Exercise-2

Marked Questions may have for Revision Questions. PART - I : OBJECTIVE QUESTIONS

1. Sol.

$$\begin{aligned}
& \lim_{x \to 0} \frac{\lim_{x \to 0} \frac{\sin(6x^{2})}{\ln\cos(2x^{2} - x)}}{\lim_{x \to 0} \frac{\sin(6x^{2})}{\ln\cos(2x^{2} - x)}} \\
&= \lim_{x \to 0} \frac{\sin 6x^{2}}{6x^{2}} \cdot \frac{6x^{2}}{\ln(\cos(2x^{2} - x))} \\
&= 1. \xrightarrow{x \to 0} \frac{6x^{2}}{\ln(\cos(2x^{2} - x))} \quad \left(\frac{0}{0} \quad \text{form}\right) \left(\frac{0}{0} \quad \text{i} \right) \\
&= 1. \xrightarrow{x \to 0} \frac{12x}{(-\sin(2x^{2} - x))(4x - 1)}}{\cos(2x^{2} - x)} \\
&= \lim_{x \to 0} \frac{-12 \cos(2x^{2} - x)}{4x - 1} \cdot \lim_{x \to 0} \frac{x}{\sin(2x^{2} - x)} = \left(\frac{-12}{-1}\right) \cdot \lim_{x \to 0} \frac{1}{(\cos(2x^{2} - x))(4x - 1)} = 12 \cdot \left(\frac{1}{-1}\right) = 1$$

2.

Sol.

$$\lim_{x \to 0^{+}} \frac{\cos^{-1} (1-x)}{\sqrt{x}}$$

$$= \lim_{h \to 0} \frac{\cos^{-1} [1-(0+h)]}{\sqrt{0+h}}$$

$$= \lim_{h \to 0} \frac{\cos^{-1} (1-h)}{\sqrt{h}}$$
Let ekuk 1 - h = cos θ
sin $\theta = \sqrt{1-(1-h)^2}$

$$\therefore \quad \theta = \sin_{-1} \sqrt{2h-h^2}$$

$$= \lim_{h \to 0} \frac{\sin^{-1} \sqrt{2h-h^2}}{\sqrt{h}}$$

$$= \lim_{h \to 0} \frac{\sin^{-1} \sqrt{2h-h^2}}{\sqrt{2h-h^2}} \cdot \frac{\sqrt{2h-h^2}}{\sqrt{h}}$$

$$= 1 \times \sqrt{2} = \sqrt{2}$$

3.

Sol.

$$\begin{cases}
\lim_{n \to \infty} n \cos \left(\frac{\pi}{4n}\right) \sin \left(\frac{\pi}{4n}\right) \\
\sin \left(\frac{\pi}{4n}\right) \sin \left(\frac{\pi}{4n}\right) \\
\frac{\sin \left(\frac{\pi}{4n}\right)}{\left(\frac{\pi}{4n}\right)} \times \frac{\pi}{4} \\
= 1 \cdot 1 \cdot \frac{\pi}{4} = \frac{\pi}{4}
\end{cases}$$

lim

 $\sum_{k=1}^{100} x^{k} - 100$ $\lim_{x \to 1}$ x-1 4. Sol. $\left[\frac{x-1}{x-1} + \frac{x^2-1}{x-1} + \dots + \frac{x^{100}-1}{x-1}\right]$ €im (100).(101) 2 = 1 + 2+ 3 ++ 100 = = 5050 **Sol.** L.H.L. = $\lim_{h\to 0^-} f(2-h).g(2+h)$ 5. $= \lim_{h \to 0^-} ((2-h)_2 + 4).(2-h)_2$ = 8.4 = 32 R.H.L. = $\lim_{h \to 0^-} (2 + h + 2).8$ = 32 $\Rightarrow \lim_{x \to 2} = 32$ **Sol.** Case - 1 when $h \in Q$ 6. $L.H.L. = \lim_{h \to 0^-} f(0 - h)$ $\lim_{h\to 0^-} (-h)$ = 0 R.H.L. = $\lim_{h \to 0^{-}} f(0 + h)$ lim =^{h→0⁻} (+h) = 0 Case-2 : When $h \in Qc$ $\lim_{h\to 0^-} f(0-h)$ lim = ^{h→0⁻} h = 0 $R.H.L. = \lim_{h \to 0^-} f(0 + h)$ $= \lim_{h \to 0^-} (-h)$ = 0 Hence $\lim_{x\to 0} f(x) = 0$ 7. **Sol.** \therefore f(-x) = -f(x)so $\lim_{x\to 0} f(x) = 0$ √x $\sqrt{\frac{1}{x}}$ $\frac{1}{x^3}$ /x | lim Sol. 8.. $x \rightarrow \infty$



 $\frac{1}{2} + \frac{c}{2} = 0 \qquad \Rightarrow \qquad \qquad$ c = -1 b = 1, a = 2so x**ℓ**n(1+b²) $\lim_{x \to 0} e^{-x} = 1 + b_2 = 2b \sin_2 \theta$ 13. Sol. $sin_2\theta = \frac{1}{2}\left(b + \frac{1}{b}\right)$ \Rightarrow 1 We know b + $\overline{b} \ge 2$ π $\Rightarrow \quad \sin_2 \theta \ge 1 \qquad \text{but} \quad \sin_2 \theta \le 1 \quad \Rightarrow \quad \sin_2 \theta = 1 \quad \Rightarrow \quad \theta = \pm \quad \overline{2}$ **Sol.** For $x \to 0$ $\frac{\lim_{x \to 0} \frac{1 + a \cos x}{x^2}}{x^2}$ 14. (0) for $\left(\overline{\overline{0}}\right)$ form ⇒ a=-1 1 + a = 0 for $\underset{x \to 0}{\overset{\text{lim}}{x \to 0}} \frac{b \sin x}{x^3} = \underset{x \to 0}{\overset{\text{lim}}{x^2}} \frac{b}{x^2}$ so b = 0**Sol.** $f(x) = \frac{x^2 - 9x + 20}{x - [x]}$ 15. $\lim_{x \to 5^{-}} \frac{x^2 - 9x + 20}{x - [x]} = \frac{25 - 45 + 20}{1} = 0$ $\lim_{x \to 5^+} \frac{x^2 - 9x + 20}{x - [x]} = \lim_{h \to 0} \frac{(5+h)^2 - 9(5+h) + 20}{5+h - [5+h]}$ $= \lim_{h \to 0} \frac{25 + 10h + h^2 - 45 - 9h + 20}{h}$ $= \lim_{h \to 0} \frac{h^2 + h}{h}$ $= \lim_{h \to 0} \frac{h(h+1)}{h} = 1$ $\lim_{x \to 5^{-}} \int_{f(x)}^{-} f(x) \neq f(x)$ ÷ $\lim_{x \to 5} f(x) \text{ does not exist} \qquad \lim_{x \to 5} f(x) \text{ fo} | eku ugha g \Delta$ SO 16. Sol. By L.H. rule $1 + \frac{\sin (\sin^{-1} x)}{\sqrt{1-x^2}}$ $\sqrt{1-x^2}$ $\lim_{x \to \infty} 0 - \frac{\sec^2(\sin^{-1}x)}{-}$ 1 $\lim_{x \to 1/\sqrt{2}} 0 - \frac{\sec(\sin x)}{\sqrt{1 - x^2}} = -\frac{1}{\sqrt{2}}$ lim 17. Sol. $x \to \infty$ $(x + e_x)_{2/x}$

 $lny = \frac{2}{x} \lim_{x \to \infty} ln(x + e_x)$ $\lim_{x \to \infty} \frac{2}{x} \left[\ln \left(1 + \frac{x}{e^x} \right) + \ln e^x \right]$ $\lim_{x \to e^{x}} \left[2 + 2 \cdot \frac{\ln \left(1 + \frac{x}{e^{x}}\right)}{x/e^{x}} \cdot \frac{1}{e^{x}} \right]$ x→∞ = $\Rightarrow y = e_{2}$ $\lim_{x \to \infty} \frac{e^{x} \left(\left(2^{x^{n}} \right)^{\frac{1}{e^{x}}} - \left(3^{x^{n}} \right)^{\frac{1}{e^{x}}} \right)}{x^{n}} , n \in \mathbb{N}$ $\frac{(2)^{\frac{x^{n}}{e^{x}}} - (3)^{\frac{x^{n}}{e^{x}}}}{\frac{x^{n}}{e^{x}}}$ $= x \rightarrow \infty 2 + 2.1.0$ Sol. 18. lim when $x \to \infty$, $\frac{x^n}{e^x} \to 0$ Put $\frac{x^n}{e^x} = t$ $\lim_{t \to 0} \left(\frac{2^t - 3^t}{t}\right) = \ell n 2 - \ell n 3 = \ell n \left(\frac{2}{3}\right)$ $\underset{x \to \infty}{\underset{x \to \infty}{\underbrace{ x \ln \left(1 + \frac{\ell n x}{x} \right) }}}$ $\lim_{x \to \infty} \frac{\ln x}{\left(\frac{\ln x}{x}\right)} \frac{\ln \left(1 + \frac{\ln x}{x}\right)}{\left(\frac{\ln x}{x}\right)} \quad (\because x \to \infty) \frac{\ln x}{x} = 0 \quad \text{and} \quad \frac{\ln (1 + f(x))}{f(x)} = 1)$ 19. Sol. = 1 $e^{\lim_{x\to 0} \left(\frac{1^x+2^x+\ldots+n^x}{n}-1\right)\cdot\frac{1}{x}}$ Sol. 20. $= e^{\lim_{x\to 0} \frac{1}{n} \left(\left(\frac{1^x-1}{x}\right) + \left(\frac{2^x-1}{x}\right) + \dots + \left(\frac{n^x-1}{x}\right) \right)}$ $= e^{\frac{1}{n} (\ln 1 + \ln 2 + \dots + \ln n)}$ $= e^{\frac{1}{n} (\ln(1.2.3....n))} = (n!) 1/n$ **Sol.** = $e^{\lim_{x\to 1} \left(\frac{\ln 5x}{\ln 5} - 1\right) \cdot \frac{\ln 5}{\ln x}}$ 21.

$$= e^{\lim_{n \to \infty} \left(\frac{\ln 6 + 6 + 6 + 5}{\ln 6}\right) \left(\frac{\ln 6}{\ln 8}\right)} = e^{\lim_{n \to \infty} \frac{\ln(\frac{5 + 5}{6})}{\ln 6 + 8}} = e$$
22. Sol. $\sin \theta < \theta < \tan \theta$, $\theta \in \left(0, \frac{\pi}{2}\right)$
 $\frac{\sin \theta}{\theta} < 1 < \frac{\tan \theta}{\theta}$
 $\left(\left[\frac{\ln \sin \theta}{\theta}\right] + \left[\frac{\ln \tan \theta}{\theta}\right]\right)$; $n \in \mathbb{N}$
L.H.L. $= \lim_{n \to \infty} \left(\left[\frac{\ln \sin \theta}{\theta}\right] + \left[\frac{\ln \tan \theta}{\theta}\right]\right)$ $= n - 1 + n = 2n - 1$
R.H.L. $= \lim_{n \to \infty} \left(\left[\frac{\ln \sin \theta}{\theta}\right] + \left[\frac{\ln \tan \theta}{\theta}\right]\right)$ $= n - 1 + n = 2n - 1$
 \therefore L.H.L. $= R.H.L = 2n - 1$
23. Sol. $\lim_{n \to 0} \frac{\sin(\pi x) + \pi}{2} \sin\left(\frac{x - \sin x}{2}\right) = a$
 $\frac{2}{\pi^2} \sin\left(\frac{\sin x + x}{2}\right) \sin\left(\frac{x - \sin x}{2}\right) = a$
 $\frac{2}{\pi^2} x^2 \sin\left(\frac{\sin x + x}{2}\right) \sin\left(\frac{x - \sin x}{2}\right) = a$
 $\frac{1}{4} \left(\frac{\sin x + x}{x}\right) \left(\frac{x - \sin x}{x}\right)$
 $= 2.1 \cdot 1 \cdot \frac{1}{4} (1 + 1) (1 - 1) = 0$
24. Sol. for contunity $\left(\frac{\pi}{2}\right) = \lim_{n \to 0} \frac{\ln ((1 - h)^2 - 2(1 - h) + 5)}{\ln (1 - 4h)}$
 $\left(\ln e^{-h \to 0} \text{ from both side}\right)$
 $\left(\frac{\pi}{t}(\frac{\pi}{2}) = \lim_{n \to 0} \frac{1 - \cosh (\ln (\cosh h)}{t^{h/2} \ln (1 + 4h^2)}\right)$
 $= \frac{1}{64} \cdot 1 \cdot 1 \cdot (-1) \cdot 1 = \frac{-1}{64}$
25. Sol. (1) f(x) is continuous no where
(2) g(x) is continuous at x = 0

1-{x}, $X\not\in I$ Since $\{-x\} =$ 0, $\textbf{X} \in I$ 26. Sol. $f(x) = x + \{-x\} + [x]$ $x + 1 - \{x\} + [x]$, X∉I x + [x] $\mathsf{X} \in I$ 1 + 2[x] $X \not\in I$ 2x $\mathsf{X} \in I$ = Curve of y = f(x)discontinuous at all integers in [-2, 2] $f(x) = [x] + \sqrt{\{x\}}$ Sol. 27. for n∈I $f(n_{+}) = [n_{+}] + \sqrt{\{n^{+}\}} = n$ $f(n_{-}) = [n_{-}] + \sqrt{\{n^{-}\}} = n - 1 + 1 = n$ \Rightarrow continuous for all $x \in I$ Hence continuous for all $x \in R$ 7 f(x) is continuous and $\overline{3} \in [f(-2), f(2)]$, by intermediate value theorem (IVT), there exists a point Sol. 28._ 7 $c \in (-2, 2)$ such that $f(c) = \overline{3}$ 29. $f(1_{-}) = f(1) \Rightarrow a - b = -1$ Sol. $f'(1_{-}) = \lim_{h \to 0} \frac{a(1-h)^2 - b + 1}{-h} = \lim_{h \to 0} \frac{(a-b+1)}{-h} + 2a = 2a$ -1 +1 $f'(1_{+}) = \lim_{h \to 0} \frac{\frac{1}{1+h} + 1}{h} = \lim_{h \to 0} \frac{h}{h(1+h)}$ = 1 3 1 $a = \overline{2}, b = \overline{2}$ ⇒ a + b = 2so 30. Sol. f(0) = 0 $\frac{h}{\sqrt{h+1}-\sqrt{h}}=1$ lim h→0 $f'(0_{+}) =$ f(x) is not defined for x < 0f(x) is differentiable at x = 0 \Rightarrow

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	$\int x \tan^{-1} \left(\frac{1}{x}\right) , x \neq 0$
31.	Sol. $f(x) = \begin{bmatrix} 0 & , & x = 0 \\ 0 & , & x = 0 \end{bmatrix}$ L.H.L. = R.H.L. = 0 continuous at x = 0
	$\begin{cases} \sin 2x , & 0 < x \le \frac{\pi}{6} \end{cases}$
32.	Sol. $f(x) = \begin{bmatrix} ax + b & , & \frac{\pi}{6} < x < 1 \\ \lim_{x \to \frac{\pi}{6}} \sin 2x = \frac{\sqrt{3}}{2} \end{bmatrix}$
	$R.H.L. = \frac{4\pi}{6} + b$ $\frac{4\pi}{6} + b = \frac{\sqrt{3}}{2} \qquad \dots (1)$
	$\lim_{X \to \frac{\pi}{6}} 2 \cos 2x = 1$ R.H.D. = a a = 1(2) $= \frac{\sqrt{3}}{2} - \frac{\pi}{6}$
33.	Sol. $f(1_{-}) = f(1) = 1$ $f(1_{+}) = a + b + c$ $\Rightarrow a + b + c = 1$ $f'(1_{-}) = 1$ $f'(1_{+}) = \frac{\lim_{h \to 0} \frac{a(1+h)^2 + b(1+h) + c - 1}{h}}{h}$ = 2a + b $\Rightarrow 2a + b = 1$ so $(a, b, c) \equiv (a, 1 - 2a, a), a \in \mathbb{R}, a \neq 0$
34.	Sol. L.H.L. = $\lim_{x \to -1-h} f(x) = a(-1 - h)^2 + b = a + b$ R.H.L. = $\lim_{x \to -1+h} f(x) = b - a + 4$ a = 2
35.	Sol. $f(x) = [x] [\sin \pi x], x \in (-1, 1)$ $\begin{cases} 1 & , x \in (-1, 0) \\ 0 & , x \in [0, 1) \\ f(x) \text{ is continuous in } (-1, 0) \end{cases}$
36.	$\textbf{Sol.} f(x) = \begin{cases} a - x & , x < a - b \\ b & , a - b < x < b - a \\ a + x & , b - a < x \end{cases}$ so f(x) cannot be differentiable at maximum two points.
37.	Sol. By use of graph $f(x) = max \{x, x_3\}$



From graph its clear that points {-1, 0, 1} are points of non differentiability due to kink points.



therefore function is discontinuous at x = -1therefore function is not differentiable at x = -1π At x = 1, LHL = 4 RHL = 0therefore LHL \neq RHL so function is discontinuous at x = 1 therefore function is not differentiable at x = 1f(x) is discontinuous at x = -1 and x = 1. 41. Sol. Let h(x) = |x|, then g(x) = |f(x)| = h(f(x))since, composition of two continuous functions is continuous, g is continuous if f is continuous. so answer is (c). (a) is wrong answer. Let $f(x) = x \Rightarrow g(x) = |x|$ Now, f(x) is an onto function. since co-domain of x is R and range of x is R. But g(x) is not onto function. since range of g(x) is $[0, \infty)$ but co-domain is given R. (b) Let $f(x) = x \Rightarrow g(x) = |x|$. Now f(x) is one-one function but g(x) is many - one function. Hence (b) is wrong. (d) Let $f(x) = x \Rightarrow g(x) = |x|$. Now, f(x) is differentiable for all $x \in R$ but g(x) = |x| is not differentiable at x = 0 Hence (d) is wrong. $\lim_{x\to 0} \frac{2f(x)-3f(2x)+f(4x)}{x^2}$ 42. Sol. $\lim_{x \to 0} \frac{2f'(x) - 6f'(2x) + 4f'(4x)}{2x}$ $\frac{2f''(x) - 12f''(2x) + 16f''(4x)}{2}$ lim = x→0 $\frac{2f''(0) - 12f''(0) + 16f''(0)}{2} = 3f''(0) = 12$ $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)}{3}$ (1) f(0) = 0, f'(0) = 343. Sol. Put x = 3x and y = 0f(3x) f(x) = 3 $f(x) = \begin{array}{ccc} 3 & \dots & \dots & (2) \\ \lim_{h \to 0} f(x+h) = & \lim_{h \to 0} f & \left(\frac{3x+3h}{3}\right) = & \lim_{h \to 0} f & \frac{f(3x)+f(3h)}{3} = & \frac{f(3x)}{3} = f(x) \end{array}$ Similarly we can prove $\lim_{h\to 0} f(x-h) = f(x)$ \Rightarrow f(x) is continuous for all x in R Given that f'(0) = 3 $\Rightarrow \lim_{h \to 0} \frac{f(h)}{h} = \lim_{h \to 0} \frac{f(-h)}{-h} = 3$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{f(x) + f(x) + xh - f(x)}{h}$ Sol. 44.

		$= \lim_{h \to 0} \frac{f(h)}{h} + x = 3 + x$	
	⇒	$f(x) = 3x + \frac{x^2}{2} + C$	
		$f(0) = 0 \qquad \Rightarrow C = 0$	
	⇒	f(x) = 3x + 2 $f(3x+3h) - f(3x+0)$	
45.	Sol.	$ \frac{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\int_{h \to 0} \frac{f(x+h) - f(x)}{h}}{h} = -\frac{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}{h} $ $ \frac{4 - 2(f(3x) + f(3h)) - 4 - 2(f(3x) + f(0))}{h} $	
	$= \lim_{h\to 0} \frac{h}{h}$	<u>3</u> <u>3</u> h	
	$= \lim_{h \to 0} \frac{1}{2}$	$\frac{-2}{3} \frac{(f(3h) - f(0))}{h} = -2 f'(0)$	
	$\Rightarrow f(x) =$ put v =	= –2f'(0) x + C 0 in the given equation	
	$f\left(\frac{x}{3}\right)_{=}$	$\frac{4-2(f(x)+f(0))}{3}$	
	$\Rightarrow \frac{1}{3} f'$	$\left(\frac{x}{3}\right)_{=} \frac{-2}{3} f'(x) \qquad \text{put } x = 0 \Rightarrow f'(0) = 0 \& f(0) = \frac{4}{7} = 0$	2
	$\Rightarrow f(x) :$	<u>4</u> 7	

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS:

(3)

2:

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

- (1) Both the statements are true.
- (2) Statement-I is true, but Statement-II is false.
- (3) Statement-I is false, but Statement-II is true.
- (4) Both the statements are false.

A-1. Ans

1:
$$\lim_{x\to 0} \left(\tan\left(\frac{\pi}{4} + x\right) \right)^{\frac{1}{x}}$$

$$\lim_{x \to a} (1 + f(x))_{g(x)} = e^{\lim_{x \to a} f(x) \cdot g(x)}, \quad \lim_{x \to a} f(x) = 0 \quad \lim_{x \to a} g(x) = \infty$$

Ans. (3)

Statement-1:
$$\lim_{x \to 0} \left(\frac{1 + \tan x}{1 - \tan x} \right)^{\frac{1}{x}} = \lim_{x \to 0} \left(1 + \frac{2\tan x}{1 - \tan x} \right)^{\frac{1}{x}} = \lim_{x \to 0} \exp \left[\frac{2\tan x}{x(1 - \tan x)} \right]_{x} = e_2$$

Sol. Statement-1 : $x \to 0$ $(1 - \tan x) = x \to 0$ $(1 - \tan x)$ Hence statement-1 is false and statement-2 is true.

A-2. Ans. (2)

Sol. Statement-1 is true

 $\frac{2x^2+3x+1}{3x^4+5} = 0 \neq \frac{2}{3}$ lim Statement-2 is false. Consider $x \to \infty$ A-3. Ans. (2) Sol. Statement - 1 $\begin{bmatrix} x & \cos x & , & x \ge 0 \end{bmatrix}$ $f(x) = |x| \cos x = \begin{bmatrix} -x \cos x & , & x < 0 \end{bmatrix}$ f (x) is not differentiable at x = 0. (True) Statement - 2 Consider $\begin{bmatrix} x^3 & x \ge 0 \end{bmatrix}$ $g(x) = \mid x_3 \mid = \begin{bmatrix} -x^3 & , & x < 0 \end{bmatrix}$ but g(x) is differentiable at x = 0 (false) A-4. Ans. (1) Statement - 1 Sol. $f(x) = \{tanx\} - [tan x]$ $\tan x \quad , \quad 0 \leq x < \frac{\pi}{4}$ $\tan x - 2$, $\frac{\pi}{4} \le x < \tan^{-1} 2$ f(x) = tanx - 2 [tan x] =π obviously at $x = \frac{3}{3} f(x)$ is continuous. (True) Statement-2 y = f(x) & y = g(x) both are continuous at x = athen $y = f(x) \pm (g(x))$ will also be continuous at x = a (True) Statement-1 can be explained with the help of statement-2.

Section (B) : MATCH THE COLUMN

Note : Only one answer type (1 × 1)

B-1_. Sol. (P)
$$\lim_{x \to 0} \frac{\tan x - \sin x}{x^3} = \lim_{x \to 0} \left(\frac{\tan x}{x}\right) \left(\frac{1 - \cos x}{x^2}\right)_{=} \frac{1}{2}$$
(Q)
$$\lim_{x \to \infty} \sqrt{\frac{x - \sin x}{x + \cos^2 x}} = \lim_{x \to \infty} \sqrt{\frac{1 - \frac{1}{x} \sin x}{1 + \frac{1}{x} \cos^2 x}} = \sqrt{\frac{1 - 0}{1 + 0}}$$
(R)
$$\lim_{x \to 1} \frac{-\sqrt{25 - x^2} + \sqrt{24}}{x - 1} = \lim_{x \to 1} \frac{(x^2 - 1)}{(x - 1)(\sqrt{25 - x^2} + \sqrt{24})}$$

$$= \lim_{x \to 1} \frac{(x + 1)}{(\sqrt{25 - x^2} + \sqrt{24})} = \frac{2}{2\sqrt{24}} = \frac{1}{\sqrt{24}} = \frac{1}{2\sqrt{6}}$$
(S)
$$\frac{\left(\frac{\tan x}{x}\right)^{1/x^2}}{x} = e^{\lim_{x \to 0} \left(\frac{\tan x}{x^2}\right)}$$



Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. Sol. Obviously (1), (2), (3) are true.

C-2. Sol. $\lim_{x \to 0} \frac{\left(x + \frac{x^3}{3} + \frac{2x^5}{15} + \dots - x\right) \left[1 + x + \frac{x^2}{2!} + \dots - \left(x - \frac{x^3}{3!} + \dots\right) - 1\right]}{x^n}$ $= \lim_{x \to 0} \frac{x^5 \left[\frac{1}{3} + \frac{2x^2}{5} + \dots\right] \left[\frac{1}{2!} + \left(\frac{x}{3!}\right) \dots\right]}{x^n}$ for finite values of x = 5 and $x = \frac{1}{6}$

for finite value of $~\ell,~n$ = 5 and ℓ = 6

C-3. Sol. From graph it is clear.



C-4. Sol.
$$\lim_{x \to 0} \frac{2 \sin x \cos x + a \sin x}{x^3} = p \text{ (finite if jfer)}$$
$$\Rightarrow \quad \lim_{x \to 0} \frac{\sin x}{x} \cdot \left(\frac{2 \cos x + a}{x^2}\right) = p$$

 $\lim_{x \to 0} \frac{\sin x}{x} \lim_{x \to 0} \frac{2\cos x}{x^2} +$ = p ⇒ $\lim_{x \to 0} \frac{2\cos x + a}{x^2}$ = p ⇒ $\frac{1}{10}$ $\frac{1}{10}$ ds fy, 0` For form 2 + a = 0a = – 2 $2\left(2\sin^2\frac{x}{2}\right)$ $p = \lim_{x \to 0} \frac{2(\cos x - 1)}{x^2}$ $p = \lim_{x \to 0} \frac{p}{2}$ ⇒ \Rightarrow p = - 1 ⇒ $\begin{cases} 1 + \frac{2x}{a}, & 0 \le x < 1 \\ ax, & 1 \le x < 2 \end{cases}$ C-5. Sol. f(x) =2 2(1–h) L.H.L. = $x \to 1^-$ f(x) = $\lim_{h \to 0} 1 + \frac{2(1-h)}{a} = 1 + \frac{2}{a}$ $\text{R.H.L.}=\overset{\text{lim}}{\overset{x\to1^+}{\xrightarrow{}}}f(x)=\overset{\text{lim}}{\overset{h\to0}{\xrightarrow{}}}a\ (1+h)=a$ **ℓim** x→1 f(x) exists ÷ L.H.L.= R.H.L. 2 1 + a = a⇒ $a_2 - a - 2 = 0$ ⇒ (a-2)(a+1) = 0⇒ ⇒ a = 2, -1

C-6. Sol. Clearly graph is so (1) and (4) is true.

C-7. Sol. Case-I : when $y \ge 0$ then y = x. х Case-II : When y < 0 then $y = \overline{3}$. Х, $x \geq 0 \\$ $\left\lfloor \frac{x}{3} \right\rfloor$ $\mathbf{x} < \mathbf{0}$

Hence (1), (2), (4) is true.

y =

C-8. **Sol.** f (x) is continuous in [0, 2]

...

 $\begin{array}{l} f'(x) = \begin{bmatrix} x, & 0 < x \leq 1 \\ 4x - 3, & 1 \leq x < 2 \\ \end{array} \\ Now, \quad f''(x) = \begin{bmatrix} 1, & 0 < x < 1 \\ 4, & 1 < x < 2 \\ \end{array} \\ So, f'' \text{ is not continuous at } x = 1 \text{ and } f'(x) \text{ is not differentiable at } x = 1. \end{array}$