Fundamental of Mathematics - I

MATHEMATICS

Exercise-1

OBJECTIVE QUESTIONS

Section (A) : Representation of sets, Types of sets, subset and power set

A-1. **Sol.** Since, intelligency is not defined for students in a class so set of intelligent students in a class is not well defined collection.

A-2. **Sol.** (i) $x^2 - 1 = 0$ $x = \pm 1$
(ii) $x^2 + 1 = 0$ $x = \pm i$ $x \in \varphi$
(iii) $x^2 - 9 = 0$ $x = \pm 3$
(iv) $x^2 - x - 2 = 0$, $x = 2, -1$

A-3. **Sol.** $x^2 = 16 \Rightarrow$ $x = \pm 4$
$2x = 6$ $x = 3$
No common value of x

A-4. **Sol.** A = {–2, –1, 0, 1, 2}
No. of subsets = $2^n = 2^5 = 32$

A-5. **Sol.** Obvious

A-6. **Sol.** P(A) = {φ, {7}, {10}, {11}, {7, 10}, {7, 11}, {10, 11}, {7, 10, 11}}

A-7. **Sol.** Collection of all beautiful women in Jalandhar is not a set as it is not a well defined collection. It is not possible to decide logically which woman is to be included in the collection and which is not to be included.

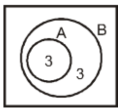
A-8. **Sol.** 2,3,5 and 7 are the only positive primes less than 10.

A-9. **Sol.** Between any two real numbers there lie infinitely many real numbers.

A-10. **Sol.** P(A) = {φ, {φ}, {{φ}}, {φ, {φ}}} = {φ, {φ}, {{φ}}, A}

Section (B) : Operations on sets, Law of Algebra of sets

B-1.

Sol. 

n(AB) = n(A) + n(B) – n(AB) $\Rightarrow$ minimum value of n(A∪B)
= 3 + 6 – 3 = 6

B-2. **Sol.** A = {1, 2, 3}
B = {3, 4}
C = {4, 5, 6}
$B \cap C$ = { 4 }
$A \cup (B \cap C)$ = {1, 2, 3, 4}

B-3. **Sol.** Obvious

B-4. **Sol.** A ∩ B = {3, 4, 10}
A ∩ C = {4}
(A ∩ B) ∪ (A ∩ C)= {3, 4, 10}

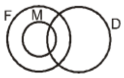
B-5. **Sol.** Obviously $A - (B \cup C)$

B-6. **Sol.** $B' = U - B = \{1, 2, 3, 4, 5, 8, 9, 10\}$
$A \cap B' = \{1,2,5\} = A$

B-7. **Sol.** $A = \{5, 9, 13, 17, 21\}$ and $B = \{3, 6, 9, 12, 15, 18, 21, 24\}$
$A - B = \{5, 13, 17\}$
$A - (A - B) = \{9, 21\}$

B-8. **Sol.** Let $A \cup B = A \cap B$
Now, $x \in A \Rightarrow x \in A \cup B$ $(\because A \subseteq A \cup B)$
$\Rightarrow x \in A \cap B$ $(\because A \cup B = A \cap B)$
$\Rightarrow x \in B$
Similarly, $x \in B$ implies $x \in A$ $\therefore A = B$
Conversly, let $A = B$
$\therefore A \cup B = A \cup A = A = A \cap A = A \cap B$
$\therefore A \cup B = A \cap B$

B-9. **Sol.** $bN \cap . cN$
(+ve integral multiple of b) $\cap$ (+ve integral multiple of c)
since b & c are relatively primes :
$= b\,c\,N$ $\therefore$ $d = bc$

B-10. **Sol.** 

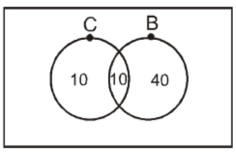
$M \equiv$ Mother; $F \equiv$ Female; $D =$ Doctor

Section (C) : Cardinal number Problems

C-1. **Sol.** (i) $A \cup B \geq A \cup B$ (ii) $A \cap B \leq A \cup B$ (iii) $A \cap B = A \cup B$ not always

C-2. **Sol.** $n(A^c \cap B^c) = n[(\{A \cup B)^c] = n(U) - n(A \cup B)$
$= n(U) - [n(A) + n(B) - n(A \cap B)] = 700 - [200 + 300 - 100] = 300.$

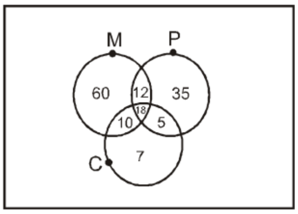
C-3. **Sol.** Let number of newspapers is x.
As every newspaper is read by 60 students
Since, every students reads 5 newspapers
$\therefore 60x = 300(5)$ $\Rightarrow$ $x = 25.$



C-4. **Sol.**
$P = 10 + 10 + 40 = 60\ \%$

C-5. **Sol.** $n(A) = 40\%$ of $10,000 = 4,000$
$n(B) = 20\%$ of $10,000 = 2,000$
$n(C) = 10\%$ of $10,000 = 1,000$
$n(A \cap B) = 5\%$ of $10,000 = 500$
$n(B \cap C) = 3\%$ of $10,000 = 300$
$n(C \cap A) = 4\%$ of $10,000 = 400$
$n(A \cap B \cap C) = 2\%$ of $10,000 = 200$
$n(A \cap B^c \cap C^c) = n[A \cap (B \cup C)^c]$
$= n(A) - n[A \cap (B \cup C)] = n(A) - n[(A \cap B) \cup (A \cap C)]$
$= n(A) - [n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)]$
$= 4000 - [500 + 400 - 200] = 4000 - 700 = 3300.$

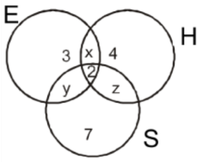
C-6.



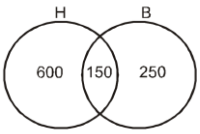
Sol. Number of students offered maths alone = 60

n (M) = 100
n(P) = 70
n (C) = 40
n (M ∩ P) = 30
n (M ∩ P) = 28
n (P ∩ C) = 23
n (M ∩ P ∩ C) = 18

C-7. **Sol.** x+ y = 10 ; x + z = 9 ; y + z = 11 ⇒ x + y + z = 15
x = 4, y = 6, z = 5



C-10



Sol.

n(H ∪ B) = n(H) + n(B) – n(H ∩ B)
1000 = 750 + 400 – n(H ∩ B) = 150
Now n(only hindi) = n(H) – n(H ∩ B) = 750 – 150 = 600
n(only bengali) = n(B) – n(H ∩ B)
400 – 150 = 250

Section (D) : Standard formulae, Polynomials & Divisional Algorithm

D-1 **Sol.** sum = 2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 = 77

D-2. **Sol.** $\frac{5+3\sqrt{7}}{5-3\sqrt{7}} = \frac{(5+3\sqrt{7})^2}{(5-3\sqrt{7})(5+3\sqrt{7})} = \frac{25+63+30\sqrt{7}}{25-63} = \frac{88+30\sqrt{7}}{-38} = -\frac{44}{19}-\frac{15}{19}\sqrt{7}$

D-3. **Sol.** The sum of three non-negative numbers is zero only when they are all zero but x – 1, x – 2 & x – 3 cannot be equal to zero simultaneously.

D-4. **Sol.** Obvious

D-5. **Sol.** $x^2 + \frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2 = 4 - 2 = 2$

D-6. **Sol.** $\frac{1}{(2-1)}\frac{(2^2-1)(2^2+1)(2^4+1)(2^8+1)}{(2^8-1)} = \frac{2^{16}-1}{2^8-1} = (2^8)+1$

$= (4^4 + 1) \quad \Rightarrow \quad n = 4$

D-7. **Sol.** $x^2 + y^2 - 2xy = 0 \quad \Rightarrow \quad (x - y)^2 = 0 \quad \Rightarrow \quad x = y$ then rc $\lambda^2 = 4$

$\Rightarrow \quad \lambda = \pm 2$

$\lambda > 0, \lambda = 2$

D-8. **Sol.** $\frac{a+3d}{a+9d} = \frac{a+d}{a+5d} = k \quad \Rightarrow \quad a + 3d = ak + 9dk$

$\Rightarrow \quad (3 - 9k)d = a(k - 1)$ ....(i)

and $a + d = ak + 5dk$

$d(1 - 5k) = a(k - 1)$ .....(ii)

$3 - 9k = 1 - 5k \quad \Rightarrow \quad 4k = 2 \quad \Rightarrow \quad k = \frac{1}{2}$

D-9 . **Sol.** x = a satisfies the given expression

$\therefore \; a^3 - a^3 + a + 2 = 0 \Rightarrow a = -2.$

D-10. **Sol.** $P(4) = 64k + 48 - 3$

$= 64k + 45$

$Q(4) = 128 - 20 + k$

$= 108 + k$

$\because \quad P(4) = Q(4)$

$64k + 45 = 108 + k$

$63k = 63 \quad \Rightarrow \quad k = 1$

D-11. **Sol.** $2x^3 - 5x^2 + x + 2 = ax^3 - (2a + b)x^2 + (2b - 1)x + 2$

$\therefore \quad a = 2$

$2a + b = 5$ and $2b - 1 = 1$

$\therefore \quad b = 1$

Section (E) : Rational Inequalities

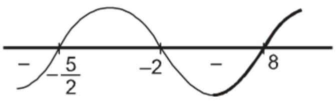
E-1. **Sol.** $-5 \le x \le 10$ means $x = -5, -4, -3, ...9, 10$

and $0 \le x \le 15$ means $x = 0, 1, 2, ...9, 10, 11, 12, ...15$

Required integers values of x are 0,1, 2, 3, 4,........9 $\Rightarrow$ Number of integers values of x = 10

E-2. **Sol.** $\frac{x^2-1}{2x+5} - 3 < 0 \quad \Rightarrow \quad \frac{x^2-1-6x-15}{2x+5} < 0 \quad \Rightarrow \quad \frac{x^2-6x-16}{2x+5} < 0 \quad \Rightarrow$

$\frac{x^2-8x+2x-16}{\left(x+\frac{5}{2}\right)} < 0 \quad \Rightarrow \quad \frac{x(x-8)+2(x-8)}{x+\frac{5}{2}} < 0 \quad \Rightarrow \quad \frac{(x-8)(x-2)}{x+\frac{5}{2}} < 0$



$x \in (-\infty, -5/2) \cup (-2, 8)$

E-3. **Sol.** $2x - 1 \le x^2 + 3 \le x - 1$

$x^2 + 3 \le x - 1$

$\Rightarrow \quad x^2 - x + 4 \le 0$ which is not true for $x \in R$.

$\Rightarrow \quad x \in \varphi$

E-4. **Sol.** $5x + 2 < 3x + 8 \quad \Rightarrow \quad 2x < 6 \quad \Rightarrow \quad x < 3$ ...(i)

$\frac{x+2}{x-1} < 4 \quad \Rightarrow \quad \frac{x+2}{x-1} - 4 < 0 \quad \Rightarrow \quad \frac{-3x+6}{x-1} < 0$

$\Rightarrow \quad \frac{x-2}{x-1} > 0 \quad \Rightarrow \quad x \in (-\infty, 1) \cup (2, \infty)$ ...(ii)

Taking intersection of (i) and (ii) $\quad x \in (-\infty, -1) \cup (2, 3)$

E-5. **Sol.** $x_2 + 9 < (x + 3)_2 < 8x + 25$

$\Rightarrow \quad (x + 3)_2 > x_2 + 9 \Rightarrow \quad x > 0$ ....(i)

and $\quad (x + 3)_2 < 8x + 25 \quad \Rightarrow \quad x_2 - 2x - 16 < 0$

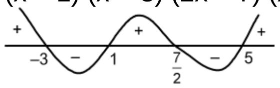
$\Rightarrow \quad x \in (1 - \sqrt{17}, 1 + \sqrt{17})$ .....(ii)

(i) $\cap$ (ii) $\Rightarrow x \in (0, 1 + \sqrt{17})$

Number of integers = 5

E-6. **Sol.** $(x - 1)(x + 3)(2x - 7)(5 - x) \le 0$

$(x - 1)(x + 3)(2x - 7)(x - 5) \ge 0$



E-7. **Sol.** $\frac{(x+2)(x^2-2x+1)}{4+3x-x^2} \ge 0$

$\frac{(x+2)(x-1)^2}{-(x+1)(x-4)} \ge 0$

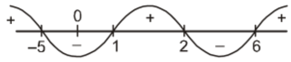
$\frac{(x+2)(x-1)^2}{(x+1)(x-4)} \le 0$

$x \in (-\infty, -2] \cup (-1,4)$

E-8. **Sol.** $\frac{x^2(x^2-3x+2)}{x^2-x-30} \ge 0 \quad \Rightarrow \quad \frac{x^2(x-1)(x-2)}{(x+5)(x-6)} \ge 0$

$x \ne -5,6$

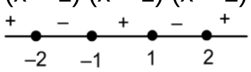
$x \in (-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$



E-9. **Sol.** $x_4 - 5x_2 + 4 \le 0$

$(x_2 - 1)(x_2 - 4) \le 0$

$(x + 1)(x - 1)(x + 2)(x - 2) \le 0$



$x \in [-2,-1] \cup [1,2]$

so number of integers = 4

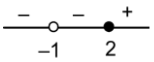
E-10. **Sol.** $x \in (-\infty, -9) \cup (-9, -3) \cup [-1, 0)(0, 2) \cup [4, 6)$

so +ve integral solutions are x = 1,4,5

E-11. **Sol.** $\frac{2}{x^2-x+1} - \frac{1}{x+1} - \frac{2x-1}{x^3+1} \ge 0$

$\frac{2x+2-x^2+x-1}{x^3+1} - \frac{(2x-1)}{x^2+1} \ge 0 \Rightarrow \frac{3x+1-x^2-2x+1}{x^3+1} \ge 0 \Rightarrow \frac{-x^2+x+2}{x^3+1} \ge 0 \Rightarrow \frac{x^2-x-2}{x^3+1} \le 0$

$\Rightarrow \dfrac{(x+1)(x-2)}{(x+1)(x^2-x+1)} \le 0 \Rightarrow \dfrac{(x-2)}{(x^2-x+1)} \le 0$



$x \in (-\infty,-1) \cup (-1,2]$

required value of x = {0, 1, 2}

Section (F) : Logarithm

F-1. **Sol.** $\dfrac{1}{1+\log_b a+\log_b c} + \dfrac{1}{1+\log_c a+\log_c b} + \dfrac{1}{1+\log_a b+\log_a c}$

$= \dfrac{1}{\log_b abc} + \dfrac{1}{\log_c abc} + \dfrac{1}{\log_a abc}$

$= \log_{abc} b + \log_{abc} c + \log_{abc} a = \log_{abc} abc = 1$

F-2. **Sol.** $\dfrac{1}{\log_{\sqrt{bc}} abc} + \dfrac{1}{\log_{\sqrt{ca}} abc} + \dfrac{1}{\log_{\sqrt{ab}} abc}$

$= \log_{abc}\sqrt{bc} + \log_{abc}\sqrt{ca} + \log_{abc}\sqrt{ab}$

$= \log_{abc} abc = 1$

F-3. **Sol.** $a^4 b^5 = 1$

take log with base 'a' both sides

$4 + 5\log_a b = \log_a 1 \quad \Rightarrow \quad \log_a b = -\dfrac{4}{5}$

Now $\log_a(a^5b^4) = 5 + 4\log_a b = 5 + 4\left(-\dfrac{4}{5}\right) = \dfrac{25-16}{5} = \dfrac{9}{5}$

F-4. **Sol.** $x = 2^{\log 3}$, $y = 3^{\log 2} = 2^{\log 3} = x$

F-5. **Sol.** $\log_a(ab) = x \quad \Rightarrow \quad 1 + \log_a b = x \quad \Rightarrow \quad \log_a b = x - 1 \quad \Rightarrow \quad \log_b a = \dfrac{1}{x-1}$

Now $\log_b(ab) = 1 + \log_b a = 1 + \dfrac{1}{x-1} = \dfrac{x-1+1}{x-1} = \dfrac{x}{x-1}$

F-6. **Sol.** $\log_2(5 \times 2).\log_2(2^4 \times 5) - (\log_2 5)\log_2(2^5 \times 5) = (\log_2 5 + 1)(4 + \log_2 5) - \log_2 5(5 + \log_2 5)$

let $\log_2 5 = t$

$(t+1)(4+t) - t(5+t) = t^2 + 5t + 4 - 5t - t^2 = 4 = 4\log_2 2 = \log_2 16$

F-7. **Sol.** $y = \dfrac{2^{\log_{2^{1/4}} a} - 3^{\log_{27}(a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1} \quad \Rightarrow \quad 2^{\log_{2^{1/4}} a} = 2^{4\log_2 a} = a^4$

$3^{\log_{27}(a^2+1)^3} = 3^{\log_3(a^2+1)} = a^2 + 1 \quad \Rightarrow \quad 7^{4\log_{49} a} = 7^{2\log_7 a} = a^2$

$\therefore \quad y = \dfrac{a^4-(a^2+1+2a)}{a^2-a-1} = \dfrac{a^4-(a+1)^2}{a^2-a-1} = a^2 + a + 1$

F-8. **Sol.** $\log_p(\log_q(\log_r x)) = 0 \quad \Rightarrow \quad \log_q(\log_r x) = b$

$\Rightarrow \quad \log_r x = q \quad \Rightarrow \quad x = r^q$ ......(i)

and $\log_q(\log_r(\log_p x)) = 0$

$\Rightarrow \quad \log_r(\log_p x) = 1 \quad \Rightarrow \quad \log_p x = r \quad \Rightarrow \quad x = p^r$ ......(ii) from (i) and (ii) $p^r = r^q$

$\Rightarrow$ $p = r^{q/r}$

F-9. **Sol.** $\log_{10}\pi$ is quantity lie between 0 to 1.

F-10. **Sol.** $\log_{10}(\log_2 3 \cdot \log_3 4 \cdot \log_4 5 .... \log_{1023} 1024) = \log_{10}(\log_2 1024) = \log_{10}(\log_2 2^{10}) = \log_{10}(10) = 1$

F-11. **Sol.** $2\log_{10}x - \log_{10}(2x - 75) = 2$ $\Rightarrow$ $\frac{x^2}{2x-75} = 10^2 = 100$

$\Rightarrow$ $x^2 - 200x + 7500 = 0$ $\Rightarrow$ $x = 50, x = 150$

sum = 200

F-12. **Sol.** $\log_x \log_{18}\left(\sqrt{2}+2\sqrt{2}\right) = \frac{1}{3}$ $\Rightarrow$ $\log_x \log_{18}\left(\sqrt{18}\right) = \frac{1}{3}$ $\Rightarrow$ $\log_x \frac{1}{2} = \frac{1}{3}$

$\Rightarrow$ $x^{1/3} = \frac{1}{2}$ $\Rightarrow$ $x = \frac{1}{8}$ $\Rightarrow$ $1000\,x = 125.$

F-13_. **Sol.** $\log_2(\log_3(x^2-1)) = 0$

$\log_3(x^2 - 1) = 1$

$x^2 - 1 = 3^1$

$x^2 = 4$

$x = \pm 2$

sum of solutions = 0

F-14. **Sol.** $- 2x - 3 = 0$ $\Rightarrow$ $x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$ $\Rightarrow$ $x = 3, x = -1$ but $x \neq -1$

$\therefore$ $x = 3.$

F-15. **Sol.** $\log_2(\log_9 x + \frac{3}{2} + 8^x) = 3x$

$\log_9 x + \frac{3}{2} + 8^x = 2^{3x}$

$\log_9 x + \frac{3}{2} + 8^x = 8^x$

$\log_9 x + \frac{3}{2} = 0$

$\log_9 x = -\frac{3}{2}$

$x = 9^{-\frac{3}{2}} = 3^{-3}$

$x = \frac{1}{27}$ $\Rightarrow$ $27x = 1$

F-16. **Sol.** Clearly Domain is $x > 0$ and $x \neq 1$

Section (G) : Logarithmic inequalities

G-1. **Sol.** $\log_{\frac{\sqrt{3}}{2}}(x^2 - 3x + 2) \geq 2$ $\Rightarrow$ $x^2 - 3x + 2 \leq \left(\frac{\sqrt{3}}{2}\right)^2$ $\Rightarrow$ $4x^2 - 12x + 8 \leq 3$

$\Rightarrow$ $4x^2 - 12x + 5 \le 0$ $\Rightarrow$ $(2x - 5)(2x - 1) \le 0$ $\Rightarrow$ $x \in \left[\frac{1}{2}, \frac{5}{2}\right]$

But domain $x^2 - 3x + 2 > 0$ $\Rightarrow$ $(x - 1)(x - 2) > 0$ $\Rightarrow$ $x \in (-\infty, 1) \cup (2, \infty)$

Hence $x \in \left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$

G-2. **Sol.** $\log_{0.3} (x - 1) < \log_{0.09} (x - 1)$; $\log_{0.3} (x - 1) < \frac{\log_{0.3} (x-1)}{2}$

$\Rightarrow$ $\log_{0.3} (x - 1) < 0 \Rightarrow$ $x - 1 > 1$ $\Rightarrow$ $x > 2$

G-3. **Sol.** $2 - \log_2 (x^2 + 3x) \ge 0$ $\Rightarrow$ $\log_2 (x^2 + 3x) \le 2$

$x^2 + 3x > 0$ $\Rightarrow$ $x \in (-\infty , - 3) \cup (0, \infty)$ .....(i)

and vkSj $x^2 + 3x \le 4$

$\Rightarrow$ $(x - 1) (x + 4) \le 0$ $\Rightarrow$ $x \in [-4, 1]$ .....(ii)

(i) $\cap$ (ii) ls $\Rightarrow x \in [-4, -3) \cup (0, 1]$

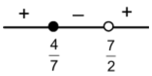
G-4. **Sol.** $\left(\frac{1}{2}\right)^{x^2-2x} < \left(\frac{1}{2}\right)^3$ $\Rightarrow x^2 - 2x > 3$ $\Rightarrow$ $x^2 - 2x - 3 > 0$ $\Rightarrow x \in (-\infty, -1) \cup (3,\infty)$

G-5. **Sol.** $\log_3 \frac{5x+3}{7-2x} \ge 0$

$\frac{5x+3}{7-2x} \ge 3^0$

$\frac{5x+3}{7-2x} - 1 \ge 0$

$\frac{7x-4}{2x-7} \le 0$



$x \in \left[\frac{4}{7}, \frac{7}{2}\right)$

G-6 **Sol.** $\log_{\frac{1}{3}} \left(\frac{3x-7}{2x}\right) \ge 0$

$0 < \frac{3x-7}{2x} \le \left(\frac{1}{3}\right)^0$

$\frac{3x-7}{2x} > 0$ and $\frac{3x-7}{2x} \le 1$

$x \in (-\infty,0) \cup \left(\frac{7}{3}, \infty\right)$ ..(i) $\frac{x-7}{2x} \le 0$ $x \in (0,7]$ ...(ii)

now vc (i) $\cap$ (ii)

$x \in \left(\frac{7}{3}, 7\right]$

so integer values of x = 3, 4, 5, 6, 7

Section (H) : Determinants

H-1. **Sol.** $\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix} = 1(-2 - 10) + 3(8 - 6) + 2(20 + 3) = -12 + 6 + 46 = 40$

H-2. **Sol.** $\begin{vmatrix} 1 & 2 & 3 \\ -1 & -3 & 2 \\ 2 & -1 & -1 \end{vmatrix} = 1\,(3 + 2) - 2\,(1 - 4) + 3\,(1 + 6) = 5 + 6 + 21 = 32$

H-3. **Sol.** $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} = 0 \quad \Rightarrow \quad k = 3$

Exercise-2

Marked questions may have for revision questions.

PART - I : OBJECTIVE QUESTIONS

1. **Sol.** A = [x : x ∈ R, – 1 < x < 1]
B = [x : x ∈ R : x ≤ 0 or x ≥ 2]
∴ A ∪ B = R – D, where D = [x : x ∈ R, 1 ≤ x < 2]

2. **Sol.** 1. (N ∪ B) ∩ Z = (N ∩ Z) ∪ (B ∩ Z) = N ∪ (B ∩ Z)
2. A = {3, 6, 9, 12, 15, 18, 21, 24}

3. **Sol.** $A_1 \cup A_2 \cup A_3$ is the smallest element containing subset of all we set A_1, A_2 and A_3

4. **Sol.** 1. ((A ∩ B) ∪ C)′ ∩ B′)′
= (A ∩ B) ∪ C) ∪ B
= (A ∩ B) ∪ B ∪ C
= B ∪ C ≠ B ∩ C
2. (A′ ∩ B′) ∩ (A ∪ B ∪ C′)
= (A ∪ B)′ ∩ ((A ∪ B) ∪ C′)
= φ ∪ ((A ∪ B)′ ∩ C′)
= ((A ∪ B) ∪ C)′
= (A ∪ (B ∪ C))′

5. **Sol.** n(A ∪ B) = 280
n(A` ∪ B`) = 2009 – n(A ∪ B)
= 2009 – 280 = 1729 = $12_3 + 1_3 = 10_3 + 9_3$
n(A – B) = 1681 – 1075 = 606
= 4 + 2 × 301 = 4 + 2 × 7 × 43
= (2) 2 + 2 × 7 × 43

6. **Sol.** n(M) = 23, n(P) = 24, n(C) = 19
n(M ∩ P) = 12, n(M ∩ C) = 9, n(P ∩ C) = 7
n(M ∩ P ∩ C) = 4
n (M ∩ P′ ∩ C′) = n[M ∩ (P ∪ C)′]
= n(M) – n(M ∩ (P ∪ C))
= n(M) – n[(M ∩ P) ∪ (M ∩ C)]
= n(M) – n(M ∩ P) – n(M ∩ C) + n(M ∩ P ∩ C)

$= 23 - 12 - 9 + 4 = 27 - 21 = 6$

$n(P \cap M' \cap C) = n[P \cap (M \cup C)']$

$= n(P) - n[P \cap (M \cup C) = n(P) - n[(P \cap M) \cup (P \cap C)]$

$= n(P) - n(P \cap M) - n(P \cap C) + n(P \cap M \cap C)$

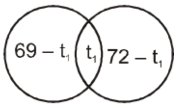
$= 24 - 12 - 7 + 4 = 9$

$n(C \cap M' \cap P') = n(C) - n(C \cap P) - n(C \cap M) + n(C \cap P \cap M)$

$= 19 - 7 - 9 + 4 = 23 - 16 = 7$

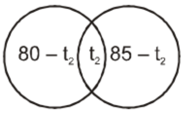
7.

Sol.

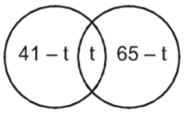


$70 + 72 - t_1 = 100$

$t_1 = 42\%$ $\Rightarrow$ min. in $P \cap C = 42\%$



$t_2 = 85\% - 20\% = 65\%$ $\Rightarrow$ min. $M \cap E = 65\%$



$t = 42 - 35 = 7\%$

min. in $((P \cap C) \cap (M \cap E)) = 7\%$

8. **Sol.** Formula lw=k $(a + b + c)^3 = a^3 + b^3 + c^3 + 3(a + b)(b + c)(c + a)$

$\Rightarrow$ $(a + b)(b + c)(c + a) = 0$

9. **Sol.** Let $x^5 + ax^3 + bx = Q(x - 1)(x + 1) + px + q$

put $x = 1$ we get $1 + a + b = p + q$ .....(1)

put $x = -1$ we get $-1 - a - b = -p + q$ .....(2)

on adding $q = 0$

Now at $x = -1$, $f(-1) = -3 \Rightarrow$ $-1 - a - b = -3$

$a + b = 2$ $\Rightarrow$ $p = 3$ $\Rightarrow$ remainder = 3x

10. **Sol.** $f(x) = (x - 1)\, g(x) + 5$; $f(x) = (x + 1)\, h(x) + 3$

$f(x) = (x + 2)\, \varphi(x) + 2$; $f(x) = (x^3 + 2x^2 - x - 2)\, p(x) + ax^2 + bx + c$ ......(1)

$f(1) = 0 + a + b + c$; $f(-1) = 0 + a - b + c$

$f(-2) = 4a - 2b + c$ $\Rightarrow$ $a + b + c = 5,\ a - b + c = 3,\ 4a - 2b + c = 2$

$\Rightarrow$ $a = 0, b = 1, c = 4$ from equation (1) remainder is $x + 4$

11. **Sol.** $P(1) = 5$ $\Rightarrow$ $2 - a + b = 5$ $\Rightarrow$ $b - a = 3$ ......(i)

$P(-1) = 6 + a + b = 19$ $\Rightarrow$ $b + a = 13$ ....(ii)

solving (i) and (ii)

$b = 8, a = 5. = P(2) = 10.$

12. **Sol.** $\dfrac{6x^2 - 5x - 3}{x^2 - 2x + 6} - 4 \le 0$ $\Rightarrow$ $\dfrac{2x^2 + 3x - 27}{x^2 - 2x + 6} \le 0$

denominator $x^2 - 2x + 6 > 0$ $\forall$ $x \in R$ $(\because D < 0)$

then $2x^2 + 3x - 27 \le 0$ $\Rightarrow$ $(2x + 9)(x - 3) \le 0$

$-\dfrac{9}{2} \le x \le 3$ $\Rightarrow$ $0 \le x^2 \le \dfrac{81}{4}$

$(4x^2)_{max} = 4 = 81$ ⇒ $(4x^2)_{min} = 4(0) = 0$

13. Sol. $\frac{14x}{x+1} \le \frac{9x-30}{x-4}$

$\frac{14x}{x+1} - \frac{9x-30}{x-4} \le 0$

$\frac{5x^2-35x+20}{(x+1)(x-4)} \le 0$

$\frac{(x-1)(x-6)}{(x+1)(x-4)} \le 0$

+ ○ – ● + ○ – ● +
–1 1 4 6

$x \in (-1, 1] \cup (4, 6]$

so integer values of x = 0,1, 5, 6

sum of integer values = 12

14. Sol. $b = a^2$, $c = b^2$, $\frac{c}{a} = 3^3$ ⇒ $c = 27a$ ⇒ $b^2 = 27a$ ⇒ $a^4 = 27a$

⇒ a = 3, a > 0

c = 81, b = 9

∴ a + b + c = 3 + 9 + 81 = 93

15. Sol. $9^x - 6.3^x + 8 = 0$

$3^x = t$ ⇒ $t^2 - 6t + 8 = 0$ ⇒ t = 2, t = 4

⇒ $x = \log_3 2$, $x = \log_3 4$ ⇒ sum = $\log_3 2 + \log_3 4 = \log_3 8$

16. Sol. $\frac{(x+1)+3x-x\log_2 8}{(x-1)(1)} = \frac{(x+1)+3x-3x}{(x-1)} = \frac{x+1}{x-1}$

17. Sol. $a^{(\log_3 7)^2} = \left(a^{\log_3 7}\right)^{\log_3 7} = 27^{\log_3 7} = 7^{\log_3 27} = 7^3 = 343$

$b^{(\log_7 11)^2} = \left(b^{\log_7 11}\right)^{\log_7 11} = 49^{\log_7 11} = 11^{\log_7 49} = 121$

$c^{(\log_{11} 25)^2} = \left(c^{\log_{11} 25}\right)^{\log_{11} 25} = \left(\sqrt{11}\right)^{\log_{11} 25} = 25^{\log_{11}\sqrt{11}} = 5$

hence the sum is vr% ;kxQy 343 + 121 + 5 = 469 gSA

18. Sol. $\log_p \log_p (p)^{\frac{1}{p^n}} = \log_p\left(\frac{1}{p}\right)^n = -\log_p p^n = -n$ independent of p. (p ls Lora=k)

19. Sol. $\frac{\sqrt{(x-8)(2-x)}}{\log_{0.3}\left(\frac{10}{7}(\log_2 5-1)\right)} \ge 0$

For $\sqrt{(x-8)(2-x)}$ to be defined

(i) $(x-8)(2-x) \ge 0$

$(x-2)(x-8) \le 0$ ⇒ $2 \le x \le 8$

Now Let say $y = \log_{0.3}\left(\frac{10}{7}(\log_2 5 - \log_2 2)\right) = \log_{0.3}\left(\frac{10}{7}\log_2 \frac{5}{2}\right)$

Let y < 0 (assume) then $\log_{0.3}\left(\frac{10}{7}\log_2 \frac{5}{2}\right) < 0$

$\Rightarrow \quad \frac{10}{7}\log_2\frac{5}{2} > 1 \quad \Rightarrow \quad \log_2\frac{5}{2} > \frac{7}{10} \quad \Rightarrow \frac{5}{2} > 2^{\frac{7}{10}}$ which is true

So $\quad y < 0$

so denominator is – ve and numerator is +ve, so inequality is not satisfied,

thus $\sqrt{(x-8)(2-x)} = 0$

$x = 2, 8 \qquad$ .....(i)

Now $\quad 2^{x-3} > 31$

$\Rightarrow \quad (x-3) > \log_2 31 \Rightarrow \quad x > 3 + \log_2 2^{4.9}$ (approx)

$\Rightarrow \quad x > 7.9 \qquad \Rightarrow \quad x = 8$

20. **Sol.** $\log_{0.5}\log_5(x^2-4) > \log_{0.5} 1$; $\log_{0.5}\log_5(x^2-4) > 0$

$\Rightarrow \quad x^2 - 4 > 0 \qquad \Rightarrow \quad x \in (-\infty, -2) \cup (2, \infty)$ ....(i)

,oa $\quad \log_5(x^2-4) > 0 \qquad \Rightarrow \quad x^2 - 5 > 0$

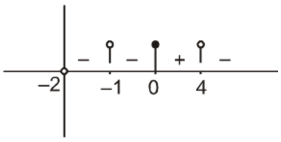
$\Rightarrow \quad x \in (-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$ ....(ii)

,oa $\quad \log_5(x^2-4) < 1$

$\Rightarrow \quad x^2 - 9 < 0 \qquad \Rightarrow \quad x \in (-3, 3)$ ....(iii)

(i) ∩ (ii) ∩ (iii) ls $\qquad \Rightarrow \quad x \in (-3, \sqrt{5}) \cup (\sqrt{5}, 3)$

21. **Sol.** $\dfrac{4^x\left(\left(\frac{3}{4}\right)^x - 1\right)\ \ln(x+2)}{(x-4)\ (x+1)} \le 0$



22. **Sol.** $\begin{vmatrix} x^2-2x+3 & 7x+2 & x+4 \\ 2x+7 & x^2-x+2 & 3x \\ 3 & 2x-1 & x^2-4x+7 \end{vmatrix}$

$= ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g$ put x = 0 (x = 0)

$\begin{vmatrix} 3 & 2 & 4 \\ 7 & 2 & 0 \\ 3 & -1 & 7 \end{vmatrix} = g \qquad \Rightarrow \quad 3(14) - 2(49) + 4(-7-6) = g$

$42 - 98 - 52 \qquad \Rightarrow \quad -108 = g$

23. **Sol.** $\sqrt{\log_4\{\log_3\{\log_2(x^2-2x+a)\}\}}$

for defined, $\log_4\log_3\log_2(x^2-2x+a) \ge 0$

$\Rightarrow \quad \log_3\log_2(x^2-2x+a) \ge 1 \Rightarrow \quad \log_2(x^2-2x+a) \ge 3$

$\Rightarrow \quad x^2 - 2x + a \ge 8 \qquad \Rightarrow \quad x^2 - 2x + (a-8) \ge 0 \qquad \Rightarrow \quad D \le 0$

$4 - 4(a-8) \le 0 \qquad \Rightarrow \quad 1 - a + 8 \le 0 \quad \Rightarrow \quad a \ge 9$

24. **Sol.** Let $\log_3 x = t$

$t^2 - 2t - 5 = 0$

$t_1 + t_2 = 2$

$\log_3 x_1 + \log_3 x_2 = 2$

$\log_3 x_1x_2 = 2$

$x_1x_2 = 9$

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

A-1. **Ans. (1)**
Sol. $X \cap (Y \cup X)' = X \cap (Y' \cap X') = X \cap X' \cap Y' = \varphi$
$\Rightarrow$ Statement – 1 true.
$X \Delta Y = (X \sim Y) \cup (Y \sim X) = (X \cup Y) \sim (X \cap Y)$
$\Rightarrow$ number of element in $X \Delta Y = m - n$.
$\Rightarrow$ Statement-2 is true but does explain statement-1

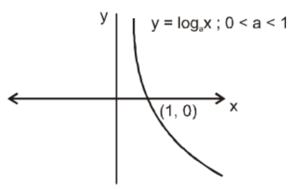
A-2. **Ans. (1)**
Sol. Let $p = q + 2 = r - 2$ where p, q, r are all prime numbers
$\Rightarrow q = p - 2$ and $r = p + 2$
$q = 3\lambda + 1$ or $3\lambda + 2$ where $\lambda \in N$ is not possible else p or r will not be prime
$\Rightarrow q = 3\lambda \Rightarrow q = 3$ $\therefore p = 5$

A-3. **Ans. (1)**
Sol. Let $P(x) = (x - 1)(x - 2) Q(x) + (ax + b)$
Given $P(1) = -1$ and $P(2) = 1$
$\therefore$ $a + b = -1$ and $2a + b = 1$ $\Rightarrow$ $a = 2, b = -3$

A-4. **Ans. (1)**
Sol. Since $0 < \sqrt{13} - \sqrt{12} < 1$ $\therefore$ $\log_{10} (\sqrt{13} - \sqrt{12}) < 0$
Since $0 < \sqrt{14} - \sqrt{13} < 1$ $\therefore$ $\log_{0.1} (\sqrt{14} - \sqrt{13}) > 0$

A-5. **Ans. (3)**
Sol. $(\log_{10} x)^2 - \log_{10} x^3 + 2 = 0$ has two roots $x = 10,100$

A-6. **Ans. (1)**
Sol. Statement-1 : $y = \log_{1/3}(x^2 - 4x + 5)$ is max.
when $x^2 - 4x + 5$ is min.
Let $f(x) = x^2 - 4x + 5$
$\Rightarrow$ $(x - 2)^2 + 1$
$f(x)_{min} = 1$
$y_{max} = \log_{1/3}1 = 0$
Statement-1 is true
Statement-2 : $\log_a x \le 0$ for $x \ge 1, 0 < a < 1$



$\therefore$ Statement-2 is true and

correct explanation for statement-1

Section (B) : MATCH THE COLUMN

B-1. **Ans.** (1) → (q), (2) → (r), (3) → (s), (4) → (p)
Sol. (1) The set $\{3^{2n} - 8n - 1 : n \in N\}$ contains 0 and every element of this set is a multiple of 64.
(2) $2^{3n} - 1$ is always divisible by 7.
(3) $3^{2n} - 1$ is always divisible by 8.
(4) $2^{2n} - 7n - 1$ is always divisible by 49 and $2^{3n} - 7n - 1 = 0$ for n = 1.

B-2. **Ans.** (A) → r, (B) → p, (C) → s, (D) → q

Sol. (A) x = 0.363636......

100x = 36.363636.......

99x = 36

⇒ $x = \frac{36}{99} = \frac{4}{11}$

sum of numerator and denominator is 4 + 11 = 15

(B) $2^{2x} - 3^{2y} = 55$, x, y ∈ I ⇒ $4^x - 9^y = 55$

only x = 3, y = 1 satisfy

There fore number of solution is one set (x, y) g.e. (3, 1)

$2^{2x} - 3^{2y} = 55$, x, y ∈ I ⇒ $4^x - 9^y = 55$

(C) $\frac{1}{\log_a 8} + \frac{1}{\log_b 8} = \frac{1}{\log_a 8.\log_b 8}$ ⇒ $\log_b 8 + \log_a 8 = 1$ (given $\log_a b = 3$)

⇒ $4\log_b 8 = 1$ $\log_b 8 = 3\log_8 a$ ....(1)

⇒ $\log_b 8 = 1/4$ $\log_a 8 = 3\log_b 8$

$8 = b^{1/4}$ ⇒ $b = 8^4$

$\log_8(8^4) = 3\log_8 a$

⇒ $\log_8 a = \frac{4}{3}$ ⇒ $a = (8^{4/3}) = (2^3) = 2^4 = 16$

(D) $3^{\sqrt{\log_3 2}} - 2^{\sqrt{\log_2 3}} = 3^{\frac{\log_3 2}{\sqrt{\log_3 2}}} - 2^{\sqrt{\log_2 3}}$

$= 3^{\log_3 2^{\frac{1}{\sqrt{\log_3 2}}}} - 2^{\sqrt{\log_2 3}} = 2^{\frac{1}{\sqrt{\log_3 2}}} - 2^{\sqrt{\log_2 3}}$

$= 2^{\sqrt{\log_2 3}} - 2^{\sqrt{\log_2 3}} = 0$

B-3. **Ans.** (A) → r, (B) → s, (C) → p, (D) → q

Sol. (A) $\text{Antilog}_{27}(0.\overline{6}) = x$ ⇒ $0.\overline{6} = \log_{27}x = \frac{2}{3}$

⇒ $x = (27)^{2/3} = (3^3)^{2/3} = 9$

(B) Since $2^{10} < 2008 < 2^{11}$ ⇒ $\log_2(2^{10}) < \log_2 2008 < \log_2(2^{11})$

⇒ $10 < \log_2 2008 < 11$

(C) $\log_e 2.\log_b 625 = \log_{10}16.\log_e 10$ ⇒ $\log_e 2.\log_b 265 = \log_e 16$ ⇒ $\log_b 625 = \log_2 16 = 4$

⇒ $625 = b^4$

(D) $x = \left(\frac{5}{6}\right)^{100} = \left(\frac{10}{2\times 6}\right)^{100} = \left(\frac{10}{2^2.3}\right)^{100}$

$\log_{10}x = 10(1 - 2\log 2 - \log 3) = 100(1 - 2(0.3010) - 0.4771) = 100(-0.0791) = 7.91$

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. **Sol.** n(A ∪ B) is minimum when n(A ∩ B) is maximum i.e. 3.

∴ minimum n(A ∪ B) = 6

n(A ∪ B) is maximum when n(A ∩ B) is minimum i.e. 0

∴ maximum n(A ∪ B) = 9

C-2_ **Sol.** n(A) = 21, n(B) = 26, n(C) = 29

$n(A \cap B) = 14$, $n(A \cap C) = 12$, $m(B \cap C) = 13$, $n(A \cap B \cap C) = 8$

$n(C \cap A' \cap B') = n(C \cap \overline{A \cup B}) = n(C) - n((C \cap A) \cup (C \cap B))$

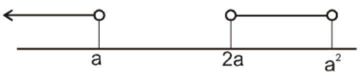
$n(C) - [n(C \cap A) + n(C \cap B) - n(A \cap B \cap C)]$

$29 - [12 + 13 - 8] = 12$

$n(A \cap B \cap C') = n(A \cap B) - n(A \cap B \cap C) = 14 - 8 = 6$

C-3. Sol. $\frac{a}{d} = \frac{b}{e} = \frac{c}{f} = \lambda \Rightarrow$ $a = d\lambda,\ b = e\lambda,\ c = f\lambda$ $\Rightarrow$ $\frac{(\lambda^k(d^k + e^k + f^k))^{\frac{1}{k}}}{(d^k + e^k + f^k)^{\frac{1}{k}}} = \lambda = \frac{a}{d} = \frac{b}{e} = \frac{c}{f}$

C-4 Sol.



so, $a - 1 + a^2 - 2a - 1 = 18$

$a = 5, -4$ $\therefore$ $a = 5$

C-5. Sol. $N = \frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405} 3} = (\log_3 27 + \log_3 5) - \log_3 5 . \log_3 405$

$= (3 + \log_3 5) - \log_3 5 \log_3 (81 \times 5) = (3 + \log_3 5)(1 + \log_3 5) - \log_3 5(4 + \log_3 5) = 3$

C-6. Sol. $(\log_5 x)^2 + \log_{5x} = 1$ $\Rightarrow$ $(\log_5 x)^2 + \log_{5x} 5 - \log_{5x} x = 1$

$\Rightarrow$ $(\log_5 x)^2 + \frac{\log_5 5}{\log_5 5 + \log_5 x} - \frac{\log_5 x}{\log_5 5 + \log_5 x} = 1$

$\Rightarrow$ $(\log_5 x)^2 + \frac{1}{1 + \log_5 x} - \frac{\log_5 x}{1 + \log_5 x} = 1$

Let $\log_5 x = t$

$\therefore$ $t^2 + \frac{1}{1+t} - \frac{t}{1+t} = 1$ $\Rightarrow$ $\frac{t^2(1+t) + 1 - t}{1+t} = 1$ $\Rightarrow$ $t^3 + t^2 + 1 - t = 1 + t$

$t^3 + t^2 - 2t = 0$; $t(t^2 + t - 2) = 0$; $t(t-1)(t+2) = 0$

$t = 0, 1, -2$

$\therefore$ $\log_5 x = 0, 1, -2$ $\therefore$ $x = 1, 5,$

C-7. Sol. $\log_{x^2} 16 + \log_{2x} 64 = 3$ $\Rightarrow$ $4 \log_{x^2} 2 + 6 \log_{2x} 2 = 3$

$\Rightarrow$ $\frac{4}{\log_2 x^2} + \frac{6}{\log_2 2x} = 3$ $\Rightarrow$ $\frac{2}{\log_2 x} + \frac{6}{1 + \log_2 x} = 3$

but $\log_2 x = t$

$\therefore$ $\frac{2}{t} + \frac{6}{1+t} = 3$ $\Rightarrow$ $2 + 2t + 6t = 3t + 3t^2$

$\Rightarrow$ $3t^2 - 5t - 2 = 0$ $\Rightarrow$ $3t^2 - 5t - 2 = 0$

$\Rightarrow$ $(3t + 1)(t - 2) = 0$

$\Rightarrow$ $t = -\frac{1}{3}$, $t = 2$

$\Rightarrow$ $\log_2 x = -\frac{1}{3}$ $\log_2 x = 2$

$\Rightarrow$ $x = 2^{-1/3}$ $x = 4$

$= \frac{1}{2^{1/3}}$.

C-8. **Sol.** $\log_3 x + \log_3 y = 2 + \log_3 2$, $\quad \log_{27}(x + y) = \frac{2}{3}$

$\Rightarrow \log_3 xy = \log_3 9 + \log_3 2, \Rightarrow \quad x + y = (27)^{2/3}$

$\Rightarrow xy = 18$, $\quad \Rightarrow x + y = 9$

$\Rightarrow x = 6$ or $x = 3$

$y = 3, y = 6$

as $x > 0, y > 0$. $\quad (x > 0 , y > 0)$

C-9 **Sol.** $\frac{1}{2} \leq \log_{1/10} x \leq 2 \quad \Rightarrow \quad \frac{1}{100} \leq x \leq \frac{1}{\sqrt{10}}$

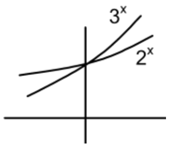
C-10. **Sol.** $\log_2 3 > 1,\ \log_{12} 10 < 1 \quad \Rightarrow \quad \log_2 3 > \log_{12} 10$

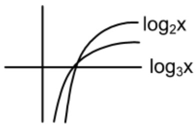
$\log_6 5 < 1,\ \log_7 8 > 1 \quad \Rightarrow \quad \log_6 5 < \log_7 8$

$\log_3 26 < 3, \log_2 9 > 3 \quad \Rightarrow \quad \log_3 26 < \log_2 9$

$\log_{16} 15 < 1, \log_{10} 11 > 1 \quad \Rightarrow \quad \log_{16} 15 < \log_{10} 11$

C-11 **Sol.**





Exercise-3

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. **Sol.** We have, $A \cup B = A \cup C \quad \Rightarrow \quad (A \cup B) \cap C = (A \cup C) \cap C$

$\Rightarrow (A \cap C) \cup (B \cap C) = C \quad [\because (A \cup C) \cap C = C]$

$\Rightarrow (A \cap B) \cup (B \cap C) = C \quad$ ...(i) $\quad [\because A \cap C = A \cap B]$

Again, $A \cup B = A \cup C$

$\Rightarrow (A \cup B) \cap B = (A \cup C) \cap B \quad \Rightarrow \quad B = (A \cap B) \cup (C \cap B)$

$\Rightarrow (A \cap B) \cup (C \cap B) = B \Rightarrow \quad (A \cap B) \cup (B \cap C) = B \quad$ ...(ii)

From (i) and (ii), we get $B = C$

2. **Sol.** Every element has 3 options. Either set Y or set Z or none

so number of ordered pairs = 3^5

3. **Sol.** $X = \{0, 9, ... 4^n - 3n - 1\}$

$Y = \{0, 9, ..., 9(n - 1)\}$

Now $4^n - 3n - 1 = (3 + 1)^n - 3n - 1$

$= 3^n + n.3^{n-1} + ... + {}^nC_2 .9.$

is a multiple of 9.

Also Y consists of all multiples of '9' from 0, 9,.......

Hence all values of X are subset of values of Y.

Thus $X \cup Y = Y$

4.* **Sol.** Number of subsets = 2^m

also number of subsets = 2^n

$2^m - 2^n = 56$

$m = 6 \quad n = 3$

$(m, n) = (6, 3)$

5. **Ans.** **(4)**

Sol. $(x^2-5x+5)^{x^2+4x-60} = 1$

$x^2 - 5x + 5 = 1$
$x^2 - 5x + 4 = 0$
$x = 1, x = 4$

$x^2 + 4x - 60 = 0$
$x = -10, x = 6$

$x^2 - 5x + 5 = -1$
$x^2 - 5x + 6 = 0$
$x = 2, 3$

at $x=2$ $x^2 + 4x - 60 = -48$ (even)
$\therefore x=2$ is valid
at $x=3$ $x^2 + 4x - 60 = -39$ (odd)
$\therefore x = 3$ is invalid
$x = 1, 2, 4, 6, -10$

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. **Sol.** $\frac{1}{2}\log_2(x-1) = \log_2(x-3) \Rightarrow \sqrt{x-1} = x-3$

$(x-1) = x^2 - 6x + 9 \Rightarrow x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$ but $x \neq 2$

$\therefore x = 5$

2. **Ans.** **(3)**

Sol. $(2x)^{\ell n2} = (3y)^{\ell n3}$

$\Rightarrow \ell n2\ \ell n(2x) = \ell n3\ \ell n(3y) = \ell n3\ (\ell n3 + \ell ny)$ .... (1)

also $3^{\ell nx} = 2^{\ell ny}$

$\Rightarrow \ell nx\ \ell n3 = \ell ny\ \ell n2$ .... (2)

by (1) $\Rightarrow \ell n2\ \ell n(2x) = \ell n3\ (\ell n3 + \ell ny) \Rightarrow \ell n2 . \ell n(2x) = \ell n3 \left\{\ell n3 + \frac{\ell nx \ell n3}{\ell n2}\right\}$

$\Rightarrow \ell n^2 2\ \ell n2x = \ell n^2 3\ (\ell n2 + \ell nx) \Rightarrow (\ell n^2 2 - \ell n^2 3)\ (\ell n2x) = 0 \Rightarrow \ell n2x = 0 \Rightarrow x = \frac{1}{2}$

Fundamental of Mathematics - I

Additional Problems For Self Practice (APSP)

1. **Sol.** **Obvious**

2. **Sol.** **Obvious**

3. **Sol.** **Obvious**

4. **Sol.** $\frac{1}{x+1} - \frac{1}{x-2} > 0$

$\frac{(x-2)-(x+1)}{(x+1)(x-2)} > 0 \Rightarrow \frac{-3}{(x+1)(x-2)} > 0 \Rightarrow \frac{3}{(x+1)(x-2)} < 0$

5. **sol.** N = 6^{50}

logN = 50 [log6] = 50 [log2 + log3] = 50[0.3010 + 0.4771] = 50 (0.7781) = 38.09

characterstic = 38

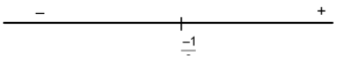
No. of digits = 39

6. **Sol.** $(x-3)^2 = 4x - 15 \Rightarrow x^2 - 10x + 24 = 0$

x = 6, 4 x = 4 (reject)

7. **Sol.** $\frac{2x-4}{2x+1} > 1 \Rightarrow \frac{(2x-4)-(2x+1)}{2x+1} > 0$

$\frac{1}{2x+1} < 0$



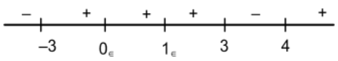
$(-\infty, -1/2)$

8. **Sol.** $x^2 - 2x - 3 = 0$

x = –1, 3 ⇒ x = –1 (reject) x = 3

9. **Sol.** $x + \frac{1}{x} = 3 \Rightarrow x^2 + \frac{1}{x^2} + 2 = 9 \Rightarrow x^4 + \frac{1}{x^4} + 2 = 49$

10. **Sol.**



$x \in (-3, 0) \cup (0, 1) \cup (1, 3) \cup (4, \infty)$

11. **Sol.** $(9x^2 + 6x + 1) + (y^2 - 2y + 1) = 0$

$(3x+1)^2 + (y-1)^2 = 0$

$x = \frac{-1}{3}$; y = 1

12. **Sol.** $\log_2\left(\frac{2^{28}.\ 5^{10}.3^{12}.}{5^{14}.2^{15}.3^5.2^{12}.\ 5^3.}\right) \Rightarrow \log_2\left(2.\left(\frac{3}{5}\right)^7\right) \Rightarrow 1 + 7\log_2(3/5)$

13. **Sol.** let $\log_3 x = t$, $t = (9t - 15)\left(\frac{1}{1+t}\right)$

$t^2 + t = 9t - 15$

$t^2 - 8t + 15 = 0$

t = 3, 5 ⇒ x = 27, 243

14. N = 3 ∈[81, 243)
N = $5^{2+\beta_2}$ ∈[25, 125)
common : N∈[81, 125)

15. **Sol.** $\log_2(3x) - 1 > 0$

$\log_2(3x) > 1 \Rightarrow (3x) > 2 \Rightarrow \log_2(3x) < 2 \Rightarrow x < 4/3$

16. **Sol.** $f\left(\frac{-2}{3}\right) = \frac{-32}{243} - 4\left(\frac{16}{81}\right) + 3\left(\frac{-8}{27}\right) - 2\left(\frac{4}{9}\right) - \frac{2}{3} + 2 = \frac{-32-192-216-216-162+486}{243}$

17. **Sol.** (2x –y) (x+2y) = 7

case-I 2x – y = 1 x + 2y = 7 ⇒ (x, y) = $\left(\frac{9}{5}, \frac{13}{5}\right)$
case-II 2x – y = 7 x + 2y = 1 ⇒ (x, y) = (3, –1)
case-III 2x – y = –1 x + 2y = –7 ⇒ (x, y) = (–9/5, –13/ 5)
case-IV 2x – y = –7 x + 2y = –1 ⇒ (x, y) = (–3, 1)

18. **Sol.** $\log_{0.3} x \geq -6 \Rightarrow x \leq (0.3)^{-6}$

$x \leq \left(\frac{10}{3}\right)^6$

$\log_{0.3} x \leq -4 \Rightarrow x \geq \left(\frac{10}{3}\right)^4$

19. **Sol.** $e^x + 1 > 0$
(Always)
$3^x + 4 > 0$

– –2 +

x∈(–2, ∞)

20. **Sol.** $\sqrt{3x+1} = 1 + \sqrt{(x+4)}$

$3x + 1 = 1 + (x + 4) + 2\sqrt{(x+4)}$
$(x-2)^2 = (x + 4)$
$x^2 - 4x - x = 0$
x(x – 5) = 0
x = 0 x = 5
Crosscheck solutions ⇒ x = 0 (reject) x = 5

21. **Sol.** x = 0 $\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix} = d$
Expanding the determinant w.r.t. first row
1(–2–10) + 3(8–6)+2 (20+3)
– 12 + 6 + 46 = 40

22. **Sol.** $\frac{3.5^{2x}}{5} - \frac{2.5^x}{5} = 0.2$
Let $5^x = t$:
$3t^2 - 2t = 1$
$3t^2 - 3t + t - 1 = 0$
(3t +1) (t –1) = 0

$t = \frac{-1}{3}$; $t = 1$

$x = 0$

23. **Sol.** + – + +

$x \in (-\infty, -1) \cup (3, 4) \cup (4, \infty)$

24. **Sol.** $M = \log_4 128 = 7/2$ $\Rightarrow$ $m = \log_8 16 = 4/3$

25. **sol.** Let $\log_5 x = t$

$t + \frac{1}{t} = \frac{10}{3}$ $\Rightarrow t = 3, \frac{1}{3}$ $\Rightarrow x = 125, 5^{\frac{1}{3}}$

26. **Sol.** $x = 2^{\log 3 - \log 4}$

$y = 3^{\log 4 - \log 2}$

$z = 4^{\log 2 - \log 3}$

$xyz = \frac{2^{\log 3} \quad 3^{\log 4} \quad 4^{\log 2}}{2^{\log 4} \quad 3^{\log 2} \quad 4^{\log 3}} = 1$

27. **Sol.** $\sqrt{14 - 6\sqrt{5}} = \left(3 - \sqrt{5}\right)$ $\Rightarrow$ $6 - 3\sqrt{5} + \sqrt{14 - 6\sqrt{5}} = 9 - 4\sqrt{5} = \left(\sqrt{5} - 2\right)^2$

$2 + \sqrt{5} - \sqrt{6 - 3\sqrt{5} + \sqrt{14 - 6\sqrt{5}}} = 4$

28. **sol.** $r(x) = ax + b$

$R(1) = 2 = a + b$

$R(-1) = -2 = -a + b$

$(a, b) = (2, 0)$

29. **Sol.** let $\log x = t$

$(t + 2)^2 + (t + 1)^2 = 14 - t$

$2t^2 + 7t - 9 = 0$

$(t - 1)(2t + 9) = 0$

$t = 1$; $t = -9/2$

30. **Sol.** $\log_7 (35.\ 14/10) = 2$

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Fundamental of Mathematics - I

MATHEMATICS

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

1.
Sol. $\log_{50} 96 = \log_{50} 32 + \log_{50}(3)$ $\Rightarrow$ $\alpha - \beta = 5\log_{50}(2)$

2. **Sol.** (ii) $7^{\log_{11}3} = 3^{\log_{11}7}$

3. **Sol.** $x^{(\log_3 4)^2} = 27^{\log_3 4}$

4. **Sol.** **Obvious**

5. **Sol.** **Obvious**

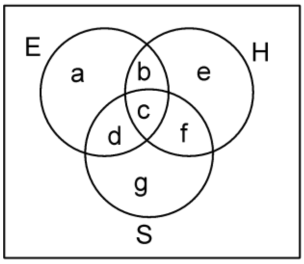
6. **Sol.** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$0 \le n(A \cap B) \le 5$
$7 \le n(A \cup B) \le 12$

7. **Sol.** A = {2,3,5,7,11, 13, 17, 19,23}
B = {4,6,8,9,10,12,14,15,16,18}
$n(A \cup B) = 19$, $n(A \cap B) = 0$
$n(A \cap B') = 9$

8. **Sol.** $A = \varphi$
$P(A) = \{\varphi\} \Rightarrow n(P(A)) = 1$
$n(P(P(\varphi))) = 2^1$
$n(P(P(P(\varphi)))) = 2^2 = 4$
$n(P(P(P(P(\varphi))))) = 2^4 = 16$

9. **Sol.** $A' \cup \{(A \cup B) \cap B'\}$
$A' \cup \{(A \cap B') \cup (B \cap B')\}$
$A' \cup (A \cap B') = (A' \cup A) \cap (A' \cup B') = A' \cup B' = (A \cap B)'$

10. **Sol.** a = 18 , a + d = 23, c + d = 8
c + f = 8, a + b + c + d = 26
c + d + f + g = 48
a + b + c + d + e + f + g = 100 – 24 = 76
a = 18, d = 5, c = 3, f = 5 , b = 0
g = 48 – (3 + 5 + 5) = 35
e = 76 – (18 + 0 + 3 + 5 + 5 + 35) = 10
Now b + c + e + f = 0 + 3 + 10 + 5 = 18



11. **Sol.** $n(2) = \left[\frac{1000}{2}\right] = 500$
$n(3) = \left[\frac{1000}{3}\right] = 333$
$n(5) = \left[\frac{1000}{5}\right] = 200$
$n(2 \cap 3) = 166, n(3 \cap 5) = 66$
$n(5 \cap 2) = 100$ $n(2 \cap 3 \cap 5) = 33$, $n(2 \cup 3 \cup 5) = 734$
$n(2' \cap 3' \cap 5') = 1000 - 734 = 266$

12. **Sol.** $80 = 40 + 50 + 60 - 2(n(A \cap B) + n(B \cap C) + n(C \cap A)) + 30$
$\Rightarrow n(A \cap B) + n(B \cap C) + n(C \cap A) = 50$
Required number of members $T = n(A \cap B) + n(B \cap C) + n(C \cap A) - 2n(A \cap B \cap C)$
$= 50 - 2 \times 10 = 30$

13. **Sol.** $0 \le n(A \cap B) \le \min.\{n(A), n(B)\}$
$0 \le n(A \cap B) \le 12$
$n(A' \cap B) = n(B) - n(A \cap B)$
$3 \le n(A' \cap B) \le 15$
$\Rightarrow x = 3, y = 15$

14. **Sol.** $X \cap (X \cup Y)' = X \cap (X' \cap Y') = (X \cap X') \cap Y'$
$= \varphi \cap Y' = \varphi$

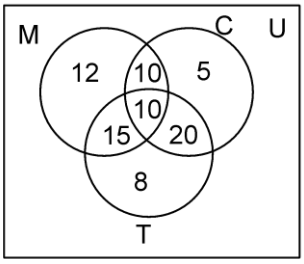
15. **Sol.** $N_3 \cap N_5 = N_{15}$ [$\because$ 3 and 5 are relatively prime numbers)

16. **Sol.** $\therefore x^2 + 4y^2 = 45$
We can see that $x = \pm 3$ $y = \pm 3$
(3,3) (3,– 3), (–3,–3) (–3,3) are 4 elements

17. **Sol.** $2^m + 2^n = 144$
$2^n\{2^{m-n} + 1\} = 2^4 \times 3^2$
$n = 4$, $m - n = 3$
$n = 4$, $m = 7$

18. **Sol.** Every element of X have 3 options
either in Y or in Z or none
so number of ordered pairs = 3^4

19. **Sol.** $n(U) = 100$
$n(M \cap C \cap T) = 10; n(M \cap C) = 20;$
$n(C \cap T) = 30; n(M \cap T) = 25;$
n(M only) = 12; n(only C) = 5; n(only T) = 8



∴ $n(M \cap C \cup T) = 12+10+5+15+10+20+8 = 80$

∴ $n(M \cap C \cup T)' = 100 - 80 = 20$

20. **Sol.** Subsets = $2^9 - 1 - 1$
= 510

21. **Sol.** $\frac{(x-6)^3(x-1)^4(3-x)}{(x+3)x^5} \geq 0$

$\frac{(x-6)^3(x-1)^4(x-3)}{(x+3)x^5} \leq 0$

$$\begin{array}{cccccccccccc} + & & - & & + & & + & & - & & + \\ & -3 & & 0 & & 1 & & 3 & & 6 & \end{array}$$

$x \in (-3, 0) \cup [3, 6] \cup \{1\}$

positive integral solutions are x = 1, 3, 4, 5, 6

/kukRed iw.kkZd gy x = 1, 3, 4, 5, 6

22. **Sol.** $(\log_{10}8)x^2 - (\log_{10}5)x + x - 2\log_{10}2 = 0$

(1) sum of roots = $\frac{-(1-\log_{10}5)}{\log_{10}8} = \frac{-\log_{10}\left(\frac{10}{5}\right)}{\log_{10}8} = \frac{-\log_{10}2}{3\log_{10}2} = \frac{-1}{3}$ =rational

(2) Product of roots = $\frac{-2\log_{10}2}{\log_{10}8} = \frac{-2}{3}$ = rational

(3) sum of cofficient = $\log_{10}8 - \log_{10}5 + 1 - \log_{10}4 = \log_{10}\left(\frac{8\times10}{5\times4}\right)$
= $\log_{10}4$ = irrational

(4) Difference of roots = $\sqrt{\left(-\frac{1}{3}\right)^2 - 4.\left(-\frac{2}{3}\right)} = \sqrt{\frac{1}{9}+\frac{8}{3}} = \sqrt{\frac{25}{9}} = \frac{5}{3}$ =rational

23. **Sol.** $2\log_2\log_2 x + \log_{1/2}\log_2\left(2\sqrt{2}\ \ x\right) = 1$

⇒ $\log_2(\log_2 x)^2 - \log_2\log_2\left(2\sqrt{2}\ \ x\right) = 1$

⇒ $\log_2\frac{(\log_2 x)^2}{\log_2\left(2\sqrt{2}\ \ x\right)} = 1$ ⇒ $\frac{(\log_2 x)^2}{\frac{3}{2}+\log_2 x} = 2$

let $\log_2 x = y$

∴ $y^2 - 2y - 3 = 0$ ⇒ $(y-3)(y+1) = 0$

∴ $y = 3, -1$ ⇒ $\log_2 x = 3, -1$,

but $\log_2 x > 0$
$\therefore$ $\log_2 x = -1$ is not possible
$\Rightarrow$ $x = 8$

NEW Type

24._ **Sol.** $\because$ a + b + c is also divisible by 7

and for least,

a = 1, b = 2, c = 4

25._ **Sol.** $f(x) = K(x-1)(x-2)(x-3) + 2x$

$f(4) = K(6) + 8 = 2 \Rightarrow K = -1$

$f(0) = -6K \Rightarrow 6$

26. **Sol.** $\log_4\left(\frac{5}{4}\cdot\frac{6}{5}\cdot\frac{7}{6}\cdot\frac{8}{7}\cdots\cdots\frac{64}{63}\right) = \log_4\left(\frac{64}{4}\right) = \log_4 16 = 2$

27. **Sol.** $\frac{x^4+x^2+1}{x^2-4x-5} < 0 \Rightarrow \frac{1}{(x-5)(x+1)} < 0$

+ – +
–1 5

$x \in (-1, 5)$

28._ **Sol.** $(x^2 + x + 1) + x(x - 1)$
$\Rightarrow 2x^2 + 1$

29._ **Sol.** $x = \frac{2\left(\sqrt[8]{3}-1\right)}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)} = \sqrt[8]{3} - 1$

30. **Sol.** $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc \text{ and } ca)$
$a^2 + b^2 + c^2 = 1 + 16 = 17$
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
$a^3 + b^3 + c^3 = 3(-12) + 1(17 + 8) = -36 + 25 = -11$

31. **Sol.** $\log_3(1 + \log_3(2_x - 7)) = 1$

$1 + \log_3(2_x - 7) = 3$

$\log_3(2_x - 7) = 2$
$2_x - 7 = 3_2$
$2_x = 16$
$x = 4$
only one solution is possible.

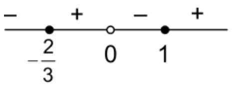
32. **Sol.** $\frac{2x+2}{x} \geq 3x + 1$

$\frac{2x+2}{x} - (3x+1) \geq 0$

$\frac{2x+2-3x^2-x}{x} \geq 0$

$$\frac{3x^2 - x - 2}{x} \le 0$$

$$\frac{(x-1)(3x+2)}{x} \le 0$$



$$x \in \left(-\infty, -\frac{2}{3}\right] \cup (0, 1]$$

positive integral solutions x = 1

sum of solutions = 1

33. **Sol.**

$3^x = 4^{x-1}$ $\Rightarrow$ $x = (x-1)\log_3 4$ $\Rightarrow$ $x(1 - 2\log_3 2) = -2\log_3 2$

$$x = \frac{2\log_3 2}{2\log_3 2 - 1}$$

34. **Sol.** $\dfrac{3}{5\log_4 1600} + \dfrac{2}{5\log_5 1600}$

$$\frac{3}{5}\log_{1600} 4 + \frac{2}{5}\log_{1600} 5 = \frac{1}{5}\left[3\log_{1600} 4 + 2\log_{1600} 5\right] = \frac{1}{5}\left(\log_{1600}(4^3 \times 5^2)\right) = \frac{1}{5}\left(\log_{1600} 1600\right) = \frac{1}{5}$$

35. **Sol.** $\log_{10}(-2x) = 2\log_{10}(x+1)$

$-2x = (x+1)^2$

$-2x = x^2 + 2x + 1$

$x^2 + 4x + 1 = 0$

$x = \dfrac{-4 \pm \sqrt{16-4}}{2} = -2 \pm \sqrt{3}$

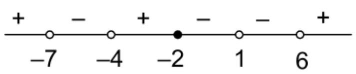
when $x = -2 - \sqrt{3}$ $\quad x + 1 = -1 - \sqrt{3} < 0$

so, $x = -2 - \sqrt{3}$ is not possible

only solution is $x = -2 + \sqrt{3}$

36. **Sol.** $\dfrac{(x-1)^3(x+2)^3}{(x+7)(x-1)(x-6)(x+4)} \le 0$

$\dfrac{(x-1)^2(x+2)^3}{(x+7)(x-6)(x+4)} \le 0$ $\qquad x \ne 1$



$x \in (-7, -4) \cup [-2, 1) \cup (1, 6)$

Integral values of x = –6, –5, –2, 0, 2, 3, 4, 5

sum = 1

37. **Sol.** $\log_4\left(\frac{(-2)^2}{4}\right) - 2\log_4(4(-2)_4) = \log_4 1 - 2\log_4 64 = 0 - 6 = -6$

38. **Sol.** $3_{72}.3_{-x}.\ 3^{-\sqrt{x}} > 3_0 \Rightarrow 3^{72-x-\sqrt{x}} > 3^0 \Rightarrow 72 - x - \sqrt{x} > 0$

$\sqrt{x} < 72 - x \Rightarrow$ Let $\sqrt{x} = t \Rightarrow t < 72 - t_2$

$t_2 + t - 72 < 0 \Rightarrow (t + 9)(t - 8) < 0 \Rightarrow t - 8 < 0$

$t < 8 \Rightarrow \sqrt{x} < 8 \Rightarrow x \in [0, 64)$

39. **Sol.** $0 < \frac{1+2x}{1+x} < 3$

$\frac{1+2x}{1+x} > 0$ and $\frac{1+2x}{1+x} - 3 < 0$

$x \in (-\infty, -1) \cup \left(-\frac{1}{2}, \infty\right)$ $\quad \frac{-2-x}{1+x} < 0$

$\frac{x+2}{x+1} > 0$

$\therefore \quad x \in (-\infty, -2) \cup \left(-\frac{1}{2}, \infty\right)$

$\alpha = -2$

$2\alpha = -4$

40. Sol. Let $\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\ldots\ldots}} = t \Rightarrow \sqrt{4 - \frac{1}{3\sqrt{2}}t} = t \Rightarrow 4 - \frac{1}{3\sqrt{2}}t = t_2 \Rightarrow$

$t_2 + \frac{1}{3\sqrt{2}}t - 4 = 0 \Rightarrow 3\sqrt{2}\ t_2 + t - 12\sqrt{2} = 0 \Rightarrow t = \frac{-1 \pm \sqrt{1 + 4 \times 3\sqrt{2} \times 12\sqrt{2}}}{2 \times 3\sqrt{2}} = \frac{-1 \pm 17}{2 \times 3\sqrt{2}}$

$t = \frac{16}{6\sqrt{2}}, \frac{-18}{6\sqrt{2}} \Rightarrow t = \frac{8}{3\sqrt{2}}, \frac{-3}{\sqrt{2}}$ and $\frac{-3}{\sqrt{2}}$ is rejected

so $6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}} \times \frac{8}{3\sqrt{2}}\right) = 6 + \log_{3/2}\left(\frac{4}{9}\right) = 6 + \log_{3/2}\left(\left(\frac{2}{3}\right)^2\right) = 6 - 2 = 4$

41. **Sol.** $x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3} \Rightarrow (\log_3 x)^3 - \frac{9}{2}\log_3 x + 5 = \log_x 3\sqrt{3}$

$\Rightarrow (\log_3 x)^2 - \frac{9}{2}\log_3 x + 5 = \frac{3}{2}\log_x 3$

Let $\log_3 x = t \Rightarrow t_2 - \frac{9}{2}t + 5 = \frac{3}{2t} \Rightarrow 2t_3 - 9t_2 + 10t - 3 = 0$

t = 1 satisfies it

$2t_3 - 9t_2 + 10t - 3 = 2t_2(t - 1) - 7t(t - 1) + 3(t - 1) = (t - 1) = (t - 1)(2t - 1)(t - 3)$

$\Rightarrow \quad t = 1 \qquad t = \frac{1}{2} \qquad t = 3$

$\Rightarrow \quad \log_3 x = 1 \qquad \log_3 x = \frac{1}{2} \qquad \log_3 x = 3$

$\Rightarrow \quad x = 3 \qquad x = 3_{1/2} \qquad x = 27.$