **Sol.** $\frac{dy}{dx} = \frac{y}{x+3y^2}$

$\frac{dx}{dy} - \frac{x}{y} = 3y$

I.F. = $e^{-\int \frac{1}{y}dy} = \frac{1}{y}$

$\frac{x}{y} = \int 3\frac{y}{y}\,dy$

$\Rightarrow \frac{x}{y} = 3y + c$

C-5. **Ans.** **(3)**

Sol. $y\,dx = (x + y^2)\,dy$

$\frac{dx}{dy} - \frac{x}{y} = y$

I.F. = $e^{\int -\frac{1}{y}dy} = e^{-\ell n\, y} = \frac{1}{y}$

so $x\left(\frac{1}{y}\right) = \int y\left(\frac{1}{y}\right)dy$

$\Rightarrow \quad \frac{x}{y} = y + c$

Put $y = 1, x = 1 \quad \Rightarrow \quad c = 0$

$\Rightarrow \quad x = y^2$

Now at $y = 4$, $x = 4^2 = 16$

C-6. **Sol.** $\frac{dy}{dx} = \frac{-(1+y+x^2y)}{x+x^3}$

$\frac{dy}{dx} + \frac{y(1+x^2)}{x(1+x^2)} = \frac{-1}{x+x^3}$

$\frac{dy}{dx} + y/x = \frac{-1}{x(1+x^2)}$

I.F. $= x$

$\frac{d}{dx}(yx) = \frac{-1}{1+x^2}$ $\Rightarrow$ $yx = -\tan^{-1}x + c$

C-7. **Sol.** $\frac{dy}{dx} = 2\cos x - y\cot x$

$\frac{dy}{dx} + y\cot x = 2\cos x$

IF $= e^{\int \cot x dx} = e^{\ell n \sin x} = \sin x$

complete solution

$y.\sin x = \int \sin x.2\cos x\, dx$

$y\sin x = \sin^2 x + C$

$y = \sin x + C\,\text{cosec}x$

C-8. **Sol.** $\frac{dy}{dx} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}$

$\frac{dy}{dx} + \frac{1}{\sqrt{x}}y = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

I.F. $= e^{\int \frac{1}{\sqrt{x}}dx} = e^{2\sqrt{x}}$

complete solution

$y.\ e^{2\sqrt{x}} = \int \frac{1}{\sqrt{x}}dx$

$y e^{2\sqrt{x}} = 2\sqrt{x} + C$

C-9. **Sol.** $\frac{dy}{dx} + y\tan x = \sin 2x$

IF $= e^{\int \tan x dx} = e^{\ell n|\sec x|} = |\sec x|$

$\therefore$ solution is

$y(\sec x) = \int \sec x.\sin 2x dx$

$y\sec x = \int 2\sin x dx$

$y\sec x = -2\cos x + c$

at $x = 0, y = 1$

$1 = -2 + c$

$c = 3$

so, $y\sec x = -2\cos x + 3$

$y = -2\cos^2 x + 3\cos x$

at $x = \pi$

$y = -2 - 3 = -5$

C-10. Sol. $\sin 2x\left(\frac{dy}{dx}-\sqrt{\tan x}\right)-y=0$

$\frac{dy}{dx}-(\text{cosec}2x)\,y=\sqrt{\tan x}$

$\text{IF}=e^{\int -\text{cosec}2x\,dx}=e^{-\frac{1}{2}\ell n\tan x}=\sqrt{\cot x}$

so solution is

$y\sqrt{\cot x}=\int 1dx$

$y\sqrt{\cot x}=x+C$

C-11. Sol. $x\frac{dy}{dx}+y=x^2y^4$

$\Rightarrow \frac{1}{y^4}\frac{dy}{dx}+\frac{y}{x}\cdot\frac{1}{y^4}=\frac{x^2}{x}$

$\Rightarrow \frac{1}{y^4}\frac{dy}{dx}+\frac{1}{xy^3}=x$ $\qquad \frac{1}{y^3}=t\Rightarrow \frac{-3}{y^4}\frac{dy}{dx}=\frac{dt}{dx}$

$\Rightarrow -\frac{1}{3}\frac{dt}{dx}+\frac{t}{x}=x$

$\Rightarrow \frac{dt}{dx}-\frac{3t}{x}+3x=0$

$\text{I.F.}=e^{-\int\frac{3}{x}dx}=e^{-\ell nx^3}=\frac{1}{x^3}$

$\Rightarrow -\frac{1}{3x^3}\frac{dt}{dx}+\frac{t}{x^4}=\frac{1}{x^2}$

$\Rightarrow d\left(\frac{-t}{3x^3}\right)=\frac{1}{x^2}\,dx$

$\Rightarrow -\frac{t}{3x^3}=-\frac{1}{x}+c$

$\Rightarrow t=3x^2-3cx^3$ $\qquad (\because\ t=\frac{1}{y^3})$

$\Rightarrow \frac{1}{y^3}=3x^2+kx^3$

C-12. Ans. (4)

Sol. $2y\frac{dy}{dx}+y^2\sec x=\tan x$

put $y^2=t \quad\Rightarrow\quad 2y\frac{dy}{dx}=\frac{dt}{dx}$

$\frac{dt}{dx}+t\sec x=\tan x$

$\text{I.F.}=e^{\int\sec x\,dx}=e^{\ell n(\sec x+\tan x)}=\sec x+\tan x$

$t.(\sec x+\tan x)=\int(\sec x+\tan x)\tan x\,dx$

$=\int\sec x\tan x\,dx+\int\tan^2 x\,dx$

$y^2(\sec x+\tan x)=\sec x+\tan x-x+c$

$y(0)=1 \quad\Rightarrow\quad c=0$

$\Rightarrow \quad y^2=1-\frac{x}{\sec x+\tan x}$

C-13. **Sol.** $2\,\dfrac{dy}{dx} = \dfrac{y^2 - x}{xy + y}$

$\Rightarrow 2y\,\dfrac{dy}{dx} = \dfrac{y^2}{x+1} - \dfrac{x}{x+1}$

$\Rightarrow \dfrac{dt}{dx} = \dfrac{t}{x+1} - \dfrac{x}{x+1}$ put $y_2 = t \Rightarrow 2y\,\dfrac{dy}{dx} = \dfrac{dt}{dx}$ $y_2 = t$ $2y\,\dfrac{dy}{dx} = \dfrac{dt}{dx}$

I.F. = $e^{-\int \frac{1}{x+1}dx} = e^{\ell n \frac{1}{1+x}} = \dfrac{1}{1+x}$ (I.F.) = $e^{-\int \frac{1}{x+1}dx} = e^{\ell n \frac{1}{1+x}} = \dfrac{1}{1+x}$

$\Rightarrow \dfrac{1}{(1+x)}\dfrac{dt}{dx} - \dfrac{t}{(1+x)^2} = \dfrac{-\,x}{(1+x)^2}$

$\Rightarrow \int d\left(\dfrac{t}{(1+x)}\right) = \int \dfrac{-x}{(1+x)^2}dx$

$\Rightarrow \dfrac{t}{1+x} = -\int\left(\dfrac{1}{1+x} - \dfrac{1}{(1+x)^2}\right) dx$

$\Rightarrow \dfrac{t}{1+x} = -\ell n(1+x) - \dfrac{1}{1+x} + c$

$\Rightarrow y_2 + (1+x)\,\ell n(1+x) + 1 = c(1+x)$

Section (D) : Exact differential equation, Geometrical and physical applications

D-1. **Sol.** $(y + 3x_2y_2\,e^{x^3})dx = x\,dy$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{y + 3x^2y^2e^{x^3}}{x}$

$\Rightarrow \dfrac{dy}{dx} - \dfrac{y}{x} = y_2(3x\,e^{x^3})$

$\Rightarrow \dfrac{1}{y^2}\dfrac{dy}{dx} - \dfrac{1}{xy} = 3x\,e^{x^3}$

put $-\dfrac{1}{y} = t \Rightarrow \dfrac{dt}{dx} + \dfrac{t}{x} = 3x\,e^{x^3}$ I.F. = $e^{\int \frac{dx}{x}} = x$

$-\dfrac{1}{y} = t$ $\dfrac{dt}{dx} + \dfrac{t}{x} = 3x\,e^{x^3}$ (I.F.) = $e^{\int \frac{dx}{x}} = x$

$\dfrac{d}{dx}(t\,x) = 3x_2\,e^{x^3} \Rightarrow tx = \int 3x^2e^{x^3}dx \Rightarrow \dfrac{-x}{y} = e^{x^3} + c$

D-2. **Sol.** $y(x_2y + e_x)\,dx = e_x\,dy$

$x_2y_2\,dx + e_xy\,dx = e_xdy$

$-x_2dx = \dfrac{e^xy\ \ dx - e^xdy}{y^2}$

$\Rightarrow -\dfrac{x^3}{3} = d\left(\dfrac{e^x}{y}\right) \Rightarrow -\dfrac{x^3}{3} = \dfrac{e^x}{y} + c$

D-3. **Sol.** $2y\sin x\,dy + (y_2\cos x + 2x)\,dx = 0$

$\Rightarrow$ $d(y_2\sin x) + 2x\,dx = 0$

$\Rightarrow \quad y^2 \sin x = -x^2 + c$

D-4. **Sol.** $(2x - y + 1)dx + (2y - x - 1)dy = 0$

$(2x + 1)dx + (2y - 1)dy - (ydx + xdy) = 0$

integrating it

$x^2 + x + y^2 - y - xy = c$

D-5. **Sol.** $(2y\,dx + 2x\,dy) + 3y\,dy - 5\,dy + 3x\,dx - 5\,dx = 0$

$\Rightarrow 2xy + \frac{3y^2}{2} - 5y + \frac{3x^2}{2} - 5x + c' = 0$

D-6. **Sol.** $(x^2y^2 - 1)\,dy + 2xy^3\,dx = 0$

$\Rightarrow x^2y^2\,dy + 2xy^3\,dx = dy$

$\Rightarrow x^2dy + 2xy\,dx = \frac{dy}{y^2}$

$\Rightarrow \int d(x^2y) = \int \frac{dy}{y^2} + c$

$\Rightarrow x^2y = + c$

$\Rightarrow x^2y^2 = \frac{y^{-1}}{-1} - 1 + cy$

$\Rightarrow 1 + x^2y^2 = cy$

D-7. **Ans.** **(2)**

Sol. $\frac{ydx - xdy}{y^2} = 2dy \Rightarrow d\left(\frac{x}{y}\right) = 2dy$

$\frac{x}{y} = 2y + c \quad \Rightarrow c = 1 \quad \Rightarrow \frac{x}{y} = 2y + 1$

put y = 1

f(1) = 3

D-8. **Ans.** **(2)**

Sol. $\frac{d(xy)}{d\left(\frac{x}{y}\right)} = \frac{x^2e^{xy}}{y^2}$

$-\int e^{-xy}d(-xy) = \int \left(\frac{x}{y}\right)^2 d\left(\frac{x}{y}\right)$

$-e^{-xy} = \frac{1}{3}\left(\frac{x}{y}\right)^3 + C$

given y(0) = 1

$-1 = 0 + C \Rightarrow C = -1$

$-e^{-xy} = \frac{x^3}{3y^3} - 1$

$3y^3(1 - e^{-xy}) = x^3$

D-9. **Sol.** $(x^3\cos y \sin^2 y - 2y \sin x)\,dy - (y^2 \cos x - x^2 \sin^3 y)\,dx = 0$

$\left(\frac{x^3}{3} \quad d \quad \sin^3 y - \sin x \quad dy^2\right) + \sin^3 y\,d\left(\frac{x^3}{3}\right) - y^2 d \sin x = 0$

$\frac{x^3}{3}\,d \sin^3 y + \sin^3 y\,d\left(\frac{x^3}{3}\right) - (\sin x\,dy^2 + y^2\,d \sin x)$

$d\left(\frac{x^3}{3} \quad \sin^3 y\right) - d(y^2 \sin x) = 0$

$\frac{x^3}{3} \sin^3 y - y^2 \sin x = c$

D-10. **Sol.** $\frac{dy}{dx} = \frac{1}{2y} \Rightarrow y^2 = x + c$

$\because$ (4, 3) satisfies

$\Rightarrow 9 = 4 + c \Rightarrow c = 5$

$\therefore\ y^2 = x + 5$

D-11. **Sol.** $L_{SN} = y\frac{dy}{dx}$

$y\frac{dy}{dx} = a$

$\Rightarrow \frac{y^2}{2} = ax + C_2 \qquad \Rightarrow \qquad y^2 = 2ax + b$

D-12. **Sol.** $\frac{dy}{dx} = \frac{ax+3}{2y+1}$

i.e. $(2y + 1)\, dy = (ax + 3)\, dx$

$\therefore \quad y^2 + y = \frac{ax^2}{2} + 3x + c$

$\therefore \quad a = -2$

D-13. **Sol.** $\frac{|y-mx|}{\sqrt{1+m^2}} = \frac{|my+x|}{\sqrt{1+m^2}}$

$|y - mx| = |my + x| \quad \Rightarrow y - mx = \pm(my + x)$ ...(i)

by taking positive sign

$y - mx = my + x$

$y - x = m(x + y)$

$\Rightarrow m = \frac{dy}{dx} = \frac{y-x}{y+x}$ let y = tx “y = tx

$t + x\frac{dt}{dx} = \frac{t-1}{t+1}$

$-\int\frac{t\ dt}{1+t^2} - \int\frac{dt}{1+t^2} = \ell n\, x + c$

$-\frac{1}{2}\ell n(1 + t^2) - \tan^{-1} t = \ell n\, x + c$

$\ell n\sqrt{x^2+y^2} = -\tan^{-1}\frac{y}{x} + c$

$\Rightarrow \sqrt{x^2+y^2} = c_1 e^{-\tan^{-1}\left(\frac{y}{x}\right)}$

from equ. (i) by taking negative sign

$$\frac{dy}{dx} = \frac{\frac{y}{x}+1}{\frac{-y}{x}+1}$$ put $y = tx$ $y = tx$

$\Rightarrow t + x\frac{dt}{dx} = \frac{t+1}{-t+1}$ $\Rightarrow \frac{1-t}{1+t^2}dt = \frac{dx}{x}$ $\Rightarrow \tan^{-1} t - \frac{1}{2}\ell n(1 + t^2) = \ell nx + c$

$\ell n\sqrt{x^2+y^2} = +\tan^{-1}\frac{y}{x} + c$ $\Rightarrow \sqrt{x^2+y^2} = c_2 e^{\tan^{-1}\left(\frac{y}{x}\right)}$

then final solution $\sqrt{x^2+y^2} = c\, e^{\tan^{-1}\left(\frac{y}{x}\right)}$

D-14. **Sol.** $Y - y = m(X - x)$

$X_{int} = \frac{-y}{m} + x = ay$

$\frac{-y}{m} = ay - x$ $\Rightarrow$ $m = \frac{y}{x-ay} = \frac{dy}{dx}$

$\frac{dx}{dy} = \frac{x-ay}{y}$ $\Rightarrow$ $\frac{dx}{dy} - \frac{x}{y} = -a$

Exercise-2

PART - I : OBJECTIVE QUESTIONS

1. **Sol.** $\left(\frac{dy}{dx}\right)^{\frac{5}{2}} = 5x\left(\frac{d^2y}{dx^2}\right)^2 - 7y$

$\left(\frac{dy}{dx}\right)^5 = \left(5x\left(\frac{d^2y}{dx^2}\right)^2 - 7y\right)^2$

so degree is 4

2. **Sol.** $(x - \alpha)^2 + (y - 0)^2 = \alpha^2$

$x^2 + y^2 - 2\alpha x = 0$ ..... (i)

$2x + 2y\frac{dy}{dx} - 2\alpha = 0$

$\alpha = x + y\frac{dy}{dx}$ .... (ii)

equation of differential equation

$x^2 + y^2 = 2x\left(x + y\frac{dy}{dx}\right)$

order = 1

degree = 1

3. **Sol.** $y^2 = 4x$

equation of normal to parabola is

$y = mx - 2m - m^3$

$m = \frac{dy}{dx}$

$y = \frac{dy}{dx}x - 2\frac{dy}{dx} - \left(\frac{dy}{dx}\right)^3$

order = 1

degree = 3

4. **Sol.** $\frac{dy}{dx} + 2\frac{y}{x} = 0$

$x^2 y = C$ put x = 1, y = 1 and we get C = 1

put x = 2 $\Rightarrow$ $y = \frac{1}{4}$

5. **Sol.** $\frac{dy}{1+y^2} + \frac{dx}{\sqrt{1-x^2}} = 0$

$\Rightarrow \tan^{-1} y + \sin^{-1} x = c$

6. **Sol.** $\frac{ydx - xdy}{y^2} = dx + \frac{dy}{y^2}$ $\Rightarrow$ $d\left(\frac{x}{y}\right) = dx + \frac{dy}{y^2}$ $\Rightarrow$ $\frac{x}{y} = x - \frac{1}{y} + k$

$\Rightarrow$ $x = xy - 1 + ky$ $\Rightarrow$ $(x + 1)(1 - y) = cy$

7. **Sol.** $(x^2 + y^2)\, dy = xy\, dx$

$\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$ put y = vx $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{v}{1+v^2}$ $\Rightarrow \int \frac{1+v^2}{v^3} dv = -\int \frac{dx}{x}$

$\Rightarrow$ $\ell n\, v - \frac{1}{2v^2} = -\ell n\, x + c$

$\Rightarrow$ put x = 1 and y = 1

$\therefore$ $c = -\frac{1}{2}$

$\therefore$ $\ell n \frac{y}{x} - \frac{1}{2}\frac{x^2}{y^2} = -\ell n\, x - \frac{1}{2}$ put y = e

$\therefore$ $x = \sqrt{3}\, e$

8. **Sol.** $\frac{dx}{dy} = x + y + 1$

$\Rightarrow -x - y - 1 = 0$ I.F. $= e^{-\int dy} = e^{-y}$

$\Rightarrow e^{-y}\frac{dx}{dy} - xe^{-y} - ye^{-y} - e^{-y} = 0$

$\Rightarrow \int d\,(xe^{-y}) = \int (e^{-y} + ye^{-y})\, dy$

$\Rightarrow xe^{-y} = -e^{-y} - ye^{-y} + \int e^{-y}\, dy$

$\Rightarrow xe^{-y} = -e^{-y} - ye^{-y} - e^{-y} + C$

$\Rightarrow x = -1 - y - 1 + Ce^{y}$

$\Rightarrow x + y + 2 = Ce^{y}$

9. **Sol.** $\frac{dy}{dt} - \frac{t}{t+1} y = \frac{1}{t+1}$

I.F = $e^{-\int \frac{t+1-1}{t+1} dt} = e^{-t + \ell n(t+1)} = (t+1) e^{-t}$

solution is $(t + 1)e^{-t}\ y = -e^{-t} + c$ put t = 0 , and y = – 1 ⇒ c = 0

∴ $2e^{-1}\ y = -e^{-1}$ put t = 1

$y = -\frac{1}{2}$

10. **Sol.** (1) $\frac{dy_1}{dx} + f(x)\, y_1 = 0$ ⇒ $f(x) = \frac{-1}{y_1}\frac{dy_1}{dx}$

(2) $\frac{dy}{dx} - \frac{1}{y_1}\frac{dy_1}{dx} . y = r(x)$

$e^{-\int \frac{1}{y_1}\frac{dy_1}{dx} dx} = e^{-\int \frac{dy_1}{y_1}} = \frac{1}{y_1}$

$\frac{d}{dx}\left(\frac{y}{y_1}\right) = \frac{r(x)}{y_1}$ ⇒ $\frac{y}{y_1} = \int \frac{r(x)dx}{y_1} + c$

$y = y_1 \int \frac{r(x)\ dx}{y_1} + cy_1$

11. **Sol.** $\frac{dy_1}{dx} + fy_1 = r$

$\frac{dy_2}{dx} + fy_2 = r$

Add $\frac{d}{dx}(y_1 + y_2) + f(y_1 + y_2) = 2r$

here $\frac{dy}{dx} + f(x)y = 2r$

12. **Sol.** Given DE can be written as $\frac{dy}{dx} - \left(1 + \frac{f'(x)}{f(x)}\right) y = f(x)$

Which is L.D.E.

I.F. = $e^{-x-\ell n\, f(x)} = \frac{e^{-x}}{f(x)}$

General solution $y \frac{e^{-x}}{f(x)} = \int f(x)\frac{e^{-x}}{f(x)} dx + c = -e^{-x} + c$

⇒ $y = -f(x) + ce^{x} f(x)$

13. **Sol.** $(2x - 10y^3)\frac{dy}{dx} + y = 0$ ⇒ $\frac{dy}{dx} = \frac{y}{10\ y^3 - 2x}$ ⇒ $\frac{dx}{dy} = \frac{10\ y^3 - 2x}{y}$

$\frac{dx}{dy} = 10y^2 - 2\frac{x}{y}$ ⇒ $\frac{dx}{dy} + \frac{2}{y}x = 10y^2$ ⇒ $xy^2 = 10\frac{y^5}{5} + c$

⇒ $xy^2 = 2y^5 + c$

14. **Sol.** $\sin y \frac{dy}{dx} = \cos y\,(1 - x \cos y)$

put cos y = t $-\sin y \dfrac{dy}{dx} = \dfrac{dt}{dx}$

$\Rightarrow \quad -\dfrac{dt}{dx} = t(1 - tx) \quad \Rightarrow \quad \dfrac{dt}{dx} = t^2x - t$

$\Rightarrow \quad \dfrac{dt}{dx} + t = t^2x$

$\dfrac{1}{t^2}\dfrac{dt}{dx} + \dfrac{1}{t} = x \qquad \dfrac{1}{t} = v$

$-\dfrac{dv}{dx} + v = x \quad \Rightarrow \quad \dfrac{dv}{dx} - v = -x$

I.F. = $e^{\int -dx} = e^{-x}$

Now solution $v.e^{-x} = \int -xe^{-x}dx + C$

$ve^{-x} = xe^{-x} - \int e^{-x}dx + c$

$ve^{-x} = xe^{-x} + e^{-x} + C$

$\Rightarrow \quad 1/t = x + 1 + Ce^{x}$

15. **Sol.** $\sec^2 y \dfrac{dy}{dx} + \tan y = 1$ put tan y = t $\sec^2 y \dfrac{dy}{dx} = \dfrac{dt}{dx}$

$\Rightarrow \quad \dfrac{dt}{dx} = 1 - t \quad \Rightarrow \quad \ell n(1 - t) = -cx$

$\Rightarrow \quad 1 - t = e^{-cx} \quad \Rightarrow \quad t = 1 - e^{-cx} \quad \Rightarrow \quad \tan y = 1 + ce^{-x}$

16. **Sol.** $xdy = ydx + y^2 dy \quad \Rightarrow \quad \dfrac{x\ dy - y\ dx}{y^2} = dy \quad \Rightarrow \quad -d\left(\dfrac{x}{y}\right) = dy$

$-\dfrac{x}{y} = y + c$ put x = 1 y = 1 $\Rightarrow$ c = –2

$-\dfrac{x}{y} = y - 2$ put y = –3 $\therefore \quad \dfrac{x}{3} = -5 \quad \Rightarrow \quad x = -15$

17. **Sol.** $2x^3dx + 2y^3dy - (xy^2 dx + x^2y\, dy) = 0$

$d\left(\dfrac{x^4}{2}\right) + d\left(\dfrac{y^4}{2}\right) - \dfrac{1}{2}d(x^2y^2) = 0$

$\Rightarrow \quad d(x^4 + y^4 - x^2y^2) = 0 \quad \Rightarrow \quad x^4 + y^4 - x^2y^2 = c$

18. **Sol.** $\dfrac{xdy - ydx}{x^2 + y^2} + dx = 0 \quad \Rightarrow \quad \dfrac{\dfrac{xdy - ydx}{x^2}}{1 + \left(\dfrac{y}{x}\right)^2} + dx = 0 \quad \Rightarrow \quad \dfrac{d\left(\dfrac{y}{x}\right)}{1 + \left(\dfrac{y}{x}\right)^2} + dx = 0$

$\Rightarrow \quad d\left(\tan^{-1}\left(\dfrac{y}{x}\right)\right) + dx = 0 \quad \Rightarrow \quad \tan^{-1}\left(\dfrac{y}{x}\right) + x = c$

19. **Sol.** $e^y dx + xe^y dy - 2y dy = 0$

$d(xe^y) - d(y^2) = 0$

Solution is $xe^y - y^2 = c$

$xe^y - y^2 = c$

PART - II : MISCELLANEOUS QUESTIONS

A-1 Ans. (2)

Sol. Statement-1 is true because differential equation of y = A sin x + B cos x is $\frac{d^2y}{dx^2} + y = 0$

Statement-2 $\sec^2 y \frac{dy}{dx} + x \tan y = x^2$

Put tan y = z, then

$\frac{dz}{dx} + xz = x^2$

I.F. = $e^{\int x dx} = e^{x^2/2}$

∴ the solution is $z \cdot e^{x^2/2} = \int x^2 \; e^{x^2/2} dx + c$

i.e. $\tan y \cdot e^{x^2/2} = \int x \; e^{x^2/2} \cdot x \, dx + c$

∴ statement is false.

A-2. Ans. (3)

Sol. $\tan^{-1} \frac{y}{x} = \frac{mx^2}{2} + C$

$\frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{\left(x\frac{dy}{dx} - y\right)}{x^2} = mx \Rightarrow \frac{x^2}{x^2+y^2} \cdot \frac{\left(x\frac{dy}{dx} - y\right)}{x^2} = mx \Rightarrow \frac{x\frac{dy}{dx} - y}{x^2+y^2} = mx$

statement-1 is false

statement-2 is linear form of differential equation

I.F. = $e^{\int \frac{1}{x} dx} = x$

∴ $y.x = \int x \sin x + c$

$xy = -x \cos x + \int \cos x + c$

$xy = -x \cos x + \sin x + c$

$x(y + \cos x) = \sin x + c$

∴ statement-2 is true.

A-3. Ans. (1)

Sol. $y - \frac{xdy}{dx} = y^2 + \frac{dy}{dx}$

⇒ $y - y^2 = (x + 1) \frac{dy}{dx}$

⇒ $\int \frac{dx}{x+1} = \int \frac{dy}{y(1-y)}$

⇒ $\ell n (x + 1) = -\ell n (1 - y) + \ell n y + \ell n c$

⇒ $(x + 1)(1 - y) = cy$

statement-1 is true

statement-2 is true and it explains statement-1

Section (B) : MATCH THE COLUMN

B-1. Ans. A → s, B → p, C → q, D → r

Sol. (A) $\frac{d^2y}{dx^2} = \left(y + \left(\frac{dy}{dx}\right)^6\right)^{\frac{1}{4}}$

$\left(\frac{d^2y}{dx^2}\right)^4 = y + \left(\frac{dy}{dx}\right)^6$

order = 2 , degree = 4

sum = 6

(B) $\left(\frac{d^4y}{dx^4}\right)^3 + 3\left(\frac{d^2y}{dx^2}\right)^6 + \sin x = 2\cos x$

order = 4, degree = 3

sum = 7

(C) $\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$

$\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} = 3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 4$

$\left(\frac{d^3y}{dx^3}\right)^2 = \left(3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 4\right)^3$

order = 3, degree = 2

sum = 5

(D) $\frac{dy}{dx} + y = \frac{1}{\left(\frac{dy}{dx}\right)}$

$\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx} = 1$

order = 1, degree = 2

sum = 3

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. **Sol.** We have $(x-h)^2 + (y - k)^2 = a^2$ ....(1)

Differentiating w.r.t. x, we get

$2(x - h) + 2(y - k)\frac{dy}{dx} = 0$

$(x - h) + (y - k)\dfrac{dy}{dx} = 0$ ....(2)

Differentiating w.r.t. x, we get

$1 + \left(\dfrac{dy}{dx}\right)^2 + (y - k)\dfrac{d^2y}{dx^2} = 0$ ....(3)

From equation (3),

$y - k = -\left(\dfrac{1+p^2}{q}\right)$, where $p = \dfrac{dy}{dx}$, $q = \dfrac{d^2y}{dx^2}$

Putting the value of y – k in equation (2), we get

$x - h = \dfrac{(1+p^2)p}{q}$

Substituting the values of x – h and y – k in equation (1), we get

$\left(\dfrac{1+p^2}{q}\right)^2 (1 + p^2) = a^2$ or $\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^3 = a^2\left(\dfrac{d^2y}{dx^2}\right)^2$

which is the required differential equation

C-2. **Sol.** $y\left(\dfrac{dy}{dx}\right)^2 + (x - y)\dfrac{dy}{dx} - x = 0$

or $\dfrac{dy}{dx} = \dfrac{(y-x)\pm\sqrt{(x-y)^2+4xy}}{2y}$

or $\dfrac{dy}{dx} = 1$ which gives straight line

or $\dfrac{dy}{dx} = -\dfrac{x}{y}$ which gives circle.

C-3. **Sol.** Obviously (1) is linear differential equation with $P = \dfrac{1}{x}$ and Q = log x

$y\left(\dfrac{dy}{dx}\right) + 4x = 0 \Rightarrow \dfrac{dy}{dx} + \dfrac{4x}{y} = 0$

Hence, it is not linear

$(2x + y^3)\left(\dfrac{dy}{dx}\right) = 3y$

$\Rightarrow \dfrac{dx}{dy} - \dfrac{2x}{3y} = \dfrac{y^2}{3}$ which is linear with $P = -\dfrac{2}{3y}$ and $Q = \dfrac{y^2}{3}$

C-4. **Sol.** $\dfrac{dy}{dx} + y\cos x = \cos x$ (linear)

I.F. = $e^{\int \cos x dx} = \int e^{\sin x}$

Thus, solution is $y.e^{\sin x} = \int e^{\sin x}\cos x dx$

$ye^{\sin x} = e^{\sin x} + c$

when x = 0, y = 1, then c = 0

Thus, y = 1, Hence , option (1), (2), (4) are true

C-5. **Sol.** $x = \sin\left(\frac{dy}{dx} - 2y\right) \Rightarrow \frac{dy}{dx} - 2y = \sin^{-1}x$

$x - 2y = \log \Rightarrow \frac{dy}{dx} = e^{x-2y}$

C-6. **Sol.** Equation of normal at P(x, y) is

$Y - y = -\frac{dx}{dy}\ (X - x)$

so coordinate of Q is $\left(x + y\frac{dy}{dx},\ 0\right)$

Thus $(PQ)^2 = (X - x)^2 + (0 - y)^2$

$\Rightarrow \quad k^2 = \left(y\frac{dy}{dx}\right)^2 + y^2 \quad \Rightarrow \quad y\frac{dy}{dx} = \pm\sqrt{k^2 - y^2} \quad \Rightarrow \quad \frac{ydy}{\sqrt{k^2 - y^2}} = \pm\int dx$

$\Rightarrow \quad = \pm x + c$

it passes through (0, k) $\Rightarrow$ c = 0 $\Rightarrow -\sqrt{k^2 - y^2} = \pm x$

$\Rightarrow \quad k^2 - y^2 = x^2$

$x^2 + y^2 = k^2$

C-7. **Sol.** $\frac{dy}{dx} - y\tan x = 2x\sec x$

y(0) = 0

I.F. $= e^{-\int \tan x\ dx} = e^{-\ell n \sec x}$

I.F. = cos x

$\cos x \cdot y = \int 2x\ \sec\ x.\text{cox}\ dx$

$\cos x.\ y = x^2 + c$

c = 0

$y = x^2 \sec x$

$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16}.\sqrt{2} = \frac{\pi^2}{8\sqrt{2}}$

$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2}.\sqrt{2} + \frac{\pi^2}{16}.\sqrt{2}$

$y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}.2 = \frac{2\pi^2}{9}$

$y'\left(\frac{\pi}{3}\right) = 2\frac{\pi}{2}.2 + \frac{\pi^2}{9}.2.\sqrt{3}$

$\frac{4\pi}{3} + \frac{2\pi^2\sqrt{3}}{9}$

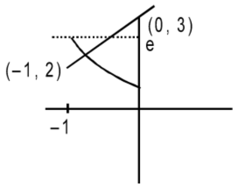
C-8. **Sol.** $(1 + e^x)\frac{dy}{dx} + ye^x = 1$

$\frac{dy}{dx} + \frac{e^x}{1+e^x}y = \frac{1}{1+e^x}$

I.F = $e^{\int \frac{e^x}{1+e^x}dx} = e^{\ell n(1+e^x)}$ = 1 + e^x
complete solution
y.(1 + e^x) = $\int 1dx$
(1 + e^x)y = x + c
x = 0, y = 2 ⇒ c = 4
(1 + e^x)y = x + 4
y = $\frac{x+4}{e^x+1}$
x = –4, y = 0
x = –2, y = $\frac{2}{x^{-2}+1}$

$$\frac{dy}{dx} = \frac{(e^x+1).1-(x+4)e^x}{(e^x+1)^2}$$

$$\frac{e^x(-x-3)+1}{(e^x+1)^2}$$



$\frac{dy}{dx}$ = 0 ⇒ x + 3 = e^{-x}
e^x = $\frac{1}{x+3}$