Exercise-1

Marked Questions may have for Revision Questions.

OBJECTIVE QUESTIONS

A-1. **Sol.** By definition of 'statement'.

A-2. **Sol.** By definition of 'statement'.

A-3. **Sol.** Obvious

A-4. **Sol.** A

A-5. **Sol.** Componant statements are (i) 100 is divisible by 5
(ii) 100 is divisible by 10 (iii) 100 is divisible by 11

A-6. **Sol.** If it is a holiday as well as sunday than also the office can be closed.

A-7. **Sol.** Statement (1) All prime numbers are even.
Statement (2) All prime numbers are odd.
Both false

A-8_. **Sol.** Obviously

A-9_. **Sol.** Obviously

A-10. **Sol.** The given statement can be written as :
"If it stays warm for a month, then the Banana trees will bloom"

A-11. **Sol.** $p \Rightarrow q$ means p is sufficient for q and q is necessary for p.

A-12.
Sol. Obvious

A-13. **Sol.** If p, then q
$\Rightarrow$ q is necessary for p and p is sufficient for q.

A-14. **Sol.** $T \Rightarrow F$ is F

A-15_ **Sol.** Obviously

A-16. **Sol.** (1) $p \wedge q = T \wedge T = T$
$(p \wedge q) \rightarrow S = T \rightarrow T = T$
(2) $(q \vee r) = T \vee T = T$, $\sim s = F$
$(q \vee r) \rightarrow \sim s = T \rightarrow F = F$
(3) $(p \wedge \sim q) = T \wedge F = F$
$q \rightarrow s = T \rightarrow T = T$
$(p \wedge \sim q) \wedge (q \rightarrow s) = F \wedge T = F$

A-17. **Sol.** Converse of statement $p \Rightarrow q$ is $q \Rightarrow p$.

A-18. **Sol.** $(\sim T \vee F) \wedge \sim T \Rightarrow T$
$\therefore$ $(F \vee F) \wedge F \Rightarrow T$
$\therefore$ $F \wedge F \Rightarrow T$ $\therefore$ $F \Rightarrow T$

A-19. Sol.

p	q	~q	p ↔ ~q
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	F

A-20. Sol. "p only if q" is equivalent to "if p then q"

A-21. Sol.

p	q	~p	p ↔ q	~(p↔q)	~p↔q
T	T	F	T	F	F
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	T	F	F

A-22. Sol. $\sim p \rightarrow q$

$(\sim(\sim p) \vee q) \wedge ((\sim p) \vee \sim q)$

$(p \vee q) \wedge (\sim p \vee \sim q)$

$(\sim p \rightarrow q) \wedge (\sim q \rightarrow p)$

A-23. Ans (2)

Sol.

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$(p \wedge (p \rightarrow q)) \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Hence tautology

A-24. Sol. $(p \rightarrow q) \wedge (q \rightarrow p) \equiv (p \leftrightarrow q)$

$(p \rightarrow \sim p) \wedge (\sim p \rightarrow p) \equiv (p \leftrightarrow \sim p) \equiv$ Fallacy/contradiction

A-25. Sol. By using truth table

A-26. Sol. (1)

$p \wedge p = p$

$p \vee (\sim p) = t$

$p \wedge (\sim p) = f$

$p \vee (\sim p) = t$

Section (B) : Negation of compound statements, Contrapositive of conditional statements, Quantifiers

B-1. Sol. Let P : Ramesh is cruel

q : He is strict

∴ $p \vee q$: Ramesh is cruel or he is strict

$\sim(p \vee q) = \sim p \wedge \sim q$

∴ So negation is Ramesh is neither cruel nor strict.

B-2. Sol. The component statements of the given statement are :

p : The sand heats up quickly in the sun.

q : The sand does not cool down fast at night.

The given statement is (p and q). So its negation is

~p or ~q = Either the sand does not heat up quickly in the sun or it cools down fast at night.

B-3. Sol. Let p and q be the statements as given below

p : a quadrilateral is a square

q : a quadrilateral is a rhombus

the given statement is $p \rightarrow q$

$\therefore \sim(p \rightarrow q) \equiv p \wedge \sim q$

Therefore the negation of the given statement is a quadrilateral is a square and it is not a rhombus.

B-4. **Sol.** Negation of $(p \rightarrow q)$ is $(p \wedge \sim q)$

B-5. **Sol.** Let "p : Two lines are parallel" and "q : they have the same slope".

Then, the given statement in symbolic form is $p \leftrightarrow q$.

Now, $\sim(p \leftrightarrow q)$ $(p \wedge \sim q) \wedge (\sim p \wedge q)$

$\Rightarrow$ $\sim(p \leftrightarrow q)$: Either two lines are parallel and they have different slopes or two lines are not parallel and they have the same slope.

B-6. **Sol.** Negation of $p \leftrightarrow q$ is $p \leftrightarrow \sim q$

B-7. **Sol.** $\sim(p \rightarrow (q \wedge r)) \equiv p \wedge \sim(q \wedge r)$ $(\because \sim(p \rightarrow q) \equiv p \wedge \sim q)$

$\equiv p \wedge (\sim q \vee \sim r)$

B-8. **Sol.**

p	q	$\sim(\sim p \rightarrow q)$	$(p \leftrightarrow q)$	$\sim(p \vee q)$	$p \vee \sim p$	$\sim p \vee q$	$p \vee \sim q$	$(\sim p \vee q) \wedge (p \vee \sim q)$	$(p \wedge \sim q) \vee (\sim p \wedge q)$
T	F	F	F	F	T	F	T	F	T
F	T	F	F	F	T	T	F	F	T
T	T	F	T	F	T	T	T	T	F
F	F	T	T	T	T	T	T	T	F

B-9. **Sol.**

A	B	$\sim B$	$A \vee \sim B$	$A \rightarrow (A \vee \sim B)$	$\sim(A \rightarrow (A \vee \sim B))$	$(A \vee B) \rightarrow A$	$A \wedge \sim B$	$A \rightarrow (A \wedge \sim B)$
T	T	F	T	T	F	T	F	F
T	F	T	T	T	F	T	T	T
F	T	F	F	T	F	T	F	T
F	F	T	T	T	F	T	F	T

B-10. **Sol.** We know $\sim(p \rightarrow q) \equiv p \wedge \sim q$

Given statement is $p \wedge q$

$= p \wedge \sim(\sim q) \equiv \sim(p \rightarrow \sim q)$

B-11. **Sol.** $(p \wedge r) \wedge (\sim r \wedge \sim q) = p \wedge (r \wedge \sim r) \wedge \sim q = p \wedge f \wedge \sim q = f$

B-12. **Sol.** You watch television if and only if your mind is free.

B-13. **Sol.** If something does not have low temperature, then it is not cold.

B-14. **Sol.** Let p, q, r be the three statements such that

p : x = 5, q : y = –2 and r : x – 2y = 9

Here given statement is $(p \wedge q) \rightarrow r$ and its contrapositive is $\sim r \rightarrow \sim(p \wedge q)$

i.e. $\sim r \rightarrow (\sim p \vee \sim q)$

i.e. if $x - 2y \neq 9$ then $x \neq 5$ or $y \neq -2$

B-15. (**Ans.** **(1)**

Sol. p ≡ The side of a square doubles

q ≡ Area of square increases four time

so the contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim P$

B-16. **Ans.** **(4)**

Sol. Let p : It is raining

q : I will not come

contrapositive of $p \rightarrow q$

is $\sim q \rightarrow \sim p$
$\Rightarrow$ If I will come then if is not raining

B-17. Sol. $\sim q \rightarrow \sim p$ (Given statement) $\Rightarrow$ then $p \rightarrow q$

B-18. Sol. Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

B-19 Sol. Contrapositive of $(p \wedge q) \Rightarrow r$ is $\sim r \Rightarrow \sim(p \wedge q) = \sim p \vee \sim q$ [De Morgan's law]

B-20. Sol. $\sim(\sim q \rightarrow \sim r) \rightarrow \sim p \equiv (\sim q \wedge q) \rightarrow \sim p$

B-21. Sol. At least one natural number is not greater than 0.

B-22. Sol. The negation of ''Everyone in Germany speaks German'' is - there is at least one person in Germany who does not speak German.

Exercise-2

Marked Questions may have for Revision Questions.

PART - I : OBJECTIVE QUESTIONS

1. Sol. Product of two irrational numbers may or may not be irrational.

2. Sol. p is false, q is false so $p \rightarrow q$ is true.

3. Sol. Given statement is : Let p : it is a national holiday, q : kids go to picnic with their parents.
$\therefore$ Given statement is $p \rightarrow q$
The converse of $p \rightarrow q$ is $q \rightarrow p$
i.e. "If kids go to picnic with their parents, then it is a national holiday".

4. Sol. converse of $p \Rightarrow (q \wedge \sim r)$ is $(q \wedge \sim r) \Rightarrow p$
$\equiv \sim(\sim q \vee r) \Rightarrow p$
$\equiv \sim(r \vee \sim q) \Rightarrow p$

5. Sol. If p then q means p only if q

6. Sol. If p then q
$\Rightarrow$ p is sufficient for q

7. Sol. Let p be the statement "Traders do not reduce the prices" and q be the statement "Government takes action against them"
The first statement in symbolic form is $p \rightarrow q$ and the second statement is $\sim(p \wedge \sim q)$.
In order to check the equivalence of the above statements let us prepare the following truth table.

p	q	$\sim q$	$p \wedge \sim q$	$\sim(p \wedge \sim q)$	$p \rightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	T

Clearly, $\sim q \rightarrow \sim p$ and $\sim(p \wedge \sim q)$ have same truth values for all the values of p and q. Hence, the two statements are equivalent.
Aliter : We have,
$\sim(p \wedge \sim q) \equiv (\sim p \vee q) \equiv (p \rightarrow q)$
Hence the two statements are equivalent.

8. **Sol.** $p \wedge (\sim p \vee q) = (p \wedge \sim p) \vee (p \wedge q)$
$f \vee (p \wedge q) = p \wedge q$

9. **Sol.** $(p \wedge q) \vee \sim p = (p \vee \sim q) \wedge (q \vee \sim p)$
$= t \wedge (q \vee \sim p) = q \vee \sim p = \sim p \vee q$

10. **Sol.**

p	q	~p	~q	~q ∧ p	pv~p	(~q ∧ p)v(pv~p)
T	T	F	F	F	T	T
T	F	F	T	T	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

11. **Sol.**

p	q	r	p↔q	q→r	(q→r)∧r	[(p↔q)∧((q→r)∧r)]	[(p↔q)∧((q→r)∧r)]→r
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	F	T	T	F	T
F	T	F	F	F	F	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	F	F	T

12. **Sol.**

p	q	~q	p ∨ ~q	~(p ∨ ~q)	p ∨ q	~(p ∨ q)	~(p ∨ ~q) ∨ ~(p ∨ q)
T	T	F	T	F	T	F	F
T	F	T	T	F	T	F	F
F	T	F	F	T	T	F	T
F	F	T	T	F	F	T	T

13. **Sol.** $p \Rightarrow (q \vee r) \equiv (p \Rightarrow q) \vee (p \Rightarrow r)$

14. **Sol.** $\sim [p \vee (\sim p \vee q)] = (\sim p) \wedge [\sim(\sim p \vee q)] = (\sim p) \wedge (p \wedge \sim q) = (\sim p \wedge p) \wedge (\sim q) = f \wedge (\sim q) = f$

15. **Sol.** Given Statement is
(ΔABC is right angled at B) $\Rightarrow$ ($AB_2 + BC_2 = AC_2$)
Its negation is
(In ΔABC, $AB_2 + BC_2 = AC_2$) $\Rightarrow$ ΔABC is not right angled at B.

16. **Sol.** $\sim(p \vee q) \equiv \sim p \rightarrow q$

17. **Sol.** $\sim (\sim p \rightarrow q) \equiv \sim (\sim(\sim p) \vee q)$
$\equiv \sim (p \vee q)$
clearly option (3) is correct
$\sim (p \vee q) \wedge (p \vee (\sim p))$
$\equiv \sim (p \vee q) \wedge T$
$\equiv \sim(p \vee q)$

18. **Sol.** Statement $p \rightarrow q$ and its contrapositive $\sim q \rightarrow \sim p$ are logically equivalent and give same meaning.

19. **Sol.** Statement P is False
Statement Q is True.
$V_1 \equiv F$
$V_2 \equiv T$

20. **Sol.** Contrapositive of statement $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$

21. **Sol.** Contrapositive of $(p \rightarrow q)$ is $(\sim q \rightarrow \sim p)$

22. **Sol.** By using concept of quantifiers

PART - II : MISCELLANEOUS QUESTIONS

A-1. **Ans.** (1)
Sol. Statement - 1 :

A	B	~B	A ↔B	A ↔ ~ B	~ (A ↔ B)
T	F	T	F	T	F
F	T	F	F	T	F
T	T	F	T	F	T
F	F	T	T	F	T

so statement - 1 is true
Statement -2
A ∨ (~(A ∧ ~B))
A ∨ (~A ∨ B)
= (A ∨ ~ A) ∨ B
= t ∨ B
= t
Statement (2) is true.

A-2. **Ans.** (2)
Sol. S1 : (p → q) ↔ (p ∨ ~q) ≡ (~p ∨ q) ↔ (~p ∨ q) ≡ tautology so S1 is true
S2 : ~(~p ∧ q) ∧ (p ∨ q) ↔ p
≡ (p ∨ ~q) ∧ p) ∨ ((p ∨ ~q) ∧ q) ↔ p ≡ p ∨ q ↔ p
true when p is true & q is false
S2 is incorrect

A-3. **Ans.** (2)
Sol. ~ (pvq) = ~ p ^ ~ q = Sachin Tendulakar is not a good cricketer and Mukesh Ambani is not a rich person in india So statement-1 is correct and statement 2 is false.

A-4. **Ans.** (2)
Sol. Truth table for the logical statements in statement-1

p	q	~q	p ↔ ~q	~(p ↔ ~q)	p ↔ q
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	F

∴ ~(p ↔ ~q) and p ↔ q are identical
Also ~ (p ↔ ~q) is not a tautology as all entries in its column are not T.
∴ statement-1 in true but statement-2 is false

A-5. **Ans.** (1)
Sol. Statement-2 is correct definition of inclusive OR and also the correct reason of statement-1

Section (B) : MATCH THE COLUMN

B-1_. **Ans.** (A) → (r), (B) → (p), (C) → (s), (D) → (q)
Sol. (A) ~(~p ∧ q) = ~(~p) ∨ ~ q = p ∨ ~q (using Demorgan law)
(B) p ∧ (p ∨ q) = (p ∧ p) ∨ (p ∧ q) = p ∨ (p ∧ q)
(C) (p ∧ q) ∨ [~ p ∨ (p ∧ ~ q)]
= (p ∧ q) ∨ [(~ p ∨ p) ∧ (~ p ∨ ~q)]
= (p ∧ q) ∨ [t ∧ (~ p ∨ ~ q)]

$= (p \wedge q) \vee (\sim p \vee \sim q)$
$= (p \wedge q) \vee [\sim (p \wedge q)] = t$
also $(\sim p \wedge q) \vee t = t$

(D)

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. Sol. By definition of 'statement'.

C-2. Sol. Polygon cannot be both concave and convex

C-3. Sol. x cannot be both rational and irrational

C-4_ Sol. (1)

p	q	$p \Rightarrow q$	$\sim(p \Rightarrow q)$	~p	~q	~p v ~q	$\sim(p \Rightarrow q) \Leftrightarrow (\sim p v \sim q)$
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	T	F	T	F
F	F	T	F	T	T	T	F

so statement is not a fallacy blfy, dFku fojksf/kf}r ugha gS&

(2) $(p \wedge \sim q) \wedge (\sim p \vee q) = ((p \wedge \sim q) \wedge \sim p) \vee ((p \wedge \sim q) \wedge q) = ((p \wedge \sim p) \vee \sim q) \vee (p \wedge (\sim q \wedge q))$
$= (f \wedge \sim q) \vee (p \wedge f) = f \vee f = f$

(3) $p \wedge \sim p = f$

(4) $(p \wedge \sim q) \wedge (\sim p \wedge q) = (p \wedge \sim p) \wedge (\sim q \wedge q) = f \wedge f = f$

C-5_ Sol. Obviously

Exercise-3

Marked Questions may have for Revision Questions.

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Sol. Given statement $r = p \Leftrightarrow q$
Statement - 1 : r_1 = (pq) is not equilvalent to $p \Leftrightarrow q$
Statement - 2 :

p	q	~q	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$	$p \leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

Hence statement - 1 is false and statement -2 is true

2.

Sol.

p	q	$p \vee q$	$p \wedge q$	$p \to q$	$p \leftrightarrow q$	$(q \to p)$	$p \to (q \to p)$	$p \to (p \leftrightarrow q)$	$p \to (p \to q)$	$p \to (p \vee q)$	$p \to p \wedge q$
T	T	T	T	T	T	T	T	T	T	T	T
T	F	T	F	F	F	T	T	F	F	T	F
F	T	T	F	T	F	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T	T	T	T

3. Sol. Statement-1 :

p	q	$p\leftrightarrow\sim q$	$\sim(p\leftrightarrow\sim q)$	$p\leftrightarrow q$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

Statement-2 : False.

4.
Ans. **(2)**
Sol. P : At least one rational number $x \in S$ such that $x > 0$
Negation : all rational numbers $x \in S$ are $x \le 0$

5. **Sol.** Negation of $(P\wedge\sim R) \leftrightarrow Q$ is $((P\wedge\sim R) \leftrightarrow Q)$ ~ $((P\wedge\sim R)\leftrightarrow Q)$
It may also be written as ~ $(Q\leftrightarrow(P\wedge\sim R))$

6. **Sol.**

A	B	$A\vee B$	$A\wedge B$	$A\wedge(A\vee B)$	$A\vee(A\wedge B)$	$A\to B$	$A\wedge(A\to B)$	$A\wedge(A\to B)\to B$	$B\to[A\wedge(A\to B)]$
T	F	T	F	T	T	F	F	T	T
F	T	T	F	F	F	T	F	T	F
T	T	T	T	T	T	T	T	T	T
F	F	F	F	F	F	T	F	T	T

7. **Sol.** Let p : I become a teacher
q : I will open a school
Negation of $p \to q$ is $\sim(p\to q) = p \wedge \sim q$
i.e. I will become a teacher and I will not open a school.

8. **Sol.** Statement-II : $(p\to q)\leftrightarrow(\sim q\to\sim p)$
$\equiv (p\to q)\leftrightarrow(p\to q)$
which is always true
so statement -II is true
Statement-I : $(p\wedge\sim q)\wedge(\sim p\wedge q)$
$= p\wedge\sim q\wedge\sim p\wedge q$
$= p\wedge\sim p\wedge\sim q\wedge q$
$= f\wedge f$
$= f$
so statement -I is true
Alternate
Statement-II : $(p\to q)\leftrightarrow(\sim q\to\sim p)$
$\sim q\to\sim p$ is contrapositive
of $p\to q$ hence $(p\to q)\leftrightarrow(p\to q)$
will be a tautology
statement -II (p ~q) (~ p q)

p	q	$P\wedge\sim q$	$\sim p\wedge q$	$(p\wedge\sim q)\wedge(\sim p\wedge q)$
T	T	F	F	F
T	F	T	F	F
F	T	F	T	F
F	F	F	F	F

9. **Sol.** Method-1 : $\sim(p\leftrightarrow\sim q)$

p	q	~q	p ~q	~(p ~q)	~p	~p ~q	p q
T	T	F	F	T	F	F	T
T	F	T	T	F	F	T	F
F	T	F	T	F	T	T	F
F	F	T	F	T	T	F	T

Method-2 : logically equivalent of $\sim(p\leftrightarrow\sim q)$ dk rkfdZd laxr is $p\leftrightarrow q$ gS

10. **Sol.** $\sim s \vee (\sim r \wedge s) = (\sim s \vee \sim r) \wedge (\sim s \vee s)$
$= \sim (s \wedge r) \wedge t$
$= \sim (s \wedge r)$
So negation is s r.

11. **Ans.** **(2)**
Sol. $[(p \ \sim q) \vee q] \vee (\sim p \wedge q)$
$= (p \wedge q) \wedge (\sim q \vee q) \vee (\sim p \wedge q)$
$= (p \wedge q) \wedge [t \vee (\sim p \wedge q)]$
$= (p \wedge q) \wedge t$
$= p \wedge q$

12. **Ans.** **(1)**
Sol. $(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$
$(p \rightarrow q) \rightarrow ((p \vee q) \rightarrow q)$
$(p \rightarrow q) \rightarrow ((\sim p \wedge \sim q) \vee q)$
$(p \rightarrow q) \rightarrow ((\sim p \vee q) \wedge (\sim q \vee q))$
$(p \rightarrow q) \rightarrow (p \rightarrow q)$ which is tautology (tautology)

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

1. **Sol.** (4) $4 + 3 \geq 2 \quad 4 + 3 \leq 9$

2. **Sol.** (3) $\Rightarrow$ can be true or false

3. **Sol.** (3) If both passport and voter ID is present, you can visit America.

4. **Sol.** $p \rightarrow q$
F F $\Rightarrow$ True

5. **Sol.** (4) It is not necessary condition

6. **Sol.** $p \rightarrow q$: q is necessary for p
p is sufficient for q

7. **Sol.** De - Moivre's law

8. **Sol.** By Algebra $(\sim p \vee q) \wedge (\sim p \ \ \sim q) = \sim p \wedge \ \sim q$

9. **Sol.** "There exist" should be used for complete negation.

10. **Sol.** "And" connect different component statement

11. **Sol.** $(\sim q \wedge \ p) \vee t = t$

12. **Sol.** $(\sim p) \wedge \sim(\sim p \vee q)$
$(\sim p) \wedge (p \wedge \sim q)$
$(\sim p \wedge p) \wedge (\sim q) = f \ \wedge (\sim q) = f$

13. **Sol.** $\sim (\sim p \rightarrow q) \equiv \sim p \wedge \sim q$

14. **Sol.**

p	q	~ p v q
T	F	F
F	T	T
T	T	T
F	F	T

15. **Sol.** $\sim(p \wedge q) = \sim p \vee \sim q$

16. **Sol.** $\sim (q \vee q) = \sim p \wedge \sim q$

17. **Sol.** Truth table

18. **Sol.** And $= \wedge$

19. **Sol.** If .... then ....$\Rightarrow \sim p \rightarrow \sim q$

20. **Sol.** For $p \rightarrow q \equiv F$, $\Rightarrow p \equiv$ true ; q = false

21. **Sol.** Contrapositive convey same meaning

22. Sol.

(3)	A	B	$A \rightarrow B$	$A(A \rightarrow B)$	$[A(A \rightarrow B)] \rightarrow B$
T	T	T	T	T	
T	F	F	F	T	
F	T	T	F	T	
F	F	T	F	T	

23. Sol. $(p \wedge \sim q) \leftrightarrow Q = Q \leftrightarrow (p \wedge \sim q)$

24. Sol.

	p	q	$q \rightarrow p$	$p \rightarrow q \rightarrow p$	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T	T	T	
T	F	T	T	T	T	
F	T	F	T	T	T	
F	F	T	T	F	T	

25. Sol. $(\sim p \wedge \sim q) \vee (\sim p \wedge q) \equiv \sim p$ (By Algebra

26. Sol.

	p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$
T	T	F	F	T	
T	F	T	T	F	
F	T	F	T	F	
F	F	T	F	T	

27. Sol.

	p	q	r	$p \wedge q$	$(q \wedge r)$	$(p \wedge q)\ (q \wedge r)$
F	F	F	F	F	F	
T	T	T	T	T	T	
T	T	F	T	F	F	
T	F	F	F	F	F	

28. Sol. By truth table

29. Sol. Negation of "If p then q" is $p \wedge \sim q$

30. Sol. $\sim p$: Rohan is not smart
$\sim q$: he is not poor

PART - II : PRACTICE QUESTIONS

1. Sol. Both statements p and q are true
so $p \Rightarrow q$ and $q \Rightarrow p$
$\therefore$ $p \Leftrightarrow q$

2. Sol.

A	B	$A \rightarrow B$	$A \wedge (A \rightarrow B)$	$[A \wedge (A \rightarrow B)] \rightarrow B$
T	F	F	F	T
F	T	T	F	T
T	T	T	T	T
F	F	T	F	F

3. Sol.

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$	$p \wedge q$	$p \rightarrow (p \wedge q)$	$p \leftrightarrow q$	$p \rightarrow (p \leftrightarrow q)$	$p \rightarrow q$	$p \rightarrow (p \rightarrow q)$
T	T	T	T	T	T	T	T	T	T
F	F	T	T	F	T	T	T	T	T
T	F	T	T	F	F	F	F	F	F
F	T	F	T	F	T	F	T	T	T

4. Sol. $((A \cap B) \cup C)' = (A \cap B)' \cap C' = (A' \cap C') \cup (B' \cap C')$

5. Sol. converse of $p \rightarrow q$ is $q \rightarrow p$

6. **Sol.** By using definition of negation

7. **Sol.** Given that $(p \leftrightarrow q) \wedge r$ is true

$\Rightarrow (p \leftrightarrow q)$ is true and r is true

$\Rightarrow$ p and q are both true or both false

Therefore, the truth values of p, q and r are respectively T, T or F, F and T.

As $p \rightarrow \sim q$ is given to be false, p is true and ~ q is false

$\Rightarrow$ p is true and ~q is false $\Rightarrow$ p is true and q is true.

8. **Sol.**

p	~ p	p ∧ ~ p	p ∨ ~ p	~ (~ p) ↔ p
T	F	F	T	T
F	T	F	T	T

p	q	p → q	~ p →~ q	(p → q) ↔ (~ q → ~ p)
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

9. **Sol.** S_1 : f(x) is not continuous in [a, b]

$\sim S_1$: f(x) is continuous in [a, b]

given statement $(\sim S_1 \wedge S_2) \rightarrow S_3$

10. **Sol.** By using truth table

11. **Sol.** $\sim(\sim p \wedge q) \wedge (p \wedge q) \equiv [\sim(\sim p) \vee (-q)] \wedge (p \vee q)$

$\equiv \equiv [p \vee (\sim q) \wedge (p \vee q]$

$\equiv \equiv p \vee [(\sim q) \wedge q]$

$\equiv p \vee f \equiv p$

12. **Sol.** By using concept of quantifiers